

# Continuity

## 4 Marks Questions

1. Find the value of  $k$ , so that the function  $f$  defined below, is continuous at  $x = 0$ , where

$$f(x) = \begin{cases} \left(\frac{1 - \cos 4x}{8x^2}\right), & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases} \quad \text{All India 2014C}$$

Given function is

$$f(x) = \begin{cases} \left(\frac{1 - \cos 4x}{8x^2}\right), & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$

Also, given  $f(x)$  is continuous at  $x = 0$ .

$$\therefore (\text{LHL})_{x=0} = (\text{RHL})_{x=0} = f(0) \quad \dots(i)$$

$$\text{Now, LHL} = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} \frac{1 - \cos 4x}{8x^2}$$

$$= \lim_{h \rightarrow 0} \frac{1 - \cos(-4h)}{8h^2}$$

[put  $x = 0 - h = -h$ , when  $x \rightarrow 0$ ,  $h \rightarrow 0$ ](1)

$$= \lim_{h \rightarrow 0} \frac{1 - \cos 4h}{8h^2}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin^2 2h}{8h^2}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{\sin^2 2h}{4h^2} = \lim_{h \rightarrow 0} \left(\frac{\sin 2h}{2h}\right)^2 = 1 \quad (1)$$

At  $x = 0$ ,  $f(0) = k$

Now, from Eq. (i), we have

$$\text{LHL} = f(0)$$

$$\Rightarrow 1 \cdot 1 = k \Rightarrow k = 1 \quad (1)$$

Hence, for  $k = 1$ , the given function  $f(x)$  is continuous at  $x = 0$ . (1)

$$2. \text{ If } f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & \text{when } x < 0 \\ a, & \text{when } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & \text{when } x > 0 \end{cases}$$

and  $f$  is continuous at  $x = 0$ , then find the value of  $a$ .

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Given,

$$f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2}, & \text{when } x < 0 \\ a, & \text{when } x = 0 \\ \frac{\sqrt{x}}{(\sqrt{16 + \sqrt{x}}) - 4}, & \text{when } x > 0 \end{cases}$$

Since,  $f(x)$  is continuous at  $x = 0$ .

$$\therefore \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$\begin{aligned} \Rightarrow \lim_{x \rightarrow 0^-} \frac{1 - \cos 4x}{x^2} \\ = \lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{(\sqrt{16 + \sqrt{x}}) - 4} = f(0) \quad (1) \end{aligned}$$

$$\begin{aligned} &\Rightarrow \lim_{h \rightarrow 0} \frac{1 - \cos 4(0 - h)}{(0 - h)^2} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{0 + h}}{(\sqrt{16 + \sqrt{0 + h}} - 4)} = f(0) \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{1 - \cos 4h}{h^2} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{h}}{\sqrt{16 + \sqrt{h}} - 4} \times \frac{\sqrt{16 + \sqrt{h}} + 4}{\sqrt{16 + \sqrt{h}} + 4} = a \quad (1) \\ &\Rightarrow \lim_{h \rightarrow 0} \frac{2 \sin^2 2h}{h^2} = \lim_{h \rightarrow 0} \frac{\sqrt{h}(\sqrt{16 + \sqrt{h}} + 4)}{(16 + \sqrt{h}) - 16} = a \\ &[\because 1 - \cos 2\theta = 2 \sin^2 \theta \text{ and } (a + b)(a - b) \\ &= a^2 - b^2] \end{aligned}$$

$$\begin{aligned} &\Rightarrow 2 \lim_{h \rightarrow 0} \left( \frac{\sin 2h}{2h} \right)^2 \times 4 \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{h}(\sqrt{16 + \sqrt{h}} + 4)}{\sqrt{h}} = a \quad (1) \\ &[\because \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1] \end{aligned}$$

$$\begin{aligned} &\Rightarrow 2 \times (1)^2 \times 4 = \sqrt{16 + \sqrt{0}} + 4 = a \\ &\Rightarrow 8 = 4 + 4 = a \\ &\therefore a = 8 \quad (1) \end{aligned}$$

3. Find the value of  $k$ , for which

$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \leq x < 1 \end{cases}$$

is continuous at  $x=0$ .

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$$\text{Given, } f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \leq x < 1 \end{cases}$$

is continuous at  $x = 0$ .

$$\text{Now, } f(0) = \frac{2 \cdot 0 + 1}{0 - 1} = \frac{1}{-1} = -1 \quad (1)$$

$$\text{LHL} = \lim_{h \rightarrow 0} f(0 - h)$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{1-kh} - \sqrt{1+kh}}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{1-kh} - \sqrt{1+kh} \times (\sqrt{1-kh} + \sqrt{1+kh})}{-h(\sqrt{1-kh} + \sqrt{1+kh})} \quad (1)$$

$$= \lim_{h \rightarrow 0} \frac{(1-kh) - (1+kh)}{-h(\sqrt{1-kh} + \sqrt{1+kh})}$$

$$[\because (a+b)(a-b) = a^2 - b^2]$$

$$= \lim_{h \rightarrow 0} \frac{-2kh}{-h(\sqrt{1-kh} + \sqrt{1+kh})} \quad (1)$$

$$= \lim_{h \rightarrow 0} \frac{2k}{\sqrt{1-kh} + \sqrt{1+kh}} = \frac{2k}{1+1} = \frac{2k}{2} = k$$

$\therefore f(x)$  is continuous at  $x = 0$ .

$$\therefore f(0) = \text{LHL} \Rightarrow -1 = k$$

$$\Rightarrow k = -1 \quad (1)$$

4. Find the value of  $k$ , so that the function  $f$  defined by

$$f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases} \text{ is continuous at}$$

$$x = \frac{\pi}{2}$$

HOTS; Delhi 2012C; Foreign 2011



A function  $f(x)$  is said to be continuous at point  $x = a$ , if  $\text{LHL at } (x = a) = \text{RHL at } (x = a) = f(a)$ . So,

to find the value of  $k$ , we equate any one of LHL or RHL to  $f(a)$  and simplify it.

$$\text{Given function is } f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$$

Also, given that function is continuous at  $x = \pi/2$ .

$$\therefore \text{At } x = \frac{\pi}{2}, \text{ LHL} = \text{RHL} = f\left(\frac{\pi}{2}\right) \quad \dots(i) \quad (1)$$

$$\text{Now, LHL} = \lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{k \cos x}{\pi - 2x}$$

$$\text{LHL} = \lim_{h \rightarrow 0} \frac{k \cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = \lim_{h \rightarrow 0} \frac{k \sin h}{\pi - \pi + 2h}$$

$$\left[ \text{put } x = \frac{\pi}{2} - h, \text{ when } x \rightarrow \frac{\pi}{2}, \text{ then } h \rightarrow 0 \right]$$

$$\left[ \because \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta \right]$$

$$= \lim_{h \rightarrow 0} \frac{k \sin h}{2h}$$

$$= \frac{k}{2} \lim_{h \rightarrow 0} \frac{\sin h}{h} = \frac{k}{2} \left[ \because \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \right]$$

$$\Rightarrow \text{LHL} = k/2 \quad (1\frac{1}{2})$$

Also, from the given function, we get

$$f\left(\frac{\pi}{2}\right) = 3 \quad (1/2)$$

Now, from Eq. (i), we have

$$\text{LHL} = f\left(\frac{\pi}{2}\right) \Rightarrow \frac{k}{2} = 3$$

$$\text{Hence, } k = 6 \quad (1)$$

5. Find the value of  $a$  for which the function  $f$  is

$$\text{defined as } f(x) = \begin{cases} a \sin \frac{\pi}{2} (x + 1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$$

is continuous at  $x = 0$ .

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Given function is

$$f(x) = \begin{cases} a \sin \frac{\pi}{2} (x + 1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$$

Also, given that  $f(x)$  is continuous at  $x = 0$ .

$$\therefore \text{LHL} = \text{RHL} = f(0) \quad \dots(i) \quad (1)$$

$$\text{Now, LHL} = \lim_{x \rightarrow 0^-} a \sin \frac{\pi}{2} (x + 1)$$

$$\text{LHL} = \lim_{h \rightarrow 0} a \sin \frac{\pi}{2} (-h + 1)$$

[put  $x = 0 - h = -h$ ]

$$= \lim_{h \rightarrow 0} \frac{a \sin \left[ \frac{\pi(-h+1)}{2} \right]}{\frac{\pi(-h+1)}{2}} \times \frac{\pi(-h+1)}{2}$$

$$\left[ \because \text{multiplying numerator and denominator by} \right. \\ \left. \frac{\pi(-h+1)}{2} \right]$$

$$= \lim_{h \rightarrow 0} \frac{a \pi(-h+1)}{2} \left[ \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$\Rightarrow \text{LHL} = \frac{a\pi}{2} \quad [\text{put } h = 0] \quad (1)$$

$$\text{Now, RHL} = \lim_{x \rightarrow 0^+} \frac{\tan x - \sin x}{x^3}$$

Put  $x = 0 + h = h$ , we get

$$\dots \dots \tan h - \sin h \quad \dots \quad \frac{\sin h}{\cos h} - \sin h$$

$$\begin{aligned} \text{RHL} &= \lim_{h \rightarrow 0} \frac{\sin h - \sin h \cosh h}{h^3} = \lim_{h \rightarrow 0} \frac{\sin h(1 - \cosh h)}{h^3 \cosh h} \\ &= \lim_{h \rightarrow 0} \frac{\sin h - \sin h \cosh h}{h^3 \cosh h} = \lim_{h \rightarrow 0} \frac{\sin h(1 - \cosh h)}{h^3 \cosh h} \end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{\sin h}{h} \cdot \lim_{h \rightarrow 0} \frac{1 - \cosh h}{h^2} \cdot \lim_{h \rightarrow 0} \frac{1}{\cosh h}$$

$$= 1 \times \lim_{h \rightarrow 0} \frac{1 - \cosh h}{h^2} \times 1$$

$$\left[ \because \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \text{ and } \lim_{h \rightarrow 0} \frac{1}{\cosh h} = \frac{1}{\cosh 0} = \frac{1}{1} = 1 \right]$$

$$= \lim_{h \rightarrow 0} \frac{1 - \cosh h}{h^2}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{h}{2}}{h^2} \left[ \because 1 - \cos x = 2 \sin^2 \frac{x}{2} \right]$$

$$= \lim_{h \rightarrow 0} \frac{2 \times \sin^2 \frac{h}{2}}{\left(\frac{h^2}{4} \times 4\right)} = \lim_{h \rightarrow 0} \frac{2}{4} \times \lim_{h \rightarrow 0} \frac{\sin^2 \frac{h}{2}}{\frac{h^2}{4}}$$

[ $\because$  multiplying numerator and denominator by 4]

$$= \frac{1}{2} \times \lim_{h \rightarrow 0} \left( \frac{\sin \frac{h}{2}}{\frac{h}{2}} \right)^2 = \frac{1}{2} \times 1 \times 1$$

$$\left[ \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \quad (1)$$

$$\Rightarrow \text{RHL} = \frac{1}{2}$$

Now, from Eq. (i), we have LHL = RHL

$$\therefore \frac{a\pi}{2} = \frac{1}{2} \Rightarrow a\pi = 1 \Rightarrow a = \frac{1}{\pi} \quad (1)$$

6. If the function  $f(x)$  given by

$$f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$$

is continuous at  $x = 1$ , then find the values of  $a$  and  $b$ .

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The given function is

$$f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$$

Given that  $f(x)$  is continuous at  $x = 1$ .

$$\therefore \text{LHL} = \text{RHL} = f(x) \quad \dots(i)$$

$$\begin{aligned} \text{Now, LHL} &= \lim_{x \rightarrow 1^-} f(x) \\ &= \lim_{x \rightarrow 1^-} (5ax - 2b) \quad \text{(1)} \end{aligned}$$

$$\begin{aligned} \text{LHL} &= \lim_{h \rightarrow 0} [5a(1-h) - 2b] \\ &\quad [\text{put } x = 1 - h, \text{ when } x \rightarrow 1, h \rightarrow 0] \\ &= \lim_{h \rightarrow 0} (5a - 5ah - 2b) \end{aligned}$$

$$\Rightarrow \text{LHL} = 5a - 2b \quad [\text{put } h = 0] \text{ (1)}$$

$$\text{Now, RHL} = \lim_{x \rightarrow 1^+} (3ax + b)$$

$$\begin{aligned} \text{RHL} &= \lim_{h \rightarrow 0} [3a(1+h) + b] \\ &\quad [\text{put } x = 1 + h, \text{ when } x \rightarrow 1, h \rightarrow 0] \\ &= \lim_{h \rightarrow 0} (3a + 3ah + b) \end{aligned}$$

$$\Rightarrow \text{RHL} = 3a + b \quad [\text{put } h = 0]$$

$$\text{Also, given that } f(1) = 11 \quad \text{(1)}$$

Now, from Eq. (i), we have

$$\text{RHL} = f(1)$$

$$\Rightarrow 3a + b = 11 \quad \dots(\text{ii})$$

and  $LHL = f(1)$

$$\Rightarrow 5a - 2b = 11 \quad \dots(\text{iii})$$

On multiplying Eq. (ii) by 5 and Eq. (iii) by 3 and then subtracting, we get

$$\begin{array}{r} 15a + 5b = 55 \\ 15a - 6b = 33 \\ \hline - \quad + \quad - \\ 11b = 22 \end{array}$$

$$\Rightarrow b = 2$$

On putting the value of  $b$  in Eq. (ii), we get

$$3a + 2 = 11 \Rightarrow 3a = 9 \Rightarrow a = 3$$

Hence,  $a = 3$  and  $b = 2$ . (1)

7. Find the values of  $a$  and  $b$  such that the following function  $f(x)$  is a continuous function.

$$f(x) = \begin{cases} 5, & x \leq 2 \\ ax + b, & 2 < x < 10 \\ 21, & x \geq 10 \end{cases}$$

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The given function is continuous that means  $f(x)$  is continuous in its domain  $[2,10]$ , so we take  $x = 2$  and  $x = 10$  to check continuity and then find  $a$  and  $b$ .

The given function is

$$f(x) = \begin{cases} 5, & x \leq 2 \\ ax + b, & 2 < x < 10 \\ 21, & x \geq 10 \end{cases}$$

Also, given that  $f(x)$  is continuous at  $x = 2$  and at  $x = 10$ .

$\therefore$  By definition,

$$\text{LHL} = \text{RHL} = f(2) \quad \dots(i)$$

and  $\text{LHL} = \text{RHL} = f(10) \quad \dots(ii) \quad \mathbf{(1)}$

Now, first we calculate LHL and RHL at  $x = 2$ .

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} 5$$

$$\Rightarrow \text{LHL} = 5$$

$$\text{and } \text{RHL} = \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (ax + b)$$

$$\text{RHL} = \lim_{h \rightarrow 0} a(2 + h) + b = \lim_{h \rightarrow 0} (2a + ah + b)$$

$$\quad \quad \quad [\text{put } x = 2 + h, \text{ when } x \rightarrow 2, h \rightarrow 0]$$

$$= 2a + b \quad \quad \quad [\text{put } h = 0]$$

From Eq. (i), we have

$$\text{LHL} = \text{RHL}$$

$$\Rightarrow 2a + b = 5 \quad \dots(iii) \quad \mathbf{(1)}$$

Now, we find LHL and RHL at  $x = 10$ .

$$\text{LHL} = \lim_{x \rightarrow 10^-} f(x) = \lim_{x \rightarrow 10^-} (ax + b)$$

$$\text{LHL} = \lim_{h \rightarrow 0} [a(10 - h) + b]$$

$$[\text{put } x = 10 - h, \text{ when } x \rightarrow 10, h \rightarrow 0]$$

$$= \lim_{h \rightarrow 0} (10a - ah + b)$$

$$\Rightarrow \text{LHL} = 10a + b \quad [\text{put } h = 0]$$

$$\text{and RHL} = \lim_{x \rightarrow 10^+} f(x) = \lim_{x \rightarrow 10^+} 21 = 21$$

Now, from Eq. (ii), we have

$$\text{LHL} = \text{RHL}$$

$$\Rightarrow 10a + b = 21 \quad \dots(\text{iv}) \quad (1)$$

On subtracting Eq. (iv) from Eq. (iii), we get

$$-8a = -16 \Rightarrow a = 2$$

On putting  $a = 2$  in Eq. (iv), we get

$$20 + b = 21$$

$$\therefore b = 1$$

Hence,  $a = 2$  and  $b = 1$ . (1)

8. Find the relationship between  $a$  and  $b$ , so that the function  $f$  defined by

$$f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$$

is continuous at  $x = 3$ .

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The given function is  $f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$

Also, given that  $f(x)$  is continuous at point  $x = 3$ .

$$\therefore \text{LHL} = \text{RHL} = f(3) \quad \dots(i) \quad (1)$$

$$\text{Now,} \quad \text{LHL} = \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (ax + 1)$$

$$\text{LHL} = \lim_{h \rightarrow 0} [a(3 - h) + 1]$$

$$[\text{put } x = 3 - h, \text{ when } x \rightarrow 3, h \rightarrow 0]$$

$$= \lim_{h \rightarrow 0} (3a - ah + 1)$$

$$\Rightarrow \text{LHL} = 3a + 1 \quad [\text{put } h = 0] \quad (1)$$

$$\text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (bx + 3)$$

$$\text{RHL} = \lim_{h \rightarrow 0} [b(3 + h) + 3]$$

$$[\text{put } x = 3 + h, \text{ when } x \rightarrow 3, h \rightarrow 0]$$

$$= \lim_{h \rightarrow 0} (3b + bh + 3)$$

$$\Rightarrow \text{RHL} = 3b + 3 \quad [\text{put } h = 0] \quad (1)$$

From Eq. (i), we have

$$\text{LHL} = \text{RHL} \Rightarrow 3a + 1 = 3b + 3$$

$\Rightarrow 3a - 3b = 2$ , which is the required relation between  $a$  and  $b$ . (1)

**9.** Find the value of  $k$ , so that the function  $f$

$$\text{defined by } f(x) = \begin{cases} kx + 1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$$

is continuous at  $x = \pi$ .

Foreign 2011

$$\text{The given function is } f(x) = \begin{cases} kx + 1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$$

Also, given that  $f(x)$  is continuous at  $x = \pi$ .

$$\therefore \text{LHL} = \text{RHL} = f(\pi) \quad \dots(i) \quad (1)$$

$$\text{Now, LHL} = \lim_{x \rightarrow \pi^-} f(x) = \lim_{x \rightarrow \pi^-} (kx + 1)$$

$$\begin{aligned} \text{LHL} &= \lim_{h \rightarrow 0} [k(\pi - h) + 1] \\ &\quad [\text{put } x = \pi - h, \text{ when } x \rightarrow \pi, h \rightarrow 0] \\ &= \lim_{h \rightarrow 0} (k\pi - kh + 1) \\ &= k\pi + 1 \quad [\text{put } h = 0] \quad (1) \end{aligned}$$

$$\text{and RHL} = \lim_{x \rightarrow \pi^+} f(x) = \lim_{x \rightarrow \pi^+} \cos x$$

$$\begin{aligned} \text{RHL} &= \lim_{h \rightarrow 0} \cos(\pi + h) \\ &\quad [\text{put } x = \pi + h, \text{ when } x \rightarrow \pi, h \rightarrow 0] \\ &= \cos \pi \quad [\text{put } h = 0] \quad (1) \\ &= -1 \quad [\because \cos \pi = -1] \end{aligned}$$

Now, from Eq. (i), we have

$$\begin{aligned} \text{LHL} &= \text{RHL} \\ \Rightarrow k\pi + 1 &= -1 \\ \Rightarrow k\pi &= -2 \\ \therefore k &= \frac{-2}{\pi} \quad (1) \end{aligned}$$

**10.** For what values of  $\lambda$ , is the function

$$f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$$

is continuous at  $x = 0$ ?

Foreign 2011

The given function is

$$f(x) = \begin{cases} \lambda (x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$$

Also, given that  $f(x)$  is continuous at  $x = 0$ .

$$\therefore \text{LHL} = \text{RHL} = f(0) \quad \dots(i)$$

$$\text{Now, LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \lambda (x^2 - 2x) \quad (1)$$

$$\text{LHL} = \lim_{h \rightarrow 0} \lambda (h^2 + 2h)$$

$$[\text{put } x = 0 - h = -h, \text{ when } x \rightarrow 0, h \rightarrow 0]$$

$$= \lambda (0) \quad [\text{put } h = 0] \quad (1)$$

$$= 0$$

$$\text{and RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (4x + 1)$$

$$\text{RHL} = \lim_{h \rightarrow 0} (4h + 1)$$

$$[\text{put } x = 0 + h = h, \text{ when } x \rightarrow 0, h \rightarrow 0]$$

$$\Rightarrow \text{RHL} = 1 \quad [\text{put } h = 0] \quad (1)$$

Thus,  $\text{LHL} \neq \text{RHL}$


But it is given that  $\text{LHL} = \text{RHL}$  [from Eq. (i)]

Therefore, we get a contradiction.

Hence, there doesn't exist any real value of  $\lambda$  for which  $f(x)$  is continuous at  $x = 0$ . (1)

**11.** Discuss the continuity of the function  $f(x)$  at  $x = 1/2$ , when  $f(x)$  is defined as follows:

$$f(x) = \begin{cases} 1/2 + x, & 0 \leq x < 1/2 \\ 1, & x = 1/2 \\ 3/2 + x, & 1/2 < x \leq 1 \end{cases} \quad \text{Delhi 2011C}$$

 Here, we find LHL, RHL and  $f\left(\frac{1}{2}\right)$ . If

$\text{LHL} = \text{RHL} = f\left(\frac{1}{2}\right)$ , then we say that  $f(x)$  is

continuous at  $x = \frac{1}{2}$ , otherwise  $f(x)$  is

discontinuous at  $x = \frac{1}{2}$ .

The given function is

$$f(x) = \begin{cases} \frac{1}{2} + x, & 0 \leq x < \frac{1}{2} \\ 1, & x = \frac{1}{2} \\ \frac{3}{2} + x, & \frac{1}{2} < x \leq 1 \end{cases}$$

We have to check continuity of  $f(x)$  at  $x = \frac{1}{2}$ .  
(1)

$$\text{Now, LHL} = \lim_{x \rightarrow \frac{1}{2}^-} f(x) = \lim_{x \rightarrow \frac{1}{2}^-} \left( \frac{1}{2} + x \right)$$

$$\text{LHL} = \lim_{h \rightarrow 0} \left( \frac{1}{2} + \frac{1}{2} - h \right)$$

$$\left[ \text{put } x = \frac{1}{2} - h, \text{ when } x \rightarrow \frac{1}{2}, h \rightarrow 0 \right]$$

$$= \lim_{h \rightarrow 0} (1 - h)$$

$$= 1 \quad [\text{put } h = 0] \quad (1)$$

$$\text{RHL} = \lim_{x \rightarrow \frac{1}{2}^+} f(x) = \lim_{x \rightarrow \frac{1}{2}^+} \left( \frac{3}{2} + x \right)$$

$$\text{RHL} = \lim_{h \rightarrow 0} \left( \frac{3}{2} + \frac{1}{2} + h \right)$$

$$\left[ \text{put } x = \frac{1}{2} + h, \text{ when } x \rightarrow \frac{1}{2}, h \rightarrow 0 \right]$$

$$= \lim_{h \rightarrow 0} (2 + h) = 2 \quad [\text{put } h = 0] \quad (1)$$

Now, we know that a function  $f(x)$  is said to be continuous at point  $x = a$  if

continuous at point  $x = a$ , if

$$\text{LHL} = \text{RHL} = f(a).$$

Here,  $\text{LHL} \neq \text{RHL}$  at  $x = 1/2$ .

Hence,  $f(x)$  is discontinuous at  $x = \frac{1}{2}$ . (1)

**12.** Find the value of  $a$ , if the function  $f(x)$  defined by

$$f(x) = \begin{cases} 2x - 1, & x < 2 \\ a, & x = 2 \\ x + 1, & x > 2 \end{cases}$$

is continuous at  $x = 2$ .

All India 2011C; Delhi 2009C

$$\text{Given, } f(x) = \begin{cases} 2x - 1, & x < 2 \\ a, & x = 2 \\ x + 1, & x > 2 \end{cases}$$

is continuous at point  $x = 2$ .

$\therefore$  By definition of continuity of a function at a point, we have

$$\text{LHL} = \text{RHL} = f(2) \quad \dots(i) \quad (1)$$

$$\text{Now, LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (2x - 1)$$

$$\Rightarrow \text{LHL} = \lim_{h \rightarrow 0} [2(2 - h) - 1]$$

$$\quad \quad \quad [\text{put } x = 2 - h, \text{ when } x \rightarrow 2, h \rightarrow 0]$$

$$= \lim_{h \rightarrow 0} (4 - 2h - 1) = \lim_{h \rightarrow 0} (3 - 2h)$$

$$= 3 \quad \quad \quad [\text{put } h = 0] \quad (1\frac{1}{2})$$

Also, from the given function, we have

$$f(2) = a \quad \quad \quad (1/2)$$

On putting the values of  $f(2)$  and LHL in Eq. (i), we get

$$3 = a$$

$$\Rightarrow a = 3 \quad \quad \quad (1)$$

- 13.** Find the values of  $a$  and  $b$  such that the function defined as follows is continuous.

$$f(x) = \begin{cases} x + 2, & x \leq 2 \\ ax + b, & 2 < x < 5 \\ 3x - 2, & x \geq 5 \end{cases} \quad \text{Delhi 2010, 2010C}$$

• Given function is  $f(x) = \begin{cases} x + 2, & x \leq 2 \\ ax + b, & 2 < x < 5 \\ 3x - 2, & x \geq 5 \end{cases}$

Also, given that  $f(x)$  is continuous at  $x = 2$  and  $x = 5$ .

∴ By definition of continuity, we get

$$\text{LHL} = \text{RHL} = f(2) \quad \dots(i)$$

and  $\text{LHL} = \text{RHL} = f(5) \quad \dots(ii)$

First, we find LHL and RHL at  $x = 2$ . (1)

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (x + 2)$$

$$\Rightarrow \text{LHL} = \lim_{h \rightarrow 0} (2 - h + 2)$$

$$\text{[put } x = 2 - h, \text{ when } x \rightarrow 2, h \rightarrow 0]$$

$$= \lim_{h \rightarrow 0} (4 - h)$$

$$\Rightarrow \text{LHL} = 4 \quad \text{[put } h = 0]$$

$$\text{Now, RHL} = \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (ax + b)$$

$$\Rightarrow \text{RHL} = \lim_{h \rightarrow 0} [a(2 + h) + b]$$

$$\text{[put } x = 2 + h, \text{ when } x \rightarrow 2, h \rightarrow 0]$$

$$= \lim_{h \rightarrow 0} (2a + ah + b)$$

$$\Rightarrow \text{RHL} = 2a + b \quad \text{[put } h = 0]$$

From Eq. (i), we have

$$\text{LHL} = \text{RHL}$$

$$\therefore 2a + b = 4 \quad \dots(\text{iii}) \quad (1)$$

Now, we find LHL and RHL at  $x = 5$ .

$$\text{LHL} = \lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^-} (ax + b)$$

$$\Rightarrow \text{LHL} = \lim_{h \rightarrow 0} [a(5 - h) + b]$$

$$[\text{put } x = 5 - h, \text{ when } x \rightarrow 5, h \rightarrow 0]$$

$$= \lim_{h \rightarrow 0} (5a - ah + b)$$

$$\Rightarrow \text{LHL} = 5a + b \quad [\text{put } h = 0]$$

$$\text{and } \text{RHL} = \lim_{x \rightarrow 5^+} f(x) = \lim_{x \rightarrow 5^+} (3x - 2)$$

$$= \lim_{h \rightarrow 0} [3(5 + h) - 2]$$

$$[\text{put } x = 5 + h, \text{ when } x \rightarrow 5, h \rightarrow 0]$$

$$= \lim_{h \rightarrow 0} (15 + 3h - 2)$$

$$\Rightarrow \text{RHL} = 13 \quad [\text{put } h = 0]$$

$\therefore$  From Eq. (ii), we have

$$\text{LHL} = \text{RHL}$$

$$\Rightarrow 5a + b = 13 \quad \dots(\text{iv}) \quad (1)$$

On subtracting Eq. (iv) from Eq. (iii), we get

$$-3a = -9 \Rightarrow a = 3$$

Put  $a = 3$  in Eq. (iv), we get

$$15 + b = 13 \Rightarrow b = -2$$

$$\text{Hence, } a = 3 \text{ and } b = -2 \quad (1)$$

**14.** For what value of  $k$ , is the function defined by

$$f(x) = \begin{cases} k(x^2 + 2), & \text{if } x \leq 0 \\ 3x + 1, & \text{if } x > 0 \end{cases}, \text{ continuous at } x = 0?$$

Also, write whether the function is

continuous at  $x = 1$ .

Delhi 2010, 2010C

The given function is

$$f(x) = \begin{cases} k(x^2 + 2), & \text{if } x \leq 0 \\ 3x + 1, & \text{if } x > 0 \end{cases}$$

Also, given that  $f(x)$  is continuous at  $x = 0$ .

$$\therefore \text{LHL} = \text{RHL} = f(0) \quad \dots(i) \quad (1/2)$$

$$\text{Now, LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} k(x^2 + 2)$$

$$\Rightarrow \text{LHL} = \lim_{h \rightarrow 0} k(h^2 + 2)$$

$$\begin{aligned} & [\text{put } x = 0 - h = -h, \text{ when } x \rightarrow 0, h \rightarrow 0] \\ & = \lim_{h \rightarrow 0} (kh^2 + 2k) \end{aligned}$$

$$\Rightarrow \text{LHL} = 2k \quad [\text{put } h = 0] \quad (1/2)$$

$$\text{and RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (3x + 1)$$

$$\Rightarrow \text{RHL} = \lim_{h \rightarrow 0} (3h + 1)$$

$$[\text{put } x = 0 + h = h, \text{ when } x \rightarrow 0, h \rightarrow 0]$$

$$\Rightarrow \text{RHL} = 1 \quad [\text{put } h = 0] \quad (1/2)$$

Now, from Eq. (i), we have

$$\begin{aligned} \text{LHL} = \text{RHL} &\Rightarrow 2k = 1 \\ \Rightarrow k &= \frac{1}{2} \end{aligned} \quad (1/2)$$

Now, we check the continuity of the given function  $f(x)$  at  $x = 1$ .

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (3x + 1) \\ \Rightarrow \text{LHL} &= \lim_{h \rightarrow 0} [3(1 - h) + 1] \\ &\quad [\text{put } x = 1 - h, \text{ when } x \rightarrow 1, h \rightarrow 0] \\ &= \lim_{h \rightarrow 0} (3 - 3h + 1) \\ &= 4 \quad [\text{put } h = 0] \quad (1/2) \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (3x + 1) \\ \Rightarrow \text{RHL} &= \lim_{h \rightarrow 0} [3(1 + h) + 1] \\ &\quad [\text{put } x = 1 + h, \text{ when } x \rightarrow 1, h \rightarrow 0] \\ &= \lim_{h \rightarrow 0} [3 + 3h + 1] \\ &= 4 \quad [\text{put } h = 0] \quad (1/2) \end{aligned}$$

and  $f(1) = 4$  from given function.

$\therefore$  At  $x = 1$ ,  $\text{LHL} = \text{RHL} = f(1)$

Hence,  $f(x)$  is continuous at  $x = 1$ . (1)

- 15.** Find all points of discontinuity of  $f$ , where  $f$  is defined as follows:

$$f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$$

Delhi 2010



Firstly, verify continuity of the given function at  $x = -3$  and  $x = 3$ . Then, point at which the given function is discontinuous will be the point of discontinuity.

$$\text{The given function is } f(x) = \begin{cases} |x| + 3, & x \leq -3 \\ -2x, & -3 < x < 3 \\ 6x + 2, & x \geq 3 \end{cases}$$

First, we verify continuity at  $x = -3$  and then at  $x = 3$ .

**Continuity at  $x = -3$**

$$\text{LHL} = \lim_{x \rightarrow (-3)^-} f(x) = \lim_{x \rightarrow (-3)^-} (|x| + 3)$$

$$\Rightarrow \text{LHL} = \lim_{h \rightarrow 0} (|-3 - h| + 3)$$

[put  $x = -3 - h$ ,  
when  $x \rightarrow -3$ ,  $h \rightarrow 0$ ]

$$= |-3| + 3$$

$$= 3 + 3$$

$$= 6$$

[put  $h = 0$ ]

[ $\because |-x| = x, \forall x \in \mathbb{R}$ ]

(1/2)

$$\text{RHL} = \lim_{x \rightarrow (-3)^+} f(x) = \lim_{x \rightarrow (-3)^+} (-2x)$$

$$\Rightarrow \text{RHL} = \lim_{h \rightarrow 0} [-2(-3 + h)]$$

$$\begin{aligned} & [\text{put } x = -3 + h, \text{ when } x \rightarrow -3, h \rightarrow 0] \\ & = \lim_{h \rightarrow 0} (6 - 2h) \end{aligned}$$

$$\Rightarrow \text{RHL} = 6 \quad [\text{put } h = 0] \quad (1/2)$$

Also,  $f(-3) = \text{value of } f(x) \text{ at } x = -3$

$$= |-3| + 3$$

$$= 3 + 3 = 6 \quad [\because |-x| = x, \forall x \in \mathbb{R}] \quad (1/2)$$

$$\therefore \text{LHL} = \text{RHL} = f(-3)$$

$\therefore f(x)$  is continuous at  $x = -3$ . So,  $x = -3$  is the point of continuity. (1/2)

**Continuity at  $x = 3$**

$$\text{LHL} = \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} -2x$$

$$\Rightarrow \text{LHL} = \lim_{h \rightarrow 0} -2(3 - h)$$

$$[\text{put } x = 3 - h, \text{ when } x \rightarrow 3, h \rightarrow 0]$$

$$= \lim_{h \rightarrow 0} (-6 + 2h)$$

$$\Rightarrow \text{LHL} = -6 \quad [\text{put } h = 0] \quad (1/2)$$

$$\text{and } \text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (6x + 2)$$

$$\Rightarrow \text{RHL} = \lim_{h \rightarrow 0} [6(3 + h) + 2]$$

$$[\text{put } x = 3 + h, \text{ when } x \rightarrow 3, h \rightarrow 0]$$

$$= \lim_{h \rightarrow 0} (18 + 6h + 2)$$

$$\Rightarrow \text{RHL} = 20 \quad [\text{put } h = 0] \quad (1/2)$$

$$\therefore \text{LHL} \neq \text{RHL}$$

As  $f(x)$  is a polynomial function in a given interval, so it is continuous in a given interval but  $f(x)$  is not continuous at  $x = 3$ . So,  $x = 3$  is the point of discontinuity of  $f(x)$ . (1)

**16.** Show that the function  $f(x)$  defined by

$$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1 - \sqrt{1-x})}{x}, & x < 0 \end{cases}$$

is continuous at  $x = 0$ .

All India 2009C

To show that the given function is continuous at  $x = 0$ , we show that

$$(\text{LHL})_{x=0} = (\text{RHL})_{x=0} = f(0) \quad \dots(\text{i}) \quad (1)$$

Now, given function is

$$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1 - \sqrt{1-x})}{x}, & x < 0 \end{cases}$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{4(1 - \sqrt{1-x})}{x}$$

$$\begin{aligned}
\Rightarrow \text{LHL} &= \lim_{h \rightarrow 0} \frac{4[1 - \sqrt{1 - (0 - h)}]}{0 - h} \\
&\quad [\text{put } x = 0 - h, \text{ when } x \rightarrow 0, h \rightarrow 0] \\
&= \lim_{h \rightarrow 0} \frac{4[1 - \sqrt{1 + h}]}{-h} \\
&= \lim_{h \rightarrow 0} \frac{4[1 - \sqrt{1 + h}]}{-h} \times \frac{1 + \sqrt{1 + h}}{1 + \sqrt{1 + h}} \\
&= \lim_{h \rightarrow 0} \frac{4[(1)^2 - (\sqrt{1 + h})^2]}{-h[1 + \sqrt{1 + h}]} \\
&\quad [\because (a - b)(a + b) = a^2 - b^2] \\
&= \lim_{h \rightarrow 0} \frac{4[1 - (1 + h)]}{-h[1 + \sqrt{1 + h}]} \\
&= \lim_{h \rightarrow 0} \frac{-h \times 4}{-h[1 + \sqrt{1 + h}]} = \lim_{h \rightarrow 0} \frac{4}{1 + \sqrt{1 + h}} \\
&= \frac{4}{1 + \sqrt{1}} = \frac{4}{2} = 2 \quad [\text{put } h = 0]
\end{aligned}$$

$$\Rightarrow \text{LHL} = 2 \quad (1)$$

$$\text{Now, RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \left( \frac{\sin x}{x} + \cos x \right)$$

$$\begin{aligned}
\Rightarrow \text{RHL} &= \lim_{h \rightarrow 0} \left( \frac{\sin h}{h} + \cos h \right) \\
&\quad [\text{put } x = 0 + h = h, \text{ when } x \rightarrow 0, h \rightarrow 0] \\
&= \lim_{h \rightarrow 0} \frac{\sin h}{h} + \lim_{h \rightarrow 0} \cos h \quad [\text{put } h = 0] \\
&= 1 + \cos 0^\circ \quad \left[ \because \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \right] \\
&= 1 + 1 \quad [\because \cos 0 = 1] \\
&= 2 \quad (1)
\end{aligned}$$

Also, given that at  $x = 0$ ,  $f(x) = 2 \Rightarrow f(0) = 2$

Since,  $(\text{LHL})_{x=0} = (\text{RHL})_{x=0} = f(0) = 2$

Hence,  $f(x)$  is continuous at  $x = 0$ . (1)

17. For what value of  $k$ , is the following function continuous at  $x = 2$ ?

$$f(x) = \begin{cases} 2x + 1, & x < 2 \\ k, & x = 2 \\ 3x - 1, & x > 2 \end{cases}$$

Delhi 2008

Do same as Que 12.

[Ans.  $k = 5$ ]

18. If  $f(x)$  defined by the following, is continuous at  $x = 0$ , then find the values of  $a, b$  and  $c$ .

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & \text{if } x < 0 \\ c, & \text{if } x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, & \text{if } x > 0 \end{cases}$$

HOTS; All India 2008

Given function is

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & \text{if } x < 0 \\ c, & \text{if } x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, & \text{if } x > 0 \end{cases}$$

Also, given that  $f(x)$  is continuous at  $x = 0$ .

$$\therefore (\text{LHL})_{x=0} = (\text{RHL})_{x=0} = f(0) \quad \dots(i)$$

$$\text{Now, LHL} = \lim_{x \rightarrow 0^-} f(x)$$

$$= \lim_{x \rightarrow 0^-} \frac{\sin(a+1)x + \sin x}{x}$$

$$\Rightarrow \text{LHL} = \lim_{h \rightarrow 0} \frac{\sin[-(a+1)h] + \sin(-h)}{-h}$$

$$[\text{put } x = 0 - h = -h, \text{ when } x \rightarrow 0, h \rightarrow 0]$$

$$= \lim_{h \rightarrow 0} \frac{-\sin(a+1)h - \sin h}{-h}$$

$$[\because \sin(-\theta) = -\sin \theta]$$

$$= \lim_{h \rightarrow 0} \frac{\sin(a+1)h + \sin h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin \left[ \frac{(a+1)h+h}{2} \right] \cos \left[ \frac{(a+1)h-h}{2} \right]}{h}$$

$$\left[ \because \sin C + \sin D = 2 \sin \frac{(C+D)}{2} \cdot \cos \frac{(C-D)}{2} \right]$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin \left[ \frac{(a+1)h+h}{2} \right] \cos \left[ \frac{(a+1)h-h}{2} \right]}{\left[ \frac{(a+1)h+h}{2} \right] \times h}$$

$$\times \frac{(a+1)h+h}{2}$$

$$\left[ \text{multiplying and dividing by } \frac{(a+1)h+h}{2} \right]$$

$$= \lim_{h \rightarrow 0} \frac{2 \cos \left[ \frac{(a+1)h-h}{2} \right]}{h} \times \frac{h[a+1+1]}{2}$$

$$\left[ \begin{array}{l} \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \\ \therefore \lim_{h \rightarrow 0} \frac{\sin[(a+1)h+h]}{\frac{(a+1)h+h}{2}} = 1 \end{array} \right]$$

$$= \lim_{h \rightarrow 0} \frac{2(a+2)}{2} \cos \left[ \frac{(a+1)h-h}{2} \right]$$

$$= (a+2) \cos 0^\circ \quad [\text{put } h=0]$$

$$= (a+2) \times 1 \quad [\because \cos 0^\circ = 1]$$

$$= a+2 \quad (1\frac{1}{2})$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}$$

$$\Rightarrow \text{RHL} = \lim_{h \rightarrow 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{bh^{3/2}}$$

[put  $x = 0 + h = h$ , when  $x \rightarrow 0, h \rightarrow 0$ ]

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{\sqrt{h(1+bh)} - \sqrt{h}}{bh^{3/2}} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{h} [\sqrt{1+bh} - 1]}{b\sqrt{h} \cdot h} \quad [\because h^{3/2} = \sqrt{h} \cdot h] \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{1+bh} - 1}{bh} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{1+bh} - 1}{bh} \times \frac{\sqrt{1+bh} + 1}{\sqrt{1+bh} + 1} \end{aligned}$$

[multiplying and dividing by  $\sqrt{1+bh} + 1$ ]

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{(\sqrt{1+bh})^2 - (1)^2}{bh[\sqrt{1+bh} + 1]} \\ &\quad [\because (a+b)(a-b) = a^2 - b^2] \\ &= \lim_{h \rightarrow 0} \frac{1+bh-1}{bh[\sqrt{1+bh} + 1]} \\ &= \lim_{h \rightarrow 0} \frac{bh}{bh[\sqrt{1+bh} + 1]} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{1+bh} + 1} \quad (1\frac{1}{2}) \end{aligned}$$

$$\Rightarrow \text{RHL} = \frac{1}{1+1} = \frac{1}{2} \quad [\text{put } h = 0]$$

Now, from Eq. (i), we have

$$\begin{aligned} &\text{LHL} = \text{RHL} \\ \Rightarrow &a + 2 = \frac{1}{2} \\ \Rightarrow &a = \frac{1}{2} - 2 \Rightarrow a = -\frac{3}{2} \end{aligned}$$

Also, given that  $f(0) = c$

Again from Eq. (i), we have

$$\begin{aligned} &\text{RHL} = f(0) \\ \Rightarrow &c = 1/2 \end{aligned}$$

Hence, we get  $a = -\frac{3}{2}$ ,  $c = \frac{1}{2}$  and  $b$  may take any real value. (1)

**NOTE** Here, we cannot find any real and unique value of  $b$  that means  $b$  may take any real value, i.e.  $b \in R$ .

19. Show that  $f(x) = \begin{cases} 5x - 4, & \text{when } 0 < x \leq 1 \\ 4x^3 - 3x, & \text{when } 1 < x < 2 \end{cases}$  is continuous at  $x = 1$ . Delhi 2008C

Given function is

$$f(x) = \begin{cases} 5x - 4, & \text{when } 0 < x \leq 1 \\ 4x^3 - 3x, & \text{when } 1 < x < 2 \end{cases}$$

To show that  $f(x)$  is continuous at  $x = 1$ , we need to prove

$$\text{LHL}_{x=1} = \text{RHL}_{x=1} = f(1) \quad \dots(i) \quad (1)$$

$$\text{Now, LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (5x - 4)$$

$$\Rightarrow \text{LHL} = \lim_{h \rightarrow 0} [5(1-h) - 4]$$

$$[\text{put } x = 1 - h, \text{ when } x \rightarrow 1, h \rightarrow 0]$$

$$= \lim_{h \rightarrow 0} (5 - 5h - 4) = \lim_{h \rightarrow 0} (1 - 5h)$$

$$\text{LHL} = 1 \quad [\text{put } h = 0] \quad (1)$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (4x^3 - 3x)$$

$$\Rightarrow \text{RHL} = \lim_{h \rightarrow 0} [4(1+h)^3 - 3(1+h)]$$

$$\quad \quad \quad [\text{put } x = 1+h, \text{ when } x \rightarrow 1, h \rightarrow 0]$$

$$= 4(1)^3 - 3(1) \quad \quad \quad [\text{put } h = 0]$$

$$\Rightarrow \text{RHL} = 4 - 3 = 1 \quad \quad \quad (1)$$

Also, from given function,

$f(1) = \text{Value of } f(x) \text{ at } x = 1$

$$\Rightarrow f(1) = 5(1) - 4$$

$$\quad \quad \quad [\text{put } x = 1 \text{ in } f(x) = 5x - 4]$$

$$= 5 - 4 = 1$$

$$\text{Here, } (\text{LHL})_{x=1} = (\text{RHL})_{x=1} = f(1) \quad \quad \quad (1)$$

Hence,  $f(1)$  is continuous at  $x = 1$ .

**20.** If the following function  $f(x)$  is continuous at  $x = 0$ , then find the value of  $k$ .

$$f(x) = \begin{cases} \frac{1 - \cos 2x}{2x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

All India 2008C

Do same as Que 1.

[Ans.  $k = 1$ ]