

Application of Integrals

(2025)

1. The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $x = 4$ and x-axis is :

(1 Marks) (CBSE 2025 - 65/4/1)

A. $16/3$ sq. units

B. $32/3$ sq. units

C. $32/9$ sq. units

D. $16/9$ sq. units

2. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is :

(1 Marks) (CBSE 2025 - 65/6/1)

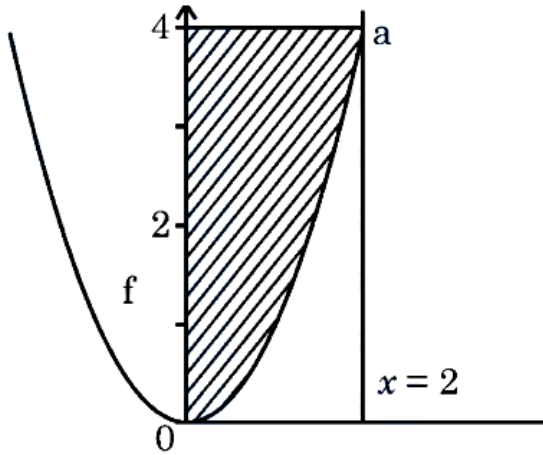
A. $2/3$ sq units

B. $3/2$ sq units

C. 3 sq units

D. $4/3$ sq units

3. The area of the shaded region (figure) represented by the curves $y = x^2$, $0 \leq x \leq 2$ and y -axis is given by



(1 Marks) (CBSE 2025 - 65/2/1)

A. $\int_0^4 x^2 dx$

B. $\int_0^2 \sqrt{y} dy$

C. $\int_0^2 x^2 dx$

D. $\int_0^4 \sqrt{y} dy$

4. The area of the region enclosed between the curve $y = x|x|$, x-axis, $x = -2$ and $x = 2$ is :

(1 Marks) (CBSE 2025 - 65/5/1)

A. 8

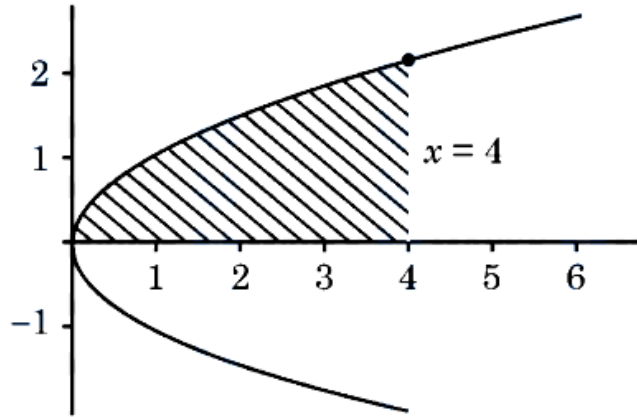
B. $16/3$

C. $8/3$

D. 0

5. The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by

(1 Marks) (CBSE 2025 - 65/1/1)



A. $\int_0^4 x \, dx$

B. $2 \int_0^4 \sqrt{x} \, dx$

C. $\int_0^2 y^2 \, dy$

D. $\int_0^4 \sqrt{x} \, dx$

6. Calculate the area of the region bounded by the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the x-axis using integration.

(2 Marks) (CBSE 2025 - 65/4/1)

7. Sketch the graph of $y = |x+3|$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.

(3 Marks) (CBSE 2025 - 65/1/1)

8. Draw a rough sketch for the curve $y = 2 + |x+1|$. Using integration, find the area of the region bounded by the curve $y = 2 + |x+1|$, $x = -4$, $x = 3$ and $y = 0$.

(5 Marks) (CBSE 2025 - 65/6/1)

9. Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x-axis and the ordinates $x = -2$ and $x = 2$.

(5 Marks) (CBSE 2025 - 65/2/1)

10. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm . She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\pi/4$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.

(5 Marks) (CBSE 2025 - 65/7/1)

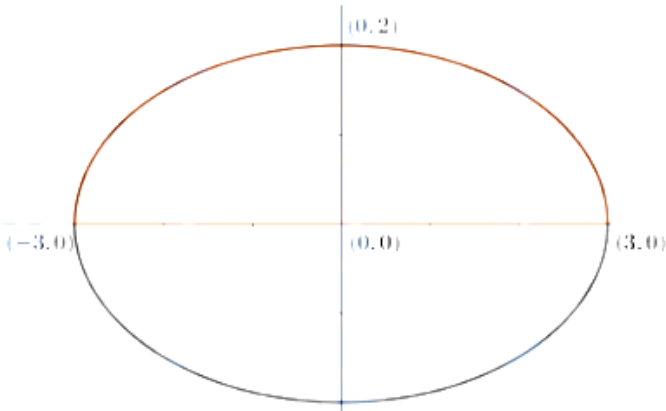
11. Sketch a graph of $y = x^2$. Using integration, find the area of the region bounded by $y = 9$, $x = 0$ and $y = x^2$.

(5 Marks) (CBSE 2025 - 65/5/1)

Answer

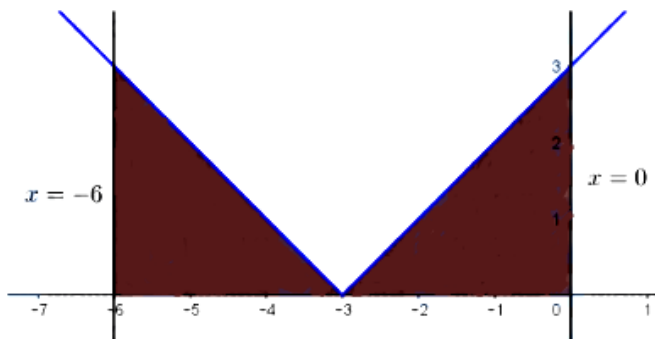
1. A 2. D 3. D 4. B 5. D

6.



$$\begin{aligned} A &= 2 \times \frac{2}{3} \int_0^3 \sqrt{9-x^2} dx \\ &= \frac{4}{3} \left[\frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \left(\frac{x}{3} \right) \right]_0^3 \\ &= \frac{4}{3} \left[\left(0 + \frac{9}{2} \sin^{-1} 1 \right) - 0 \right] \\ &= 3\pi \end{aligned}$$

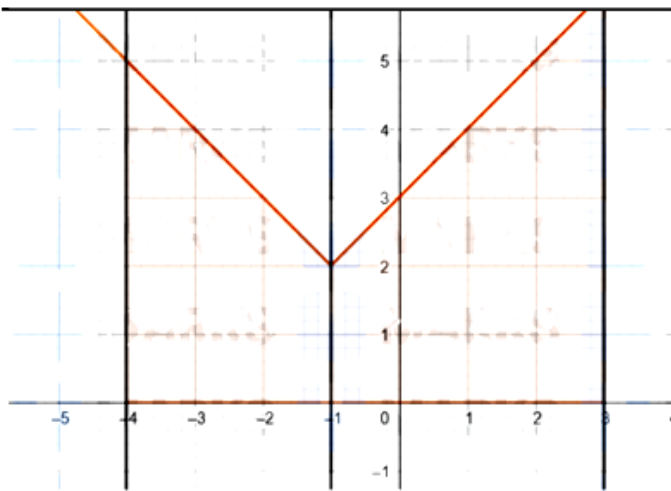
7.



Required Area

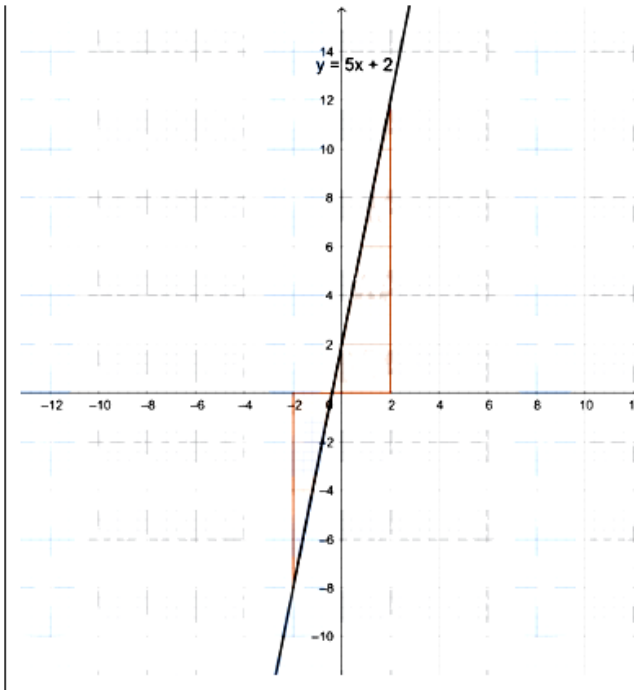
$$\begin{aligned}
&= \int_{-6}^0 y dx \\
&= 2 \int_{-3}^0 (x+3) dx \\
&= 2 \left[\frac{(x+3)^2}{2} \right]_{-3}^0 \\
&= 9
\end{aligned}$$

8.



$$\begin{aligned}
\text{Required area} &= \int_{-4}^{-1} (2 + |x+1|) dx + \int_{-1}^3 (2 + |x+1|) dx \\
&= \int_{-4}^{-1} (1-x) dx + \int_{-1}^3 (3+x) dx \\
&= -\frac{(1-x)^2}{2} \Big|_{-4}^{-1} + \frac{(3+x)^2}{2} \Big|_{-1}^3 \\
&= \frac{21}{2} + 16 = \frac{53}{2}
\end{aligned}$$

9.



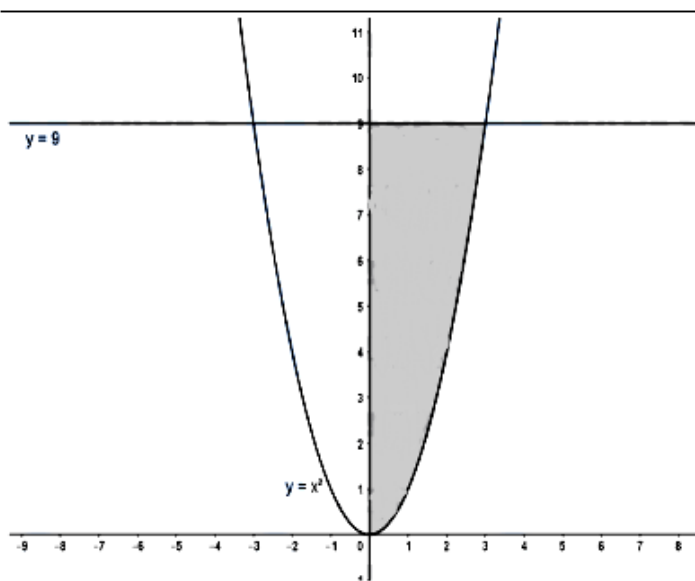
The required area

$$\begin{aligned}
 &= \left| \int_{-2}^{-\frac{2}{5}} (5x + 2) dx \right| + \int_{-\frac{2}{5}}^2 (5x + 2) dx \\
 &= \left| \left[\frac{(5x + 2)^2}{10} \right]_{-2}^{-\frac{2}{5}} \right| + \left[\frac{(5x + 2)^2}{10} \right]_{-\frac{2}{5}}^2 \\
 &= \frac{64}{10} + \frac{144}{10} = \frac{104}{5}
 \end{aligned}$$

10.

$$\begin{aligned}
&= \int_0^{4\sqrt{2}} x dx + \int_{4\sqrt{2}}^8 \sqrt{64 - x^2} dx \\
&= \left[\frac{x^2}{2} \right]_0^{4\sqrt{2}} + \left[\frac{x}{2} \sqrt{64 - x^2} + 32 \sin^{-1} \frac{x}{8} \right]_{4\sqrt{2}}^8 \\
&= \frac{32}{2} + 32 \sin^{-1} 1 - 2\sqrt{2} \cdot 4\sqrt{2} - 32 \sin^{-1} \frac{1}{\sqrt{2}} \\
&= 16 + 16\pi - 16 - 8\pi = 8\pi \text{ cm}^2
\end{aligned}$$

11.



$$\begin{aligned}
\text{Required area} &= \int_0^9 \sqrt{y} dy \\
&= \frac{2}{3} \left[y^{3/2} \right]_0^9 \\
&= 18
\end{aligned}$$

Note: If area is found in second quadrant, may be considered.

(2024)

1. Area of the region bounded by curve $y^2 = 4x$ and the X -axis between $x = 0$ and $x = 1$ is :

(1 Marks) (CBSE 2024 - 65/5/1)

A. $8/3$

B. $2/3$

C. 3

D. $4/3$

2. If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

(5 Marks) (CBSE 2024 - 65/2/1)

3. Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X -axis and the ordinates $x = -2$ and $x = 2$, using integration.

(5 Marks) (CBSE 2024 - 65/4/1)

4. Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X -axis.

(5 Marks) (CBSE 2024 - 65/4/1)

5. Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines $x = -2$ and $x = 2$.

(5 Marks) (CBSE 2024 - 65/1/1)

6. Using integration, find the area of the region enclosed between the circle $x^2 + y^2 = 16$ and the lines $x = -2$ and $x = 2$.

(5 Marks) (CBSE 2024 - 65/1/1)

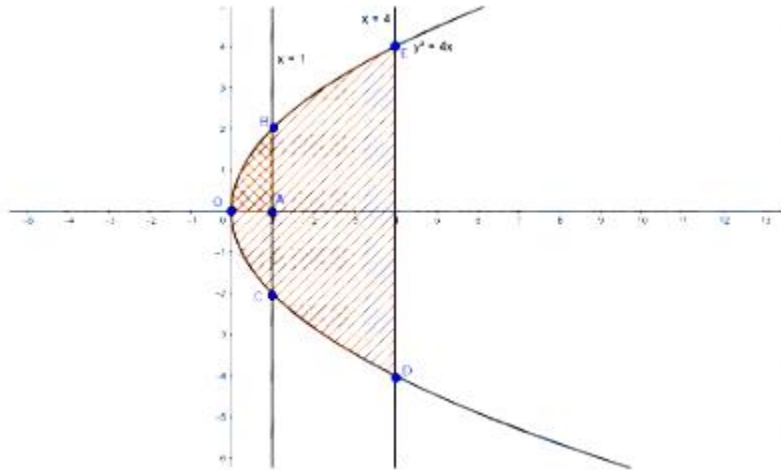
7. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

(5 Marks) (CBSE 2024 - 65/5/1)

Answer

1. D

2.

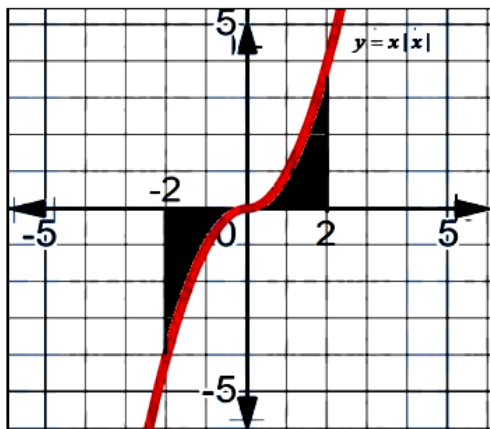


$$A_1 = \text{Area (region OABO)} = \int_0^1 2\sqrt{x} dx = 2 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 = \frac{4}{3}$$

$$A_2 = \text{Area (region ODEO)} = 2 \int_0^4 2\sqrt{x} dx = 4 \times \frac{2}{3} [2^3] = \frac{64}{3}$$

$$A_1 : A_2 = \frac{4}{3} : \frac{64}{3} = 1 : 16$$

3.

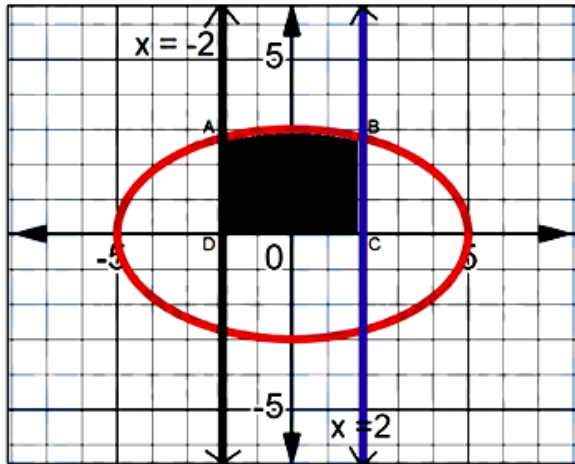


As,

$$y = x|x| = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

$$\begin{aligned} \text{Area of the shaded region} &= \int_{-2}^2 y dx = 2 \int_0^2 y dx = 2 \int_0^2 x^2 dx \\ &= 2 \left(\frac{x^3}{3} \right)_0^2 \\ &= 2 \left(\frac{8}{3} \right) = \frac{16}{3} \end{aligned}$$

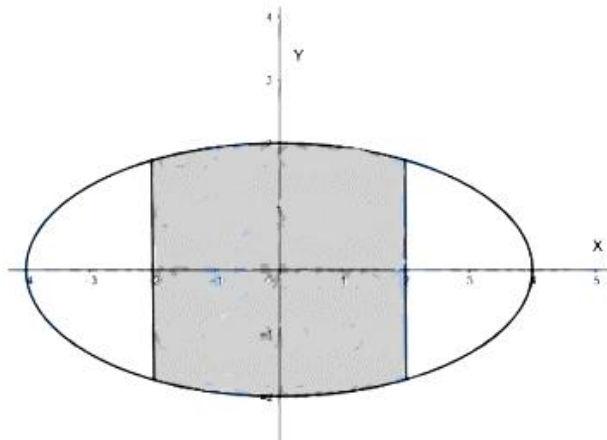
4.



$$\text{As, } 9x^2 + 25y^2 = 225 \Rightarrow y = \pm \frac{3}{5} \sqrt{5^2 - x^2}$$

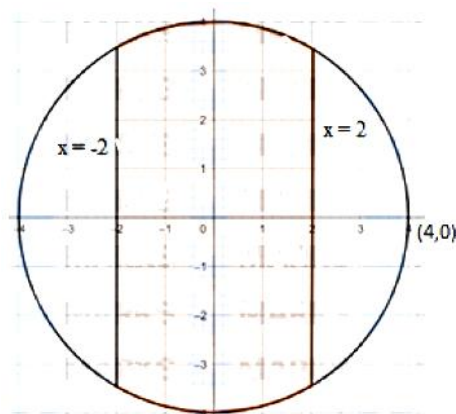
$$\begin{aligned} \text{Required Area} &= \int_{-2}^2 \frac{3}{5} \sqrt{5^2 - x^2} dx = \frac{6}{5} \int_0^2 \sqrt{5^2 - x^2} dx \\ &= \frac{6}{5} \left(\frac{x\sqrt{5^2 - x^2}}{2} + \frac{25}{2} \sin^{-1} \left(\frac{x}{5} \right) \right)_0^2 \\ &= \frac{6}{5} \left(\frac{2\sqrt{21}}{2} + \frac{25}{2} \sin^{-1} \left(\frac{2}{5} \right) \right) \\ &= \left(\frac{6\sqrt{21}}{5} + 15 \sin^{-1} \left(\frac{2}{5} \right) \right) \end{aligned}$$

5.



$$\begin{aligned}
 \text{Area} &= 4 \int_0^2 y dx \\
 &= 4 \left[\frac{1}{2} \int_0^2 \sqrt{4^2 - x^2} dx \right] \\
 &= 2 \left[\frac{x}{2} \sqrt{4^2 - x^2} + 8 \sin^{-1} \left(\frac{x}{4} \right) \right]_0^2 \\
 &= 2 \left[\sqrt{12} + \frac{8\pi}{6} \right] = 4\sqrt{3} + \frac{8\pi}{3}
 \end{aligned}$$

6.



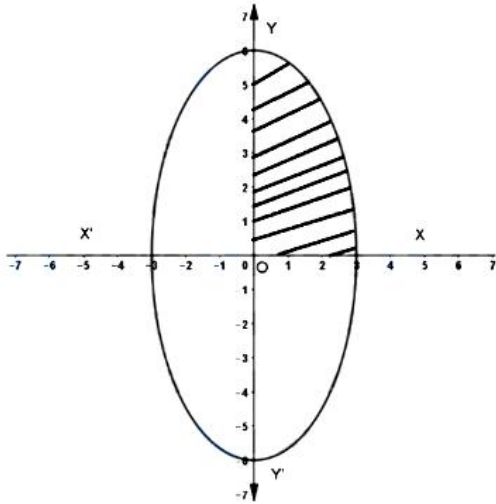
$$\begin{aligned}
 \text{Required area} &= 4 \int_0^2 \sqrt{16 - x^2} dx \\
 &= 4 \left[\frac{x}{2} \sqrt{16 - x^2} + 8 \sin^{-1} \left(\frac{x}{4} \right) \right]_0^2 \\
 &= 8\sqrt{3} + \frac{16\pi}{3}
 \end{aligned}$$

7.

The given equation can be written as:

$$\frac{x^2}{9} + \frac{y^2}{36} = 1$$

, which is an ellipse.



Area of the region bounded by the curve

$$\begin{aligned} &= 4 \times \frac{6}{3} \int_0^3 \sqrt{9 - x^2} dx \\ &= 8 \left[\frac{x}{2} \sqrt{9 - x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_0^3 \\ &= 18\pi \end{aligned}$$

Previous Years' CBSE Board Questions

8.2 Area under Simple Curves

SA I (2 marks)

1. Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y -axis. Hence, obtain its area using integration. (2023)
2. Using integration, find the area bounded by the curve $y^2 = 4x$, y -axis and $y = 3$. (2021C) (Ev)
3. Using integration, find the area of the region bounded by the line $2y = -x + 8$, x -axis, $x = 2$ and $x = 4$. (2021C) (Ev)

SA II (3 marks)

4. Find the area of the following region using integration.
 $\{(x, y) : y^2 \leq 2x \text{ and } y \geq x - 4\}$ (2023)
5. Using integration, find the area of the region bounded by $y = mx$ ($m > 0$), $x = 1$, $x = 2$ and the x -axis. (2023)
6. Using integration, find the area of the region $\{(x, y) : y^2 \leq x \leq y\}$. (Term II, 2021-22) (Ev)

LA I (4 marks)

7. Using integration, find the area of the region $\{(x, y) : 4x^2 + 9y^2 \leq 36, 2x + 3y \geq 6\}$. (Term II, 2021-22) (Ev)
8. Using integration, find the area of the region bounded by lines $x - y + 1 = 0$, $x = -2$, $x = 3$ and x -axis. (Term II, 2021-22) (Ev)
9. If the area of the region bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration find the value of a , where $a > 0$. (Term II, 2021-22) (Ev)
10. Find the area of the region bounded by curve $4x^2 = y$ and the line $y = 8x + 12$, using integration. (Term II, 2021-22) (Ev)
11. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration. (Term II, 2021-22) (Cr)

LA II (5/6 marks)

12. Using integration, find the area of the region bounded by the circle $x^2 + y^2 = 16$, line $y = x$ and y -axis, but lying in the 1st quadrant. (2023)
13. Find the area of the following region using integration:
 $\{(x, y) : y < |x| + 2, y > x^2\}$ (2020) (Ev)

14. Using integration, find the area of a triangle whose vertices are $(1, 0)$, $(2, 2)$ and $(3, 1)$. (2020) (Ev)
15. Using integration, find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$. (2020) (Ev)
16. Using integration, find the area of the region in the first quadrant enclosed by the x -axis, the line $y = x$ and the circle $x^2 + y^2 = 32$. (2020, NCERT, 2018, Delhi 2014) (Ev)
17. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a using integration. (2020C) (Ev)
18. Using the method of integration, find the area of the region bounded by the lines $3x - 2y + 1 = 0$, $2x + 3y - 21 = 0$ and $x - 5y + 9 = 0$. (2019) (Ev)
19. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration. (Delhi 2017) (Cr)
20. Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$. (AI 2017, 2015C)
21. Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and y -axis. (Foreign 2015)
22. Find the area of the region in the first quadrant enclosed by the y -axis, the line $y = x$ and the circle $x^2 + y^2 = 32$, using integration. (NCERT, Delhi 2015C) (Cr)
23. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$. (Foreign 2014) (Cr)
24. Using integration, find the area of the region bounded by the curves:
 $y = |x + 1| + 1$, $x = -3$, $x = 3$, $y = 0$ (Delhi 2014C)
25. Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$. (Delhi 2014C)
26. Using integration, find the area of the region in the first quadrant enclosed by the x -axis, the line $y = x$ and the circle $x^2 + y^2 = 18$. (AI 2014C)

8.2 Area under Simple Curves

VSA (1 mark)

1. Find the area bounded by $y = x^2$, the x -axis and the lines $x = -1$ and $x = 1$. (2020-21) **Ev**

SA I (2 marks)

2. Find the area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$. (2020-21) **Ev**

SA II (3 marks)

3. Find the area of the region bounded by the curves $x^2 + y^2 = 4$, $y = \sqrt{3}x$ and x -axis in the first quadrant. (2020-21)

4. Find the area of the ellipse $x^2 + 9y^2 = 36$ using integration. (2020-21) **Cr**

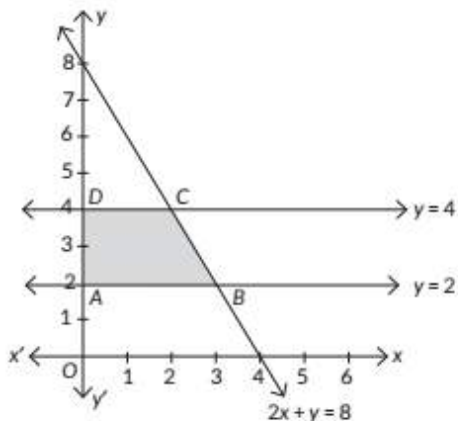
LA I (4/5 marks)

5. Make a rough sketch of the region $\{(x, y): 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 2\}$ and find the area of the region using integration. (2022-23) **Ev**
6. Using integration, find the area of the region in the first quadrant enclosed by the line $x + y = 2$, the parabola $y^2 = x$ and the x -axis. (Term II, 2021-22) **Ev**
7. Using integration, find the area of the region $\{(x, y): 0 \leq y \leq \sqrt{3}x, x^2 + y^2 \leq 4\}$. (Term II, 2021-22) **Cr**

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

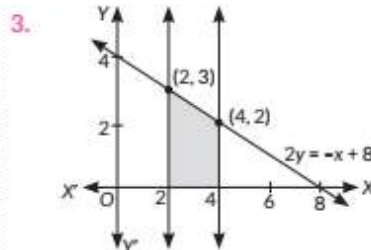
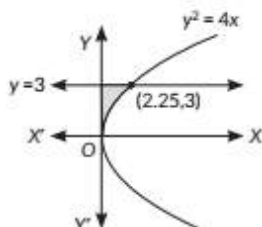
1. From the graph, ABCD is the required region.



$$\begin{aligned} \text{Now, area} &= \int_2^4 \left(\frac{8-y}{2}\right) dy = \frac{1}{2} \int_2^4 (8-y) dy \\ &= \frac{1}{2} \left[8y - \frac{y^2}{2} \right]_2^4 = \frac{1}{2} \left[\left(32 - \frac{16}{2}\right) - \left(16 - \frac{4}{2}\right) \right] \\ &= \frac{1}{2} \times 10 = 5 \text{ sq. units} \end{aligned}$$

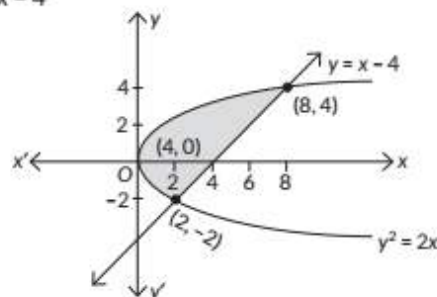
2. Required area = $\int_0^3 \frac{y^2}{4} dy$

$$= \left[\frac{y^3}{12} \right]_0^3 = \frac{9}{4} \text{ sq. units.}$$



∴ Required area
 $= \int_2^4 \left(\frac{-x+8}{2}\right) dx = \left[\frac{-x^2}{4} + 4x \right]_2^4 = 5 \text{ sq. units}$

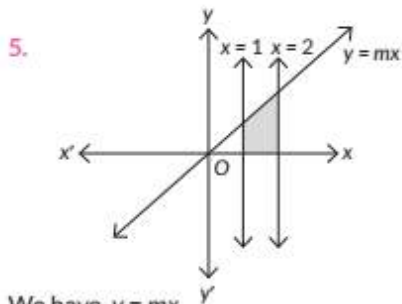
4. We have, $y^2 \leq 2x$
 and $y \geq x - 4$



Here, the shaded area represents the required area.

$$\begin{aligned} \therefore \text{Required area} &= \int_{-2}^4 (y+4) dy - \int_{-2}^4 \frac{y^2}{2} dy \\ &= \left[\frac{y^2}{2} + 4y \right]_{-2}^4 - \frac{1}{2} \left[\frac{y^3}{3} \right]_{-2}^4 \\ &= \left[\left(\frac{16}{2} + 16\right) - \left(\frac{4}{2} - 8\right) \right] - \frac{1}{2} \left[\frac{64}{3} - \left(-\frac{8}{3}\right) \right] \\ &= 20 - 12 = 8 \text{ sq. units} \end{aligned}$$

... (i)
 ... (ii)



We have, $y = mx$

From the figure it is clear that required area is the shaded region.

$$\text{Required area} = \int_1^2 mx dx = \left[\frac{mx^2}{2} \right]_1^2 = \frac{4m}{2} - \frac{m}{2} = \frac{3m}{2} \text{ sq. units.}$$

6. On solving $x = y$ and $x = y^2$, we get

$$y^2 = y$$

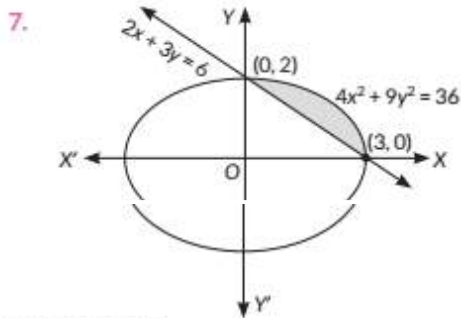
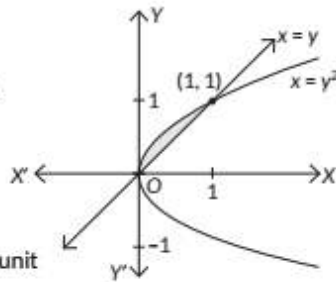
$$\Rightarrow y(y - 1) = 0$$

$$\Rightarrow y = 0 \text{ and } y = 1.$$

\therefore Required area = area of shaded region

$$= \int_0^1 (y - y^2) dy$$

$$= \left[\frac{y^2}{2} - \frac{y^3}{3} \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \text{ sq. unit}$$

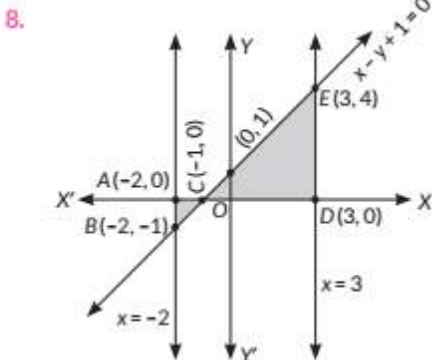


Required area

$$= \int_0^3 \left(\frac{\sqrt{36 - 4x^2}}{3} - \left(\frac{6 - 2x}{3} \right) \right) dx = \frac{2}{3} \int_0^3 (\sqrt{9 - x^2} - (3 - x)) dx$$

$$= \frac{2}{3} \left[\left(\frac{1}{2} x \sqrt{9 - x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right) + \frac{x^2}{2} - 3x \right]_0^3$$

$$= \frac{3\pi}{2} - 3 = \frac{3}{2}(\pi - 2) \text{ sq. units}$$



Required area = $ar(\triangle ABC) + ar(\triangle CDE)$

$$= \left| \int_{-2}^{-1} (x+1) dx \right| + \int_{-1}^3 (x+1) dx = \left[\frac{x^2}{2} + x \right]_{-2}^{-1} + \left[\frac{x^2}{2} + x \right]_{-1}^3$$

$$= \left| \frac{1}{2} - 1 - (-2 - 2) \right| + \left\{ \frac{9}{2} + 3 - \left(\frac{1}{2} - 1 \right) \right\} = \frac{1}{2} + 8 = \frac{17}{2} = 8.5 \text{ sq. units}$$

9. Given, area = $2 \int_0^{4a} \sqrt{4ax} dx = \frac{256}{3}$

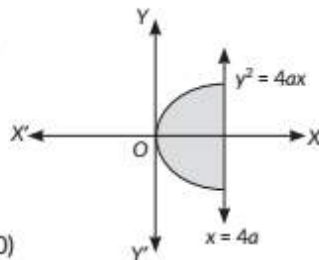
$$\Rightarrow 4\sqrt{a} \int_0^{4a} \sqrt{x} dx = \frac{256}{3}$$

$$\Rightarrow 4\sqrt{a} \frac{2}{3} [(x)^{3/2}]_0^{4a} = \frac{256}{3}$$

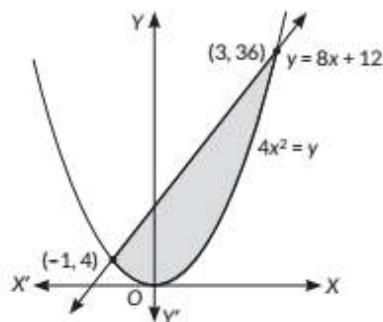
$$\Rightarrow \frac{8}{3} \sqrt{a} (4a)^{3/2} = \frac{256}{3}$$

$$\Rightarrow a^2 = \frac{256}{3} \times \frac{3}{8 \times 2 \times 4}$$

$$\Rightarrow a^2 = 4 \Rightarrow a = 2 (\because a > 0)$$



10. The graph of given region is

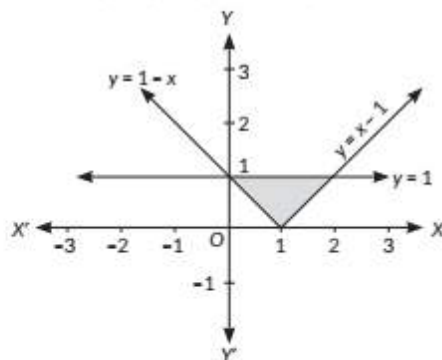


$$\therefore \text{Required area} = \int_{-1}^3 (8x + 12 - 4x^2) dx$$

$$= \left[4x^2 + 12x - \frac{4}{3}x^3 \right]_{-1}^3$$

$$= 36 + 36 - 36 - \left(4 - 12 + \frac{4}{3} \right) = 44 - \frac{4}{3} = \frac{128}{3} \text{ sq. units}$$

11. Given curve, $y = |x - 1|$ and line $y = 1$



We have, $y = \begin{cases} x - 1, & \text{if } x - 1 \geq 0 \\ -x + 1, & \text{if } x - 1 < 0 \end{cases}$

Required area = area of shaded region

$$= \int_0^2 1 dx - \left\{ \int_0^1 (1-x) dx + \int_1^2 (x-1) dx \right\}$$

$$= [x]_0^2 - \left\{ \left[x - \frac{x^2}{2} \right]_0^1 + \left[\frac{x^2}{2} - x \right]_1^2 \right\}$$

$$= 2 - \left(1 - \frac{1}{2} + 2 - 2 - \frac{1}{2} + 1 \right)$$

$$= 2 - \left(\frac{1}{2} + \frac{1}{2} \right) = 2 - 1 = 1 \text{ sq. unit}$$

12. Given, $x^2 + y^2 = 16$

$$y = x \quad \dots \text{(ii)} \quad \text{and} \quad x = 0$$

From equation (i) and (iii), we get

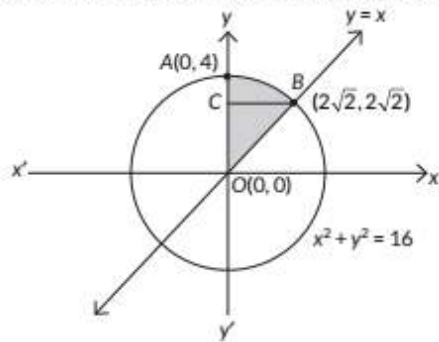
$$x = 0 \quad \text{and} \quad y = \pm 4$$

\therefore Equation (i) and (iii) intersect at $(0, 4)$ and $(0, -4)$.

From equation (i) and (ii), we get

$$x = \pm 2\sqrt{2} \quad \text{and} \quad y = \pm 2\sqrt{2}$$

Required bounded region is shown in the figure.



Let A be the required area.

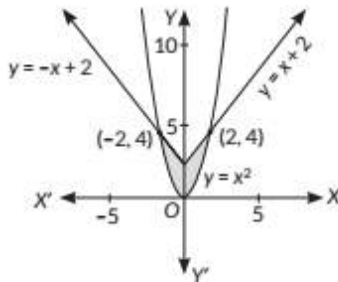
$A = \text{Area of } \triangle OBC + \text{Area of region CBAC}$

$$= \int_0^{2\sqrt{2}} y dy + \int_{2\sqrt{2}}^4 \sqrt{16-y^2} dy$$

$$= \left[\frac{y^2}{2} \right]_0^{2\sqrt{2}} + \left[\frac{y}{2} \sqrt{16-y^2} + \frac{16}{2} \sin^{-1} \left(\frac{y}{4} \right) \right]_{2\sqrt{2}}^4$$

$$= 4 + 8 \times \frac{\pi}{2} - 4 - 8 \times \frac{\pi}{4} = 2\pi \text{ sq. units}$$

13. The graph of given region is

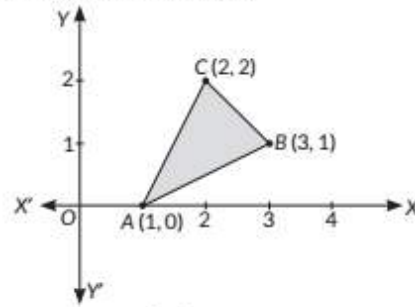


We have, $y = \begin{cases} x+2 & \text{if } x \geq 0 \\ -x+2 & \text{if } x < 0 \end{cases}$

$$\text{Required area} = 2 \int_0^2 (x+2-x^2) dx$$

$$= 2 \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_0^2 = 4 + 8 - \frac{16}{3} = \frac{20}{3} \text{ sq. units}$$

14. The graph of given region is



$$\text{Equation of AB, } y-0 = \frac{1-0}{3-1}(x-1)$$

$$\Rightarrow 2y = x-1 \Rightarrow y = \frac{x-1}{2}$$

$$\text{Equation of BC, } y-2 = \frac{1-2}{3-2}(x-2)$$

$$\Rightarrow y = -x+4$$

$$\text{Equation of AC, } y-0 = \frac{2-0}{2-1}(x-1)$$

$$\Rightarrow y = 2x-2$$

Required area

$$= \int_1^2 \left[2x-2 - \left(\frac{x-1}{2} \right) \right] dx + \int_2^3 \left[(-x+4) - \left(\frac{x-1}{2} \right) \right] dx$$

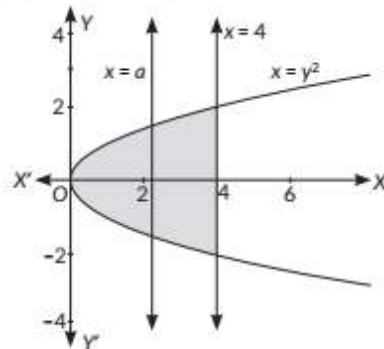
$$= \int_1^2 (3x-3) dx + \int_2^3 (-3x+9) dx$$

$$= 3 \left[\frac{x^2}{2} - x \right]_1^2 + \left[-\frac{3x^2}{2} + 9x \right]_2^3$$

$$= 3 \left(2 - 2 - \frac{1}{2} + 1 \right) + \left(-\frac{27}{2} + 27 + 6 - 18 \right)$$

$$= \frac{3}{2} + \frac{3}{2} = 3 \text{ sq. units}$$

15. The graph of given region is



$$\text{According to question, } 2 \int_0^a \sqrt{x} dx = 2 \int_a^4 \sqrt{x} dx$$

$$\Rightarrow \frac{2}{3} [x^{3/2}]_0^a = \frac{2}{3} [x^{3/2}]_a^4$$

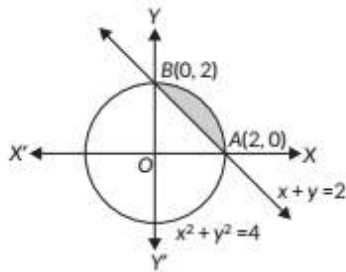
$$\Rightarrow a^{3/2} = 8 - a^{3/2}$$

$$\Rightarrow 2a^{3/2} = 8$$

$$\Rightarrow a^{3/2} = 4 \Rightarrow a = (4^2)^{1/3}$$

$$\Rightarrow a = (16)^{1/3} = (2)^{4/3}$$

16. The given curves are $x^2 + y^2 = 4$... (i) and $x + y = 2$



$$\therefore \text{Required area} = \int_0^2 [\sqrt{4-x^2} - (2-x)] dx$$

$$= \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2} \sin^{-1} \frac{x}{2} - 2x + \frac{x^2}{2} \right]_0^2$$

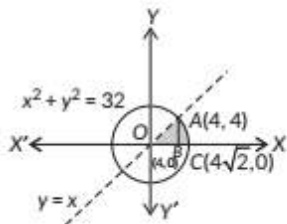
$$= 0 + 2 \sin^{-1}(1) - 4 + 2 - 0$$

$$= 2 \cdot \frac{\pi}{2} - 2 = (\pi - 2) \text{ sq. units.}$$

Key Points

$$\Rightarrow \int \sqrt{a^2 - x^2} dx = \frac{x \cdot \sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + c$$

17. The given equation of the circle is $x^2 + y^2 = 32$ and the line is $y = x$. These intersect at $A(4, 4)$ in the first quadrant. The required area is shown shaded in the figure. Points $B(4, 0)$ and $C(4\sqrt{2}, 0)$.



$$\therefore \text{Required area} = \text{Area BACB} + \text{Area OABO}$$

$$= \int_4^{4\sqrt{2}} y_1 dx + \int_0^4 y_2 dx = \int_4^{4\sqrt{2}} \sqrt{32-x^2} dx + \int_0^4 x dx$$

$$= \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2} dx + \int_0^4 x dx$$

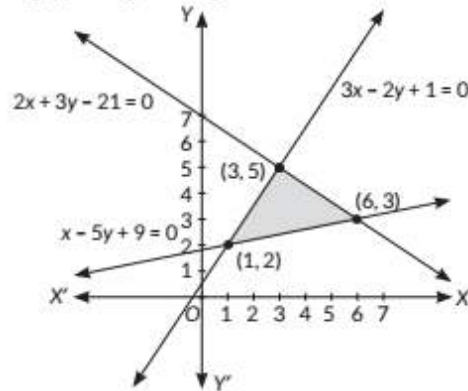
$$= \left[\frac{x\sqrt{32-x^2}}{2} + \frac{32}{2} \sin^{-1} \left(\frac{x}{4\sqrt{2}} \right) \right]_4^{4\sqrt{2}} + \left[\frac{x^2}{2} \right]_0^4$$

$$= \frac{4\sqrt{2} \times 0}{2} + 16 \sin^{-1} 1 - \left(\frac{4 \times 4}{2} + 16 \sin^{-1} \frac{1}{\sqrt{2}} \right) + \frac{1}{2} (4^2 - 0)$$

$$= 16 \cdot \frac{\pi}{2} - \left(8 + 16 \cdot \frac{\pi}{4} \right) + 8 = 16 \left(\frac{\pi}{2} - \frac{\pi}{4} \right) = 4\pi \text{ sq. units}$$

... (ii)

18. The graph of given region is



Required area

$$= \int_1^3 \left(\frac{3x+1}{2} - \frac{x+9}{5} \right) dx + \int_3^6 \left(\frac{21-2x}{3} - \frac{x+9}{5} \right) dx$$

$$= \left[\frac{3x^2}{4} + \frac{x}{2} - \frac{x^2}{10} - \frac{9}{5}x \right]_1^3 + \left[7x - \frac{x^2}{3} - \frac{x^2}{10} - \frac{9}{5}x \right]_3^6$$

$$= \frac{13}{5} + \frac{39}{10} = \frac{65}{10} = \frac{13}{2} \text{ sq. units}$$

19. We have curves, $y = \frac{1}{\sqrt{3}}x$... (i)

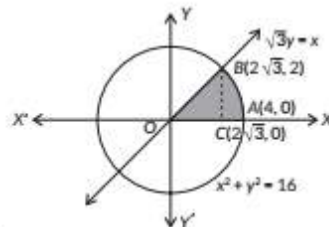
and $x^2 + y^2 = 16$... (ii)

Curves (i) and (ii) intersect at $(2\sqrt{3}, 2)$ and $(-2\sqrt{3}, -2)$.

\therefore Required area = Area of region OBAO = area of $\triangle OBC$ + area of region BCAB

$$= \int_0^{2\sqrt{3}} \frac{x}{\sqrt{3}} dx + \int_{2\sqrt{3}}^4 \sqrt{16-x^2} dx$$

$$= \left[\frac{x^2}{2\sqrt{3}} \right]_0^{2\sqrt{3}} + \left[\frac{x}{2} \sqrt{16-x^2} + \frac{16}{2} \sin^{-1} \left(\frac{x}{4} \right) \right]_{2\sqrt{3}}^4$$



$$= 2\sqrt{3} + 8 \left(\frac{\pi}{2} \right) - 2\sqrt{3} - \frac{8\pi}{3} = \frac{12\pi - 8\pi}{3} = \frac{4\pi}{3} \text{ sq. units}$$

Commonly Made Mistake

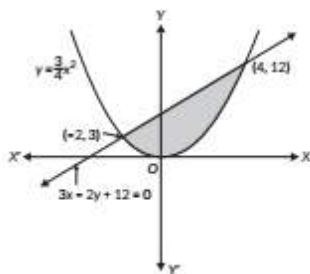
\Rightarrow Remember difference between the formula for $\int \sqrt{a^2 - x^2} dx$ and $\int \sqrt{x^2 - a^2} dx$.

20. Given equations are $y = \frac{3x^2}{4}$... (i)

and $3x - 2y + 12 = 0 \Rightarrow y = \frac{3x+12}{2}$... (ii)

Solving equations (i) and (ii), we get

$$\frac{3x^2}{4} = \frac{3x+12}{2}$$



$$\Rightarrow x^2 - 2x - 8 = 0 \Rightarrow (x+2)(x-4) = 0 \Rightarrow x = -2, 4$$

$$\text{When } x = -2 \Rightarrow y = 3$$

$$\text{When } x = 4 \Rightarrow y = 12$$

$$\therefore \text{ Required area} = \int_{-2}^4 \left(\frac{3x+12}{2} - \frac{3}{4}x^2 \right) dx$$

$$= \left[\frac{3}{4}x^2 + 6x - \frac{x^3}{4} \right]_{-2}^4$$

$$= \left[\frac{3 \times 16}{4} + 6 \times 4 - \frac{64}{4} \right] - \left[\frac{3}{4} \times 4 - 6 \times 2 + \frac{8}{4} \right] = 27 \text{ sq. units.}$$

21. We have curves $x - y + 2 = 0$ and $x = \sqrt{y}$.

$\Rightarrow y = x^2$, which is a parabola with vertex at origin.

From the given equations, we get

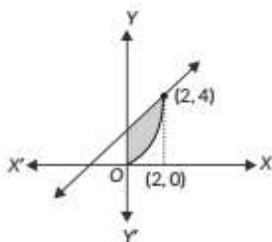
$$x - x^2 + 2 = 0 \Rightarrow (x-2)(x+1) = 0$$

$$\Rightarrow x = 2 \text{ or } x = -1 \Rightarrow x = 2$$

$[\because x \neq -1, x \text{ is positive}]$

$$\text{When } x = 2, y = 4$$

So, the point of intersection is (2, 4).



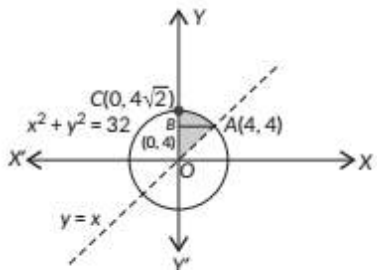
$$\therefore \text{ Required area} = \int_0^2 (x+2) dx - \int_0^2 x^2 dx = \int_0^2 (x+2-x^2) dx$$

$$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_0^2$$

$$= 2 + 4 - \frac{8}{3} = \frac{10}{3} \text{ sq. units}$$

22. The given equation of the circle is $x^2 + y^2 = 32$ and the line is $y = x$.

These intersect at A(4, 4) in the first quadrant. The required area is shown shaded in the figure. Points B(0, 4) and C(0, $4\sqrt{2}$).



\therefore Required area = Area BACB + Area OABO

$$= \int_4^{4\sqrt{2}} x_1 dy + \int_0^4 x_2 dy$$

$$= \int_4^{4\sqrt{2}} \sqrt{32-y^2} dy + \int_0^4 y dy$$

$$= \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - y^2} dy + \int_0^4 y dy$$

$$= \left[\frac{y\sqrt{32-y^2}}{2} + \frac{32}{2} \sin^{-1} \left(\frac{y}{4\sqrt{2}} \right) \right]_4^{4\sqrt{2}} + \left[\frac{y^2}{2} \right]_0^4$$

$$= \frac{4\sqrt{2} \times 0}{2} + 16 \sin^{-1} 1 - \left(\frac{4 \times 4}{2} + 16 \sin^{-1} \frac{1}{\sqrt{2}} \right) + \frac{1}{2} (4^2 - 0)$$

$$= 16 \cdot \frac{\pi}{2} - \left(8 + 16 \cdot \frac{\pi}{4} \right) + 8 = 16 \left(\frac{\pi}{2} - \frac{\pi}{4} \right) = 4\pi \text{ sq. units}$$

Concept Applied

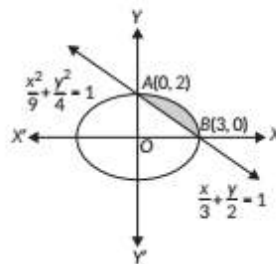
Area of the region bounded by the curve $x = g(y)$, y -axis and the lines $y = a$ and $y = b$ ($b > a$) is,

$$\text{Area} = \int_a^b x dy = \int_a^b g(y) dy$$

23. We have $\frac{x^2}{9} + \frac{y^2}{4} = 1$... (i) and $\frac{x}{3} + \frac{y}{2} = 1$... (ii)

Curve (i) is an ellipse of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

That means its major axis is along x -axis. Also this ellipse is symmetrical about the x -axis.



$$\text{Required area} = \frac{2}{3} \int_0^3 \sqrt{(3)^2 - x^2} dx - \frac{2}{3} \int_0^3 (3-x) dx$$

$$= \frac{2}{3} \left[\frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \left(\frac{x}{3} \right) \right]_0^3 - \frac{2}{3} \left[\frac{(3-x)^2}{-2} \right]_0^3$$

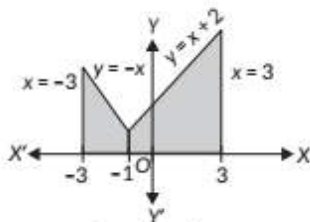
$$= \frac{2}{3} \left[\left(0 + \frac{9}{2} \sin^{-1}(1) \right) - \left(0 + \frac{9}{2} \sin^{-1}(0) \right) \right] + \frac{1}{3} [0^2 - 9]$$

$$= \frac{3\pi}{2} - 3 \text{ sq. units.}$$

24. Here, $y = |x+1| + 1$

$$y = \begin{cases} x+2 & \text{if } x \geq -1 \\ -x & \text{if } x < -1 \end{cases}$$

We know draw the lines: $y = 0, x = 3, x = -3$ and
 $y = x + 2$ if $x \geq -1$
 $y = -x$ if $x < -1$
 Lines (i) and (ii) intersect at $(-1, 1)$

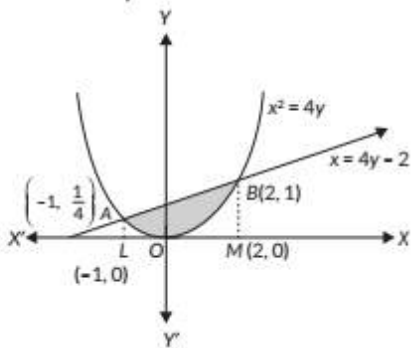


$$\therefore \text{ Required area} = \int_{-3}^{-1} (-x) dx + \int_{-1}^3 (x+2) dx$$

$$= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^3 = -\frac{1}{2}(1-9) + \frac{1}{2}(9-1) + 2(3+1)$$

$$= 4 + 4 + 8 = 16 \text{ sq. units.}$$

25. The given curve is $x^2 = 4y$
 The given line is $x = 4y - 2$



Putting $4y = (x+2)$ from (ii) in (i), we get $(x+2) = x^2$

$$\Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0 \Rightarrow x = 2, -1$$

Putting $x = 2$ in (i), we get $y = 1$

$$\text{Putting } x = -1 \text{ in (i), we get } y = \frac{1}{4}$$

Thus the points of intersection of the given curve and line are $A(-1, \frac{1}{4})$ and $B(2, 1)$.

\therefore Required area

$$= \int_{-1}^2 \left(\frac{x+2}{4}\right) dx - \int_{-1}^2 \frac{x^2}{4} dx = \int_{-1}^2 \left(\frac{x}{4} + \frac{1}{2} - \frac{x^2}{4}\right) dx$$

$$= \frac{1}{4} \left[\frac{x^2}{2}\right]_{-1}^2 + \frac{1}{2} [x]_{-1}^2 - \frac{1}{4} \left[\frac{x^3}{3}\right]_{-1}^2$$

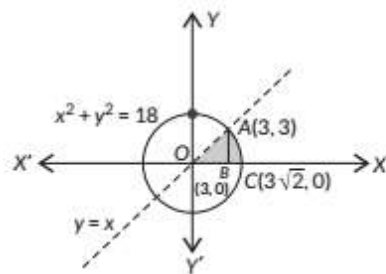
$$= \frac{1}{8} [4-1] + \frac{1}{2} [2+1] - \frac{1}{12} [8+1]$$

$$= \frac{3}{8} + \frac{3}{2} - \frac{3}{4} = \frac{3}{2} \left[\frac{1}{4} + 1 - \frac{1}{2}\right] = \frac{3}{2} \left[\frac{3}{4}\right] = \frac{9}{8} \text{ sq. units}$$

26. The given equation of the circle is $x^2 + y^2 = 18$ and the line is $y = x$.

These intersect at $A(3, 3)$ in the first quadrant. The required area is shown shaded in the figure. Points $B(3, 0)$ and $C(3\sqrt{2}, 0)$.

...(i)
 ...(ii)



\therefore Required area = Area BACB + Area OABO

$$= \int_3^{3\sqrt{2}} y_1 dx + \int_0^3 y_2 dx = \int_3^{3\sqrt{2}} \sqrt{18-x^2} dx + \int_0^3 x dx$$

$$= \int_3^{3\sqrt{2}} \sqrt{(3\sqrt{2})^2 - x^2} dx + \int_0^3 x dx$$

$$= \left[\frac{x\sqrt{18-x^2}}{2} + \frac{18}{2} \sin^{-1} \left(\frac{x}{3\sqrt{2}} \right) \right]_3^{3\sqrt{2}} + \left[\frac{x^2}{2} \right]_0^3$$

$$= \frac{3\sqrt{2} \times 0}{2} + 9 \sin^{-1} 1 - \left(\frac{3 \times 3}{2} + 9 \sin^{-1} \frac{1}{\sqrt{2}} \right) + \frac{1}{2} (9-0)$$

$$= 9 \cdot \frac{\pi}{2} - \left(\frac{9}{2} + 9 \cdot \frac{\pi}{4} \right) + \frac{9}{2} = 9 \left(\frac{\pi}{2} - \frac{\pi}{4} \right) = \frac{9\pi}{4} \text{ sq. units}$$

Answer Tips

Area of the region in the first quadrant enclosed by x-axis, the line $y = x$ and circle $x^2 + y^2 = a^2$ is $\frac{\pi a^2}{8}$.

CBSE Sample Questions

1. Required area, $A = \int_{-1}^1 x^2 dx$

$$\Rightarrow A = 2 \int_0^1 x^2 dx = \frac{2}{3} \left[x^3 \right]_0^1 = \frac{2}{3} \text{ sq. units} \quad (1)$$

2. Required area

$$= 2 \int_0^2 \sqrt{8x} dx \quad (1)$$

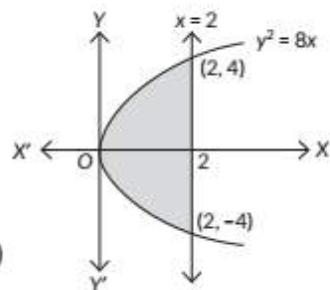
$$= 2 \times 2\sqrt{2} \int_0^2 x^{1/2} dx$$

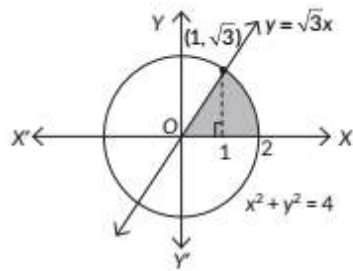
$$= 4\sqrt{2} \left[\frac{2}{3} x^{3/2} \right]_0^2 \quad (1/2)$$

$$= \frac{8}{3} \sqrt{2} [2^{3/2} - 0] = \frac{8\sqrt{2}}{3} \times 2\sqrt{2} = \frac{32}{3} \text{ sq. units} \quad (1/2)$$

3. Solving $y = \sqrt{3x}$ and $x^2 + y^2 = 4$, we get
 $x^2 + 3x = 4$

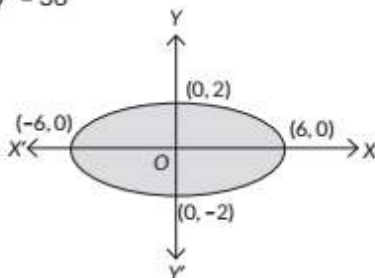
$$\Rightarrow x^2 = 1 \Rightarrow x = 1 \quad (1/2)$$





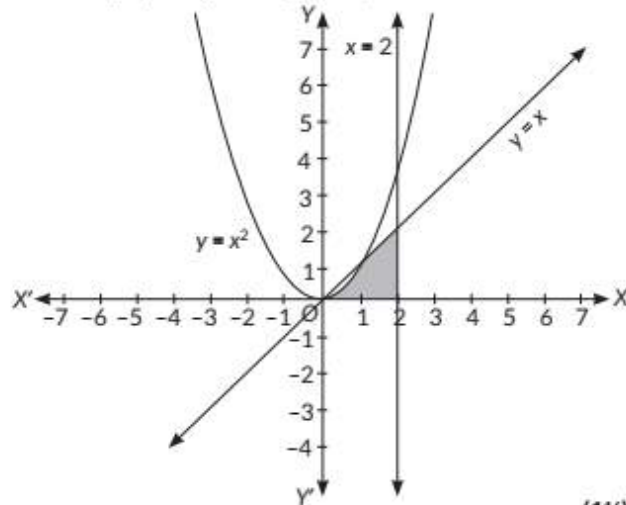
$$\begin{aligned} \therefore \text{Required area} &= \int_0^1 \sqrt{3}x dx + \int_1^2 \sqrt{2^2 - x^2} dx \\ &= \frac{\sqrt{3}}{2} [x^2]_0^1 + \left[\frac{x}{2} \sqrt{2^2 - x^2} + 2 \sin^{-1} \left(\frac{x}{2} \right) \right]_1^2 \\ &= \frac{\sqrt{3}}{2} + \left[2 \times \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \times \frac{\pi}{6} \right] = \frac{2\pi}{3} \text{ sq. units} \end{aligned}$$

4. Given equation of ellipse is $x^2 + 9y^2 = 36$



$$\begin{aligned} \therefore \text{Required area} &= 4 \int_0^6 \sqrt{\frac{36 - x^2}{9}} dx \\ &= \frac{4}{3} \int_0^6 \sqrt{6^2 - x^2} dx \\ &= \frac{4}{3} \left[\frac{x}{2} \sqrt{6^2 - x^2} + 18 \sin^{-1} \left(\frac{x}{6} \right) \right]_0^6 \\ &= \frac{4}{3} \left[18 \times \frac{\pi}{2} - 0 \right] = 12\pi \text{ sq. units} \end{aligned}$$

5. The graph of given region is;



(1½)

The points of intersection of the parabola $y = x^2$ and the line $y = x$ are

$$\begin{aligned} x^2 &= x \\ \Rightarrow x(x - 1) &= 0 \\ \Rightarrow x &= 0, 1 \end{aligned} \quad (1)$$

So, point of intersection is (0, 0) and (1, 1).

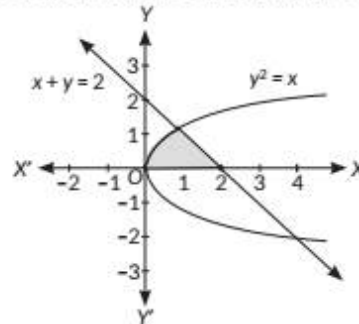
$$\text{Required area} = \int_0^1 x^2 dx + \int_1^2 x dx \quad (1)$$

(1/2)

$$(1/2) \quad = \left[\frac{x^3}{3} \right]_0^1 + \left[\frac{x^2}{2} \right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6} \text{ sq. units} \quad (1\frac{1}{2})$$

- (1) 6. By solving $x + y = 2$ and $y^2 = x$ simultaneously, we get the points of intersection as (1, 1) and (4, -2).

(1/2)



\therefore Required area = The shaded area

$$= \int_0^1 \sqrt{x} dx + \int_1^2 (2 - x) dx \quad (1)$$

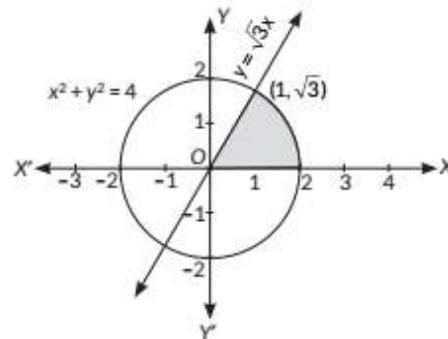
(1/2)

$$= \frac{2}{3} [x^{3/2}]_0^1 + \left[2x - \frac{x^2}{2} \right]_1^2 = \frac{2}{3} + \frac{1}{2} = \frac{7}{6} \text{ square units.} \quad (1)$$

(1/2)

7. By solving $y = \sqrt{3}x$ and $x^2 + y^2 = 4$, we get the points of intersection as $(1, \sqrt{3})$ and $(-1, -\sqrt{3})$.

(1)



(1)

\therefore Required area = The shaded area

$$= \int_0^1 \sqrt{3}x dx + \int_1^2 \sqrt{4 - x^2} dx$$

$$= \frac{\sqrt{3}}{2} [x^2]_0^1 + \frac{1}{2} \left[x \sqrt{4 - x^2} + 4 \sin^{-1} \frac{x}{2} \right]_1^2 \quad (1)$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2} \left[2\pi - \sqrt{3} - 2 \times \frac{\pi}{3} \right]$$

$$= \frac{2\pi}{3} \text{ square units.} \quad (1)$$