

## Continuity and Differentiability

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(2025)

Q.1

If  $y = \sin^{-1} x$ , then  $(1 - x^2) \frac{d^2y}{dx^2}$  is equal to :

(1 Mark) (CBSE 2025 - 65/4/1)

A.

$$-x^2 \frac{dy}{dx}$$

B.

$$x \frac{dy}{dx}$$

C.

$$-x \frac{dy}{dx}$$

D.

$$x^2 \frac{dy}{dx}$$

Q.2

If

$$f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 + ax, & 1 < x < 2 \end{cases}$$

is continuous for

$$x \in (0, 2)$$

, then  $a$  is equal to:

(1 Mark) (CBSE 2025 - 65/4/1)

A.

$$-\frac{7}{2}$$

B. -4

C. -1

D. -2

Q.3

The function  $f$  defined by

$$f(x) = \begin{cases} x & \text{for } x \leq 1 \\ 5 & \text{for } x > 1 \end{cases}$$

is not continuous at:

(1 Mark) (CBSE 2025 - 65/4/1)

A.  $x = 5$

B.  $x = 1$

C.  $x = 2$

D.  $x = 0$

Q.4

If

$$f(x) = \begin{cases} \frac{\log(1+ax) + \log(1-bx)}{x} & , \text{ for } x \neq 0 \\ k & , \text{ for } x = 0 \end{cases}$$

is continuous at

$$x = 0$$

, then the value of

$k$

is :

(1 Mark) (CBSE 2025 - 65/4/1)

A. a

B. a + b

C. b

D. a - b

Q.5

If  $\tan^{-1}(x^2 - y^2) = a$ , where 'a' is a constant, then  $\frac{dy}{dx}$  is :

(1 Mark) (CBSE 2025 - 65/6/1)

A. a/x

B. x/y

C. a/y

D. -x/y

Q.6 If  $y = a \cos(\log x) + b \sin(\log x)$ , then  $x^2y_2 + xy_1$  :

(1 Mark) (CBSE 2025 - 65/6/1)

A.  $\tan(\log x)$

B.  $\cot(\log x)$

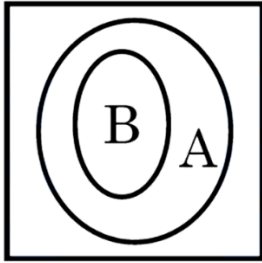
C. -y

D. y

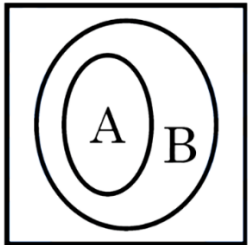
Q.7 If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B ?

(1 Mark) (CBSE 2025 - 65/2/1)

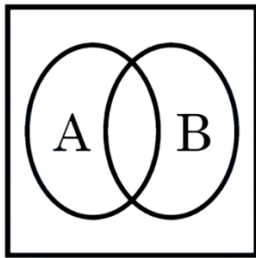
A.



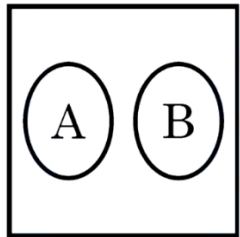
B.



C.



D.



Q.8

If

$$f(x) = \begin{cases} 1, & \text{if } x \leq 3 \\ ax + b, & \text{if } 3 < x < 5 \\ 7, & \text{if } 5 \leq x \end{cases}$$

is continuous in

$R$

, then the values of  $a$  and  $b$  are :

(1 Mark) (CBSE 2025 - 65/7/1)

A.  $a = -3, b = 8$

B.  $a = 3, b = 8$

C.  $a = -3, b = -8$

D.  $a = 3, b = -8$

Q.9 If  $f(x) = -2x^8$ , then the correct statement is :

(1 Mark) (CBSE 2025 - 65/7/1)

A.

$$-f' \left( \frac{1}{2} \right) = f \left( -\frac{1}{2} \right)$$

B.

$$f' \left( \frac{1}{2} \right) = -f' \left( -\frac{1}{2} \right)$$

C.

$$f' \left( \frac{1}{2} \right) = f' \left( -\frac{1}{2} \right)$$

D.

$$f \left( \frac{1}{2} \right) = -f \left( -\frac{1}{2} \right)$$

Q.10

Assertion (A) :

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous at  
 $x = 0$

Reason

(R) :

When

$$x \rightarrow 0, \sin \frac{1}{x}$$

is a finite value between -1 and 1 .

(1 Mark) (CBSE 2025 - 65/7/1)

- A. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- B. Assertion (A) is true, but Reason (R) is false.
- C. Assertion (A) is false, but Reason (R) is true.
- D. Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

Q.11

If

$$f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

is continuous at  
 $x = 0$

, then the value of

$a$

is :

(1 Mark) (CBSE 2025 - 65/5/1)

A. -1

B. 1

C.  $\pm 1$

D. 0

Q.12 If  $f(x) = \{[x], x \in \mathbb{R}\}$  is the greatest integer function, then the correct statement is :

(1 Mark) (CBSE 2025 - 65/5/1)

- A.  $f$  is not continuous but differentiable at  $x = 2$ .
- B.  $f$  is continuous as well as differentiable at  $x = 2$ .
- C.  $f$  is neither continuous nor differentiable at  $x = 2$ .
- D.  $f$  is continuous but not differentiable at  $x = 2$ .

Q.13 If  $f(x) = |x| + |x-1|$ , then which of the following is correct?

(1 Mark) (CBSE 2025 - 65/1/1)

- A.  $f(x)$  is both continuous and differentiable, at  $x = 0$  and  $x = 1$ .
- B.  $f(x)$  is differentiable but not continuous, at  $x = 0$  and  $x = 1$ .
- C.  $f(x)$  is continuous but not differentiable, at  $x = 0$  and  $x = 1$ .
- D.  $f(x)$  is neither continuous nor differentiable, at  $x = 0$  and  $x = 1$ .

Q.14

Assertion (A) :

$$f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$$

is continuous at

$$x = 5$$

for

$$k = \frac{5}{2}$$

Reason (R) : For a function  $f$  to be continuous at

$$x = a$$

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

(1 Mark) (CBSE 2025 - 65/1/1)

A. Assertion (A) is false, but Reason (R) is true.

B. Assertion (A) is true, but Reason (R) is false.

C. Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

D. Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

Q.15

Differentiate  $\sqrt{e^{\sqrt{2x}}}$  with respect to  $e^{\sqrt{2x}}$  for  $x > 0$ .

(2 Mark) (CBSE 2025 - 65/6/1)

Q.16

If  $(x)^y = (y)^x$ , then find  $\frac{dy}{dx}$ .

(2 Mark) (CBSE 2025 - 65/6/1)

Q.17

If  $x = e^{\frac{x}{y}}$ , then prove that  $\frac{dy}{dx} = \frac{x-y}{x \log x}$ .

(2 Mark) (CBSE 2025 - 65/2/1)

Q.18

If

$$f(x) = \begin{cases} 2x - 3, & -3 \leq x \leq -2 \\ x + 1, & -2 < x \leq 0 \end{cases}$$

Check the differentiability of

$f(x)$

at

$x = -2$

(2 Mark) (CBSE 2025 - 65/2/1)

Q.19

Differentiate  $\left(\frac{5^x}{x^5}\right)$  with respect to  $x$ .

(2 Mark) (CBSE 2025 - 65/7/1)

Q.20

If  $-2x^2 - 5xy + y^3 = 76$ , then find  $\frac{dy}{dx}$ .

(2 Mark) (CBSE 2025 - 65/7/1)

Q.21

Differentiate  $\frac{\sin x}{\sqrt{\cos x}}$  with respect to  $x$ .

(2 Mark) (CBSE 2025 - 65/5/1)

Q.22

If  $y = 5 \cos x - 3 \sin x$ , prove that  $\frac{d^2y}{dx^2} + y = 0$ .

(2 Mark) (CBSE 2025 - 65/5/1)

Q.23

Differentiate  $2^{\cos^2 x}$  w.r.t  $\cos^2 x$ .

(2 Mark) (CBSE 2025 - 65/1/1)

Q.24

If  $\tan^{-1}(x^2 + y^2) = a^2$ , then find  $\frac{dy}{dx}$ .

(2 Mark) (CBSE 2025 - 65/1/1)

Q.25 Find  $k$  so that

$$f(x) = \begin{cases} \frac{x^2-2x-3}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$$

is continuous at  $x = -1$ .

(3 Mark) (CBSE 2025 - 65/4/1)

Q.26 Check the differentiability of function  $f(x) = x|x|$  at  $x = 0$ .

(3 Mark) (CBSE 2025 - 65/4/1)

Q.27

Differentiate  $y = \sin^{-1}(3x - 4x^3)$  w.r.t.  $x$ , if  $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ .

(3 Mark) (CBSE 2025 - 65/6/1)

Q.28

Differentiate  $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$  with respect to  $x$ , when  $x \in (0, 1)$ .

(3 Mark) (CBSE 2025 - 65/6/1)

Q.29

If  $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$ , then show that  $x(x+1)^2y_2 + (x+1)^2y_1 = 2$ .

(3 Mark) (CBSE 2025 - 65/2/1)

Q.30

If  $x\sqrt{1+y} + y\sqrt{1+x} = 0$ ,  $-1 < x < 1$ ,  $x \neq y$ , then prove that  $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$ .

(3 Mark) (CBSE 2025 - 65/2/1)

Q.31

Differentiate  $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3} - 1\right)\right\}}$  with respect to  $x$ .

(3 Mark) (CBSE 2025 - 65/7/1)

Q.32 For a positive constant 'a', differentiate  $a^{t+\frac{1}{t}}$  with respect to  $\left(t + \frac{1}{t}\right)^a$ , where  $t$  is a non-zero real number.

(5 Mark) (CBSE 2025 - 65/5/1)

Q.33

Find  $\frac{dy}{dx}$  if  $y^x + x^y + x^x = a^b$ , where  $a$  and  $b$  are constants.

(5 Mark) (CBSE 2025 - 65/5/1)

Q.34

If  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ , then prove that  $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ .

(5 Mark) (CBSE 2025 - 65/1/1)

Q.35

If  $x = a \left( \cos \theta + \log \tan \frac{\theta}{2} \right)$  and  $y = \sin \theta$ , then find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{4}$ .

(5 Mark) (CBSE 2025 - 65/1/1)

## Answer

Q.1 B      Q.2 C      Q.3 B      Q.4 D      Q.5 B      Q.6 C  
Q.7 A      Q.8 D      Q.9 B      Q.10 A      Q.11 C      Q.12 C  
Q.13 C      Q.14 A      Q.15

Let  $u = \sqrt{e^{\sqrt{2x}}}$  and  $v = e^{\sqrt{2x}}$  Derivative of  $\sqrt{v}$  w.r.t.  $v = \frac{1}{2\sqrt{v}}$ .

$$\text{Required derivative} = \frac{1}{2\sqrt{e^{\sqrt{2x}}}}$$

**Q.16** Taking log on both sides, we get  $y \log x = x \log y$

Differentiating both sides w.r.t.  $x$ , we get

$$\begin{aligned} \frac{y}{x} + \log x \frac{dy}{dx} &= \frac{x}{y} \cdot \frac{dy}{dx} + \log y \\ \Rightarrow \frac{dy}{dx} &= \frac{y(x \log y - y)}{x(y \log x - x)} \end{aligned}$$

**Q.17**

$$x = e^{\frac{x}{y}}$$

$$\Rightarrow \log x = \frac{x}{y}$$

$$\Rightarrow y \log x = x$$

Differentiating both sides w.r.to  $x$ , we get

$$\begin{aligned} \frac{y}{x} + \log x \frac{dy}{dx} &= 1 \\ \Rightarrow \frac{dy}{dx} &= \frac{x - y}{x \log x} \end{aligned}$$

**Q.18**

$$\begin{aligned}Lf'(-2) &= \lim_{h \rightarrow 0} \frac{f(-2-h) - f(-2)}{-h} \quad (h > 0) \\ &= \lim_{h \rightarrow 0} \frac{2(-2-h) - 3 - (-7)}{-h} \\ &= \lim_{h \rightarrow 0} 2 = 2\end{aligned}$$

$$\begin{aligned}Rf'(-2) &= \lim_{h \rightarrow 0} \frac{f(-2+h) - f(-2)}{h} \quad (h > 0) \\ &= \lim_{h \rightarrow 0} \frac{-2+h+1 - (-7)}{h} \\ &= \lim_{h \rightarrow 0} \frac{6+h}{h}\end{aligned}$$

, which does not exist, i.e., RHD does not exist.

Therefore, the function is not differentiable at -2.

**Q.19** Let,

$$\begin{aligned}y &= \frac{5^x}{x^5} = 5^x \cdot x^{-5} \Rightarrow \frac{dy}{dx} = (5^x)' \cdot x^{-5} + 5^x \cdot (x^{-5})' \\ &= \frac{5^x}{x^5} \log 5 - \frac{5^{x+1}}{x^6}\end{aligned}$$

**Q.20**

Differentiating

$$-2x^2 - 5xy + y^3 = 76$$

, with respect to '

$x$   
,

$$\begin{aligned}-4x - 5y - 5x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} &= 0 \\ \Rightarrow \frac{dy}{dx} &= \frac{4x + 5y}{3y^2 - 5x}\end{aligned}$$

Q.21

$$\text{Let } y = \frac{\sin x}{\sqrt{\cos x}}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\sqrt{\cos x} \cdot \cos x - \sin x \cdot \left(\frac{-\sin x}{2\sqrt{\cos x}}\right)}{\cos x} \\ \Rightarrow \frac{dy}{dx} &= \frac{2\cos^2 x + \sin^2 x}{2(\cos x)^{3/2}} \text{ or } \frac{1 + \cos^2 x}{2(\cos x)^{3/2}} \end{aligned}$$

Q.22

$$y = 5 \cos x - 3 \sin x, \text{ then } \frac{dy}{dx} = -5 \cdot \sin x - 3 \cdot \cos x$$

$$\Rightarrow \frac{d^2 y}{dx^2} = -5 \cdot \cos x + 3 \cdot \sin x = -y$$

$$\Rightarrow \frac{d^2 y}{dx^2} + y = 0$$

Q.23

$$\text{Let } u = 2^{\cos^2 x} \Rightarrow \frac{du}{dx} = 2^{\cos^2 x} (-2 \cos x \sin x) \log 2$$

$$\text{Let } v = \cos^2 x \Rightarrow \frac{dv}{dx} = -2 \cos x \sin x$$

$$\text{Now } \frac{du}{dV} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dV}{dx}\right)} = 2^{\cos^2 x} \log 2$$

Q.24

$$\tan^{-1} (x^2 + y^2) = a^2 \Rightarrow x^2 + y^2 = \tan a^2$$

Differentiate both sides wrt x,

$$2x + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

Q.25

$$\lim_{x \rightarrow -1} \frac{x^2 - 2x - 3}{x + 1} = \lim_{x \rightarrow -1} \frac{(x-3)(x+1)}{x+1} = \lim_{x \rightarrow -1} (x - 3) = -4$$

Also,  $f(-1) = k$

as  $f$  is continuous,  $k = -4$

Q.26

$$f(x) = x|x| = \begin{cases} -x^2 & , x \leq 0 \\ x^2 & , x > 0 \end{cases}$$

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(0 - h) - f(0)}{-h} = \lim_{h \rightarrow 0} \frac{-h^2 - 0}{-h} = 0$$

$$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^2 - 0}{h} = 0$$

Since LHD = RHD,  $f$  is differentiable at  $x = 0$

Q.27

$$x = \sin t \text{ gives } y = \sin^{-1}(\sin 3t) = 3t = 3 \sin^{-1} x$$

$$\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$$

$$\text{Aliter: } \frac{dy}{dx} = \frac{3 - 12x^2}{\sqrt{1 - (3x - 4x^3)^2}}$$

Q.28

$$x = \tan t \text{ gives } y = \cos^{-1}(\cos 2t) = 2t = 2 \tan^{-1} x$$

$$\frac{dy}{dx} = \frac{2}{1+x^2}$$

$$\text{Aliter: } \frac{dy}{dx} = \frac{-1}{\sqrt{1 - \left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-4x}{(1+x^2)^2}$$

Q.29 The given function can be written as

$$y = 2 \log(x + 1) - \log x$$

$$\Rightarrow y_1 = \frac{2}{x+1} - \frac{1}{x} = \frac{x-1}{x(x+1)}$$

$$\Rightarrow (x+1)y_1 = \frac{x-1}{x} = 1 - \frac{1}{x}$$

$$\Rightarrow (x+1)y_2 + y_1 = \frac{1}{x^2}$$

$$\Rightarrow x(x+1)^2 y_2 + x(x+1)y_1 = 1 + \frac{1}{x}$$

$$\Rightarrow x(x+1)^2 y_2 + x(x+1)y_1 = 1 + 1 - (x+1)y_1$$

$$\Rightarrow x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$$

### Q.30

$$x\sqrt{1+y} + y\sqrt{1+x} = 0$$

$$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$$

$$\Rightarrow x^2(1+y) = y^2(1+x)$$

$$\Rightarrow (x-y)(x+y) + xy(x-y) = 0$$

$$\Rightarrow (x-y)(x+y+xy) = 0$$

$$x \neq y \Rightarrow x+y+xy = 0$$

$$\Rightarrow y = \frac{-x}{1+x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

### Q.31

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}} \cdot \frac{1}{\sin\left(\frac{x^3}{3}-1\right)} \cdot \cos\left(\frac{x^3}{3}-1\right) \cdot \frac{3x^2}{3} \\ &= \frac{x^2 \cot\left(\frac{x^3}{3}-1\right)}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}} \end{aligned}$$

Q.32 Let

$$u = a^{t+\frac{1}{t}} \Rightarrow \frac{du}{dt} = a^{t+\frac{1}{t}} \cdot \log a \cdot \left(1 - \frac{1}{t^2}\right)$$
$$v = \left(t + \frac{1}{t}\right)^a \Rightarrow \frac{dv}{dt} = a \left(t + \frac{1}{t}\right)^{a-1} \cdot \left(1 - \frac{1}{t^2}\right)$$
$$\frac{du}{dv} = \frac{du/dt}{dv/dt} = \frac{a^{t+\frac{1}{t}} \cdot \log a}{a \left(t + \frac{1}{t}\right)^{a-1}}$$

Q.33

Let

$$\mathbf{u} = \mathbf{y}^{\mathbf{x}}, \mathbf{v} = \mathbf{x}^{\mathbf{y}}$$

and

$$\mathbf{w} = \mathbf{x}^{\mathbf{x}}$$

$$\Rightarrow \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0 \dots \dots \dots (i)$$

$$u = y^x \Rightarrow \log u = x \cdot \log y \Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = \frac{x}{y} \cdot \frac{dy}{dx} + \log y$$

$$\Rightarrow \frac{du}{dx} = y^x \left( \frac{x}{y} \cdot \frac{dy}{dx} + \log y \right) = xy^{x-1} \frac{dy}{dx} + y^x \log y$$

$$v = x^y \Rightarrow \log v = y \cdot \log x \Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \frac{y}{x} + \log x \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dv}{dx} = x^y \left( \frac{y}{x} + \log x \cdot \frac{dy}{dx} \right) = yx^{y-1} + x^y \log x \frac{dy}{dx}$$

$$w = x^x \Rightarrow \log w = x \cdot \log x \Rightarrow \frac{1}{w} \cdot \frac{dw}{dx} = 1 + \log x$$

$$\Rightarrow \frac{dw}{dx} = x^x \cdot (1 + \log x)$$

$\therefore$

From (i), we get

$$xy^{x-1} \cdot \frac{dy}{dx} + y^x \cdot \log y + yx^{y-1} + x^y \cdot \log x \cdot \frac{dy}{dx} + x^x \cdot (1 + \log x) = 0$$

$$\Rightarrow \frac{dy}{dx} = - \frac{x^x \cdot (1 + \log x) + y^x \cdot \log y + yx^{y-1}}{x \cdot y^{x-1} + x^y \cdot \log x}$$

**Q.34**

Let

$$x = \sin A, y = \sin B \Rightarrow A = \sin^{-1} x, B = \sin^{-1} y$$

$$\therefore \sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

$$\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$$

$$\Rightarrow 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) = 2a \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right)$$

$$\Rightarrow \cot \left( \frac{A-B}{2} \right) = a \Rightarrow A-B = 2 \cot^{-1} a$$

$$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2 \cot^{-1} a$$

differentiate both sides wrt x.

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

**Q.35**

$$x = a \left( \cos \theta + \log \tan \frac{\theta}{2} \right)$$

$$\begin{aligned} \Rightarrow \frac{dx}{d\theta} &= a \left( -\sin \theta + \frac{1}{\tan \frac{\theta}{2}} \times \sec^2 \frac{\theta}{2} \times \frac{1}{2} \right) \\ &= a \left( -\sin \theta + \frac{1}{\sin \theta} \right) = a \left( \frac{1 - \sin^2 \theta}{\sin \theta} \right) \end{aligned}$$

$$\frac{dx}{d\theta} = a \cot \theta \cos \theta$$

Also,

$$y = \sin \theta \Rightarrow \frac{dy}{d\theta} = \cos \theta$$

$$\therefore \frac{dy}{dx} = \frac{\tan \theta}{a}$$

Differentiating wrt x.

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{\sec^2 \theta}{a} \times \frac{d\theta}{dx} \\ &= \frac{\sec^3 \theta \tan \theta}{a^2} \end{aligned}$$

$$\left. \frac{d^2y}{dx^2} \right]_{\text{at } \theta = \frac{\pi}{4}} = \frac{2\sqrt{2}}{a^2}$$

**(2024)**

**Q.1 The number of points of discontinuity of**

$$f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$$

**is :**

**(1 Mark) (CBSE 2024 - 65/2/1)**

**A. 1**

**B. infinite**

**C. 2**

**D. 0**

**Q.2 The derivative of  $\tan^{-1}(x^2)$  w.r.t.  $x$  is :**

**(1 Mark) (CBSE 2024 - 65/2/1)**

**A.**

$$\frac{1}{1+x^4}$$

**B.**

$$\frac{2x}{1+x^4}$$

**C.**

$$-\frac{2x}{1+x^4}$$

**D.**

$$\frac{x}{1+x^4}$$

**Q.3 Derivative of  $e^{2x}$  with respect to  $e^x$ , is :**

**(1 Mark) (CBSE 2024 - 65/4/1)**

A.  $2e^{2x}$

B.  $e^x$

C.  $2e^{3x}$

D.  $2e^x$

Q.4 For what value of k, the function given below is continuous at  $x = 0$  ?

$$f(x) = \begin{cases} \frac{\sqrt{4+x}-2}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

(1 Mark) (CBSE 2024 - 65/4/1)

A. 4

B.  $1/4$

C. 0

D. 1

Q.5 Which of the following statements is true for the function

$$f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

?

(1 Mark) (CBSE 2024 - 65/1/1)

A.  $f(x)$  is continuous and differentiable  $\forall x \in \mathbb{R} - \{0\}$

B.  $f(x)$  is continuous  $\forall x \in \mathbb{R}$

C.  $f(x)$  is continuous and differentiable  $\forall x \in \mathbb{R}$

D.  $f(x)$  is discontinuous at infinitely many points

Q.6 The derivative of  $\sin(x^2)$  w.r.t.  $x$ , at  $x = \sqrt{\pi}$  is :

(1 Mark) (CBSE 2024 - 65/1/1)

A. -1

B.  $-2\sqrt{\pi}$

C.  $2\sqrt{\pi}$

D. 1

Q.7 A function  $f(x) = |1 - x + |x||$  is :

(1 Mark) (CBSE 2024 - 65/3/1)

A. discontinuous at  $x = 0$

B. discontinuous at  $x = 0, 1$

C. discontinuous at  $x = 1$  only

D. continuous everywhere

Q.8 The derivative of  $2^x$  w.r.t.  $3^x$  is :

(1 Mark) (CBSE 2024 - 65/3/1)

A.

$$\left(\frac{3}{2}\right)^x \frac{\log 3}{\log 2}$$

B.

$$\left(\frac{2}{3}\right)^x \frac{\log 3}{\log 2}$$

C.

$$\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$$

D.

$$\left(\frac{3}{2}\right)^x \frac{\log 2}{\log 3}$$

Q.9 If  $x^{e^y} = 1$ , then the value of  $dy/dx$  at  $x = 1$  is :

(1 Mark) (CBSE 2024 - 65/5/1)

A. 1

B.

$$-\frac{1}{e}$$

C. -e

D. -1

Q.10

Derivative of  $e^{\sin^2 x}$  with respect to  $\cos x$  is :

(1 Mark) (CBSE 2024 - 65/5/1)

A.

$$-2 \sin^2 x \cos x e^{\sin^2 x}$$

B.

$$\sin x e^{\sin^2 x}$$

C.

$$\cos x e^{\sin^2 x}$$

D.

$$-2 \cos x e^{\sin^2 x}$$

Q.11 If  $f(x) = |\tan 2x|$ , then find the value of  $f'(x)$  at  $x = \pi/3$ .

(2 Mark) (CBSE 2024 - 65/2/1)

Q.12

If  $y = \operatorname{cosec}(\cot^{-1} x)$ , then prove that  $\sqrt{1+x^2} \frac{dy}{dx} - y = 0$ .

(2 Mark) (CBSE 2024 - 65/2/1)

Q.13

If  $y = \cos^3(\sec^2 2t)$ , find  $\frac{dy}{dt}$ .

(2 Mark) (CBSE 2024 - 65/4/1)

Q.14

If  $x^y = e^{x-y}$ , prove that  $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$ .

(2 Mark) (CBSE 2024 - 65/4/1)

Q.15 Check whether the function  $f(x) = x^2 |x|$  is differentiable at  $x = 0$  or not.

(2 Mark) (CBSE 2024 - 65/1/1)

Q.16

If  $y = \sqrt{\tan \sqrt{x}}$ , prove that  $\sqrt{x} \frac{dy}{dx} = \frac{1+y^4}{4y}$ .

(2 Mark) (CBSE 2024 - 65/1/1)

Q.17

If  $x = e^{x/y}$ , prove that  $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$

(2 Mark) (CBSE 2024 - 65/3/1)

Q.18

Check the differentiability of

$$f(x) = \begin{cases} x^2 + 1, & 0 \leq x < 1 \\ 3 - x, & 1 \leq x \leq 2 \end{cases}$$

at

$$x = 1$$

(2 Mark) (CBSE 2024 - 65/3/1)

Q.19 Verify whether the function  $f$  defined by

$$f(x) = \begin{cases} x \sin \left( \frac{1}{x} \right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous at  $x = 0$  or not.

(2 Mark) (CBSE 2024 - 65/5/1)

Q.20 Check for differentiability of the function  $f$  defined by  $f(x) = |x - 5|$ , at the point  $x = 5$ .

(2 Mark) (CBSE 2024 - 65/5/1)

Q.21

If  $x = e^{\cos 3t}$  and  $y = e^{\sin 3t}$ , prove that  $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$ .

(3 Mark) (CBSE 2024 - 65/2/1)

Q.22

Show that:

$$\frac{d}{dx} (|x|) = \frac{x}{|x|}, x \neq 0$$

(3 Mark) (CBSE 2024 - 65/2/1)

Q.23

Given that  $y = (\sin x)^x \cdot x^{\sin x} + a^x$ , find  $\frac{dy}{dx}$ .

(3 Mark) (CBSE 2024 - 65/4/1)

Q.24

If  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ , prove that  $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ .

(3 Mark) (CBSE 2024 - 65/1/1)

Q.25

If  $y = (\tan x)^x$ , then find  $\frac{dy}{dx}$ .

(3 Mark) (CBSE 2024 - 65/1/1)

Q.26 If  $x \cos(p+y) + \cos p \sin(p+y) = 0$ , prove that  $\cos p \frac{dy}{dx} = -\cos^2(p+y)$ , where  $p$  is a constant.

(3 Mark) (CBSE 2024 - 65/3/1)

Q.27

Find the value of  $a$  and  $b$  so that function  $f$  defined as :

$$f(x) = \begin{cases} \frac{x-2}{|x-2|} + a, & \text{if } x < 2 \\ a + b, & \text{if } x = 2 \\ \frac{x-2}{|x-2|} + b, & \text{if } x > 2 \end{cases}$$

is a continuous function.

(3 Mark) (CBSE 2024 - 65/3/1)

Q.28

Find  $\frac{dy}{dx}$ , if  $(\cos x)^y = (\cos y)^x$ .

(3 Mark) (CBSE 2024 - 65/5/1)

Q.29

If  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ , prove that  $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ .

(3 Mark) (CBSE 2024 - 65/5/1)

Q.30

If  $x = a \sin^3 \theta$ ,  $y = b \cos^3 \theta$ , then find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{4}$ .

(3 Mark) (CBSE 2024 - 65/5/1)

## Answer

Q.1 A      Q.2 B      Q.3 D      Q.4 B      Q.5 A      Q.6 B

Q.7 D      Q.8 C      Q.9 D      Q.10 D      Q.11

$$f(x) = -\tan 2x, \frac{\pi}{4} < x < \frac{\pi}{2}$$

$$f'(x) = -2 \sec^2 2x, \frac{\pi}{4} < x < \frac{\pi}{2}$$

$$f' \left( \frac{\pi}{3} \right) = -2(-2)^2 = -8$$

Q.12

$$y = \sqrt{1 + \cot^2 (\cot^{-1} x)} = \sqrt{1 + x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{\sqrt{1 + x^2}}$$

$$\Rightarrow \sqrt{1 + x^2} \frac{dy}{dx} - x = 0$$

Q.13

$$y = \cos^3 (\sec^2 2t)$$

$$\Rightarrow \frac{dy}{dt} = 3 \cos^2 (\sec^2 2t) [-\sin (\sec^2 2t)] \times \frac{d(\sec^2 2t)}{dt}$$

$$\Rightarrow \frac{dy}{dt} = -3 \cos^2 (\sec^2 2t) \cdot \sin (\sec^2 2t) \times 2 \sec 2t \cdot \sec 2t \tan 2t \cdot 2$$

$$\therefore \frac{dy}{dt} = -12 \cos^2 (\sec^2 2t) \times \sin (\sec^2 2t) \times \sec^2 2t \times \tan 2t$$

Q.14

$$\text{As, } x^y = e^{x-y} \Rightarrow \log (x^y) = \log (e^{x-y})$$

$$\Rightarrow y \log x = (x - y) \Rightarrow y = \frac{x}{1 + \log x}$$

Now, Differentiating both the sides wrt x

$$\frac{dy}{dx} = \frac{(\log x + 1) \cdot 1 - x \left(\frac{1}{x}\right)}{(\log x + 1)^2} = \frac{\log x}{(1 + \log x)^2}$$

**Q.15**

$$f(x) = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x \leq 0 \end{cases}$$

$$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} h^2 = 0$$

$$\text{LHD} = \lim_{h \rightarrow 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h \rightarrow 0} (-h^2) = 0$$

$\therefore \text{RHD} = \text{LHD} = 0$ , So  $f(x)$  is differentiable at  $x = 0$

**Q.16**

$$y = \sqrt{\tan \sqrt{x}}$$

$$\frac{dy}{dx} = \frac{\sec^2 \sqrt{x}}{2\sqrt{\tan \sqrt{x}}} \times \frac{1}{2\sqrt{x}}$$

$$\begin{aligned} \sqrt{x} \frac{dy}{dx} &= \frac{\sec^2 \sqrt{x}}{4\sqrt{\tan \sqrt{x}}} \\ &= \frac{1 + (\tan \sqrt{x})^2}{4\sqrt{\tan \sqrt{x}}} = \frac{1 + y^4}{4y} \end{aligned}$$

**Q.17**

$$x = e^{\frac{x}{y}} \Rightarrow \log x = \frac{x}{y} \Rightarrow y = \frac{x}{\log x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(\log x)(1) - x \left(\frac{1}{x}\right)}{(\log x)^2} = \frac{\log x - 1}{(\log x)^2}$$

**Q.18**

LHD at  $x = 1$

$$= \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{[(1-h)^2 + 1] - 2}{-h} = 2$$

RHD at  $x = 1$

$$= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[3 - (1+h)] - 2}{h} = -1$$

as LHD  $\neq$  RHD, so  $f(x)$  is not differentiable at  $x = 1$

Q.19

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} x \cdot \sin \frac{1}{x} = 0 \times \text{Finite value in } [-1, 1] = 0 = f(0)$$

$\therefore f$  is a continuous function.

Q.20

$$\text{LHD} = \lim_{x \rightarrow 5^-} \frac{|x - 5| - 0}{x - 5} = \lim_{x \rightarrow 5^-} \frac{-(x - 5)}{x - 5} = -1$$

$$\text{RHD} = \lim_{x \rightarrow 5^+} \frac{|x - 5| - 0}{x - 5} = \lim_{x \rightarrow 5^+} \frac{(x - 5)}{x - 5} = 1$$

LHD

$\neq$

RHD,

$\therefore f$

is not differentiable at

$x = 5$

Q.21

$$\frac{dx}{dt} = e^{\cos 3t} \times (-\sin 3t) \times 3$$

$$\frac{dy}{dt} = e^{\sin 3t} \times (\cos 3t) \times 3$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^{\sin 3t} \times (\cos 3t)}{-e^{\cos 3t} \times (\sin 3t)}$$

$$x = e^{\cos 3t} \Rightarrow \cos 3t = \log x$$

$$y = e^{\sin 3t} \Rightarrow \sin 3t = \log y$$

$$\therefore \frac{dy}{dx} = \frac{-y \log x}{x \log y}$$

**Q.22**

$$\begin{aligned} \frac{d(|x|)}{dx} &= \frac{d(\sqrt{x^2})}{dx}, x \neq 0 \\ &= \frac{1}{2} (x^2)^{-\frac{1}{2}} \times \frac{d(x^2)}{dx} \\ &= \frac{1}{2\sqrt{x^2}} 2x = \frac{x}{|x|} \end{aligned}$$

**Q.23**

$$\text{As, } y = (\sin x)^x \cdot x^{\sin x} + a^x = u + a^x \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{d(a^x)}{dx}$$

where  $u = (\sin x)^x \cdot x^{\sin x} \Rightarrow \log u = x \log(\sin x) + \sin x \cdot \log x$  on differentiating both sides with respect to  $x$ , we get

$$\Rightarrow \frac{du}{dx} = (\sin x)^x \cdot x^{\sin x} \left[ \log(\sin x) + x \cot x + \frac{\sin x}{x} + \log x \cdot \cos x \right]$$

$$\text{Thus, } \frac{dy}{dx} = (\sin x)^x \cdot x^{\sin x} \left[ \log(\sin x) + x \cot x + \frac{\sin x}{x} + \log x \cdot \cos x \right] + a^x \log a$$

**Q.24**

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

$$\text{Put } x = \sin \theta, y = \sin \phi$$

$$\Rightarrow \cos \theta + \cos \phi = a(\sin \theta - \sin \phi)$$

$$\Rightarrow 2 \cos \left( \frac{\theta + \phi}{2} \right) \cos \left( \frac{\theta - \phi}{2} \right) = 2a \sin \left( \frac{\theta - \phi}{2} \right) \cos \left( \frac{\theta + \phi}{2} \right)$$

$$\Rightarrow \cot \left( \frac{\theta - \phi}{2} \right) = a$$

$$\Rightarrow \theta - \phi = 2 \cot^{-1} a$$

$$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2 \cot^{-1} a$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

**Q.25**

$$y = (\tan x)^x$$

$$\log y = x \log(\tan x)$$

$$\frac{1}{y} \frac{dy}{dx} = x \left( \frac{\sec^2 x}{\tan x} \right) + \log(\tan x)$$

$$\frac{dy}{dx} = (\tan x)^x \left[ \left( \frac{x \sec^2 x}{\tan x} \right) + \log(\tan x) \right]$$

**Q.26**

$$x \cos(p+y) + \cos p \sin(p+y) = 0$$

$$\Rightarrow x = \frac{-\cos p \sin(p+y)}{\cos(p+y)} \Rightarrow x = -\cos p \cdot \tan(p+y)$$

$$\Rightarrow \frac{dx}{dy} = -\cos p \cdot \sec^2(p+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{\cos p \cdot \sec^2(p+y)}$$

$$\Rightarrow \cos p \frac{dy}{dx} = -\cos^2(p+y)$$

**Q.27**

$$f(x) = \begin{cases} \frac{x-2}{-(x-2)} + a & ; x < 2 \\ a + b & ; x = 2 \\ \frac{x-2}{(x-2)} + b & ; x > 2 \end{cases} \Rightarrow f(x) = \begin{cases} -1 + a & ; x < 2 \\ a + b & ; x = 2 \\ 1 + b & ; x > 2 \end{cases}$$

$$\lim_{x \rightarrow 2^-} f(x) = -1 + a, \lim_{x \rightarrow 2^+} f(x) = 1 + b \text{ and } f(2) = a + b$$

as  $f$  is continuous at  $x = 2 \therefore -1 + a = 1 + b = a + b$

$$\Rightarrow a = 1, b = -1$$

**Q.28**

Taking 'log' on both sides of

$$(\cos x)^y = (\cos y)^x$$

, we get

$$y \log \cos x = x \log \cos y$$

$$\Rightarrow \frac{dy}{dx} \log \cos x + y(-\tan x) = \log \cos y + x(-\tan y) \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\log \cos y + y \tan x}{\log \cos x + x \tan y}$$

**Q.29**

Let

$$x = \sin A, y = \sin B \therefore A = \sin^{-1} x, B = \sin^{-1} y$$

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

$$\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$$

$$\Rightarrow 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) = 2a \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right)$$

$\Rightarrow \cot\left(\frac{A-B}{2}\right) = a \Rightarrow A - B = 2 \cot^{-1} a, \therefore \sin^{-1} x - \sin^{-1} y = 2 \cot^{-1} a$   
 , differentiating with respect to ' $x$ '

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

**Q.30**

$$\frac{dx}{d\theta} = 3a \sin^2 \theta \cos \theta, \frac{dy}{d\theta} = -3b \cos^2 \theta \sin \theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{-3b \cos^2 \theta \sin \theta}{3a \sin^2 \theta \cos \theta} = -\frac{b}{a} \cot \theta$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{b}{a} \operatorname{cosec}^2 \theta \frac{d\theta}{dx} = \frac{b}{a} \operatorname{cosec} \theta \cdot \frac{1}{3a \sin^2 \theta \cos \theta} = \frac{b}{3a^2} \sec \theta \operatorname{cosec}^4 \theta$$

$$\Rightarrow \left. \frac{d^2y}{dx^2} \right]_{\theta=\frac{\pi}{4}} = \frac{4\sqrt{2}b}{3a^2}$$

## Previous Years' CBSE Board Questions

### 5.2 Continuity

#### MCQ

1. The function  $f(x) = [x]$ , where  $[x]$  denotes the greatest integer less than or equal to  $x$ , is continuous at  
 (a)  $x = 1$  (b)  $x = 1.5$  (c)  $x = -2$  (d)  $x = 4$   
 (2023)

2. If the function  $f(x) = \begin{cases} 3x-8, & \text{if } x \leq 5 \\ 2k, & \text{if } x > 5 \end{cases}$  is continuous, then the value of  $k$  is  
 (a)  $2/7$  (b)  $7/2$  (c)  $3/7$  (d)  $4/7$   
 (Term I, 2021-22) (Ap)

3. The function  $f(x) = [x]$ , where  $[x]$  is the greatest integer function that is less than or equal to  $x$ , is continuous at  
 (a)  $4$  (b)  $-2$  (c)  $1.5$  (d)  $1$   
 (Term I, 2021-22) (U)

#### VSA (1 mark)

4. The value of  $\lambda$  so that the function  $f$  defined by  $f(x) = \begin{cases} \lambda x, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$  is continuous at  $x = \pi$  is \_\_\_\_\_  
 (2020) (Ap)

5. Determine the value of the constant 'k' so that the function  $f(x) = \begin{cases} kx, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$  is continuous at  $x = 0$ .  
 (Delhi 2017) (Ap)

6. Determine the value of 'k' for which the following function is continuous at  $x = 3$ .

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases} \quad (\text{AI 2017}) \text{ (Ap)}$$

#### SA I (2 marks)

7. Find the value(s) of ' $\lambda$ ', if the function  $f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$  is continuous at  $x = 0$ .  
 (2023)

8. Find the relationship between  $a$  and  $b$  so that the function  $f$  defined by  $f(x) = \begin{cases} ax+1 & \text{if } x \leq 3 \\ bx+3 & \text{if } x > 3 \end{cases}$  is continuous at  $x = 3$ .  
 (2021C) (Ap)

#### LA I (4 marks)

9. Find the values of  $p$  and  $q$ , for which  $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x < \pi/2 \\ p, & \text{if } x = \pi/2 \\ \frac{q(1-\sin x)}{(\pi-2x)^2}, & \text{if } x > \pi/2 \end{cases}$  is continuous at  $x = \pi/2$ .  
 (Delhi 2016) (Ap)

10. Find the value of the constant  $k$  so that the function  $f$ , defined below, is continuous at  $x = 0$ , where

$$f(x) = \begin{cases} \left( \frac{1-\cos 4x}{8x^2} \right), & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases} \quad (\text{AI 2014C}) \text{ (Ap)}$$

### 5.3 Differentiability

#### MCQ

11. The function  $f(x) = |x|$  is  
 (a) continuous and differentiable everywhere.  
 (b) continuous and differentiable nowhere.  
 (c) continuous everywhere, but differentiable everywhere except at  $x = 0$ .  
 (d) continuous everywhere, but differentiable nowhere.  
 (2023)

12. The derivative of  $x^{2x}$  w.r.t.  $x$  is  
 (a)  $x^{2x-1}$  (b)  $2x^{2x} \log x$   
 (c)  $2x^{2x}(1+\log x)$  (d)  $2x^{2x}(1-\log x)$  (2023)

13. If  $y^2(2-x) = x^3$ , then  $\left(\frac{dy}{dx}\right)_{(1,1)}$  is equal to  
 (a)  $2$  (b)  $-2$  (c)  $3$  (d)  $-3/2$   
 (Term I, 2021-22) (Ap)

14. The function  $f(x) = \begin{cases} x^2 & \text{for } x < 1 \\ 2-x & \text{for } x \geq 1 \end{cases}$  is  
 (a) not differentiable at  $x = 1$   
 (b) differentiable at  $x = 1$   
 (c) not continuous at  $x = 1$   
 (d) neither continuous nor differentiable at  $x = 1$   
 (Term I, 2021-22) (Ev)

15. If  $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$ , then  $\frac{dy}{dx}$  is equal to  
 (a)  $\frac{x-1}{y-1}$  (b)  $\frac{x-1}{y+1}$   
 (c)  $\frac{y-1}{x+1}$  (d)  $\frac{y+1}{x-1}$  (2020C) (Ap)

#### VSA (1 mark)

16. If  $y = \tan^{-1} x + \cot^{-1} x$ ,  $x \in R$ , then  $\frac{dy}{dx}$  is equal to \_\_\_\_\_  
 (2020) (Ev)
17. If  $\cos(xy) = k$ , where  $k$  is a constant and  $xy \neq n\pi$ ,  $n \in Z$ , then  $\frac{dy}{dx}$  is equal to \_\_\_\_\_  
 (2020) (Ev)
18. Differentiate  $\sin^2(\sqrt{x})$  with respect to  $x$ . (2020) (Ap)
19. Let  $f(x) = x|x|$ , for all  $x \in R$  check its differentiability at  $x = 0$ . (2020) (Ev)
20. If  $y = f(x^2)$  and  $f'(x) = e^{\sqrt{x}}$ , then find  $\frac{dy}{dx}$ . (2020) (Ev)

21. If  $f(x) = x + 1$ , find  $\frac{d}{dx}(f \circ f)(x)$ . (Delhi 2019) (U)

**SA I (2 marks)**

22. If  $(x^2 + y^2)^2 = xy$ , then find  $\frac{dy}{dx}$ . (2023)

23. If  $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$ , then show that  $f$  is not differentiable at  $x = 1$ . (2023)

24. Check the differentiability of  $f(x) = |x - 3|$  at  $x = 3$ . (2021C) (Ev)

25. If  $y = \sqrt{a + \sqrt{a + x}}$ , then find  $\frac{dy}{dx}$ . (2020C) (Ap)

26. Differentiate  $\tan^{-1}\left(\frac{1 + \cos x}{\sin x}\right)$  with respect to  $x$ . (2018) (Ev)

27. Find  $\frac{dy}{dx}$  at  $x = 1, y = \frac{\pi}{4}$  if  $\sin^2 y + \cos xy = K$ . (Delhi 2017) (U)

**LA I (4 marks)**

Here, question 28(i) to (iii) is a case study based question of 4 marks.

28. Let  $f(x)$  be a real valued function. Then its

- Left Hand Derivative (L.H.D.):

$$Lf'(a) = \lim_{h \rightarrow 0} \frac{f(a-h) - f(a)}{-h}$$

- Right Hand Derivative (R.H.D.):

$$Rf'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Also, a function  $f(x)$  is said to be differentiable at  $x = a$  if its L.H.D. and R.H.D. at  $x = a$  exist and both are equal.

$$\text{For the function } f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$$

answer the following questions:

- What is R.H.D. of  $f(x)$  at  $x = 1$ ?
- What is L.H.D. of  $f(x)$  at  $x = 1$ ?
- Check if the function  $f(x)$  is differentiable at  $x = 1$ .

OR

(iii) Find  $f'(2)$  and  $f'(-1)$ . (2023)

29. Find the values of  $a$  and  $b$ , if the function  $f$  defined by

$$f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$$

is differentiable at  $x = 1$ . (Foreign 2016) (Ap)

30. If  $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right)$ ,  $x^2 \leq 1$

then find  $\frac{dy}{dx}$ . (NCERT Exemplar, Delhi 2015) (Ap)

31. If  $f(x) = \sqrt{x^2 + 1}$ ;  $g(x) = \frac{x+1}{x^2+1}$  and  $h(x) = 2x-3$ , then find  $f'(h'(g'(x)))$ . (AI 2015) (Ap)

32. Show that the function  $f(x) = |x - 1| + |x + 1|$ , for all  $x \in \mathbb{R}$ , is not differentiable at the points  $x = -1$  and  $x = 1$ .

(AI 2015) (Ev)

33. Find whether the following function is differentiable at  $x = 1$  and  $x = 2$  or not.

$$f(x) = \begin{cases} x, & x < 1 \\ 2-x, & 1 \leq x \leq 2 \\ -2+3x-x^2, & x > 2 \end{cases}$$

(Foreign 2015) (Ap)

34. For what value of  $\lambda$  the function defined by

$$f(x) = \begin{cases} \lambda(x^2+2), & \text{if } x \leq 0 \\ 4x+6, & \text{if } x > 0 \end{cases}$$

is continuous at  $x = 0$ ? Hence check the differentiability of  $f(x)$  at  $x = 0$ .

(AI 2015C) (Ap)

35. If  $\cos y = x \cos(a + y)$ , where  $\cos a \neq \pm 1$ , prove that

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

(Foreign 2014) (Ev)

36. If  $y = \sin^{-1}\{x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\}$  and  $0 < x < 1$ , then

find  $\frac{dy}{dx}$ . (AI 2014C) (Ap)

## 5.4 Exponential and Logarithmic Functions

**MCQ**

37. If  $y = \log(\sin e^x)$ , then  $\frac{dy}{dx}$  is

- $\cot e^x$
- $\operatorname{cosec} e^x$
- $e^x \cot e^x$
- $e^x \operatorname{cosec} e^x$

(2023)

38. If  $y = \tan^{-1}(e^{2x})$ , then  $\frac{dy}{dx}$  is equal to

- $\frac{2e^{2x}}{1+e^{4x}}$
- $\frac{1}{1+e^{4x}}$
- $\frac{2}{e^{2x} + e^{-2x}}$
- $\frac{1}{e^{2x} - e^{-2x}}$

(Term I, 2021-22) (Ev)

**VSA (1 mark)**

39. If  $y = \log(\operatorname{cosec}^x)$ , then find  $\frac{dy}{dx}$ . (NCERT, AI 2019) (Ap)

**LA I (4 marks)**

40. If  $y = e^{x^2 \cos x} + (\cos x)^x$ , then find  $\frac{dy}{dx}$ . (2020) (Ev)

41. If  $\log(x^2 + y^2) = 2 \tan^{-1}\left(\frac{y}{x}\right)$ , show that  $\frac{dy}{dx} = \frac{x+y}{x-y}$ .

(Delhi 2019) (Ev)

42. If  $y = \frac{x \cos^{-1} x}{\sqrt{1-x^2}} - \log \sqrt{1-x^2}$ , then prove that

$$\frac{dy}{dx} = \frac{\cos^{-1} x}{(1-x^2)^{3/2}}$$

(Delhi 2015C) (Ap)

43. If  $e^x + e^y = e^{x+y}$ , prove that  $\frac{dy}{dx} + e^{y-x} = 0$

(Foreign 2014) (Ev)

44. If  $y = \tan^{-1}\left(\frac{a}{x}\right) + \log\sqrt{\frac{x-a}{x+a}}$ , prove that

$$\frac{dy}{dx} = \frac{2a^3}{x^4 - a^4}. \quad (\text{AI 2014C}) \quad (\text{Ap})$$

## 5.5 Logarithmic Differentiation

SA II (3 marks)

45. If  $e^{y-x} = y^x$ , prove that  $\frac{dy}{dx} = \frac{y(1+\log y)}{x \log y}$ . (2021C) (Ev)

LA I (4 marks)

46. If  $y = x^3 (\cos x)^x + \sin^{-1} \sqrt{x}$ , find  $\frac{dy}{dx}$ . (2020) (Ev)

47. If  $y = (\log x)^x + x^{\log x}$ , then find  $\frac{dy}{dx}$ . (NCERT, 2020) (Ap)

48. Find  $\frac{dy}{dx}$ , if  $x^y \cdot y^x = x^x$ . (2019C) (Ap)

49. If  $x^y - y^x = a^b$ , find  $\frac{dy}{dx}$ . (Delhi 2019) (Ev)

50. If  $y = (x)^{\cos x} + (\cos x)^{\sin x}$ , then find  $\frac{dy}{dx}$ . (AI 2019)

51. Differentiate the function  $(\sin x)^x + \sin^{-1} \sqrt{x}$  with respect to  $x$ . (Delhi 2017) (Ap)

OR

If  $y = (\sin x)^x + \sin^{-1} \sqrt{x}$ , then find  $\frac{dy}{dx}$ . (Delhi 2015C) (Ev)

52. If  $x^y + y^x = a^b$ , then find  $\frac{dy}{dx}$ . (AI 2017) (Ev)

53. Differentiate  $x^{\sin x} + (\sin x)^{\cos x}$  with respect to  $x$ . (AI 2016) (Ap)

54. If  $x^m y^n = (x+y)^{m+n}$ , prove that  $\frac{dy}{dx} = \frac{y}{x}$ . (Foreign 2014) (Ev)

55. If  $(x-y) \cdot e^{\frac{x}{x-y}} = a$ , prove that  $y \frac{dy}{dx} + x = 2y$ . (Delhi 2014C) (Ev)

56. If  $(\tan^{-1} x)^y + y^{\cot x} = 1$ , then find  $\frac{dy}{dx}$ . (AI 2014C) (Ev)

## 5.6 Derivatives of Functions in Parametric Forms

VSA (1 mark)

57. If  $x = e^t \sin t$ ,  $y = e^t \cos t$ , then the value of  $\frac{dy}{dx}$  at  $t = \frac{\pi}{4}$  is \_\_\_\_\_. (2020C) (Ev)

SA I (2 marks)

58. If  $x = a \sec \theta$ ,  $y = b \tan \theta$ , then find  $\frac{dy}{dx}$  at  $\theta = \frac{\pi}{3}$ . (2020) (Ev)

59. Find the differential of  $\sin^2 x$  w.r.t.  $e^{\cos x}$ . (NCERT, 2020) (Ap)

SA II (3 marks)

60. Differentiate  $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$  w.r.t.  $\sin^{-1}(2x\sqrt{1-x^2})$ . (2023)

LA I (4 marks)

61. Differentiate  $\tan^{-1}\left[\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right]$  with respect to  $\cos^{-1} x^2$ . (AI 2019) (Ev)

62. If  $x = a(2\theta - \sin 2\theta)$  and  $y = a(1 - \cos 2\theta)$ , find  $\frac{dy}{dx}$  when  $\theta = \frac{\pi}{3}$ . (2018) (Ev)

63. If  $x = a \sin 2t(1 + \cos 2t)$  and  $y = b \cos 2t(1 - \cos 2t)$ , find the values of  $\frac{dy}{dx}$  at  $t = \frac{\pi}{4}$  and  $t = \frac{\pi}{3}$ . (Delhi 2016, AI 2016) (Ap)

OR

If  $x = a \sin 2t(1 + \cos 2t)$  and  $y = b \cos 2t(1 - \cos 2t)$ , show that at  $t = \frac{\pi}{4}$ ,  $\left(\frac{dy}{dx}\right) = \frac{b}{a}$ . (NCERT Exemplar, AI 2014) (Ev)

64. Differentiate  $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$  w.r.t.  $\sin^{-1} \frac{2x}{1+x^2}$ , if  $x \in (-1, 1)$ . (Foreign 2016, Delhi 2014) (Ev)

65. If  $x = ae^t(\sin t + \cos t)$  and  $y = ae^t(\sin t - \cos t)$ , prove that  $\frac{dy}{dx} = \frac{x+y}{x-y}$ . (AI 2015C) (Ap)

66. Differentiate  $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$  with respect to  $\cos^{-1}(2x\sqrt{1-x^2})$ , when  $x \neq 0$ . (Delhi 2014) (Ev)

67. Differentiate  $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$  with respect to  $\sin^{-1}(2x\sqrt{1-x^2})$ . (Delhi 2014) (Ev)

68. Find the value of  $\frac{dy}{dx}$  at  $\theta = \frac{\pi}{4}$ , if  $x = ae^{\theta}(\sin \theta - \cos \theta)$  and  $y = ae^{\theta}(\sin \theta + \cos \theta)$ . (AI 2014) (Ap)

69. If  $x = \cos t(3 - 2 \cos^2 t)$  and  $y = \sin t(3 - 2 \sin^2 t)$ , find the value of  $\frac{dy}{dx}$  at  $t = \frac{\pi}{4}$ . (AI 2014) (Ev)

## 5.7 Second Order Derivative

MCQ

70. If  $x = A \cos 4t + B \sin 4t$ , then  $\frac{d^2x}{dt^2}$  is equal to (a)  $x$  (b)  $-x$  (c)  $16x$  (d)  $-16x$  (2023)

71. If  $y = e^{-x}$ , then  $\frac{d^2y}{dx^2}$  is equal to  
 (a)  $-y$  (b)  $y$  (c)  $x$  (d)  $-x$   
 (Term I, 2021-22) (U)

72. If  $x = t^2 + 1$ ,  $y = 2at$ , then  $\frac{d^2y}{dx^2}$  at  $t = a$  is  
 (a)  $-\frac{1}{a}$  (b)  $-\frac{1}{2a^2}$  (c)  $\frac{1}{2a^2}$  (d)  $0$   
 (Term I, 2021-22) (Ev)

73. If  $y = \sin(2 \sin^{-1}x)$ , then  $(1 - x^2) y_2$  is equal to  
 (a)  $-xy_1 + 4y$  (b)  $-xy_1 - 4y$   
 (c)  $xy_1 - 4y$  (d)  $xy_1 + 4y$   
 (Term I, 2021-22) (Ap)

74. If  $y = \log_e \left( \frac{x^2}{e^2} \right)$ , then  $\frac{d^2y}{dx^2}$  equals  
 (a)  $-\frac{1}{x}$  (b)  $-\frac{1}{x^2}$  (c)  $\frac{2}{x^2}$  (d)  $-\frac{2}{x^2}$   
 (2020) (Ev)

**SA I (2 marks)**

75. If  $x = at^2$ ,  $y = 2at$ , then find  $\frac{d^2y}{dx^2}$ . (2020) (Ev)

76. If  $x = a \cos \theta$ ;  $y = b \sin \theta$ , then find  $\frac{d^2y}{dx^2}$ . (2020) (Ap)

**SA II (3 marks)**

77. If  $y = \tan x + \sec x$ , then prove that  
 $\frac{d^2y}{dx^2} = \frac{\cos x}{(1 - \sin x)^2}$ . (2023)

78. If  $x = a \cos \theta + b \sin \theta$ ,  $y = a \sin \theta - b \cos \theta$ , then show that  $\frac{dy}{dx} = -\frac{x}{y}$  and hence show that  
 $y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$ . (2021C) (Ap)

**LA I (4 marks)**

79. If  $x = a \sec^3 \theta$ ,  $y = a \tan^3 \theta$ , then find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{4}$ .  
 (2020, Delhi 2015C) (Ev)

80. If  $y = a \cos(\log x) + b \sin(\log x)$ , show that  
 $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ . (2019C) (Ap)

81. If  $y = (\sin^{-1}x)^2$ , prove that  
 $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$ . (Delhi 2019) (Ap)

82. If  $x = \sin t$ ,  $y = \sin pt$ , prove that  
 $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0$ . (AI 2019, Foreign 2016) (Ap)

83. If  $y = \sin(\sin x)$ , prove that  
 $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$ . (2018) (Ap)

84. If  $x^m y^n = (x+y)^{m+n}$ , prove that  $\frac{d^2y}{dx^2} = 0$ .  
 (Delhi 2017) (Ev)

85. If  $e^y(x+1) = 1$ , then show that  $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ .  
 (AI 2017) (Ap)

86. If  $y = x^x$ , prove that  $\frac{d^2y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$ .  
 (Delhi 2016, 2014) (Ap)

87. If  $y = 2\cos(\log x) + 3\sin(\log x)$ , prove that  
 $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$  (AI 2016) (Ap)

88. If  $x = a \cos \theta + b \sin \theta$ ,  $y = a \sin \theta - b \cos \theta$ , show that  
 $y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$  (Delhi 2015, Foreign 2014) (Ap)

89. If  $y = e^{m \sin^{-1} x}$ ,  $-1 \leq x \leq 1$ , then show that  
 $(1-x^2) \frac{d^2y}{dx^2} - \frac{xy}{dx} - m^2 y = 0$ . (AI 2015) (Ap)

90. If  $y = (x + \sqrt{1+x^2})^n$ , then show that  
 $(1+x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = n^2 y$ . (Foreign 2015) (Ap)

91. If  $y = Ae^{mx} + Be^{nx}$ , show that  
 $\frac{d^2y}{dx^2} - (m+n) \frac{dy}{dx} + mny = 0$ . (AI 2015C, 2014) (Ap)

92. If  $x = a(\cos t + t \sin t)$  and  $y = a(\sin t - t \cos t)$ , then find the value of  $\frac{d^2y}{dx^2}$  at  $t = \frac{\pi}{4}$ . (Delhi 2014C) (Ev)

93. If  $x = a \left( \cos t + \log \tan \frac{t}{2} \right)$ ,  $y = a \sin t$ , evaluate  
 $\frac{d^2y}{dx^2}$  at  $t = \frac{\pi}{3}$ . (Delhi 2014C) (Ev)

**CBSE Sample Questions**

**5.2 Continuity**

**MCQ**

1. The value of 'k' for which the function

$$f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$

is continuous at  $x = 0$  is

- (a) 0 (b) -1 (c) 1 (d) 2  
 (2022-23) (Ap)

2. The value of  $k(k < 0)$  for which the function  $f$  defined as  $f(x) = \begin{cases} 1 - \cos kx & , x \neq 0 \\ x \sin x & \\ \frac{1}{2} & , x = 0 \end{cases}$  is continuous at  $x = 0$  is
- (a)  $\pm 1$  (b)  $-1$  (c)  $\pm \frac{1}{2}$  (d)  $\frac{1}{2}$

(Term I, 2021-22) (Ap)

3. The point(s), at which the function  $f$  given by

$$f(x) = \begin{cases} \frac{x}{|x|}, x < 0 \\ -1, x \geq 0 \end{cases}$$
 is continuous, is/are

- (a)  $x \in R$  (b)  $x = 0$   
(c)  $x \in R - \{0\}$  (d)  $x = -1$  and  $1$

(Term I, 2021-22) (Ap)

#### SA I (2 marks)

4. Find the value(s) of  $k$  so that the following function is continuous at  $x = 0$ .

$$f(x) = \begin{cases} 1 - \cos kx & \text{if } x \neq 0 \\ x \sin x & \\ \frac{1}{2} & \text{if } x = 0 \end{cases}$$
 (2020-21) (Ap)

### 5.3 Differentiability

#### SA I (2 marks)

5. If  $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$ , then prove that

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$
 (2022-23) (Ev)

#### SA II (3 marks)

6. Prove that the greatest integer function defined by  $f(x) = [x]$ ,  $0 < x < 2$  is not differentiable at  $x = 1$ .

(2020-21) (Ap)

### 5.4 Exponential and Logarithmic Functions

#### MCQ

7. If  $e^x + e^y = e^{x+y}$ , then  $\frac{dy}{dx}$  is
- (a)  $e^{y-x}$  (b)  $e^{y+x}$  (c)  $-e^{y-x}$  (d)  $2e^{x-y}$

(Term I, 2021-22) (Ev)

8. If  $y = \log(\cos e^x)$ , then  $\frac{dy}{dx}$  is

- (a)  $\cos e^{x-1}$  (b)  $e^{-x} \cos e^x$   
(c)  $e^x \sin e^x$  (d)  $-e^x \tan e^x$

(Term I, 2021-22) (Ev)

### 5.5 Logarithmic Differentiation

#### SA I (2 marks)

9. If  $y = e^{x \sin^2 x} + (\sin x)^x$ , find  $\frac{dy}{dx}$ .

(2020-21) (Ap)

### 5.6 Derivatives of Functions in Parametric Forms

#### MCQ

10. The derivative of

$$\sin^{-1}(2x\sqrt{1-x^2}) \text{ w.r.t. } \sin^{-1}x, \frac{1}{\sqrt{2}} < x < 1, \text{ is}$$

- (a)  $2$  (b)  $\frac{\pi}{2} - 2$  (c)  $\frac{\pi}{2}$  (d)  $-2$

(Term I, 2021-22) (Ev)

### 5.7 Second Order Derivative

#### MCQ

11. If  $y = 5 \cos x - 3 \sin x$ , then  $\frac{d^2y}{dx^2}$  is equal to

- (a)  $-y$  (b)  $y$  (c)  $25y$  (d)  $9y$

(Term I, 2021-22) (Ap)

12. If  $x = a \sec \theta$ ,  $y = b \tan \theta$ , then  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{6}$  is

- (a)  $\frac{-3\sqrt{3}b}{a^2}$  (b)  $\frac{-2\sqrt{3}b}{a}$  (c)  $\frac{-3\sqrt{3}b}{a}$  (d)  $\frac{-b}{3\sqrt{3}a^2}$

(Term I, 2021-22) (Ap)

#### SA II (3 marks)

13. If  $x = a \sec \theta$ ,  $y = b \tan \theta$ , find  $\frac{d^2y}{dx^2}$  at  $\theta = \frac{\pi}{6}$ .

(2020-21) (Ap)

## Detailed SOLUTIONS

#### Previous Years' CBSE Board Questions

1. (b): Let  $x = 1.5$   
 $\therefore$  L.H.L. =  $\lim_{x \rightarrow 1.5^-} f(x) = \lim_{h \rightarrow 0} [1.5 - h] = 1$   
 and R.H.L. =  $\lim_{x \rightarrow 1.5^+} f(x) = \lim_{h \rightarrow 0} [1.5 + h] = 1$   
 $\therefore$  L.H.L. = R.H.L.  
 $\therefore$   $f(x)$  is continuous at  $x = 1.5$

Also, greatest integer function is discontinuous at all integral values of  $x$ .

2. (b): Since  $f(x)$  is continuous at  $x = 5$ ,  
 $\Rightarrow \lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x) = f(5)$   
 $\Rightarrow 3(5) - 8 = 2k \Rightarrow 7 = 2k \Rightarrow k = \frac{7}{2}$

3. (c): Since, greatest integer function i.e.,  $[x]$  is continuous at all points except at integers.  
 $\therefore f(x)$  is continuous at  $1.5$ .

4.  $\therefore f(x)$  is continuous at  $x = \pi$

$$\therefore f(\pi) = \lim_{x \rightarrow \pi^-} f(x) = \lim_{x \rightarrow \pi^+} f(x)$$

Here,  $f(\pi) = \lambda\pi$

$$\text{And } \lim_{x \rightarrow \pi^+} f(x) = \lim_{h \rightarrow 0} f(\pi+h)$$

$$= \lim_{h \rightarrow 0} \cos(\pi+h) = -1$$

From (i), (ii) and (iii), we get

$$\lambda\pi = -1 \Rightarrow \lambda = -\frac{1}{\pi}$$

### Concept Applied

→ A real function  $f$  is said to be continuous if it is continuous at every point in the domain of  $f$ .

5. We have,  $f(x) = \begin{cases} kx, & x < 0 \\ 3, & x \geq 0 \end{cases}$

$$\text{L.H.L.} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} kx = -k$$

$$\text{R.H.L.} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 3 = 3$$

Since,  $f(x)$  is continuous at  $x = 0$ .

$$\therefore \text{L.H.L.} = \text{R.H.L.} = f(0)$$

$$\Rightarrow -k = 3 \Rightarrow k = -3$$

6. Given,  $f(x)$  is continuous at  $x = 3$ .

$$\text{So, } \lim_{x \rightarrow 3} f(x) = f(3) \Rightarrow \lim_{x \rightarrow 3} \frac{(x+3)^2 - 36}{x-3} = k$$

$$\Rightarrow \lim_{x \rightarrow 3} \frac{(x+3)^2 - 6^2}{x-3} = k \Rightarrow \lim_{x \rightarrow 3} \frac{(x+3+6)(x+3-6)}{x-3} = k$$

$$\Rightarrow 3+3+6 = k \Rightarrow k = 12$$

7. We have,  $f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$  is continuous at  $x = 0$

$\therefore \text{L.H.L.} = \text{R.H.L.}$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{\sin^2 \lambda x}{x^2} = 1 \Rightarrow \lim_{x \rightarrow 0} \frac{\sin^2 \lambda x}{\lambda^2 x^2} \cdot \lambda^2 = 1$$

$$\Rightarrow \lambda^2 \lim_{\lambda x \rightarrow 0} \left( \frac{\sin \lambda x}{\lambda x} \right)^2 = 1 \Rightarrow \lambda^2 \cdot 1 = 1 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$$

8. We have,  $f(x) = \begin{cases} ax+1 & \text{if } x \leq 3 \\ bx+3 & \text{if } x > 3 \end{cases}$

Since,  $f(x)$  is continuous at  $x = 3$

$$\therefore \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) \Rightarrow \lim_{x \rightarrow 3^-} ax+1 = \lim_{x \rightarrow 3^+} bx+3$$

$$\Rightarrow 3a+1 = 3b+3 \Rightarrow 3a-3b = 2 \Rightarrow a-b = 2/3$$

9.  $\therefore f(x)$  is continuous at  $\pi/2$ .

$$\therefore \lim_{x \rightarrow \pi/2^-} f(x) = \lim_{x \rightarrow \pi/2^+} f(x) = f(\pi/2)$$

$$\text{Now, } \lim_{x \rightarrow \pi/2^-} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{2}-h\right)$$

$$= \lim_{h \rightarrow 0} \frac{1 - \sin^3\left(\frac{\pi}{2}-h\right)}{3\cos^2\left(\frac{\pi}{2}-h\right)} = \lim_{h \rightarrow 0} \frac{1 - \cos^3 h}{3\sin^2 h}$$

$$\dots(i) = \lim_{h \rightarrow 0} \frac{(1 - \cosh)(1 + \cos^2 h + \cosh)}{3(1 - \cosh)(1 + \cosh)}$$

$$\dots(ii) = \lim_{h \rightarrow 0} \frac{(1 + \cos^2 h + \cosh)}{3(1 + \cosh)} = \frac{1+1+1}{3(1+1)} = \frac{1}{2}$$

$$\dots(iii) \text{ and } \lim_{x \rightarrow \pi/2^+} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{2}+h\right)$$

$$= \lim_{h \rightarrow 0} \frac{q \left[ 1 - \sin\left(\frac{\pi}{2}+h\right) \right]}{\left[ \pi - 2\left(\frac{\pi}{2}+h\right) \right]^2} = \lim_{h \rightarrow 0} \frac{q(1 - \cosh)}{4h^2}$$

$$= \frac{q}{4} \times \lim_{h \rightarrow 0} \frac{2\sin^2 \frac{h}{2}}{\frac{h^2}{4} \times 4} = \frac{q}{4} \times \frac{2}{4} = \frac{q}{8} \text{ and } f(\pi/2) = p$$

$$\therefore \frac{1}{2} = \frac{q}{8} = p$$

[From (1)]

$$\Rightarrow p = \frac{1}{2} \text{ and } q = 4$$

### Commonly Made Mistake

→ Remember the difference between left hand limit and right hand limit.

10.  $\therefore f(x)$  is continuous at  $x = 0$ .

$$\therefore f(0) = k$$

$$\text{and } \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{8x^2}$$

$$= \lim_{x \rightarrow 0} \frac{2 \cdot \sin^2 2x}{8x^2} = \lim_{x \rightarrow 0} \left( \frac{\sin 2x}{2x} \right)^2 = 1$$

$\therefore f$  is continuous at  $x = 0$

$$\therefore f(0) = \lim_{x \rightarrow 0} f(x) \Rightarrow k = 1$$

11. (c):  $f(x) = |x| = \begin{cases} x, & x > 0 \\ -x, & x < 0 \end{cases}$

The function  $f(x)$  is continuous everywhere but not differentiable at  $x = 0$  as at  $x = 0$

$$Lf'(0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^-} \frac{-x - 0}{x} = -1$$

$$Rf'(0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^+} \frac{x - 0}{x} = 1$$

$\therefore Lf'(0) \neq Rf'(0)$ , so  $f(x)$  is not differentiable at  $x = 0$ .

12. (c): Let  $y = x^{2x}$

Taking log on both sides, we get

$$\log y = 2x \log x$$

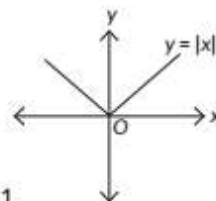
Differentiating both sides w.r.t.  $x$ , we get

$$\dots(1) \quad \frac{1}{y} \frac{dy}{dx} = 2 \left\{ x \cdot \frac{1}{x} + \log x \cdot 1 \right\}$$

$$\Rightarrow \frac{dy}{dx} = 2y \{ 1 + \log x \} = 2x^{2x} (1 + \log x)$$

13. (a): Given,  $y^2 (2-x) = x^3$

$$\Rightarrow y^2 = \frac{x^3}{2-x} \Rightarrow 2y \cdot \frac{dy}{dx} = \frac{(2-x) \times 3x^2 - x^3(-1)}{(2-x)^2}$$



$$\Rightarrow \frac{dy}{dx} = \frac{6x^2 - 2x^3}{2y(2-x)^2} \Rightarrow \left(\frac{dy}{dx}\right)_{(1,1)} = \frac{6-2}{2 \times 1} = 2$$

14. (a): At  $x = 1$   $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 = 1$

And  $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 2 - x = 1$

Also,  $f(1) = 2 - 1 = 1 \therefore \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$

$\therefore f(x)$  is continuous at  $x = 1$

Now, L.H.D. =  $\lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^-} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1^-} (x + 1) = 2$

R.H.D. =  $\lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^+} \frac{(2-x) - 1}{x - 1} = -1$

$\therefore$  L.H.D.  $\neq$  R.H.D.  $\therefore f(x)$  is not differentiable at  $x = 1$ .

**Commonly Made Mistake** 

$\Rightarrow$  Every continuous function is not differentiable.

15. (c): Given,  $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a \Rightarrow \sec a = \frac{1+x}{1-y}$

On differentiating, we get

$$\frac{(1-y) + (1+x)\frac{dy}{dx}}{(1-y)^2} = 0 \Rightarrow (1+x)\frac{dy}{dx} = y - 1 \Rightarrow \frac{dy}{dx} = \frac{y-1}{1+x}$$

16. We have,  $y = \tan^{-1} x + \cot^{-1} x$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x^2+1} - \frac{1}{x^2+1} \Rightarrow \frac{dy}{dx} = 0$$

17. We have,  $\cos(xy) = k$

$$\Rightarrow -\sin(xy) \left( y + x \frac{dy}{dx} \right) = 0 \Rightarrow y + x \frac{dy}{dx} = 0 \quad [\because xy \neq \pi]$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

18. Let  $y = \sin^2(\sqrt{x})$

$$\therefore \frac{dy}{dx} = 2\sin(\sqrt{x}) \cdot \cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}} \sin(2\sqrt{x})$$

**Key Points** 

$\Rightarrow \sin 2x = 2\sin x \cos x$

19. To check the differentiability of  $f(x) = x|x|$  at  $x = 0$ .

Consider,  $\lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$

$$= \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h|h| - 0}{h} = \lim_{h \rightarrow 0} |h| = 0$$

Hence,  $f'(0)$  exists, so  $f(x) = x|x|$  is differentiable at  $x = 0$ .

**Answer Tips** 

$\Rightarrow f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

20. We have,  $y = f(x^2) \Rightarrow \frac{dy}{dx} = f'(x^2) \cdot 2x$

$$= e^x \cdot 2x \quad [\because f'(x) = e^{\sqrt{x}}]$$

$$= 2x e^x$$

21. Given,  $f(x) = x + 1$

Now,  $(f \circ f)(x) = f(f(x)) = f(x + 1) = (x + 1) + 1 = x + 2$

$$\therefore \frac{d}{dx}(f \circ f)(x) = \frac{d}{dx}(x + 2) = 1$$

22. We have,  $(x^2 + y^2)^2 = xy$

$$\Rightarrow x^4 + y^4 + 2x^2y^2 = xy$$

On differentiating both sides w.r.t.  $x$ , we get

$$4x^3 + 4y^3 \frac{dy}{dx} + 4xy^2 + 4x^2y \frac{dy}{dx} = x \frac{dy}{dx} + y$$

$$\Rightarrow \frac{dy}{dx}(4y^3 + 4x^2y - x) = y - 4x^3 - 4xy^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - 4x^3 - 4xy^2}{4y^3 + 4x^2y - x}$$

23. We have,  $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$

R.H.D. =  $\lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^+} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 1^+} \frac{(x-1)(x+1)}{x-1}$

$$= \lim_{x \rightarrow 1^+} (x+1) = 2$$

L.H.D. =  $\lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^-} \frac{x - 1}{x - 1} = 1$

$\therefore$  L.H.D.  $\neq$  R.H.D.

$\therefore f(x)$  is not differentiable at  $x = 1$

24. We have,  $f(x) = |x - 3|$

$$f(x) = \begin{cases} x - 3, & x \geq 3 \\ -x + 3, & x < 3 \end{cases}$$

At  $x = 3$

$$f'(3^+) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{3+h-3 - (3-3)}{h} = \lim_{h \rightarrow 0} 1 = 1$$

$$f'(3^-) = \lim_{h \rightarrow 0} \frac{f(3-h) - f(3)}{-h} = \lim_{h \rightarrow 0} \frac{-3+h+3 - (3-3)}{-h} = \lim_{h \rightarrow 0} -1 = -1$$

$\therefore f'(3^+) \neq f'(3^-) \therefore f(x)$  is not differentiable at  $x = 3$ .

25. We have,  $y = \sqrt{a + \sqrt{a+x}}$

$$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{a + \sqrt{a+x}}} \times \frac{1}{2\sqrt{a+x}} = \frac{1}{4\sqrt{a}\sqrt{(a+x)+a+x}}$$

26. Let,  $y = \tan^{-1}\left(\frac{1 + \cos x}{\sin x}\right)$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \left[ \tan^{-1} \left( \frac{2\cos^2 \frac{x}{2}}{2\sin \frac{x}{2} \cos \frac{x}{2}} \right) \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \tan^{-1} \left( \frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} \right) \Rightarrow \frac{dy}{dx} = \frac{d}{dx} \tan^{-1} \left( \cot \frac{x}{2} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \tan^{-1} \left[ \tan \left( \frac{\pi}{2} - \frac{x}{2} \right) \right] \Rightarrow \frac{dy}{dx} = \frac{d}{dx} \left( \frac{\pi}{2} - \frac{x}{2} \right) = -\frac{1}{2}$$

27. We have,  $\sin^2 y + \cos xy = K$

Differentiating w.r.t.  $x$  on both sides, we get

$$2\sin y \cos y \frac{dy}{dx} + (-\sin xy) \left( x \frac{dy}{dx} + y \right) = 0$$

( $\because$  Product rule:  $(uv)' = u'v + uv'$ )

$$\Rightarrow \frac{dy}{dx} = \frac{y \sin xy}{\sin 2y - x \sin xy}$$

$$\Rightarrow \left[ \frac{dy}{dx} \right]_{\left(1, \frac{\pi}{4}\right)} = \frac{\frac{\pi}{4} \sin \frac{\pi}{4}}{\sin \frac{\pi}{2} - \sin \frac{\pi}{4}} = \frac{\pi}{4(\sqrt{2}-1)}$$

28. Given  $f(x) = \begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$

$$f(x) = \begin{cases} -(x-3), & 1 \leq x < 3 \\ x-3, & x \geq 3 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$$

(i) R.H.D. at  $x = 1$

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{-(1+h-3) - [-(1-3)]}{h} \\ = \lim_{h \rightarrow 0} \frac{-h+2-2}{h} = \lim_{h \rightarrow 0} \frac{-h}{h} = -1$$

(ii) L.H.D. at  $x = 1$

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} \\ = \lim_{h \rightarrow 0} \frac{\left[ \frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4} \right] - \left[ \frac{1}{4} - \frac{3}{2} + \frac{13}{4} \right]}{-h} \\ = \lim_{h \rightarrow 0} \frac{\left[ \frac{1+h^2-2h}{4} - \frac{3}{2} + \frac{3h}{2} + \frac{13}{4} - \frac{1}{4} + \frac{3}{2} - \frac{13}{4} \right]}{-h} \\ = \lim_{h \rightarrow 0} \frac{\left[ \frac{h^2}{4} - \frac{h}{2} + \frac{3h}{2} \right]}{-h} = \lim_{h \rightarrow 0} \left[ \frac{h^2}{-4h} + \frac{h}{-h} \right] = 0 - 1 = -1$$

(iii) (a) We know, that function  $f(x)$  is differentiable at  $x = 1$  if L.H.D. = R.H.D. =  $f'(1)$

$$\Rightarrow -1 = -1 = -1$$

Hence, the given function  $f(x)$  is differentiable at  $x = 1$ .

OR

(b) Differentiate  $f(x)$  w.r.t.  $x$ , we get

$$f'(x) = \begin{cases} -1, & 1 \leq x < 3 \\ 1, & x \geq 3 \\ \frac{x}{2} - \frac{3}{2}, & x < 1 \end{cases}$$

$$f'(2) = -1 \text{ and } f'(-1) = \frac{-1}{2} - \frac{3}{2} = \frac{-4}{2} = -2$$

29. Given that  $f(x)$  is differentiable at  $x = 1$ . Therefore,  $f(x)$  is continuous at  $x = 1$

$$\Rightarrow \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$$

$$\Rightarrow \lim_{x \rightarrow 1} (x^2 + 3x + a) = \lim_{x \rightarrow 1} (bx + 2)$$

$$\Rightarrow 1 + 3 + a = b + 2 \Rightarrow a - b + 2 = 0$$

Again,  $f(x)$  is differentiable at  $x = 1$ . So, (L.H.D. at  $x = 1$ ) = (R.H.D. at  $x = 1$ )

$$\Rightarrow \lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x-1} = \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x-1}$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{(x^2 + 3x + a) - (4 + a)}{x-1} = \lim_{x \rightarrow 1} \frac{(bx + 2) - (4 + a)}{x-1}$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{x-1} = \lim_{x \rightarrow 1} \frac{bx - 2 - a}{x-1}$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{(x+4)(x-1)}{x-1} = \lim_{x \rightarrow 1} \frac{bx - b}{x-1} \quad \text{[From (1)]}$$

$$\Rightarrow \lim_{x \rightarrow 1} (x+4) = \lim_{x \rightarrow 1} \frac{b(x-1)}{x-1} \Rightarrow 5 = b$$

Putting  $b = 5$  in (1), we get  $a = 3$ . Hence,  $a = 3$  and  $b = 5$

#### Key Points

⇒ If a function  $f$  is differentiable at a point  $c$ , then it is also continuous at that point.

30. We have,

$$y = \tan^{-1} \left( \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right), x^2 \leq 1$$

Putting  $x^2 = \cos \theta \Rightarrow \theta = \cos^{-1}(x^2)$  we get

$$y = \tan^{-1} \left( \frac{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}} \right)$$

$$= \tan^{-1} \left( \frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}} \right) = \tan^{-1} \left( \frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} \right)$$

$$= \tan^{-1} \left( \tan \left( \frac{\pi}{4} + \frac{\theta}{2} \right) \right) = \frac{\pi}{4} + \frac{\theta}{2} \Rightarrow y = \frac{\pi}{4} + \frac{1}{2} \cos^{-1}(x^2)$$

Differentiating w.r.t.  $x$  on both sides, we get

$$\frac{dy}{dx} = -\frac{1 \times 2x}{2\sqrt{1-x^4}} = \frac{-x}{\sqrt{1-x^4}}$$

31. Here  $f(x) = \sqrt{x^2+1} = (x^2+1)^{\frac{1}{2}}$

$$\Rightarrow f'(x) = \frac{1}{2} \cdot (x^2+1)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2+1}} \quad \dots(1)$$

and  $g(x) = \frac{x+1}{x^2+1}$

$$\Rightarrow g'(x) = \frac{(x^2+1) \cdot 1 - (x+1) \cdot 2x}{(x^2+1)^2} = \frac{-x^2-2x+1}{(x^2+1)^2} \quad \dots(2)$$

and  $h(x) = 2x - 3 \Rightarrow h'(x) = 2 \quad \dots(3)$

$$\therefore f[h'(g'(x))] = f' \left[ h' \left( \frac{-x^2-2x+1}{(x^2+1)^2} \right) \right] \quad \text{[Using (2)]}$$

$$= f'(2) \quad \text{[Using (3)]}$$

$$= \frac{2}{\sqrt{2^2+1}} = \frac{2}{\sqrt{5}} \quad \text{[Using (1)]}$$

#### Concept Applied

⇒ Let  $y = f(t)$ ,  $t = g(u)$  and  $u = m(x)$ , then  $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{du} \times \frac{du}{dx}$

32. The given function is  $f(x) = |x-1| + |x+1|$

$$= \begin{cases} -(x-1) - (x+1), & x < -1 \\ -(x-1) + x + 1, & -1 \leq x \leq 1 \\ x-1 + x + 1, & x > 1 \end{cases} = \begin{cases} -2x, & x < -1 \\ 2, & -1 \leq x \leq 1 \\ 2x, & x > 1 \end{cases}$$

At  $x = 1$ ,

$$f'(1^-) = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{2-2}{-h} = 0$$

$$f'(1^+) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{2(1+h) - 2}{h} = \lim_{h \rightarrow 0} \frac{2h}{h} = 2$$

$$\therefore f'(1^-) \neq f'(1^+)$$

$\Rightarrow f$  is not differentiable at  $x = 1$ .

At  $x = -1$ ,

$$f'(-1^-) = \lim_{h \rightarrow 0} \frac{f(-1-h) - f(-1)}{-h}$$

$$= \lim_{h \rightarrow 0} \frac{-2(-1-h) - (-2)}{-h} = \lim_{h \rightarrow 0} \frac{2h}{-h} = -2$$

$$f'(-1^+) = \lim_{h \rightarrow 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h \rightarrow 0} \frac{2-2}{h} = 0$$

$$\therefore f'(-1^-) \neq f'(-1^+)$$

$\Rightarrow f$  is not differentiable at  $x = -1$ .

33. At  $x = 1$

$$f'(1^-) = \lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^-} \frac{x - 1}{x - 1} = 1$$

$$f'(1^+) = \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^+} \frac{2 - x - 1}{x - 1} = \lim_{x \rightarrow 1^+} \frac{1 - x}{x - 1} = -1$$

Since,  $f'(1^-) \neq f'(1^+)$

$\therefore f(x)$  is not differentiable at  $x = 1$ .

At  $x = 2$

$$f'(2^-) = \lim_{x \rightarrow 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2^-} \frac{2 - x - 0}{x - 2} = -1$$

$$f'(2^+) = \lim_{x \rightarrow 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \rightarrow 2^+} \frac{-2 + 3x - x^2 - 0}{x - 2} = \lim_{x \rightarrow 2^+} \frac{(1 - x)(x - 2)}{x - 2} = -1$$

Since,  $f'(2^-) = f'(2^+)$

$\therefore f(x)$  is differentiable at  $x = 2$ .

34. Here  $f(x) = \begin{cases} \lambda(x^2 + 2), & \text{if } x \leq 0 \\ 4x + 6, & \text{if } x > 0 \end{cases}$

At  $x = 0$ ,  $f(0) = \lambda(0^2 + 2) = 2\lambda$

$$\text{L.H.L.} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \lambda[(0 - h)^2 + 2] = 2\lambda$$

$$\text{R.H.L.} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} [4(0 + h) + 6] = 6$$

$\therefore$  For  $f$  to be continuous at  $x = 0$

$$2\lambda = 6 \Rightarrow \lambda = 3.$$

Hence the function becomes

$$f(x) = \begin{cases} 3(x^2 + 2), & \text{if } x \leq 0 \\ 4x + 6 & \text{if } x > 0 \end{cases}$$

$$f'(0^-) = \lim_{h \rightarrow 0} \frac{f(0 - h) - f(0)}{0 - h} = \lim_{h \rightarrow 0} \frac{3(h^2 + 2) - 6}{-h} = \lim_{h \rightarrow 0} (-3h) = 0$$

$$\text{and } f'(0^+) = \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{0 + h} = \lim_{h \rightarrow 0} \frac{4h + 6 - 6}{h} = 4$$

$\Rightarrow f'(0^-) \neq f'(0^+) \therefore f$  is not differentiable at  $x = 0$ .

35. We have  $\cos y = x \cos(a + y)$

$$\Rightarrow x = \frac{\cos y}{\cos(a + y)}$$

Differentiating w.r.t.  $y$  on both sides, we get

$$\frac{dx}{dy} = \frac{\cos(a + y) \left( \frac{d}{dy} \cos y \right) - \cos y \left( \frac{d}{dy} \cos(a + y) \right)}{\cos^2(a + y)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\cos(a + y)(-\sin y) + \cos y \sin(a + y)}{\cos^2(a + y)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\cos y \sin(a + y) - \cos(a + y) \sin y}{\cos^2(a + y)}$$

$$= \frac{\sin[(a + y) - y]}{\cos^2(a + y)} = \frac{\sin a}{\cos^2(a + y)} \therefore \frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}$$

### Concept Applied

$$\Rightarrow \text{Quotient rule: } \left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2}$$

36. We have,  $y = \sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})$

$$\Rightarrow y = \sin^{-1}(x\sqrt{1-(\sqrt{x})^2} - \sqrt{x}\sqrt{1-x^2})$$

$$\Rightarrow y = \sin^{-1} x - \sin^{-1} \sqrt{x}$$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{d}{dx}(\sqrt{x}) \quad \left( \because \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \right)$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x}\sqrt{1-x^2}}$$

37. (c):  $y = \log(\sin e^x)$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = \frac{1}{\sin e^x} \cdot \frac{d}{dx}(\sin e^x) = \frac{1}{\sin e^x} \cos e^x \cdot \frac{d}{dx} e^x = \frac{1}{\sin e^x} \cos e^x \cdot e^x = e^x \cot e^x$$

38. (a): Given,  $y = \tan^{-1}(e^{2x})$

$$\therefore \frac{dy}{dx} = \frac{1}{1 + e^{4x}} \times 2e^{2x} = \frac{2e^{2x}}{1 + e^{4x}}$$

### Answer Tips

$$\Rightarrow \frac{d}{dx}(e^x) = e^x$$

39. Given  $y = \log(\csc e^x)$

On differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = \frac{1}{\csc e^x} (-\sin e^x \cdot e^x) = -e^x \tan e^x$$

40. We have,  $y = e^{x^2 \cos x} + (\cos x)^x$

$$= e^{x^2 \cos x} + e^{x(\ln \cos x)}$$

$$\therefore \frac{dy}{dx} = e^{x^2 \cos x} \frac{d}{dx}(x^2 \cos x) + e^{x \ln \cos x} \frac{d}{dx}(x \ln \cos x)$$

$$= e^{x^2 \cos x} (2x \cos x - x^2 \sin x)$$

$$+ e^{x \ln \cos x} \left( \ln \cos x - \frac{x}{\cos x} \sin x \right)$$

$$= e^{x^2 \cos x} (2x \cos x - x^2 \sin x) + (\cos x)^x (\ln \cos x - x \tan x)$$

41. Given,  $\log(x^2 + y^2) = 2 \tan^{-1} \left( \frac{y}{x} \right)$

On differentiating w.r.t.  $x$  on both sides, we get

$$\frac{1}{x^2 + y^2} (2x + 2y \frac{dy}{dx}) = 2 \times \frac{1}{1 + \frac{y^2}{x^2}} \frac{d}{dx} \left( \frac{y}{x} \right)$$

$$\Rightarrow \frac{2x}{x^2 + y^2} + \frac{2y}{x^2 + y^2} \frac{dy}{dx} = \frac{2x^2}{x^2 + y^2} \left( \frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2} \right)$$

$$\Rightarrow \frac{dy}{dx} \left[ \frac{2y}{x^2 + y^2} - \frac{2x}{x^2 + y^2} \right] = \frac{2x^2}{x^2 + y^2} \left[ \frac{-y}{x^2} - \frac{1}{x} \right]$$

$$\Rightarrow \frac{2(y-x) dy}{x^2+y^2 dx} = \frac{-2x^2}{x^2+y^2} \left( \frac{y+x}{x^2} \right) \Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$$

### Key Points

→ To differentiate an implicit function, consider  $y$  as a function of  $x$  then apply derivative rules.

42. Here  $y = \frac{x \cos^{-1} x}{\sqrt{1-x^2}} - \log \sqrt{1-x^2}$

Differentiating w.r.t.  $x$ , we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{\sqrt{1-x^2} \cdot \frac{d}{dx}(x \cos^{-1} x) - x \cos^{-1} x \cdot \frac{d}{dx} \sqrt{1-x^2}}{1-x^2} - \frac{1}{\sqrt{1-x^2}} \cdot \frac{d}{dx}(\sqrt{1-x^2}) \\ &= \frac{\sqrt{1-x^2} \cdot \left( 1 \cdot \cos^{-1} x - \frac{x}{\sqrt{1-x^2}} \right) - x \cos^{-1} x \left( \frac{-x}{\sqrt{1-x^2}} \right)}{1-x^2} - \frac{1}{\sqrt{1-x^2}} \left( \frac{-x}{\sqrt{1-x^2}} \right) \\ &= \frac{\sqrt{1-x^2} \cos^{-1} x - x + \frac{x^2 \cos^{-1} x}{\sqrt{1-x^2}} + \frac{x}{1-x^2}}{1-x^2} \\ &= \frac{(1-x^2) \cos^{-1} x + x^2 \cos^{-1} x}{(1-x^2) \sqrt{1-x^2}} = \frac{\cos^{-1} x}{(1-x^2)^{3/2}} \end{aligned}$$

### Concept Applied

→ Product rule of derivative:  $(uv)' = u'v + uv'$

43. Given  $e^x + e^y = e^{x+y} \Rightarrow 1 + e^{y-x} = e^y$  ... (1)

Differentiating (1) w.r.t.  $x$ , we get

$$\begin{aligned} e^{y-x} \cdot \frac{d}{dx}(y-x) &= e^y \frac{dy}{dx} \\ \Rightarrow e^{y-x} \left( \frac{dy}{dx} - 1 \right) &= e^y \frac{dy}{dx} \Rightarrow \frac{dy}{dx} (e^{y-x} - e^y) = e^{y-x} \\ \Rightarrow \frac{dy}{dx} (-1) = e^{y-x} &\Rightarrow \frac{dy}{dx} + e^{y-x} = 0 \quad \text{[Using (1)]} \end{aligned}$$

44. Here,  $y = \tan^{-1} \left( \frac{a}{x} \right) + \log \sqrt{\frac{x-a}{x+a}}$

$$\begin{aligned} &= \tan^{-1} \left( \frac{a}{x} \right) + \frac{1}{2} \log \left( \frac{x-a}{x+a} \right) \\ &= \tan^{-1} \left( \frac{a}{x} \right) + \frac{1}{2} [\log(x-a) - \log(x+a)] \end{aligned}$$

Differentiating w.r.t.  $x$ , we get

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{1+\frac{a^2}{x^2}} \cdot \frac{d}{dx} \left( \frac{a}{x} \right) + \frac{1}{2} \left[ \frac{1}{x-a} - \frac{1}{x+a} \right] \quad \left( \because \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1+x^2} \right) \\ &= \frac{x^2}{x^2+a^2} \cdot a \cdot \left( -\frac{1}{x^2} \right) + \frac{1}{2} \left[ \frac{(x+a) - (x-a)}{x^2-a^2} \right] \\ &= \frac{-a}{x^2+a^2} + \frac{a}{x^2-a^2} = \frac{-a(x^2-a^2) + a(x^2+a^2)}{x^4-a^4} = \frac{2a^3}{x^4-a^4} \end{aligned}$$

### Concept Applied

→  $\log \left( \frac{a}{b} \right) = \log a - \log b$

45. We have,  $e^{y-x} = y^x$   
Taking log on both sides  
 $(y-x) \log e = x \log y \Rightarrow y-x = x \log y$  ... (i)

On differentiating, we get  
 $\frac{dy}{dx} - 1 = x \cdot \frac{1}{y} \frac{dy}{dx} + \log y \Rightarrow \frac{dy}{dx} - \frac{x}{y} \frac{dy}{dx} = 1 + \log y$   
 $\Rightarrow \frac{dy}{dx} \left( 1 - \frac{x}{y} \right) = 1 + \log y \Rightarrow \frac{dy}{dx} = \frac{y(1+\log y)}{y-x}$   
 $\Rightarrow \frac{dy}{dx} = \frac{y(1+\log y)}{x \log y}$  [From (i)]

46. We have  $y = x^3(\cos x)^x + \sin^{-1} \sqrt{x}$  ... (i)

Let  $z = (\cos x)^x = e^{x \log \cos x}$   
 $\therefore \frac{dz}{dx} = e^{x \log \cos x} \left[ \frac{x(-\sin x)}{\cos x} + \log \cos x \right]$   
 $= (\cos x)^x \times [-x \tan x + \log \cos x]$  ... (ii)

Now, differentiating (i) w.r.t.  $x$ , we get  
 $\frac{dy}{dx} = 3x^2(\cos x)^x + x^3(\cos x)^x [-x \tan x + \log \cos x]$   
 $+ \frac{1}{\sqrt{1-x}} \left( \frac{1}{2\sqrt{x}} \right)$  [Using (ii)]  
 $= x^2(\cos x)^x [3 - x^2 \tan x + x \log \cos x] + \frac{1}{2\sqrt{x}} \left( \frac{1}{\sqrt{1-x}} \right)$

47. Let  $y = (\log x)^x + x^{\log x} \therefore y = e^{x \log(\log x)} + e^{(\log x)^2}$   
Differentiating w.r.t.  $x$ , we get

$$\begin{aligned} \frac{dy}{dx} &= (\log x)^x \frac{d}{dx} [x \log(\log x)] + x^{\log x} \frac{d}{dx} \{ (\log x)^2 \} \\ &= (\log x)^x \left\{ x \left( \frac{1}{\log x} \right) \frac{1}{x} + \log(\log x) \right\} + x^{\log x} \left( 2(\log x) \frac{1}{x} \right) \\ &= (\log x)^x \left\{ \frac{1}{\log x} + \log(\log x) \right\} + 2 \left( \frac{\log x}{x} \right) x^{\log x} \end{aligned}$$

48. Given,  $x^y \cdot y^x = x^x$   
Taking log on both sides, we get  
 $y \log x + x \log y = x \log x$

$$\begin{aligned} \Rightarrow y \cdot \frac{1}{x} + \log x \frac{dy}{dx} + \frac{x}{y} \frac{dy}{dx} + \log y &= x \cdot \frac{1}{x} + \log x \\ \Rightarrow \frac{dy}{dx} \left( \frac{x}{y} + \log x \right) &= 1 + \log x - \frac{y}{x} - \log y = 1 - \frac{y}{x} + \log \frac{x}{y} \\ \Rightarrow \frac{dy}{dx} &= \frac{1 - \frac{y}{x} + \log \frac{x}{y}}{\frac{x}{y} + \log x} \end{aligned}$$

49. We have,  $x^y - y^x = a^b$   
Taking log on both sides, we get  
 $y \log x - x \log y = b \log a$  ... (i)

Now, differentiating (i) w.r.t.  $x$  on both sides, we get  
 $\frac{y}{x} + (\log x) \frac{dy}{dx} - \log y - \frac{x}{y} \frac{dy}{dx} = 0$   
 $\Rightarrow \left( \frac{y}{x} - \log y \right) = \frac{dy}{dx} \left( \frac{x}{y} - \log x \right)$   
 $\Rightarrow \frac{1}{x} (y - x \log y) = \frac{dy}{dx} \left( \frac{x - y \log x}{y} \right) \Rightarrow \frac{dy}{dx} = \frac{y(y - x \log y)}{x(x - y \log x)}$

50. Given  $y = (x)^{\cos x} + (\cos x)^{\sin x}$   
 Let  $u = (x)^{\cos x}$ ,  $v = (\cos x)^{\sin x} \therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

Now,  $u = x^{\cos x}$

$$\Rightarrow \log u = \cos x \log x$$

Differentiating with respect to  $x$ , we get

$$\frac{1}{u} \frac{du}{dx} = \cos x \cdot \frac{1}{x} + \log x (-\sin x)$$

$$\Rightarrow \frac{du}{dx} = x^{\cos x} \left[ \frac{\cos x}{x} - \sin x \log x \right]$$

Now,  $v = (\cos x)^{\sin x}$

$$\Rightarrow \log v = \sin x \log \cos x$$

Differentiating with respect to  $x$ , we get

$$\frac{1}{v} \frac{dv}{dx} = \sin x \cdot \frac{1}{\cos x} (-\sin x) + \log \cos x \cdot \cos x$$

$$\Rightarrow \frac{dv}{dx} = (\cos x)^{\sin x} \left[ \frac{-\sin^2 x + \cos^2 x \log \cos x}{\cos x} \right]$$

Putting value of (ii) and (iii) into (i), we get

$$\frac{dy}{dx} = x^{\cos x} \left[ \frac{\cos x}{x} - \sin x \log x \right] + (\cos x)^{\sin x} \left[ \frac{-\sin^2 x + \cos^2 x \log \cos x}{\cos x} \right]$$

51. Let  $y = (\sin x)^x + \sin^{-1} \sqrt{x}$

$$\Rightarrow y = u + v \quad [\text{where } u = (\sin x)^x \text{ and } v = \sin^{-1} \sqrt{x}]$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(i)$$

Now,  $u = (\sin x)^x$

Taking logarithm on both sides, we get  $\log u = x \log \sin x$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = x \cdot \frac{1}{\sin x} (\cos x) + \log \sin x$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x (x \cot x + \log \sin x) \quad \dots(ii)$$

and  $v = \sin^{-1} \sqrt{x}$

$$\Rightarrow \frac{dv}{dx} = \left( \frac{1}{\sqrt{1-x}} \right) \cdot \frac{1}{2\sqrt{x}} \quad \dots(iii)$$

From (i), (ii) and (iii), we get

$$\frac{dy}{dx} = (\sin x)^x (x \cot x + \log \sin x) + \frac{1}{2\sqrt{x}} \left( \frac{1}{\sqrt{1-x}} \right)$$

### Answer Tips

$$\Rightarrow \frac{d}{dx} (\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

52. We have,  $x^y + y^x = a^b$

Taking log on both sides, we get

$$y \log x + x \log y = b \log a \quad \dots(i)$$

Now, differentiating (i) w.r.t.  $x$  on both sides, we get

$$\frac{y}{x} + \log x \left( \frac{dy}{dx} \right) + \log y + x \cdot \frac{1}{y} \frac{dy}{dx} = 0 \quad \left( \because \frac{d}{dx} (\log x) = \frac{1}{x} \right)$$

$$\Rightarrow \frac{y}{x} + \log y = - \left( \frac{x}{y} + \log x \right) \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{-y}{x} \left( \frac{y + x \log y}{x + y \log x} \right)$$

53. We have,  $y = x^{\sin x} + (\sin x)^{\cos x}$

Let  $u = x^{\sin x}$ ,  $v = (\sin x)^{\cos x}$

$$\therefore y = u + v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(i)$$

Now,  $u = x^{\sin x} \Rightarrow \log u = \sin x \cdot (\log x)$

Differentiating w.r.t.  $x$ , we get

$$\frac{1}{u} \frac{du}{dx} = \cos x \cdot (\log x) + \frac{1}{x} \cdot \sin x$$

$$\Rightarrow \frac{du}{dx} = x^{\sin x} \left[ \cos x \cdot (\log x) + \frac{1}{x} \sin x \right] \quad \dots(ii)$$

Also,  $v = (\sin x)^{\cos x} \Rightarrow \log v = \cos x (\log \sin x)$

Differentiating w.r.t.  $x$ , we get

$$\frac{1}{v} \frac{dv}{dx} = -\sin x (\log \sin x) + \cos x \cdot \frac{1}{\sin x} \cdot \cos x$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x} \left[ \frac{\cos^2 x - \sin^2 x (\log \sin x)}{\sin x} \right] \quad \dots(iii)$$

From (i), (ii) & (iii), we get

$$\frac{dy}{dx} = x^{\sin x} \left[ \frac{x \cos x \cdot (\log x) + \sin x}{x} \right] + (\sin x)^{\cos x} \left[ \frac{\cos^2 x - \sin^2 x (\log \sin x)}{\sin x} \right]$$

### Key Points

For function  $y = (f(x))^{g(x)}$ , we must take 'log' on both sides of the function to remove  $g(x)$  as the power of  $f(x)$ .

54. Given  $x^m y^n = (x+y)^{m+n}$

Taking log on both the sides, we get

$$\log x^m + \log y^n = (m+n) \log (x+y)$$

$$\Rightarrow m \log x + n \log y = (m+n) \log (x+y)$$

Differentiating w.r.t.  $x$ , we get

$$m \cdot \frac{1}{x} + n \cdot \frac{1}{y} \frac{dy}{dx} = (m+n) \cdot \frac{1}{x+y} \left( 1 + \frac{dy}{dx} \right)$$

$$\Rightarrow \frac{dy}{dx} \left( \frac{n}{y} - \frac{m+n}{x+y} \right) = \frac{m+n}{x+y} - \frac{m}{x}$$

$$\Rightarrow \frac{dy}{dx} \left( \frac{nx + ny - my - ny}{y(x+y)} \right) = \frac{mx + nx - mx - my}{x(x+y)}$$

$$\Rightarrow \frac{dy}{dx} \left( \frac{nx - my}{y(x+y)} \right) = \frac{nx - my}{x(x+y)} \therefore \frac{dy}{dx} = \frac{y}{x}$$

55. Here,  $(x-y) \cdot e^{\frac{x}{x-y}} = a$

Taking log on both sides, we get

$$\log \left\{ (x-y) \cdot e^{\frac{x}{x-y}} \right\} = \log a \Rightarrow \log(x-y) + \frac{x}{x-y} = \log a$$

Differentiating w.r.t.  $x$ , we get

$$\frac{1}{x-y} \left( 1 - \frac{dy}{dx} \right) + \frac{(x-y) \cdot 1 - x \left( 1 - \frac{dy}{dx} \right)}{(x-y)^2} = 0$$

$$\Rightarrow (x-y) \left( 1 - \frac{dy}{dx} \right) + x - y - x \left( 1 - \frac{dy}{dx} \right) = 0$$

$$\Rightarrow -y \left( 1 - \frac{dy}{dx} \right) + x - y = 0 \Rightarrow y \frac{dy}{dx} + x = 2y$$

56. Here,  $(\tan^{-1}x)^y + y^{\cot x} = 1$   
 $\Rightarrow u + v = 1$  where  $u = (\tan^{-1}x)^y$  and  $v = y^{\cot x}$   
 Differentiating w.r.t.  $x$ , we get

$$\frac{du}{dx} + \frac{dv}{dx} = 0 \quad \dots(1)$$

Now,  $u = (\tan^{-1}x)^y$   
 $\Rightarrow \log u = y \log(\tan^{-1}x)$

Differentiating w.r.t.  $x$ , we get

$$\frac{1}{u} \cdot \frac{du}{dx} = \frac{dy}{dx} \log(\tan^{-1}x) + y \cdot \frac{1}{\tan^{-1}x} \cdot \frac{1}{1+x^2}$$

$$\Rightarrow \frac{du}{dx} = (\tan^{-1}x)^y \times \left[ \frac{dy}{dx} \log(\tan^{-1}x) + \frac{y}{(1+x^2)\tan^{-1}x} \right] \quad \dots(2)$$

And  $v = y^{\cot x}$

$\Rightarrow \log v = \cot x \cdot \log y$

Differentiating w.r.t.  $x$ , we get

$$\frac{1}{v} \frac{dv}{dx} = \cot x \cdot \frac{1}{y} \frac{dy}{dx} - \operatorname{cosec}^2 x \cdot \log y$$

$$\Rightarrow \frac{dv}{dx} = y^{\cot x} \left[ \frac{\cot x}{y} \frac{dy}{dx} - \operatorname{cosec}^2 x \cdot \log y \right] \quad \dots(3)$$

From (1), (2) and (3), we get

$$(\tan^{-1}x)^y \left[ \frac{dy}{dx} \log(\tan^{-1}x) + \frac{y}{(1+x^2)\tan^{-1}x} \right] + y^{\cot x} \left[ \frac{\cot x}{y} \frac{dy}{dx} - \operatorname{cosec}^2 x \cdot \log y \right] = 0$$

$$\Rightarrow \frac{dy}{dx} [(\tan^{-1}x)^y \cdot \log(\tan^{-1}x) + y^{\cot x - 1} \cdot \cot x]$$

$$= y^{\cot x} \cdot \operatorname{cosec}^2 x \cdot \log y - (\tan^{-1}x)^{y-1} \cdot \frac{y}{(1+x^2)}$$

$$\therefore \frac{dy}{dx} = \frac{y^{\cot x} \cdot \operatorname{cosec}^2 x \cdot \log y - (\tan^{-1}x)^{y-1} \cdot \frac{y}{(1+x^2)}}{(\tan^{-1}x)^y \log(\tan^{-1}x) + y^{\cot x - 1} \cot x}$$

57. We have,  $x = e^t \sin t$

$$\Rightarrow \frac{dx}{dt} = e^t \sin t + e^t \cos t$$

and  $y = e^t \cos t \Rightarrow \frac{dy}{dt} = e^t \cos t - e^t \sin t$

$$\frac{dy}{dx} = \frac{e^t \cos t - e^t \sin t}{e^t (\cos t + \sin t)}$$

$$\left. \frac{dy}{dx} \right|_{t=\frac{\pi}{4}} = \frac{\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)} = \frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = 0$$

58. We have,  $x = a \sec \theta$ ,  $y = b \tan \theta$

$$\therefore \frac{dx}{d\theta} = a \sec \theta \tan \theta \text{ and } \frac{dy}{d\theta} = b \sec^2 \theta$$

$$\text{Now, } \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b \sec^2 \theta}{a \sec \theta \tan \theta} = \frac{b}{a} \cdot \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} = \frac{b}{a} \cdot \frac{1}{\sin \theta}$$

$$\therefore \left. \frac{dy}{dx} \right|_{\theta=\frac{\pi}{3}} = \frac{b}{a} \cdot \frac{1}{\sin \frac{\pi}{3}} = \frac{b}{a} \cdot \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2b}{a\sqrt{3}}$$

59. Let  $u = \sin^2 x$ ,  $v = e^{\cos x}$

$$\therefore \frac{du}{dx} = 2 \sin x \cos x \text{ and } \frac{dv}{dx} = e^{\cos x} (-\sin x)$$

$$\text{Now, } \frac{du}{dv} = \frac{du}{dx} \cdot \frac{dx}{dv} = \frac{2 \sin x \cos x}{e^{\cos x} (-\sin x)} = \frac{-2 \cos x}{e^{\cos x}}$$

$$60. \text{ Let } u = \sec^{-1} \left( \frac{1}{\sqrt{1-x^2}} \right) \quad \dots(i)$$

$$\text{and } v = \sin^{-1} (2x\sqrt{1-x^2}) \quad \dots(ii)$$

Put  $x = \sin t$  in (i) and (ii), we get

$$u = \sec^{-1} \left( \frac{1}{\sqrt{1-\sin^2 t}} \right) = \sec^{-1} \left( \frac{1}{\cos t} \right) = \sec^{-1} (\sec t) = t$$

and  $v = \sin^{-1} (2 \sin t \sqrt{1-\sin^2 t}) = \sin^{-1} (\sin 2t) = 2t$

$$\therefore \frac{du}{dt} = 1 \text{ and } \frac{dv}{dt} = 2. \text{ So, } \frac{du}{dv} = \frac{du}{dt} \times \frac{dt}{dv} = \frac{1}{2}$$

$$61. \text{ Let } u = \tan^{-1} \left[ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right] \quad \dots(i)$$

Putting  $x^2 = \cos 2\theta$  in (i), we get

$$u = \tan^{-1} \left[ \frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}} \right]$$

$$= \tan^{-1} \left[ \frac{\sqrt{2\cos^2 \theta} - \sqrt{2\sin^2 \theta}}{\sqrt{2\cos^2 \theta} + \sqrt{2\sin^2 \theta}} \right]$$

$$= \tan^{-1} \left[ \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \right] = \tan^{-1} \left[ \frac{1 - \tan \theta}{1 + \tan \theta} \right]$$

$$= \tan^{-1} \left[ \tan \left( \frac{\pi}{4} - \theta \right) \right] = \frac{\pi}{4} - \theta$$

$$= \frac{\pi}{4} - \frac{\cos^{-1}(x^2)}{2} \quad \left( \because x^2 = \cos 2\theta \Rightarrow \theta = \frac{\cos^{-1} x^2}{2} \right)$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{2} \cdot \frac{1}{\sqrt{1-x^4}} \cdot 2x = \frac{x}{\sqrt{1-x^4}} \quad \dots(ii)$$

$$\text{Let } v = \cos^{-1}(x^2) \Rightarrow \frac{dv}{dx} = \frac{-2x}{\sqrt{1-x^4}} \quad \dots(iii)$$

Dividing (ii) by (iii), we get  $\frac{du}{dv} = -\frac{1}{2}$ .

### Key Points

$$\Rightarrow 1 + \cos 2\theta = 2\cos^2 \theta, 1 - \cos 2\theta = 2\sin^2 \theta.$$

62. We have,  $x = a(2\theta - \sin 2\theta)$  ... (i)

and  $y = a(1 - \cos 2\theta)$  ... (ii)

Differentiating (i) w.r.t.  $\theta$ , we get

$$\frac{dx}{d\theta} = a(2 - 2\cos 2\theta) \quad \dots(iii)$$

Differentiating (ii) w.r.t.  $\theta$ , we get

$$\frac{dy}{d\theta} = 2a \sin 2\theta \quad \dots(iv)$$

$$\text{Now, } \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2a \sin 2\theta}{a(2 - 2\cos 2\theta)} = \frac{\sin 2\theta}{1 - \cos 2\theta}$$

$$\begin{aligned} \therefore \left. \frac{dy}{dx} \right|_{\theta=\frac{\pi}{3}} &= \frac{\sin 2\left(\frac{\pi}{3}\right)}{1-\cos \frac{2\pi}{3}} = \frac{\sin\left(\pi-\frac{\pi}{3}\right)}{1-\cos\left(\pi-\frac{\pi}{3}\right)} \\ &= \frac{\sin\left(\frac{\pi}{3}\right)}{1+\cos\left(\frac{\pi}{3}\right)} = \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} \end{aligned}$$

### Concept Applied

⇒ If  $x = f(t)$  and  $y = g(t)$ , then  $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}, f'(t) \neq 0$

63.  $x = a \sin 2t(1 + \cos 2t), y = b \cos 2t(1 - \cos 2t)$

Now,  $\frac{dx}{dt} = 2a \cos 2t(1 + \cos 2t) + a \sin 2t(-2 \sin 2t)$   
 $= 2a \cos 2t + 2a[\cos^2 2t - \sin^2 2t]$   
 $= 2a \cos 2t + 2a \cos 4t \quad (\because \cos^2(a) - \sin^2(a) = \cos(2a))$

Also,  $\frac{dy}{dt} = -2b \sin 2t(1 - \cos 2t) + b \cos 2t(2 \sin 2t)$   
 $= -2b \sin 2t + 4b(\sin 2t \cos 2t)$   
 $= -2b \sin 2t + 2b \sin 4t \quad (\because 2 \sin(a) \cos(a) = \sin(2a))$

So,  $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2b(\sin 4t - \sin 2t)}{2a(\cos 4t + \cos 2t)}$

$\therefore \left. \left( \frac{dy}{dx} \right)_{at t=\pi/4} = \frac{b}{a} \left[ \frac{\sin \pi - \sin(\pi/2)}{\cos \pi + \cos(\pi/2)} \right] = \frac{b}{a} \left[ \frac{0-1}{-1+0} \right] = \frac{b}{a}$

$\left. \left( \frac{dy}{dx} \right)_{at t=\pi/3} = \frac{b}{a} \left[ \frac{\sin(4\pi/3) - \sin(2\pi/3)}{\cos(4\pi/3) + \cos(2\pi/3)} \right] \right.$   
 $= \frac{b}{a} \left[ \frac{-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}}{-\frac{1}{2} - \frac{1}{2}} \right] = \frac{\sqrt{3}b}{a}$

64. Let  $u = \tan^{-1} \left( \frac{\sqrt{1+x^2}-1}{x} \right)$

Put  $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$

$\therefore u = \tan^{-1} \left( \frac{\sqrt{1+\tan^2 \theta}-1}{\tan \theta} \right)$

$\Rightarrow u = \tan^{-1} \left( \frac{\sec \theta - 1}{\tan \theta} \right) \Rightarrow u = \tan^{-1} \left( \frac{1-\cos \theta}{\sin \theta} \right)$

$\Rightarrow u = \tan^{-1} \left( \frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right) \Rightarrow u = \tan^{-1} \left( \tan \frac{\theta}{2} \right)$

$\therefore u = \frac{\theta}{2} \Rightarrow u = \frac{1}{2} \tan^{-1} x$

$(\because 2 \sin^2 \frac{x}{2} = 1 - \cos x, 2 \sin \frac{x}{2} \cos \frac{x}{2} = \sin x)$

Differentiating w.r.t.  $x$ , we get  $\frac{du}{dx} = \frac{1}{2(1+x^2)}$

Also, let  $v = \sin^{-1} \left( \frac{2x}{1+x^2} \right) \Rightarrow v = 2 \tan^{-1} x$

Differentiating w.r.t.  $x$ , we get

$$\frac{dv}{dx} = \frac{2}{1+x^2} \quad \therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{\frac{1}{2(1+x^2)}}{\frac{2}{1+x^2}} \Rightarrow \frac{du}{dv} = \frac{1}{4}$$

65. We have  $x = ae^t(\sin t + \cos t)$

$\Rightarrow \frac{dx}{dt} = ae^t(\sin t + \cos t) + ae^t(\cos t - \sin t) = 2ae^t \cos t$

and  $y = ae^t(\sin t - \cos t)$

$\Rightarrow \frac{dy}{dt} = ae^t(\sin t - \cos t) + ae^t(\cos t + \sin t) = 2ae^t \sin t$

$\therefore \text{L.H.S.} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2ae^t \sin t}{2ae^t \cos t} = \tan t$

Also, R.H.S.  $= \frac{x+y}{x-y} = \frac{ae^t(\sin t + \cos t) + ae^t(\sin t - \cos t)}{ae^t(\sin t + \cos t) - ae^t(\sin t - \cos t)}$

$= \frac{2ae^t \sin t}{2ae^t \cos t} = \tan t = \text{L.H.S.}$

66. Let  $u = \tan^{-1} \left( \frac{\sqrt{1-x^2}}{x} \right)$

Put  $x = \cos \theta$

$\therefore u = \tan^{-1} \left[ \frac{\sqrt{1-\cos^2 \theta}}{\cos \theta} \right] = \tan^{-1} \left( \frac{\sin \theta}{\cos \theta} \right)$

$= \tan^{-1}(\tan \theta) = \theta \Rightarrow \frac{du}{d\theta} = 1$

Also let,

$v = \cos^{-1}(2x\sqrt{1-x^2}) \Rightarrow v = \cos^{-1}(2 \cos \theta \sqrt{1-\cos^2 \theta})$

$= \cos^{-1}(2 \cos \theta \sin \theta) = \cos^{-1}(\sin 2\theta)$

$= \cos^{-1} \left( \cos \left( \frac{\pi}{2} - 2\theta \right) \right) = \frac{\pi}{2} - 2\theta \Rightarrow \frac{dv}{d\theta} = -2$

Now  $\frac{du}{dv} = \frac{du/d\theta}{dv/d\theta} = \frac{-1}{2}$

67. Let  $u = \tan^{-1} \left( \frac{x}{\sqrt{1-x^2}} \right)$

Put  $x = \sin \theta \Rightarrow \theta = \sin^{-1} x$

$\therefore u = \tan^{-1} \left( \frac{\sin \theta}{\sqrt{1-\sin^2 \theta}} \right) = \tan^{-1}(\tan \theta)$

$\Rightarrow u = \theta \Rightarrow u = \sin^{-1} x$

Differentiating w.r.t.  $x$ , we get  $\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \dots(i)$

Again, let  $v = \sin^{-1}(2x\sqrt{1-x^2})$

Put  $x = \sin \theta$

$\therefore v = \sin^{-1}(2 \sin \theta \cos \theta) = \sin^{-1}(\sin 2\theta) = 2\theta$

$\Rightarrow v = 2 \sin^{-1} x$

Differentiating w.r.t.  $x$ , we get  $\frac{dv}{dx} = \frac{2}{\sqrt{1-x^2}} \quad \dots(ii)$

From (i) & (ii), we get  $\frac{du}{dv} = \frac{1}{2}$

### Concept Applied

⇒ Power rule:  $\frac{d}{dx}(x^n) = nx^{n-1}, n \neq 0$

68. We have,  $x = ae^{\theta}(\sin \theta - \cos \theta)$

Differentiating w.r.t.  $\theta$ , we get

$$\frac{dx}{d\theta} = ae^{\theta}(\sin \theta - \cos \theta) + ae^{\theta}(\cos \theta + \sin \theta)$$

$$\Rightarrow \frac{dx}{d\theta} = ae^{\theta}(2\sin \theta)$$

Also  $y = ae^{\theta}(\sin \theta + \cos \theta)$

Differentiating w.r.t.  $\theta$ , we get

$$\frac{dy}{d\theta} = ae^{\theta}(\sin \theta + \cos \theta) + ae^{\theta}(\cos \theta - \sin \theta)$$

$$\Rightarrow \frac{dy}{d\theta} = ae^{\theta}(2\cos \theta)$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2ae^{\theta}\cos \theta}{2ae^{\theta}\sin \theta} = \cot \theta \quad [\text{From (i) \& (ii)}]$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{\theta = \frac{\pi}{4}} = \cot \frac{\pi}{4} = 1$$

69. Here,  $x = \cos t(3 - 2\cos^2 t)$ ,  $y = \sin t(3 - 2\sin^2 t)$

$$\Rightarrow \frac{dx}{dt} = -\sin t(3 - 2\cos^2 t) + \cos t[2 \cdot 2\cos t \sin t]$$

$$= -3\sin t + 6\cos^2 t \sin t$$

and  $\frac{dy}{dt} = \cos t(3 - 2\sin^2 t) + \sin t(-2 \cdot 2\sin t \cos t)$

$$= 3\cos t - 6\sin^2 t \cos t$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\cos t - 6\sin^2 t \cos t}{-3\sin t + 6\cos^2 t \sin t} = \frac{3\cos t \cdot \cos 2t}{3\sin t \cdot \cos 2t} = \cot t$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{t = \frac{\pi}{4}} = \cot \frac{\pi}{4} = 1$$

70. (d): We have,  $x = A\cos 4t + B\sin 4t$

Differentiating both sides w.r.t.  $t$ , we get

$$\frac{dx}{dt} = A(-\sin 4t) \cdot 4 + B\cos 4t \cdot 4$$

Again differentiating both sides of (i) w.r.t.  $t$ , we get

$$\frac{d^2x}{dt^2} = -4A(\cos 4t) \cdot 4 + 4B(-\sin 4t) \cdot 4$$

$$= -16A\cos 4t - 16B\sin 4t = -16(A\cos 4t + B\sin 4t) = -16x$$

71. (b): Given,  $y = e^{-x}$

$$\Rightarrow \frac{dy}{dx} = -e^{-x} \Rightarrow \frac{d^2y}{dx^2} = e^{-x} = y$$

72. (b): Given,  $x = t^2 + 1$  and  $y = 2at$

$$\Rightarrow \frac{dx}{dt} = 2t \Rightarrow \frac{dy}{dt} = 2a \therefore \frac{dy}{dx} = \frac{a}{t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-a}{t^2} \cdot \frac{dt}{dx} = \frac{-a}{2t^3} \therefore \left( \frac{d^2y}{dx^2} \right)_{at = a} = \frac{-a}{2a^3} = \frac{-1}{2a^2}$$

73. (c): We have,  $y = \sin(2\sin^{-1}x)$

$$\Rightarrow y = \sin\left[\sin^{-1}\left(2x\sqrt{1-x^2}\right)\right]$$

$$\left[ \because 2\sin^{-1}x = \sin^{-1}2x\sqrt{1-x^2} \right]$$

$$\Rightarrow y = 2x\sqrt{1-x^2}$$

$$\Rightarrow y_1 = 2x \times \frac{-2x}{2\sqrt{1-x^2}} + 2\sqrt{1-x^2} = \frac{-4x^2+2}{\sqrt{1-x^2}} \quad \dots(ii)$$

$$\therefore y_2 = \frac{\sqrt{1-x^2}(-8x) - (-4x^2+2) \times \frac{-2x}{2\sqrt{1-x^2}}}{1-x^2}$$

$$= \frac{4x^3-6x}{(1-x^2)\sqrt{1-x^2}} \Rightarrow (1-x^2)y_2 = \frac{4x^3-6x}{\sqrt{1-x^2}}$$

Now, consider  $xy_1 - 4y$

$$= \frac{-4x^3+2x}{\sqrt{1-x^2}} - 8x\sqrt{1-x^2} \quad [\text{Using (i) and (ii)}]$$

$$= \frac{4x^3-6x}{\sqrt{1-x^2}}$$

Thus,  $(1-x^2)y_2 = xy_1 - 4y$

74. (d): We have,  $y = \log_e\left(\frac{x^2}{e^2}\right)$

$$\therefore \frac{dy}{dx} = \frac{e^2}{x^2} \cdot \frac{1}{e^2} \cdot 2x = \frac{2}{x} \Rightarrow \frac{d^2y}{dx^2} = -\frac{2}{x^2}$$

75. Given,  $x = at^2$ ,  $y = 2at$

$$\therefore \frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a \therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

$$\text{So, } \frac{d^2y}{dx^2} = \frac{1}{t^2} \cdot \frac{dt}{dx} = \frac{1}{t^2} \cdot \frac{1}{2at} = \frac{1}{2at^3}$$

76. We have,  $x = a \cos \theta$ ,  $y = b \sin \theta$

$$\therefore \frac{dx}{d\theta} = -a \sin \theta, \frac{dy}{d\theta} = b \cos \theta$$

$$\text{So, } \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{b \cos \theta}{-a \sin \theta} = -\frac{b}{a} \cot \theta$$

$$\text{Now, } \frac{d^2y}{dx^2} = -\frac{b}{a}(-\operatorname{cosec}^2 \theta) \frac{d\theta}{dx}$$

$$= \frac{b}{a} \operatorname{cosec}^2 \theta \left( -\frac{1}{a} \operatorname{cosec} \theta \right) = -\frac{b}{a^2} \operatorname{cosec}^3 \theta$$

77. We have,  $y = \tan x + \sec x$

Differentiating both sides w.r.t.  $x$ , we get

$$\frac{dy}{dx} = \sec^2 x + \sec x \tan x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos^2 x} + \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \frac{1+\sin x}{\cos^2 x}$$

Now, again differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = \frac{\cos^2 x(\cos x) - (1+\sin x)(-2\cos x \sin x)}{(\cos^2 x)^2}$$

$$= \frac{\cos^3 x + (1+\sin x) \cdot 2\sin x \cdot \cos x}{(1-\sin^2 x)^2}$$

$$= \frac{\cos^3 x + 2\sin x \cos x + 2\sin^2 x \cos x}{(1-\sin x)^2(1+\sin x)^2}$$

$$= \frac{\cos x(\cos^2 x + \sin^2 x) + \sin^2 x \cos x + 2 \sin x \cos x}{(1 - \sin x)^2 (1 + \sin x)^2}$$

$$= \frac{\cos x(1 + \sin^2 x + 2 \sin x)}{(1 - \sin x)^2 (1 + \sin x)^2} = \frac{\cos x \cdot (1 + \sin x)^2}{(1 - \sin x)^2 (1 + \sin x)^2} = \frac{\cos x}{(1 - \sin x)^2}$$

Hence proved.

78. We have,  $x = a \cos \theta + b \sin \theta$ ,  $y = a \sin \theta - b \cos \theta$

$$\therefore \frac{dx}{d\theta} = -a \sin \theta + b \cos \theta, \frac{dy}{d\theta} = a \cos \theta + b \sin \theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{a \cos \theta + b \sin \theta}{-a \sin \theta + b \cos \theta} = \frac{x}{-y} \cdot \frac{d^2 y}{dx^2} = \frac{-y \cdot 1 - x \left(-\frac{dy}{dx}\right)}{(-y)^2}$$

$$\Rightarrow y^2 \frac{d^2 y}{dx^2} = -y + x \frac{dy}{dx} \Rightarrow y^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0$$

79. Here  $x = a \sec^3 \theta$

$$\Rightarrow \frac{dx}{d\theta} = a \cdot 3 \sec^2 \theta \cdot \sec \theta \tan \theta = 3a \sec^3 \theta \tan \theta \text{ and } y = a \tan^3 \theta$$

$$\Rightarrow \frac{dy}{d\theta} = a \cdot 3 \tan^2 \theta \cdot \sec^2 \theta$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3a \tan^2 \theta \sec^2 \theta}{3a \sec^3 \theta \tan \theta} = \frac{\tan \theta}{\sec \theta} = \sin \theta$$

On differentiating w.r.t.  $x$ , we get

$$\frac{d^2 y}{dx^2} = \cos \theta \cdot \frac{d\theta}{dx} = \frac{\cos \theta}{3a \sec^3 \theta \tan \theta} = \frac{1}{3a} \cos^4 \theta \cdot \cot \theta$$

$$\therefore \left. \frac{d^2 y}{dx^2} \right|_{\theta = \frac{\pi}{4}} = \frac{1}{3a} \cos^4 \frac{\pi}{4} \cdot \cot \frac{\pi}{4} = \frac{1}{3a} \left(\frac{1}{\sqrt{2}}\right)^4 \cdot 1 = \frac{1}{3a} \cdot \frac{1}{4} = \frac{1}{12a}$$

80. We have,  $y = a \cos(\log x) + b \sin(\log x)$

$$\Rightarrow \frac{dy}{dx} = -a \sin(\log x) \cdot \frac{1}{x} + b \cos(\log x) \cdot \frac{1}{x}$$

$$= \frac{-a \sin(\log x) + b \cos(\log x)}{x}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{x \left[ -a \cos(\log x) \cdot \frac{1}{x} - b \sin(\log x) \cdot \frac{1}{x} \right] - (-a \sin(\log x) + b \cos(\log x))}{x^2}$$

$$\Rightarrow x^2 \frac{d^2 y}{dx^2} = -y - \frac{xy}{dx} \Rightarrow x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

81. We have,  $y = (\sin^{-1} x)^2$

$$\Rightarrow \frac{dy}{dx} = 2(\sin^{-1} x) \times \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{4(\sin^{-1} x)^2}{(1-x^2)}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{4y}{1-x^2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 (1-x^2) = 4y$$

Again, differentiating (ii) w.r.t.  $x$  on both sides, we get

$$2 \frac{dy}{dx} \frac{d^2 y}{dx^2} (1-x^2) + \left(\frac{dy}{dx}\right)^2 (-2x) = 4 \left(\frac{dy}{dx}\right)$$

$$\Rightarrow (1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 2 = 0$$

**Concept Applied** 

Let  $y = f(x)$ , then  $\frac{dy}{dx} = f'(x)$ . If  $f'(x)$  is differentiable,

$$\text{then } \frac{d}{dx} \left( \frac{dy}{dx} \right) = f''(x) \text{ or } \frac{d^2 y}{dx^2} = f''(x)$$

82. We have,  $x = \sin t$  and  $y = \sin pt$

$$\frac{dx}{dt} = \cos t \text{ and } \frac{dy}{dt} = p \cos pt \Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{p \cos pt}{\cos t}$$

Differentiating both sides w.r.t.  $x$ , we get

$$\frac{d^2 y}{dx^2} = \frac{-p^2 \sin pt \cos t + p \cos pt \sin t}{\cos^2 t} \times \frac{dt}{dx}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{-p^2 \sin pt \cos t + p \cos pt \sin t}{\cos^3 t}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = -\frac{p^2 \sin pt \cos t}{\cos^3 t} + \frac{p \cos pt \sin t}{\cos^3 t}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{-p^2 y}{\cos^2 t} + \frac{x \frac{dy}{dx}}{\cos^2 t} \Rightarrow \cos^2 t \frac{d^2 y}{dx^2} = -p^2 y + x \frac{dy}{dx}$$

$$\Rightarrow (1 - \sin^2 t) \frac{d^2 y}{dx^2} = -p^2 y + x \frac{dy}{dx}$$

$$\Rightarrow (1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0$$

83. Here,  $y = \sin(\sin x)$

Differentiating both sides w.r.t.  $x$ , we get

$$\frac{dy}{dx} = \cos(\sin x) \cdot \cos x$$

Again, differentiating w.r.t.  $x$  both sides, we get

$$\frac{d^2 y}{dx^2} = -\sin(\sin x) \cdot \cos x \cdot \cos x + (-\sin x) \cos(\sin x)$$

$$= -\cos^2 x \cdot \sin(\sin x) - \sin x \cdot \cos(\sin x)$$

$$\text{Now, L.H.S.} = \frac{d^2 y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x$$

$$= -\cos^2 x \cdot \sin(\sin x) - \sin x \cdot \cos(\sin x) + \tan x (\cos x \cdot \cos(\sin x)) + \cos^2 x \cdot \sin(\sin x)$$

$$= -\sin x \cdot \cos(\sin x) + \sin x \cdot \cos(\sin x)$$

$$= 0 = \text{R.H.S.}$$

84. Given  $x^m y^n = (x+y)^{m+n}$

Taking log on both the sides, we get

$$\log x^m + \log y^n = (m+n) \log(x+y)$$

$$\Rightarrow m \log x + n \log y = (m+n) \log(x+y)$$

Differentiating w.r.t.  $x$ , we get

$$m \cdot \frac{1}{x} + n \cdot \frac{1}{y} \frac{dy}{dx} = (m+n) \cdot \frac{1}{x+y} \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} \left( \frac{n}{y} - \frac{m+n}{x+y} \right) = \frac{m+n}{x+y} - \frac{m}{x}$$

$$\Rightarrow \frac{dy}{dx} \left( \frac{nx + ny - my - ny}{y(x+y)} \right) = \frac{mx + nx - mx - my}{x(x+y)}$$

...(i)

[From (i)]

...(ii)

$$\Rightarrow \frac{dy}{dx} \left( \frac{nx-my}{y(x+y)} \right) = \frac{nx-my}{x(x+y)} \therefore \frac{dy}{dx} = \frac{y}{x}$$

Again, differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = \frac{x \frac{dy}{dx} - y}{x^2} = \frac{x \left( \frac{y}{x} \right) - y}{x^2} = 0 \therefore \frac{d^2y}{dx^2} = 0$$

85. Given that  $e^y \cdot (x+1) = 1$

Differentiating (i) w.r.t.  $x$ , we get

$$e^y \frac{d}{dx}(x+1) + (x+1) \frac{d}{dx} e^y = \frac{d}{dx}(1) \Rightarrow e^y + (x+1)e^y \frac{dy}{dx} = 0$$

$$\Rightarrow e^y \left[ 1 + (x+1) \frac{dy}{dx} \right] = 0 \Rightarrow (x+1) \frac{dy}{dx} = -1$$

$$\text{and } \frac{dy}{dx} = \frac{-1}{x+1}$$

$$\text{or } \left( \frac{dy}{dx} \right)^2 = \frac{1}{(x+1)^2}$$

Again, differentiating (ii) w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = \frac{1}{(x+1)^2} \Rightarrow \frac{d^2y}{dx^2} = \left( \frac{dy}{dx} \right)^2$$

86. We have,  $y = x^x \Rightarrow y = e^{x \log x}$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = e^{x \log x} \left( x \times \frac{1}{x} + \log x \right)$$

$$\Rightarrow \frac{dy}{dx} = x^x (1 + \log x) \Rightarrow \frac{dy}{dx} = y(1 + \log x)$$

Again, differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = (1 + \log x) \cdot \frac{dy}{dx} + y \times \frac{1}{x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{y} \left( \frac{dy}{dx} \right)^2 + \frac{y}{x}$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y} \left( \frac{dy}{dx} \right)^2 - \frac{y}{x} = 0$$

87. We have,  $y = 2 \cos(\log x) + 3 \sin(\log x)$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = -2 \sin(\log x) \times \frac{1}{x} + 3 \cos(\log x) \times \frac{1}{x}$$

$$\Rightarrow x \frac{dy}{dx} = -2 \sin(\log x) + 3 \cos(\log x)$$

Again, differentiating w.r.t.  $x$ , we get

$$x \frac{d^2y}{dx^2} + \frac{dy}{dx} = -2 \cos(\log x) \times \frac{1}{x} - 3 \sin(\log x) \times \frac{1}{x}$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -[2 \cos(\log x) + 3 \sin(\log x)]$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -y \Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$$

### Answer Tips

$$\Rightarrow \frac{d}{dx}(\log(x)) = \frac{1}{x}$$

88. Given,  $x = a \cos \theta + b \sin \theta$ ,  $y = a \sin \theta - b \cos \theta$

$$\Rightarrow x^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta$$

$$\text{and } y^2 = a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cos \theta \quad \dots(2)$$

Adding (1) and (2), we get  $x^2 + y^2 = a^2 + b^2$

Differentiating w.r.t.  $x$ , we get

$$2x + 2y \frac{dy}{dx} = 0 \Rightarrow x + y \frac{dy}{dx} = 0 \quad \dots(3)$$

Again, differentiating w.r.t.  $x$ , we get

$$1 + y \frac{d^2y}{dx^2} + \left( \frac{dy}{dx} \right)^2 = 0 \quad \dots(i)$$

Multiplying by  $y$  on both sides, we get

$$y^2 \frac{d^2y}{dx^2} + \left( y \cdot \frac{dy}{dx} \right) \frac{dy}{dx} + y = 0$$

$$\Rightarrow y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0 \quad \text{[From (3)]}$$

89. We have,  $y = e^{m \sin^{-1} x}$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = e^{m \sin^{-1} x} \left( \frac{m}{\sqrt{1-x^2}} \right) = \frac{my}{\sqrt{1-x^2}} \quad \dots(1)$$

[From (iii)] Again, differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = m \left[ \frac{\sqrt{1-x^2} \frac{dy}{dx} - \frac{y}{2\sqrt{1-x^2}} \cdot (-2x)}{(1-x^2)} \right]$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} = m \left[ my + \frac{xy}{\sqrt{1-x^2}} \right] \quad \text{[From (1)]}$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} = m \left[ my + x \cdot \left( \frac{1}{m} \cdot \frac{dy}{dx} \right) \right]$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} = m^2 y + x \frac{dy}{dx} \Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$$

[From (1)]

90. We have,  $y = (x + \sqrt{1+x^2})^n$

Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1} \left( 1 + \frac{2x}{2\sqrt{1+x^2}} \right)$$

$$\Rightarrow \frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1} \left( \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{n(x + \sqrt{1+x^2})^n}{\sqrt{1+x^2}} = \frac{ny}{\sqrt{1+x^2}} \quad \dots(1)$$

Again, differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = n \left[ \frac{\sqrt{1+x^2} \cdot \frac{dy}{dx} - \frac{2(xy)}{2\sqrt{1+x^2}}}{1+x^2} \right]$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} = n \left[ \sqrt{1+x^2} \times \frac{ny}{\sqrt{1+x^2}} - \frac{xy}{\sqrt{1+x^2}} \right]$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} = n^2 y - \frac{nxy}{\sqrt{1+x^2}}$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} = n^2 y - x \frac{dy}{dx} \quad \text{[From (1)]}$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = n^2 y$$

91. Given  $y = Ae^{mx} + Be^{nx}$   
Differentiating w.r.t.  $x$ , we get

$$\frac{dy}{dx} = Ae^{mx} \cdot m + Be^{nx} \cdot n \Rightarrow \frac{d^2y}{dx^2} = m^2 Ae^{mx} + n^2 Be^{nx}$$

$$\text{Now, L.H.S.} = \frac{d^2y}{dx^2} - (m+n) \frac{dy}{dx} + mny$$

$$= m^2 Ae^{mx} + n^2 Be^{nx} - (m+n)(mAe^{mx} + nBe^{nx}) + mn(Ae^{mx} + Be^{nx})$$

$$= Ae^{mx}[m^2 - (m+n)m + mn] + Be^{nx}[n^2 - (m+n)n + mn]$$

$$= Ae^{mx} \times 0 + Be^{nx} \times 0 = 0 = \text{R.H.S.}$$

92. Here,  $x = a(\cos t + t \sin t)$

$$\Rightarrow \frac{dx}{dt} = a[-\sin t + 1 \cdot \sin t + t \cos t] = at \cos t \quad \dots(1)$$

and  $y = a(\sin t - t \cos t)$

$$\Rightarrow \frac{dy}{dt} = a[\cos t - (1 \cdot \cos t - t \sin t)] = a t \sin t$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{a t \sin t}{a t \cos t} = \tan t$$

$$\Rightarrow \frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{dt}{dx} = \frac{\sec^2 t}{a t \cos t}$$

$$= \frac{1}{a} \cdot \frac{1}{t \cos^3 t}$$

$$\left. \frac{d^2y}{dx^2} \right|_{t=\frac{\pi}{4}} = \frac{1}{a} \cdot \frac{1}{\frac{\pi}{4} \cos^3 \frac{\pi}{4}} = \frac{4}{\pi a} \cdot (\sqrt{2})^3 = \frac{8\sqrt{2}}{\pi a}$$

93. Here,  $x = a(\cos t + \log \tan \frac{t}{2})$

$$\Rightarrow \frac{dx}{dt} = a \left( -\sin t + \frac{1}{\tan \frac{t}{2}} \cdot \frac{1}{2} \sec^2 \frac{t}{2} \right)$$

$$= a \left( -\sin t + \frac{\cos \frac{t}{2}}{\sin \frac{t}{2}} \cdot \frac{1}{2 \cos^2 \frac{t}{2}} \right) = a \left( -\sin t + \frac{1}{\sin t} \right)$$

$$= a \frac{(-\sin^2 t + 1)}{\sin t} = a \frac{\cos^2 t}{\sin t}$$

$$\text{Also, } y = a \sin t \Rightarrow \frac{dy}{dt} = a \cos t$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = a \cos t \cdot \frac{\sin t}{a \cos^2 t} = \tan t$$

Again, differentiating w.r.t.  $x$ , we get

$$\frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{dt}{dx} = \frac{\sec^2 t}{a \cos^2 t / \sin t} = \frac{\sin t}{a \cos^4 t}$$

$$\therefore \left. \frac{d^2y}{dx^2} \right|_{t=\frac{\pi}{3}} = \frac{\sin \frac{\pi}{3}}{a \cos^4 \frac{\pi}{3}} = \frac{\sqrt{3}/2}{a(1/2)^4} = \frac{8\sqrt{3}}{a}$$

### CBSE Sample Questions

1. (c): Given, the function  $f$  is continuous at  $x = 0$ .

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0).$$

Now,  $\lim_{x \rightarrow 0} f(x)$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos 4x}{8x^2} = \lim_{x \rightarrow 0} \frac{2 \sin^2 2x}{8x^2} = \lim_{x \rightarrow 0} \frac{\sin^2 2x}{4x^2}$$

$$= \lim_{x \rightarrow 0} \left( \frac{\sin 2x}{2x} \right)^2 = 1 \quad (1)$$

Also,  $f(0) = k$

Hence,  $k = 1$

$$2. (b): \text{ We have, } f(x) = \begin{cases} \frac{1 - \cos kx}{x \sin x}, & x \neq 0 \\ \frac{1}{2}, & x = 0 \end{cases}$$

$\therefore f(x)$  is continuous at  $x = 0$ .

$$\therefore \lim_{x \rightarrow 0} \frac{1 - \cos kx}{x \sin x} = \frac{1}{2} \Rightarrow \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{kx}{2}}{x^2 \frac{\sin x}{x}} = \frac{1}{2}$$

$$\Rightarrow \lim_{x \rightarrow 0} 2 \cdot \frac{k^2 \left[ \frac{\sin \left( \frac{kx}{2} \right)}{\frac{kx}{2}} \right]^2}{\frac{1}{x}} = \frac{1}{2}$$

$$\Rightarrow \frac{k^2}{2} = \frac{1}{2} \Rightarrow k = \pm 1$$

But  $k < 0 \therefore k = -1$  (1)

$$3. (a): \text{ We have, } f(x) = \begin{cases} \frac{x}{|x|}, & x < 0 \\ -1, & x \geq 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{x}{-x} = -1, & x < 0 \\ -1, & x \geq 0 \end{cases}$$

$\Rightarrow f(x) = -1 \forall x \in R$

$\Rightarrow f(x)$  is continuous  $\forall x \in R$  as it is a constant function. (1)

$$4. \text{ We have, } \lim_{x \rightarrow 0} \frac{1 - \cos kx}{x \sin x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \left( \frac{kx}{2} \right)}{x \sin x}$$

$$= \frac{\lim_{x \rightarrow 0} \frac{2 \sin^2 \left( \frac{kx}{2} \right)}{\left( \frac{kx}{2} \right)^2} \times \left( \frac{k}{2} \right)^2}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} = \frac{2 \times 1 \times \frac{k^2}{4}}{1} = \frac{k^2}{2} \quad (1\frac{1}{2})$$

$\therefore f(x)$  is continuous at  $x = 0$ .

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

$$\Rightarrow \frac{k^2}{2} = \frac{1}{2} \Rightarrow k^2 = 1 \Rightarrow k = \pm 1 \quad (1/2)$$

$$5. \text{ We have, } y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1 \quad \dots(i)$$

Let  $\sin^{-1} x = A$  and  $\sin^{-1} y = B$ .

Then,  $x = \sin A$  and  $y = \sin B$

From (i)  $\sin B \cos A + \sin A \cos B = 1$

$$\Rightarrow \sin(A+B) = 1 \quad (1)$$

$$\Rightarrow A + B = \sin^{-1} 1 = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$$

Differentiating w.r.t.  $x$ , we obtain  $\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

6. We have,  $f(x) = [x]$ ,  $0 < x < 2$ .

$$\begin{aligned} \text{R.H.D. (at } x = 1) &= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[1+h] - [1]}{h} \\ &= \lim_{h \rightarrow 0} \frac{(1-1)}{h} = 0 \end{aligned}$$

$$\begin{aligned} \text{L.H.D. (at } x = 1) &= \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{[1-h] - [1]}{-h} \\ &= \lim_{h \rightarrow 0} \frac{0-1}{-h} = \lim_{h \rightarrow 0} \frac{1}{h} = \infty \end{aligned}$$

Since, R.H.D.  $\neq$  L.H.D.

Therefore  $f(x)$  is not differentiable at  $x = 1$ .

7. (c): We have,  $e^x + e^y = e^{x+y}$

$$\Rightarrow e^{-y} + e^{-x} = 1$$

Differentiating w.r.t.  $x$ , we get

$$-e^{-y} \frac{dy}{dx} - e^{-x} = 0 \Rightarrow \frac{dy}{dx} = -e^{y-x}$$

8. (d): We have,  $y = \log(\cos e^x)$

Differentiating both sides w.r.t.  $x$ , we get

$$\frac{dy}{dx} = \frac{1}{\cos e^x} \cdot (-\sin e^x) \cdot e^x$$

$$\Rightarrow \frac{dy}{dx} = -e^x \tan e^x$$

9. We have,  $y = e^{x \sin^2 x} + (\sin x)^x \Rightarrow y = u + v$ ,

where  $u = e^{x \sin^2 x}$  and  $v = (\sin x)^x$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

Now, consider  $u = e^{x \sin^2 x}$

Differentiating both sides with respect to  $x$ , we get

$$\begin{aligned} \frac{du}{dx} &= e^{x \sin^2 x} [x \cdot (2 \sin x \cos x) + \sin^2 x] \\ &= e^{x \sin^2 x} [x \sin 2x + \sin^2 x] \end{aligned}$$

Also,  $v = (\sin x)^x$

$$\Rightarrow \log v = x \log(\sin x)$$

Differentiating both sides with respect to  $x$ , we get

$$\frac{1}{v} \frac{dv}{dx} = x \cdot \frac{1}{\sin x} \cdot \cos x + \log(\sin x)$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^x [x \cot x + \log(\sin x)] \quad \dots \text{(iii)} \quad (1/2)$$

From (i), (ii) and (iii), we get

$$\frac{dy}{dx} = e^{x \sin^2 x} [x \sin 2x + \sin^2 x] + (\sin x)^x [x \cot x + \log(\sin x)] \quad (1/2)$$

$$10. \text{ (a): Let } u = \sin^{-1}(2x\sqrt{1-x^2}) \quad \dots \text{(i)}$$

$$\text{and } v = \sin^{-1} x, \frac{1}{\sqrt{2}} < x < 1$$

$$\Rightarrow \sin v = x \quad \dots \text{(ii)}$$

(1) From (i) and (ii), we get

$$u = \sin^{-1}(2 \sin v \cos v) = \sin^{-1}(\sin 2v)$$

$$\Rightarrow u = 2v$$

Differentiating with respect to  $v$  both sides, we get

$$(1) \quad \frac{du}{dv} = 2 \quad (1)$$

11. (a): We have,  $y = 5 \cos x - 3 \sin x$

$$\Rightarrow \frac{dy}{dx} = -5 \sin x - 3 \cos x$$

$$(1) \quad \Rightarrow \frac{d^2 y}{dx^2} = -5 \cos x + 3 \sin x = -y \quad (1)$$

(1) 12. (a): We have,  $x = a \sec \theta$

$$\Rightarrow \frac{dx}{d\theta} = a \tan \theta \sec \theta \quad \text{and } y = b \tan \theta \Rightarrow \frac{dy}{d\theta} = b \sec^2 \theta$$

$$(1) \quad \therefore \frac{dy}{dx} = \frac{b \sec^2 \theta}{a \tan \theta \sec \theta} = \frac{b}{a} \operatorname{cosec} \theta$$

$$\begin{aligned} \Rightarrow \frac{d^2 y}{dx^2} &= \frac{-b}{a} \operatorname{cosec} \theta \cot \theta \frac{d\theta}{dx} \\ &= \frac{-b}{a} \operatorname{cosec} \theta \cot \theta \frac{1}{a \tan \theta \sec \theta} = \frac{-b}{a^2} \cot^3 \theta \end{aligned}$$

$$(1) \quad \therefore \left[ \frac{d^2 y}{dx^2} \right]_{\theta = \frac{\pi}{6}} = \frac{-3\sqrt{3}b}{a^2} \quad (1)$$

$$13. \text{ We have, } y = b \tan \theta \Rightarrow \frac{dy}{d\theta} = b \sec^2 \theta \quad \dots \text{(i)}$$

$$\text{and } x = a \sec \theta \Rightarrow \frac{dx}{d\theta} = a \sec \theta \tan \theta \quad \dots \text{(ii)}$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{b \sec^2 \theta}{a \sec \theta \tan \theta} = \frac{b}{a} \operatorname{cosec} \theta \quad (1\frac{1}{2})$$

(1) Differentiating both sides with respect to  $x$ , we get

$$\frac{d^2 y}{dx^2} = \frac{-b}{a} \operatorname{cosec} \theta \cot \theta \times \frac{d\theta}{dx}$$

$$= -\frac{b}{a} \operatorname{cosec} \theta \cot \theta \times \frac{1}{a \sec \theta \tan \theta} \quad [\text{using (ii)}] \quad (1)$$

$$= -\frac{b}{a^2} \cot^3 \theta$$

$$\therefore \left[ \frac{d^2 y}{dx^2} \right]_{\theta = \pi/6} = \frac{-b}{a^2} \left[ \cot \frac{\pi}{6} \right]^3 = \frac{-b}{a^2} (\sqrt{3})^3 = -\frac{3\sqrt{3}b}{a^2} \quad (1/2)$$