


Algebra of Vectors

1 Mark Questions

1. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units.

Foreign 2014

 To find a vector in the direction of given vector, first of all we find unit vector in the direction of given vector and then multiply it with given magnitude.

$$\text{Let } \vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$$

$$\begin{aligned} |\vec{a}| &= \sqrt{(2)^2 + (-3)^2 + (6)^2} \\ &= \sqrt{4 + 9 + 36} \\ &= \sqrt{49} = 7 \text{ units} \end{aligned} \quad (1/2)$$

The unit vector in the direction of the given vector \vec{a} is

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{7}(2\hat{i} - 3\hat{j} + 6\hat{k}) = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

Therefore, the vector of magnitude equal to 21 units and in the direction of \vec{a} is

$$\begin{aligned} 21\hat{a} &= 21\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) \\ &= 6\hat{i} - 9\hat{j} + 18\hat{k} \end{aligned} \quad (1/2)$$

2. Find a vector \vec{a} of magnitude $5\sqrt{2}$, making an angle of $\frac{\pi}{4}$ with X-axis, $\frac{\pi}{2}$ with Y-axis and an acute angle θ with Z-axis. All India 2014

Given, a vector \vec{a} makes an angle $\frac{\pi}{4}$ with X-axis and $\frac{\pi}{2}$ with Y-axis.

$$\text{So, } l = \cos \frac{\pi}{4} \text{ and } m = \cos \frac{\pi}{2} \Rightarrow l = \frac{1}{\sqrt{2}}, m = 0$$

$$\text{We know that, } l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + (0)^2 + n^2 = 1 \Rightarrow \frac{1}{2} + n^2 = 1$$

$$\Rightarrow n^2 = 1 - \frac{1}{2} \Rightarrow n = \pm \frac{1}{\sqrt{2}} \Rightarrow n = \frac{1}{\sqrt{2}}$$

$$\therefore \cos \theta = \frac{1}{\sqrt{2}}$$

[: θ is an acute angle with Z-axis]

$$\Rightarrow \theta = \frac{\pi}{4}$$

Thus, direction cosines of a line are

$$\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \quad (1/2)$$

\therefore Vector \vec{a}

$$= |\vec{a}| \left(\cos \frac{\pi}{4} \hat{i} + \cos \frac{\pi}{2} \hat{j} + \cos \frac{\pi}{4} \hat{k} \right)$$

$$= 5\sqrt{2} \left(\frac{1}{\sqrt{2}} \hat{i} + (0) \hat{j} + \frac{1}{\sqrt{2}} \hat{k} \right) = 5\hat{i} + 5\hat{k} \quad (1/2)$$

3. Write a unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and

$$\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$$

Delhi 2014C

Given, $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$

$$\begin{aligned}\text{Now, } \vec{a} + \vec{b} &= 2\hat{i} + 2\hat{j} - 5\hat{k} + 2\hat{i} + \hat{j} - 7\hat{k} \\ &= 4\hat{i} + 3\hat{j} - 12\hat{k}\end{aligned}$$

$$\begin{aligned}\text{and } |\vec{a} + \vec{b}| &= \sqrt{(4)^2 + (3)^2 + (-12)^2} \\ &= \sqrt{16 + 9 + 144} \\ &= \sqrt{25 + 144} \\ &= \sqrt{169} = 13 \text{ units} \quad (1/2)\end{aligned}$$

\therefore Required unit vector = Unit vector along
the direction of $\vec{a} + \vec{b}$

$$= \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{4}{13}\hat{i} + \frac{3}{13}\hat{j} - \frac{12}{13}\hat{k} \quad (1/2)$$

4. Find the value of p for which the vectors
 $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel.

All India 2014

We have, $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are two
parallel vectors, so their direction ratios will be
proportional.

$$\begin{aligned}\therefore \frac{3}{1} &= \frac{2}{-2p} = \frac{9}{3} \Rightarrow \frac{2}{-2p} = \frac{3}{1} \\ \Rightarrow -6p &= 2 \Rightarrow p = \frac{2}{-6} \Rightarrow p = -\frac{1}{3} \quad (1)\end{aligned}$$

5. Write the value of cosine of the angle which the
vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ makes with Y-axis.

Delhi 2014C

$$\text{Given, } \vec{a} = \hat{i} + \hat{j} + \hat{k}$$

Now, unit vector in the direction of \vec{a} is

$$\begin{aligned}\hat{a} &= \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{(1)^2 + (1)^2 + (1)^2}} \\ &= \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} + \frac{1}{\sqrt{3}} \hat{k}\end{aligned}$$

\therefore Cosine of angle which given vector makes with Y-axis is $\frac{1}{\sqrt{3}}$. (1)

6. Find the angle between X-axis and the vector $\hat{i} + \hat{j} + \hat{k}$. All India 2014C

$$\text{Let } \vec{a} = \hat{i} + \hat{j} + \hat{k}$$

Now, unit vector in the direction of \vec{a} is

$$\begin{aligned}\hat{a} &= \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{1^2 + 1^2 + 1^2}} \Rightarrow \hat{a} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} \\ \Rightarrow \hat{a} &= \frac{1}{\sqrt{3}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} + \frac{1}{\sqrt{3}} \hat{k}\end{aligned}$$

So, angle between X-axis and the vector

$$\hat{i} + \hat{j} + \hat{k} \text{ is } \cos \alpha = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \alpha = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$[\because \hat{a} = l\hat{i} + m\hat{j} + n\hat{k} \text{ and } \cos \alpha = l \Rightarrow \alpha = \cos^{-1} l] \quad (1)$$

7. Write a vector in the direction of the vector

$$\hat{i} - 2\hat{j} + 2\hat{k} \text{ that has magnitude 9 units.}$$

Delhi 2014C

$$\text{Let } \vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$$

Comparing with $X = x\hat{i} + y\hat{j} + z\hat{k}$, we get

$$x = 1, y = -2, z = 2$$

$$\text{Magnitude } |\vec{a}| = \sqrt{x^2 + y^2 + z^2}$$

$$= \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{1 + 4 + 4} = 3 \text{ units}$$

∴ Unit vector in the direction of given vector

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} \quad (1/2)$$

Hence, the vector in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$ which has magnitude 9 units is given by

$$9\hat{a} = 9 \left(\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} \right) = 3\hat{i} - 6\hat{j} + 6\hat{k} \quad (1/2)$$

- 8.** Write a unit vector in the direction of vector \vec{PQ} , where \vec{P} and \vec{Q} are the points $(1, 3, 0)$ and $(4, 5, 6)$ respectively. **Foreign 2014**

💡 Firstly, find the vector \vec{PQ} by using the formula $(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$,
 then required unit vector is given by $\frac{\vec{PQ}}{|\vec{PQ}|}$.

The given points are \vec{P} (1, 3, 0) and \vec{Q} (4, 5, 6).

Here, $x_1 = 1, y_1 = 3, z_1 = 0$

and $x_2 = 4, y_2 = 5, z_2 = 6$

So, vector \vec{PQ}

$$\begin{aligned} &= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \\ &= (4 - 1)\hat{i} + (5 - 3)\hat{j} + (6 - 0)\hat{k} \\ &= 3\hat{i} + 2\hat{j} + 6\hat{k} \end{aligned} \quad (1/2)$$

∴ Magnitude of given vector

$$\begin{aligned} |\vec{PQ}| &= \sqrt{3^2 + 2^2 + 6^2} \\ &= \sqrt{9 + 4 + 36} \\ &= \sqrt{49} = 7 \text{ units} \end{aligned}$$

Hence, the unit vector in the direction of \vec{PQ}

$$\frac{\vec{PQ}}{|\vec{PQ}|} = \frac{3\hat{i} + 2\hat{j} + 6\hat{k}}{7} = \frac{3}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k} \quad (1/2)$$

9. Write the value of the following:

$$\hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j}) \quad \text{Foreign 2014}$$

$$\begin{aligned}
& \hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j}) \\
&= \hat{i} \times \hat{j} + \hat{i} \times \hat{k} + \hat{j} \times \hat{k} + \hat{j} \times \hat{i} + \hat{k} \times \hat{i} + \hat{k} \times \hat{j} \\
&= \hat{k} - \hat{j} + \hat{i} - \hat{k} + \hat{j} - \hat{i} = 0 \\
& [\because \hat{i} \times \hat{j} = \hat{k}, \hat{i} \times \hat{k} = -\hat{j}, \hat{j} \times \hat{k} = \hat{i}, \hat{j} \times \hat{i} = -\hat{k}, \\
& \quad \hat{k} \times \hat{i} = \hat{j}, \hat{k} \times \hat{j} = -\hat{i}] \quad (1)
\end{aligned}$$

- 10.** Write a unit vector in the direction of the sum of vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$.

Delhi 2013

$$\begin{aligned}
\text{Let } \vec{c} = \vec{a} + \vec{b} &= (2\hat{i} - \hat{j} + 2\hat{k}) + (-\hat{i} + \hat{j} + 3\hat{k}) \\
&= \hat{i} + 0\hat{j} + 5\hat{k}
\end{aligned}$$

$$\text{Now, } |\vec{c}| = |\vec{a} + \vec{b}| = \sqrt{1^2 + 5^2} = \sqrt{26}$$

$$\begin{aligned}
\therefore c &= \frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} \left[\because \hat{a} = \frac{\vec{a}}{|\vec{a}|} \right] \quad (1/2) \\
&= \frac{1}{\sqrt{26}} \hat{i} + \frac{5}{\sqrt{26}} \hat{k} \quad (1/2)
\end{aligned}$$

which is the required unit vector.

- 11.** If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then write the value of $x + y + z$.

HOTS; Delhi 2013

Two vectors are equal, if coefficients of their components are equal. (1/2)

$$\begin{aligned}
\text{Given, } \vec{a} &= \vec{b} \\
\Rightarrow x\hat{i} + 2\hat{j} - z\hat{k} &= 3\hat{i} - y\hat{j} + \hat{k}
\end{aligned}$$

$$\therefore x = 3, y = -2, z = -1$$

$$\text{Now, } x + y + z = 3 - 2 - 1 = 0 \quad (1/2)$$

Given, L and M are two points with position vectors $2\vec{a} - \vec{b}$ and $\vec{a} + 2\vec{b}$, respectively. Also, point N divides the line segment LM in the ratio 2:1 externally.

∴ Position vector of a point N

$$= \frac{2 \times (\vec{a} + 2\vec{b}) - 1 \times (2\vec{a} - \vec{b})}{2 - 1}$$

[by section formula]

$$= \frac{2\vec{a} + 4\vec{b} - 2\vec{a} + \vec{b}}{1} = 5\vec{b} \quad (1)$$

- 14.** A and B are two points with position vectors $2\vec{a} - 3\vec{b}$ and $6\vec{b} - \vec{a}$, respectively. Write the position vector of a point P which divides the line segment AB internally in the ratio 1 : 2.

All India 2013

Given, A and B are two points with position vectors $2\vec{a} - 3\vec{b}$ and $6\vec{b} - \vec{a}$, respectively. Also, point P divides the line segment AB in the ratio 1 : 2 internally.

∴ Position vector of a point P

$$= \frac{1 \times (6\vec{b} - \vec{a}) + 2 \times (2\vec{a} - 3\vec{b})}{1 + 2}$$

[by section formula]

$$= \frac{6\vec{b} - \vec{a} + 4\vec{a} - 6\vec{b}}{3} = \frac{3\vec{a}}{3} = \vec{a} \quad (1)$$

- 15.** Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$,
 $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

Delhi 2012

Given vectors are $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$,


$$\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k} \quad \text{and} \quad \vec{c} = \hat{i} - 6\hat{j} + 7\hat{k}$$

Sum of the vectors \vec{a} , \vec{b} and \vec{c} is

$$\begin{aligned}\vec{a} + \vec{b} + \vec{c} &= (\hat{i} - 2\hat{j} + \hat{k}) + (-2\hat{i} + 4\hat{j} + 5\hat{k}) \\ &\quad + (\hat{i} - 6\hat{j} - 7\hat{k}) \\ &= -4\hat{j} - \hat{k} \quad (1)\end{aligned}$$

- 16.** Find the scalar components of \vec{AB} with initial point A (2, 1) and terminal point B (-5, 7).

HOTS ; All India 2012

 Scalar components of $\vec{AB} = \vec{OB} - \vec{OA}$
= Position vector of B - Position vector of A .

Given, initial point is A (2, 1) and terminal point is B (-5, 7) then scalar component of

\vec{AB} are $x_2 - x_1$ and $y_2 - y_1$.

$$\Rightarrow \quad -5 - 2 = -7 \quad \text{and} \quad 7 - 1 = 6 \quad (1)$$

- 17.** For what values of a , the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear?

HOTS ; Delhi 2011

💡 If \vec{a} and \vec{b} are collinear, then $\vec{a} = \pm \lambda \vec{b}$ or $|\vec{a}| = \lambda \cdot |\vec{b}|$

Let given vectors are $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = a\hat{i} - 6\hat{j} - 8\hat{k}$

Vectors \vec{a} and \vec{b} are said to be collinear, if

$$\vec{a} = k \cdot \vec{b}, \text{ where } k \text{ is a scalar}$$

$$\therefore 2\hat{i} - 3\hat{j} + 4\hat{k} = k(a\hat{i} + 6\hat{j} - 8\hat{k})$$

Above equation is satisfied, when $a = -4$.

$$\therefore a = -4 \quad (1)$$

18. Write the direction cosines of vector $-2\hat{i} + \hat{j} - 5\hat{k}$. Delhi 2011

💡 Direction cosines of the vector $a\hat{i} + b\hat{j} + c\hat{k}$ are $\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}}$

Let $\vec{a} = -2\hat{i} + \hat{j} - 5\hat{k}$

\therefore Direction cosines of \vec{a} are

$$\frac{-2}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}, \frac{1}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}$$

and $\frac{-5}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}$

$$\text{i.e. } \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}} \quad (1)$$

19. Write the position vector of mid-point of the vector joining points $P(2, 3, 4)$ and $Q(4, 1, -2)$.

Foreign 2011

💡 Mid-point of the position vectors

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \text{ and}$$

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} \text{ is } \frac{\vec{a} + \vec{b}}{2}$$

$$\text{or } \frac{(a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}}{2}$$

Given, points are $P(2, 3, 4)$ and $Q(4, 1, -2)$
whose position vectors $\vec{OP} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and
 $\vec{OQ} = 4\hat{i} + \hat{j} - 2\hat{k}$.

Now, position vector of mid-point of vector joining points $P(2, 3, 4)$ and $Q(4, 1, -2)$ is

$$\vec{OR} = \frac{\vec{OP} + \vec{OQ}}{2} = \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{2}$$

$$\therefore \vec{OR} = \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k} \quad (1)$$

20. Write a unit vector in the direction of vector

$$\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}. \quad \text{Delhi 2009; All India 2011}$$

We know that, unit vector in the direction of \vec{a}
is $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$

\therefore Required unit vector in the direction of

$$\text{vector } \vec{a} = 2\hat{i} + \hat{j} + 2\hat{k} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{(2)^2 + (1)^2 + (2)^2}}$$

$$= \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{9}}$$

$$= \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k} \quad (1)$$

21. Find the magnitude of the vector

$$\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}. \quad \text{All India 2011C; Delhi 2008}$$

💡 Magnitude of a vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

Given, vector is $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

$$\begin{aligned}\text{Magnitude of } \vec{a} &= |\vec{a}| = \sqrt{(3)^2 + (-2)^2 + (6)^2} \\ &= \sqrt{9 + 4 + 36} = \sqrt{49} = 7 \text{ units}\end{aligned}\quad (1)$$

22. Find a unit vector in the direction of vector

$$\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}.$$

All India 2011C

Given vector is $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

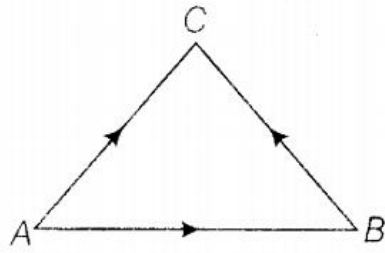
$$\begin{aligned}\text{Unit vector in the direction of } \vec{a} &= \frac{\vec{a}}{|\vec{a}|} \\ &= \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{\sqrt{(2)^2 + (3)^2 + (6)^2}} = \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{\sqrt{49}} \\ &= \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\end{aligned}\quad (1)$$

23. If A, B and C are the vertices of a ΔABC , then

what is the value of $\vec{AB} + \vec{BC} + \vec{CA}$?

Delhi 2011C

Let ΔABC be the given triangle.



Now, by triangle law of vector addition, we

have $\vec{AB} + \vec{BC} = \vec{AC}$

$$\Rightarrow \vec{AB} + \vec{BC} + \vec{CA} = \vec{CA} + \vec{AC}$$

[adding \vec{CA} on both sides]

$$\Rightarrow \vec{AB} + \vec{BC} + \vec{CA} = \vec{CA} - \vec{CA} \quad [\because \vec{AC} = -\vec{CA}]$$

$$\Rightarrow \vec{AB} + \vec{BC} + \vec{CA} = \vec{0} \quad (1)$$

24. Find a unit vector in the direction of

$$\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}.$$

Delhi 2011C

Given vector is $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$$\begin{aligned} \text{Unit vector in the direction of } \vec{a} &= \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{(2)^2 + (-3)^2 + (6)^2}} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{49}} \\ &= \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k} \end{aligned} \quad (1)$$

25. Find a vector in the direction of $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, which has magnitude 6 units. Delhi 2010C

Do same as Que. 7. [Ans. $4\hat{i} - 2\hat{j} + 4\hat{k}$]

26. Find a position vector of mid-point of the line segment AB , where A is point $(3, 4, -2)$ and B is point $(1, 2, 4)$. Delhi 2010

Do same as Que. 19. [Ans. $2\hat{i} - 3\hat{j} + \hat{k}$]

27. Write a vector of magnitude 9 units in the direction of vector $-2\hat{i} + \hat{j} + 2\hat{k}$. **All India 2010**

Do same as Que. 7. [**Ans.** $-6\hat{i} - 3\hat{j} + 6\hat{k}$]

28. Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$. **Delhi 2010**

Do same as Que. 7. [**Ans.** $5\hat{i} - 10\hat{j} + 10\hat{k}$]

29. What is the cosine of angle which the vector $\sqrt{2}\hat{i} + \hat{j} + \hat{k}$ makes with Y-axis? **HOTS; Delhi 2010**

Let $\vec{a} = \sqrt{2}\hat{i} + \hat{j} + \hat{k}$

Now, unit vector in the direction of \vec{a} is

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\sqrt{2}\hat{i} + \hat{j} + \hat{k}}{\sqrt{(\sqrt{2})^2 + (1)^2 + (1)^2}} = \frac{\sqrt{2}\hat{i} + \hat{j} + \hat{k}}{2}$$

$$= \frac{\sqrt{2}}{2}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$$

\therefore Cosine of angle which given vector makes with Y-axis is $1/2$. **(1)**

Alternate Method

? The direction cosines along Y-axis are $(0, 1, 0)$, then we use the following cosine formula

$$\theta = \cos^{-1} \{l_1l_2 + m_1m_2 + n_1n_2\}$$

$$\Rightarrow \cos\theta = l_1l_2 + m_1m_2 + n_1n_2$$

Let $\vec{a} = \sqrt{2}\hat{i} + \hat{j} + \hat{k}$

Its direction cosines form

$$\vec{a} = \frac{\sqrt{2}}{2}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \frac{1}{2}\hat{k}$$

$$\left[\begin{aligned} \therefore \text{Direction cosine of vector } a_1\hat{i} + a_2\hat{j} + a_3\hat{k} \\ = \frac{a_1}{\sqrt{a_1^2 + a_2^2 + a_3^2}}, \frac{a_2}{\sqrt{a_1^2 + a_2^2 + a_3^2}}, \end{aligned} \right.$$

$$\left. \begin{aligned} & \sqrt{a_1^2 + a_2^2 + a_3^2} \quad \sqrt{a_1^2 + a_2^2 + a_3^2} \\ & \text{and } \frac{a_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}} \end{aligned} \right\}$$

and let the position along with Y-axis is

$$\vec{b} = 0\hat{i} + 1\hat{j} + 0\hat{k}$$

If θ is the angle between them.

$$\therefore \cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$\Rightarrow \theta = \cos^{-1} \left\{ \frac{1}{\sqrt{2}} \cdot 0 + \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0 \right\}$$

$$\therefore \cos\theta = \frac{1}{2}$$

So, cosine of angle which the given vector makes with Y-axis is $\frac{1}{2}$. (1)

30. Find a unit vector in the direction of vector

$$\vec{b} = 6\hat{i} - 2\hat{j} + 3\hat{k}$$

All India 2009C

Do same as Que. 24. [Ans. $\frac{6}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{3}{7}\hat{k}$]

31. Find a unit vector in the direction of vector

$$\vec{a} = -2\hat{i} + \hat{j} + 2\hat{k}$$

Delhi 2009C

Do same as Que. 24. [Ans. $-\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$]

32. Write a unit vector in the direction of

$$\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}$$

Delhi 2009

Do same as Que. 24. [Ans. $\frac{2}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{3}{7}\hat{k}$]

33. Find the magnitude of the vector

$$\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$$

All India 2008C

Given, vector is $\vec{a} = 2\hat{j} - 6\hat{j} - 3\hat{k}$

$$\begin{aligned}\text{Magnitude of } \vec{a} &= |\vec{a}| = \sqrt{(2)^2 + (-6)^2 + (-3)^2} \\ &= \sqrt{4 + 36 + 9} = \sqrt{49} = 7 \text{ units} \quad (1)\end{aligned}$$

34. Find a unit vector in the direction of

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}.$$

Delhi 2008C

Do same as Que. 24.

$$\left[\text{Ans. } \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k} \right]$$

35. If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 5\hat{k}$, then find a unit vector in the direction $\vec{a} - \vec{b}$.

All India 2008

4 Marks Questions

💡 Firstly, find $\vec{a} - \vec{b}$ by subtracting the components \hat{i} , \hat{j} and \hat{k} of \vec{a} and \vec{b} simultaneously and then use the formula $= \frac{\vec{a} - \vec{b}}{|\vec{a} - \vec{b}|}$ to get required unit vector.

Given, $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} + \hat{j} - 5\hat{k}$

$$\begin{aligned}\therefore \vec{a} - \vec{b} &= (\hat{i} + 2\hat{j} - \hat{k}) - (3\hat{i} + \hat{j} - 5\hat{k}) \\ &= -2\hat{i} + \hat{j} + 4\hat{k}\end{aligned}$$

Let $\vec{a} - \vec{b} = \vec{c} = -2\hat{i} + \hat{j} + 4\hat{k}$

Now, unit vector in the direction of \vec{c} is

$$\begin{aligned}\hat{c} &= \frac{\vec{c}}{|\vec{c}|} = \frac{-2\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{(-2)^2 + (1)^2 + (4)^2}} = \frac{-2\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{21}} \\ &= \frac{-2}{\sqrt{21}}\hat{i} + \frac{1}{\sqrt{21}}\hat{j} + \frac{4}{\sqrt{21}}\hat{k} \quad (1)\end{aligned}$$

which is the required unit vector.

- 36.** Find a vector of magnitude 5 units and parallel to the resultant of $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$. HOTS ; Delhi 2011



Firstly, find resultant of the vectors \vec{a} and \vec{b} , which is $\vec{a} + \vec{b}$. Then, find a unit vector in the direction of $\vec{a} + \vec{b}$ i.e. the unit vector multiplying by 5.

$$\text{Given } \vec{a} = 2\hat{i} + 3\hat{j} - \hat{k} \text{ and } \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\begin{aligned} \text{Now, resultant of above vectors} &= \vec{a} + \vec{b} \\ &= (2\hat{i} + 3\hat{j} - \hat{k}) + (\hat{i} - 2\hat{j} + \hat{k}) = 3\hat{i} + \hat{j} \end{aligned} \quad (1)$$

$$\text{Let } \vec{a} + \vec{b} = \vec{c}$$

$$\therefore \vec{c} = 3\hat{i} + \hat{j}$$

Now, unit vector \hat{c} in the direction of \vec{c} , i.e.

$$= \frac{\vec{c}}{|\vec{c}|} \quad (1)$$

$$= \frac{3\hat{i} + \hat{j}}{\sqrt{(3)^2 + (1)^2}} = \frac{3\hat{i} + \hat{j}}{\sqrt{10}} = \frac{3}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{10}}\hat{j} \quad (1)$$

Hence, vector of magnitude 5 units and parallel to resultant of \vec{a} and \vec{b} is


$$5 \left(\frac{3}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{10}}\hat{j} \right) \text{ or } \frac{15}{\sqrt{10}}\hat{i} + \frac{5}{\sqrt{10}}\hat{j} \quad (1)$$

37. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and

$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$. Find a vector of magnitude 6 units, which is parallel to the vector

$$2\vec{a} - \vec{b} + 3\vec{c}.$$

All India 2010

 Firstly, find the vector $2\vec{a} - \vec{b} + 3\vec{c}$, then find the vector in the direction of $2\vec{a} - \vec{b} + 3\vec{c}$, i.e. the unit vector multiplying by 6.

$$\text{Given, } \vec{a} = \hat{i} + \hat{j} + \hat{k}; \vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$$

$$\text{and } \vec{c} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\begin{aligned} \therefore 2\vec{a} - \vec{b} + 3\vec{c} &= 2(\hat{i} + \hat{j} + \hat{k}) - (4\hat{i} - 2\hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k}) \\ &= 2\hat{i} + 2\hat{j} + 2\hat{k} - 4\hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k} \end{aligned}$$

$$\Rightarrow 2\vec{a} - \vec{b} + 3\vec{c} = \hat{i} - 2\hat{j} + 2\hat{k} \quad (1\frac{1}{2})$$

Now, a unit vector in the direction of vector

$$\begin{aligned} 2\vec{a} - \vec{b} + 3\vec{c} &= \frac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|} \\ &= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (2)^2}} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{9}} \\ &= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} = \frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k} \quad (1\frac{1}{2}) \end{aligned}$$

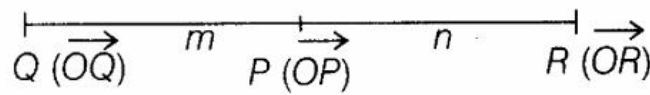
Hence, vector of magnitude 6 units parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c} = 6 \left(\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k} \right)$

$$= 2\hat{i} - 4\hat{j} + 4\hat{k} \quad (1)$$

- 38.** Find the position vector of a point R , which divides the line joining two points P and Q whose position vectors are $2\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively, externally in the ratio $1:2$. Also, show that P is the mid-point of line segment RQ .

? Here, we use the section formula for external division

$$\vec{OR} = \frac{m(\vec{OQ}) - n(\vec{OP})}{m - n}$$



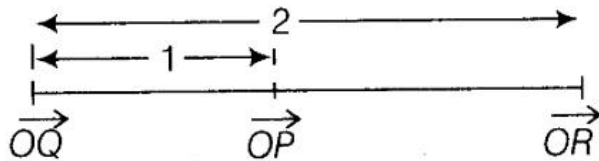
and then use the mid-point formula

$$\vec{OP} = \frac{\vec{OQ} + \vec{OR}}{2}$$

Given, \vec{OP} = Position vector of $P = 2\vec{a} + \vec{b}$

and \vec{OQ} = Position vector of $Q = \vec{a} - 3\vec{b}$

Let \vec{OR} be the position vector of point R , which divides PQ in the ratio $1 : 2$ externally



$$\therefore \vec{OR} = \frac{1(\vec{a} - 3\vec{b}) - 2(2\vec{a} + \vec{b})}{1 - 2}$$

$$\left[\therefore \vec{OR} = \frac{m(\vec{OQ}) - n(\vec{OP})}{m - n} \cdot \text{Here, } m = 1, n = 2 \right]$$

(1)

$$= \frac{\vec{a} - 3\vec{b} - 4\vec{a} - 2\vec{b}}{-1} = \frac{-3\vec{a} - 5\vec{b}}{-1} = 3\vec{a} + 5\vec{b}$$

$$\text{Hence, } \overrightarrow{OR} = 3\vec{a} + 5\vec{b} \quad (1\frac{1}{2})$$

Now, we have to show that P is the mid-point of RQ .

$$\text{i.e. } \overrightarrow{OP} = \frac{\overrightarrow{OR} + \overrightarrow{OQ}}{2}$$

$$\text{We have, } \overrightarrow{OR} = 3\vec{a} + 5\vec{b}, \overrightarrow{OQ} = \vec{a} - 3\vec{b}$$

$$\begin{aligned} \therefore \frac{\overrightarrow{OR} + \overrightarrow{OQ}}{2} &= \frac{(3\vec{a} + 5\vec{b}) + (\vec{a} - 3\vec{b})}{2} \\ &= \frac{4\vec{a} + 2\vec{b}}{2} \\ &= \frac{2(2\vec{a} + \vec{b})}{2} = 2\vec{a} + \vec{b} \\ &= \overrightarrow{OP} \quad [\because \overrightarrow{OP} = 2\vec{a} + \vec{b}, \text{ given}] \quad (1\frac{1}{2}) \end{aligned}$$

Hence, P is the mid-point of line segment RQ .