Three Dimensional Geometry

Previous Years' CBSE Board Questions

11.1 Introduction

MCQ

1. Distance of the point (p, q, r) from y-axis is
   (a) q (b) $|q|$
   (c) $|q| + |r|$ (d) $\sqrt{p^2 + r^2}$ (2023) U

2. The length of the perpendicular drawn from the point $(4, -7, 3)$ on the y-axis is
   (a) 3 units (b) 4 units
   (c) 5 units (d) 7 units (2020) U

3. The vector equation of XY-plane is
   (a) $\vec{r} \cdot \hat{k} = 0$ (b) $\vec{r} \cdot \hat{j} = 0$
   (c) $\vec{r} \cdot \hat{i} = 0$ (d) $\vec{r} \cdot \vec{n} = 1$ (2020) Ap

VSA (1 mark)

4. Write the distance of a point $P(a, b, c)$ from x-axis. (2020C, Delhi 2014C) Ev

11.2 Direction Cosines and Direction Ratios of a Line

MCQ

5. If the direction cosines of a line are $\left(\frac{1}{a}, \frac{1}{a}, \frac{1}{a}\right)$, then
   (a) $0 < a < 1$ (b) $a > 2$
   (c) $a > 0$ (d) $a = \pm\sqrt{3}$ (2023)

6. If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are
   (a) $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$
   (c) $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$ (d) $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (2023)

VSA (1 mark)

7. Find the direction cosines of a line which makes equal angles with the coordinate axes. (2019) Ev

8. If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines? (2019) Ev

9. If a line makes angles 90°, 135°, 45° with the x, y and z axes respectively, find its direction cosines. (NCERT, Delhi 2019) An

10. If a line makes angles 90° and 60° respectively with the positive directions of x and y axes, find the angle which it makes with the positive direction of z-axis. (Delhi 2017) Ev

OR

If a line makes angles 90°, 60° and θ with x, y and z-axis respectively, where θ is acute, then find θ. (Delhi 2015) Ev

11. If a line makes angles α, β, γ with the positive direction of coordinate axes, then write the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$. (Delhi 2015C) U

SA I (2 marks)

12. If a line makes an angle α, β, γ with the coordinate axes, then find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$. (Term II, 2021-22) U

13. Find all the possible vectors of magnitude $5\sqrt{3}$ which are equally inclined to the coordinate axes. (Term II, 2021-22) U

11.3 Equation of a Line in Space

VSA (1 mark)

14. The vector equation of a line which passes through the points $(3, 4, -7)$ and $(1, -1, 6)$ is ________. (2020) An

15. A line passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of the vector $\hat{i} + \hat{j} - 2\hat{k}$. Find the equation of the line in cartesian form. (2019) Ev

16. The equation of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line. (AI 2015) Ev

17. If the cartesian equation of a line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, write the vector equation for the line. (AI 2014) Ev

SA I (2 marks)

18. The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line and find the coordinates of a point through which it passes. (2023)

19. Write the cartesian equation of the line PQ passing through points $P(2, 2, 1)$ and $Q(5, 1, -2)$. Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is -2. (Term II, 2021-22) Ev

20. The Cartesian equation of a line AB is:
$$\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3}$$
Find the direction cosines of a line parallel to line AB. (Term II, 2021-22) Ev

21. The x-coordinate of a point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its z-coordinate. (AI 2017) Ap

SA II (3 marks)

22. Find the coordinates of the point where the line through the points $(1, 1, 8)$ and $(5, 2, 10)$ crosses the ZX-plane. (Term II, 2021-22C) Ap

23. If a line makes 60° and 45° angles with the positive directions of x-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line. *(Term II, 2021-22)* Ev

LA I (4 marks)

24. Prove that the line through $A(0, -1, -1)$ and $B(4, 5, 1)$ intersects the line through $C(3, 9, 4)$ and $D(-4, 4, 4)$. *(Foreign 2016)*

25. Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Also find their point of intersection. *(Delhi 2014)* Ev

26. Show that lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(3\hat{i}-\hat{j})$ and $\vec{r}=(4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k})$ intersect. Also, find their point of intersection. *(Delhi 2014)* Ev

LA II (5/6 marks)

27. A line with direction ratios < 2, 2, 1 > intersects the lines $\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}$ and $\frac{x-1}{2}=\frac{y+1}{4}=\frac{z+1}{3}$ at the points P and Q respectively. Find the length and the equation of the intercept PQ. *(2019C)*

11.4 Angle between Two Lines

MCQ

Q. no. 28 is Assertion and Reason based question carrying 1 mark. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

28. **Assertion (A) :** The lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ are perpendicular, when $\vec{b}_1\cdot\vec{b}_2=0$.
Reason (R) : The angle θ between the lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ is given by $\cos\theta=\frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$.
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true. *(2023)*

29. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is
(a) 0° (b) 30° (c) 45° (d) 90° *(2023)*

30. If the two lines $L_1: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$ $L_2: x=2, \frac{y}{-1}=\frac{z}{2-\alpha}$ are perpendicular, then the value of α is
(a) $\frac{2}{3}$ (b) 3 (c) 4 (d) $\frac{7}{3}$ *(2020C)* Ap

VSA (1 mark)

31. Find the vector equation of the line which passes through the point (3, 4, 5) and is parallel to the vector $2\hat{i}+2\hat{j}-3\hat{k}$. *(Delhi 2019)* Ev

32. Find the cartesian equation of the line which passes through the point (–2, 4, –5) and is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$. *(Delhi 2019)* Ev

33. Find the angle between the lines
$\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ and
$\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$. *(Foreign 2014)* Ap

34. Write the equation of the straight line through the point (α, β, γ) and parallel to z-axis. *(AI 2014C)* Ap

SA I (2 marks)

35. Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$; $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$. *(2023)*

36. Find the vector and the cartesian equations of a line that passes through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$. *(2023)*

37. Find the value of k so that the lines $x = -y = kz$ and $x - 2 = 2y + 1 = -z + 1$ are perpendicular to each other. *(2020)* Ev

38. Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$. *(Delhi 2017)* Ap

SA II (3 marks)

39. Find the coordinates of the foot of the perpendicular drawn from point (5, 7, 3) to the line $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$. *(2023)*

40. Find the coordinates of the foot of the perpendicular drawn from the point $P(0, 2, 3)$ to the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. *(2023)*

LA I (4 marks)

41. Find the value of λ, so that the lines $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{2}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not. *(Delhi 2019)* Ev

42. Find the vector and cartesian equations of the line through the point (1, 2, –4) and perpendicular to the two lines
$\vec{r}=(8\hat{i}-19\hat{j}+10\hat{k})+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$ and
$\vec{r}=(15\hat{i}+29\hat{j}+5\hat{k})+\mu(3\hat{i}+8\hat{j}-5\hat{k})$.
(Delhi 2016, AI 2015) Cr

43. Find the vector and cartesian equations of the line passing through the point (2, 1, 3) and perpendicular

to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$. *(AI 2014)* Ev

44. Find the value of p, so that the lines $l_1: \frac{1-x}{3}=\frac{7y-14}{p}=\frac{z-3}{2}$ and $l_2: \frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other. Also find the equation of a line passing through a point (3, 2, –4) and parallel to line l_1. *(AI 2014)* Cr

45. A line passes through (2, –1, 3) and is perpendicular to the lines
$\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-3\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$.
Obtain its equation in vector and cartesian form. *(AI 2014)* Ev

46. Find the direction cosines of the line $\frac{x+2}{2}=\frac{2y-7}{6}=\frac{5-z}{6}$. Also, find the vector equation of the line through the point $A(-1, 2, 3)$ and parallel to the given line. *(Delhi 2014C)* Ap

LA II (5/6 marks)

47. Show that the following lines do not intersect each other :
$\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{5}$; $\frac{x+2}{4}=\frac{y-1}{3}=\frac{z+1}{-2}$ *(2023)*

48. Find the angle between the lines
$2x = 3y = -z$ and $6x = -y = -4z$. *(2023)*

49. Find the vector and cartesian equations of a line passing through (1, 2, –4) and perpendicular to the two lines $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$. *(Delhi 2017)* Cr

11.5 Shortest Distance between Two Lines

VSA (1 mark)

50. The line of shortest distance between two skew lines is _______ to both the lines. *(2020)* R

SA II (3 marks)

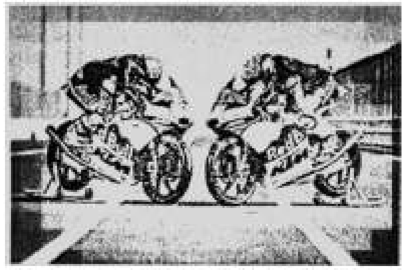
51. Find the distance between the lines $x=\frac{y-1}{2}=\frac{z-2}{3}$ and $x+1=\frac{y+2}{2}=\frac{z-1}{3}$. *(Term II, 2021-22)* Ev

52. Find the distance between the following parallel lines :
$\vec{r}=(2\hat{i}+\hat{j}-\hat{k})+\lambda(\hat{i}+\hat{j}-\hat{k})$
$\vec{r}=(\hat{i}-2\hat{j}+\hat{k})+\mu(\hat{i}+\hat{j}-\hat{k})$ *(Term II, 2021-22)* Ev

53. Check whether the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ are skew or not? *(Term II, 2021-22)* Ev

LA I (4 marks)

54. Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r}=\lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=(3\hat{i}+3\hat{j})+\mu(2\hat{i}+\hat{j}+\hat{k})$ respectively.



Based on the above information, answer the following questions.
(i) Find the shortest distance between the given lines.
(ii) Find the point at which the motorcycles may collide. *(Term II, 2021-22)* Ev

55. Find the shortest distance between the lines
$\vec{r}=(4\hat{i}-\hat{j})+\lambda(\hat{i}+2\hat{j}-3\hat{k})$ and
$\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(2\hat{i}+4\hat{j}-5\hat{k})$. *(2018)* Ev

56. Find the shortest distance between the lines
$\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and
$\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$ *(Delhi 2015C)* Ap

57. Find the shortest distance between the following lines:
$\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ and
$\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$ *(AI 2015C)* Ev

58. Find the shortest distance between the following lines:
$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$, $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ *(Foreign 2014)* Cr

59. Find the shortest distance between the lines whose vector equations are
$\vec{r}=\hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k})$
and $\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$. *(Foreign 2014)* Ev

60. Find the distance between the lines l_1 and l_2 given by
$l_1: \vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$
$l_2: \vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(4\hat{i}+6\hat{j}+12\hat{k})$. *(Foreign 2014)* Ap

61. Find the shortest distance between the two lines whose vector equations are
$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ and
$\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$. *(Delhi 2014C)* Ev

LA II (5 / 6 marks)

62. Find the vector equation of a line passing through the point (2, 3, 2) and parallel to the line $\vec{r}=(-2\hat{i}+3\hat{j})+\lambda(2\hat{i}-3\hat{j}+6\hat{k})$. Also, find the distance between these two lines. *(AI 2019)* Cr

CBSE Sample Questions

11.3 Equation of a Line in Space

MCQ

1. P is a point on the line joining the points $A(0, 5, -2)$ and $B(3, -1, 2)$. If the x-coordinate of P is 6, then its z-coordinate is
(a) 10 (b) 6 (c) −6 (d) −10
(2022-23) Ap

SA I (2 marks)

2. Find the direction ratio and direction cosines of a line parallel to the line whose equations are
$6x - 12 = 3y + 9 = 2z - 2$. *(2022-23)* Ev

3. Find the direction cosines of the following line:
$\frac{3-x}{-1}=\frac{2y-1}{2}=\frac{z}{4}$ *(Term II, 2021-22)* Ap

11.4 Angle between Two Lines

MCQ

4. **Assertion (A) :** The acute angle between the line $\vec{r}=\hat{i}+\hat{j}+2\hat{k}+\lambda(\hat{i}-\hat{j})$ and the x-axis is $\frac{\pi}{4}$.
Reason (R) : The acute angle θ between the lines $\vec{r}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}+\lambda(a_1\hat{i}+b_1\hat{j}+c_1\hat{k})$ and $\vec{r}=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}+\mu(a_2\hat{i}+b_2\hat{j}+c_2\hat{k})$ is given by
$$\cos\theta=\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}.$$
(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true. *(2022-23)* Ap

11.5 Shortest Distance between Two Lines

SA II (3 marks)

5. Find the shortest distance between the following lines:
$\vec{r}=(\hat{i}+\hat{j}-\hat{k})+s(2\hat{i}+\hat{j}+\hat{k}),\ \vec{r}=(\hat{i}+\hat{j}+2\hat{k})+t(4\hat{i}+2\hat{j}+2\hat{k})$
(Term II, 2021-22) Ev

LA II (5/6 marks)

6. An insect is crawling along the line $\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$ and another insect is crawling along the line $\vec{r}=-4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$. At what points on the lines should they reach so that the distance between them is shortest? Find the shortest possible distance between them.
(2022-23)

7. The equation of motion of a rocket are : $x = 2t$, $y = -4t$, $z = 4t$, where the time t is given in seconds, and the coordinates of a moving point in km. What is the path of the rocket? At what distances will the rocket be from the starting point $O(0, 0, 0)$ and from the following line in 10 seconds?
$\vec{r}=20\hat{i}-10\hat{j}+40\hat{k}+\mu(10\hat{i}-20\hat{j}+10\hat{k})$ *(2022-23)* U

8. Find the shortest distance between the lines
$\vec{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k})$
and $\vec{r}=5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$.
If the lines intersect, then find their point of intersection. *(2020-21)* Ap

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

1. (d): Given point is (p, q, r)
The foot of perpendicular drawn from point (p, q, r) on the y-axis is $(0, q, 0)$.
Now, distance between these two points is
$\sqrt{(p-0)^2+(q-q)^2+(r-0)^2}=\sqrt{p^2+r^2}$

2. (c) : Let $P(4, -7, 3)$ be the given point and A be a point on y-axis s.t. $PA \perp$ to y-axis.
$\therefore\ A \equiv (0, -7, 0)$
Now, $PA = \sqrt{(4-0)^2+(-7-(-7))^2+(3-0)^2}$
$= \sqrt{4^2+3^2}=\sqrt{16+9}=\sqrt{25}=5$ units

Answer Tips

- Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

3. (a) : Vector equation of XY-plane is $\vec{r}\cdot\hat{k}=0$.

4. We have equation of x-axis is $y = 0$, $z = 0$
$\therefore$ Distance of $P(a, b, c)$ from x-axis
$=\sqrt{(a-a)^2+b^2+c^2}=\sqrt{b^2+c^2}$ units.

5. (d): Given that the direction cosines of a line are $\left(\frac{1}{a},\frac{1}{a},\frac{1}{a}\right)$.

We know that the sum of squares of the direction cosines is 1.

$\Rightarrow \frac{1}{a^2}+\frac{1}{a^2}+\frac{1}{a^2}=1 \Rightarrow \frac{3}{a^2}=1 \Rightarrow a^2=3$

$\Rightarrow a=\pm\sqrt{3}$

6. (a): Direction cosines are $\langle \cos 90°, \cos 135°, \cos 45° \rangle$

$=\left\langle 0, \cos(90°+45°), \frac{1}{\sqrt{2}} \right\rangle = \left\langle 0, -\sin 45°, \frac{1}{\sqrt{2}} \right\rangle$

$=\left\langle 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

7. If a line makes α, β, γ with positive direction of x, y, z axis respectively, then direction cosines of line will be $\cos\alpha, \cos\beta, \cos\gamma$ or $-\cos\alpha, -\cos\beta, -\cos\gamma$.
and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
Since, $\alpha = \beta = \gamma$

$\therefore \quad \cos\alpha = \pm\frac{1}{\sqrt{3}}$

Therefore, direction cosines are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

8. Since, D.R'.s are $-18, 12, -4$

$\therefore$ D.C'.s are $\frac{-18}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{12}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{-4}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$

$\Rightarrow \frac{-18}{22}, \frac{12}{22}, \frac{-4}{22} \Rightarrow \frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$

9. Since the line makes angles 90°, 135° and 45° with x, y and z axes respectively.

$\therefore \quad l = \cos 90° = 0,\ m = \cos 135° = -\frac{1}{\sqrt{2}}$ and $n = \cos 45° = \frac{1}{\sqrt{2}}$

Hence, direction cosines of the line are $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$.

Concept Applied

- If a line makes angles α, β and γ with x, y and z-axes respectively, then $l = \cos\alpha$, $m = \cos\beta$ and $n = \cos\gamma$ are direction cosines of line.

10. Let the line makes an angle α, β, γ with the positive direction of x, y, z axes respectively.

$\therefore \quad \alpha = 90°, \beta = 60°$ and $\gamma = \theta$ (say)

Since, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \cos^2 90° + \cos^2 60° + \cos^2\theta = 1$

$\Rightarrow \frac{1}{4}+\cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{3}{4} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$

11. Here, the direction cosines of the given line are $\cos\alpha, \cos\beta, \cos\gamma$ and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow (1-\sin^2\alpha) + (1-\sin^2\beta) + (1-\sin^2\gamma) = 1$

$[\because \sin^2\alpha + \cos^2\alpha = 1]$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2.$

12. Here, the direction cosines of the given line are $\cos\alpha$, $\cos\beta$, $\cos\gamma$ and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

By using $\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$

$\cos^2\beta = \frac{1+\cos 2\beta}{2}$ and so on.

$\Rightarrow \frac{1}{2}[\cos 2\alpha + 1 + \cos 2\beta + 1 + \cos 2\gamma + 1] = 1$

$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

13. $\pm 5\sqrt{3}\left(\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right)$ are two possible vectors of magnitude $5\sqrt{3}$, which are equally inclined to the coordinate axes.

14. Vector equation of a line passes through the points $(3, 4, -7)$ and $(1, -1, 6)$ is given by

$\vec{r} = (3\hat{i}+4\hat{j}-7\hat{k}) + \lambda[(\hat{i}-\hat{j}+6\hat{k})-(3\hat{i}+4\hat{j}-7\hat{k})]$

$\therefore \quad \vec{r} = 3\hat{i}+4\hat{j}-7\hat{k}+\lambda(-2\hat{i}-5\hat{j}+13\hat{k})$

15. The equation of line in vector from $\vec{r} = \vec{a}+\lambda\vec{b}$.

Here, $\vec{a} = 2\hat{i}-\hat{j}+4\hat{k}$ and $\vec{b} = \hat{i}+\hat{j}-2\hat{k}$

$\therefore \quad (a_1, a_2, a_3) \equiv (2, -1, 4)$

D.R'.s. b_1, b_2, b_3 are $1, 1, -2$

The equation of line in cartesian form is given by

$\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3} \Rightarrow \frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}$

16. The given line is $5x - 3 = 15y + 7 = 3 - 10z$

$\Rightarrow \frac{x-\frac{3}{5}}{\frac{1}{5}} = \frac{y+\frac{7}{15}}{\frac{1}{15}} = \frac{z-\frac{3}{10}}{-\frac{1}{10}}$

Its direction ratios are $\frac{1}{5}, \frac{1}{15}, -\frac{1}{10}$

i.e., Its direction ratios are proportional to 6, 2, −3.

Now, $\sqrt{6^2+2^2+(-3)^2} = 7$

$\therefore \quad$ Its direction cosines are $\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}$.

17. The cartesian equation of a line is

$\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$ ...(i)

$\Rightarrow \frac{x-3}{-5} = \frac{y-(-4)}{7} = \frac{z-3}{2} = \lambda$ (say)

$\Rightarrow x = 3 - 5\lambda, y = -4 + 7\lambda, z = 3 + 2\lambda$

Take $\vec{a} = 3\hat{i}-4\hat{j}+3\hat{k}$ and $\vec{b} = -5\hat{i}+7\hat{j}+2\hat{k}$.

$\therefore$ The vector equation of the line (i) is $\vec{r} = \vec{a}+\lambda\vec{b}$

$\Rightarrow \vec{r} = (3\hat{i}-4\hat{j}+3\hat{k})+\lambda(-5\hat{i}+7\hat{j}+2\hat{k})$

Answer Tips

- If cartesian equation of a line is $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$, then its vector equation is $\vec{r} = (x_1\hat{i}+y_1\hat{j}+z_1\hat{k})+\lambda(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})$

18. The given line is $5x - 3 = 15y + 7 = 3 - 10z$

$$\Rightarrow \frac{x-\frac{3}{5}}{\frac{1}{5}} = \frac{y+\frac{7}{15}}{\frac{1}{15}} = \frac{z-\frac{3}{10}}{-\frac{1}{10}}$$

Its direction ratios are $\frac{1}{5}, \frac{1}{15}, -\frac{1}{10}$

i.e., Its direction ratios are proportional to 6, 2, −3.

Now, $\sqrt{6^2+2^2+(-3)^2} = 7$

$\therefore$ Its direction cosines are $\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}$.

19. We have $P(2, 2, 1)$ and $Q(5, 1, -2)$, then the equation of line PQ is

$$\frac{x-2}{5-2} = \frac{y-2}{1-2} = \frac{z-1}{-2-1} \text{ or } \frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3} \quad \text{...(i)}$$

Given, $z = -2$, then from (i), we have $\frac{y-2}{-1} = \frac{-2-1}{-3}$

$\Rightarrow$ $y = 1$

20. The cartesian equation of line AB is

$$\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3}$$

can be rearranged as $\frac{x-\frac{1}{2}}{6} = \frac{y+2}{2} = \frac{z-3}{3}$

So, $a = 6, b = 2, c = 3$

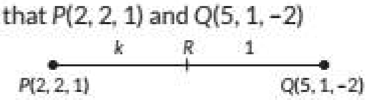
$\Rightarrow$ $\sqrt{a^2+b^2+c^2} = \sqrt{6^2+2^2+3^2} = 7$

$\therefore$ Required direction cosines are $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$.

Answer Tips

- Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

21. Given that $P(2, 2, 1)$ and $Q(5, 1, -2)$



Let the point R on the line PQ, divides the line in the ratio $k : 1$. And x-coordinate of point R on the line is 4.

So, by section formula $4 = \frac{5k+2}{k+1} \Rightarrow k = 2$

Now, z-coordinate of point R, $z = \frac{-2k+1}{k+1} = \frac{-2\times2+1}{2+1} = -1$

$\Rightarrow$ z-coordinate of point $R = -1$

22. We have the points $P(1, 1, 8)$ and $Q(5, 2, 10)$, then the equations of line is $\frac{x+1}{5+1} = \frac{y-1}{-2-1} = \frac{z-(-8)}{10-(-8)}$

or $\frac{x+1}{6} = \frac{y-1}{-3} = \frac{z+8}{18}$ ...(i)

If the line crosses the zx-plane, then $y = 0$, so from (1)

$\frac{x+1}{6} = \frac{1}{3}$ and $\frac{z+8}{18} = \frac{1}{3}$

$x = 1$ and $z = -2$

$\therefore$ The coordinates of the required point is $(1, 0, -2)$.

23. Since, α and β be the angle made by x-axis and z-axis and $\alpha = 60°$ and $\beta = 45°$ (given)

Let θ be the angle made by the line with y-axis.

Then, $\cos^2 60° + \cos^2 45° + \cos^2\theta = 1$

[$\because$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$]

$\Rightarrow$ $\frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow$ $\cos^2\theta = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

$\therefore$ Direction cosines of the line are

<cos60°, cos45°, cos60°> *i.e.*, $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$.

24. The equation of line AB is given by

$$\frac{x-0}{4-0} = \frac{y+1}{5+1} = \frac{z+1}{1+1} = \lambda \text{ (say)}$$

$\Rightarrow$ $x = 4\lambda, y = 6\lambda - 1, z = 2\lambda - 1$

The coordinates of a general point on AB are $(4\lambda, 6\lambda - 1, 2\lambda - 1)$

The equation of line CD is given by

$$\frac{x-3}{3+4} = \frac{y-9}{9-4} = \frac{z-4}{4-4} = \mu \text{ (say)}$$

$\Rightarrow$ $x = 7\mu + 3,\ y = 5\mu + 9, z = 4$

The coordinates of a general point on CD are $(7\mu + 3, 5\mu + 9, 4)$

If the line AB and CD intersect then they have a common point. So, for some values of λ and μ, we must have

$4\lambda = 7\mu + 3, 6\lambda - 1 = 5\mu + 9, 2\lambda - 1 = 4$

$\Rightarrow$ $4\lambda - 7\mu = 3$ ...(i)

$6\lambda - 5\mu = 10$ ...(ii)

and $\lambda = \frac{5}{2}$ ...(iii)

Substituting $\lambda = \frac{5}{2}$ in (ii), we get $\mu = 1$

Since $\lambda = \frac{5}{2}$ and $\mu = 1$ satisfy (i), so the given lines AB and CD intersect.

Key Points

- Equation of a line passing through points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

25. Any point on the line

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = r \text{ (say)} \quad \text{...(i)}$$

is $(3r - 1, 5r - 3, 7r - 5)$.

Any point on the line

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = k \text{ (say)} \quad \text{...(ii)}$$

is $(k + 2, 3k + 4, 5k + 6)$

For lines (i) and (ii) to intersect, we must have

$3r - 1 = k + 2, 5r - 3 = 3k + 4, 7r - 5 = 5k + 6$

On solving these, we get $r=\frac{1}{2}, k=-\frac{3}{2}$

$\therefore$ Lines (i) and (ii) intersect and their point of intersection is $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$

26. The given lines are

$\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(3\hat{i}-\hat{j})=(3\lambda+1)\hat{i}+(1-\lambda)\hat{j}-\hat{k}$ ...(i)

and $\vec{r}=(4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k})=(2\mu+4)\hat{i}+0\cdot\hat{j}+(3\mu-1)\hat{k}$ ...(ii)

If the lines (i) & (ii) intersect, then they have a common point. So, we must have

$(3\lambda+1)\hat{i}+(1-\lambda)\hat{j}-\hat{k}=(2\mu+4)\hat{i}+0\cdot\hat{j}+(3\mu-1)\hat{k}$

$\Rightarrow$ $3\lambda+1=2\mu+4, 1-\lambda=0$ and $-1=3\mu-1$

On solving last two equations, we get $\lambda=1$ and $\mu=0$. These values of λ and μ satisfy the first equation.

So, the given lines intersect.

Putting $\lambda=1$ in (i), we get the position vector of the point of intersection.

Thus, the coordinates of the point of intersection are $(4, 0, -1)$.

27. Let $\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}=\alpha$ (say)

$\Rightarrow$ Any point $P(3\alpha+7, 2\alpha+5, \alpha+3)$ lie on this line.

Let $\frac{x-1}{2}=\frac{y+1}{4}=\frac{z+1}{3}=\beta$ (say)

$\Rightarrow$ Any point $Q(2\beta+1, 4\beta-1, 3\beta-1)$ lie on this line.

D.R's. of line PQ are 2, 2, 1, then

$$\frac{2\beta+1-(3\alpha+7)}{2}=\frac{4\beta-1-(2\alpha+5)}{2}=\frac{3\beta-1-(\alpha+3)}{1}$$

$\Rightarrow$ $\alpha=\frac{-2}{3}$ and $\beta=\frac{1}{3}$

$\Rightarrow$ Point are $P\left(5, \frac{11}{3}, \frac{5}{3}\right)$ and $Q\left(\frac{5}{3}, \frac{1}{3}, 0\right)$

Length of $PQ=\sqrt{\left(\frac{5}{3}-5\right)^2+\left(\frac{1}{3}-\frac{11}{3}\right)^2+\left(0-\frac{5}{3}\right)^2}$

$=\sqrt{\frac{225}{9}}=\frac{15}{3}=5$ units

The equation of intercept PQ is

$$\frac{x-5}{\frac{5}{3}-5}=\frac{y-\frac{11}{3}}{\frac{1}{3}-\frac{11}{3}}=\frac{z-\frac{5}{3}}{0-\frac{5}{3}}$$

$$\Rightarrow \frac{x-5}{-\frac{10}{3}}=\frac{y-\frac{11}{3}}{-\frac{10}{3}}=\frac{z-\frac{5}{3}}{-\frac{5}{3}}$$

28. (a): If lines are perpendicular, then $\theta=\frac{\pi}{2}$

$\therefore$ $\cos\frac{\pi}{2}=\frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\Rightarrow\cos\frac{\pi}{2}=\frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$

$\Rightarrow$ $\vec{b}_1\cdot\vec{b}_2=0$

$\therefore$ Both A and R are true and R is the correct explanation of A.

29. (d): The given equation of lines can be rewritten as

$\frac{x-0}{1/2}=\frac{y-0}{1/3}=\frac{z-0}{-1}$ and $\frac{x-0}{1/6}=\frac{y-0}{-1}=\frac{z-0}{-1/4}$

$\therefore$ $a_1=\frac{1}{2}, b_1=\frac{1}{3}, c_1=-1$

and $a_2=\frac{1}{6}, b_2=-1, c_2=\frac{-1}{4}$

Now, $\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$$=\frac{\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{3}\cdot(-1)+(-1)\cdot\left(\frac{-1}{4}\right)}{\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^2+(-1)^2}\sqrt{\left(\frac{1}{6}\right)^2+(-1)^2+\left(\frac{-1}{4}\right)^2}}=0$$

$\Rightarrow$ $\cos\theta=0\Rightarrow\theta=90°$

30. (d): The given lines are perpendicular, if

$a_1a_2+b_1b_2+c_1c_2=0$ ...(i)

Here, $L_1: \frac{x-5}{0}=\frac{y-0}{3-\alpha}=\frac{z-0}{-2}$

$L_2: \frac{x-2}{0}=\frac{y-0}{-1}=\frac{z-0}{2-\alpha}$

Here, a_1, b_1, c_1 are $0, 3-\alpha, -2$, and a_2, b_2, c_2 are $0, -1, 2-\alpha$ respectively.

$\therefore$ $0\times0-(3-\alpha)-2(2-\alpha)=0$

$\Rightarrow$ $\alpha=\frac{7}{3}$ [from (i)]

31. We know that vector equation of a line passing through point $\vec{a}$ and parallel to vector $\vec{b}$ is given by $\vec{r}=\vec{a}+\lambda\vec{b}$

Here $\vec{a}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{b}=2\hat{i}+2\hat{j}-3\hat{k}$

$\therefore$ Required equation is

$\vec{r}=(3\hat{i}+4\hat{j}+5\hat{k})+\lambda(2\hat{i}+2\hat{j}-3\hat{k})$

32. Equation of the line can be written as

$\frac{x+3}{3}=\frac{y-4}{-5}=\frac{z+8}{6}$

Direction ratios of this line are 3, –5, 6.

The required line passes through (–2, 4, –5) and its direction ratios are proportional to 3, –5, 6. So, its equation is $\frac{x+2}{3}=\frac{y-4}{-5}=\frac{z+5}{6}$.

33. For $\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$, we have

$\vec{b}_1=3\hat{i}+2\hat{j}+6\hat{k}$

Also for

$\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$, we have $\vec{b}_2=\hat{i}+2\hat{j}+2\hat{k}$

Let θ be the angle between the lines.

So, $\theta=\cos^{-1}\left|\frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

$\Rightarrow\ \theta=\cos^{-1}\left|\dfrac{(3\hat{i}+2\hat{j}+6\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{3^2+2^2+6^2}\ \sqrt{1^2+2^2+2^2}}\right|$

$\Rightarrow\ \theta=\cos^{-1}\left|\dfrac{3+4+12}{7\times 3}\right| \Rightarrow \theta=\cos^{-1}\left(\dfrac{19}{21}\right)$

34. Any line parallel to z-axis has direction ratios proportional to 0, 0, 1.

$\therefore$ The equation of a line through (α, β, γ) and parallel to z-axis is $\dfrac{x-\alpha}{0}=\dfrac{y-\beta}{0}=\dfrac{z-\gamma}{1}$

35. Let the equation of line passing through (2, 1, 3) and perpendicular to the lines

$\dfrac{x-1}{1}=\dfrac{y-2}{2}=\dfrac{z-3}{3}$ and $\dfrac{x}{-3}=\dfrac{y}{2}=\dfrac{z}{5}$ be

$\dfrac{x-2}{l}=\dfrac{y-1}{m}=\dfrac{z-3}{n}$ ...(i)

$\therefore\ l\cdot 1+m\cdot 2+n\cdot 3=0$ and $l\cdot(-3)+m\cdot 2+n\cdot 5=0$

$\Rightarrow\ \dfrac{l}{10-6}=\dfrac{m}{-9-5}=\dfrac{n}{2+6} \Rightarrow \dfrac{l}{2}=\dfrac{m}{-7}=\dfrac{n}{4}.$

$\therefore$ The equation of the required line is

$\dfrac{x-2}{2}=\dfrac{y-1}{-7}=\dfrac{z-3}{4}.$

Also its vector equation is

$\vec{r}=(2\hat{i}+\hat{j}+3\hat{k})+\lambda(2\hat{i}-7\hat{j}+4\hat{k}).$

36. Let the equation of line passing through $A(1, 2, -1)$ be

$\dfrac{x-1}{a}=\dfrac{y-2}{b}=\dfrac{z+1}{c}$ ...(i)

Now, given equation of line is,

$5x-25=14-7y=35z$

$\Rightarrow\ \dfrac{x-5}{1/5}=\dfrac{y-2}{-1/7}=\dfrac{z-0}{1/35}$

$\Rightarrow\ \dfrac{x-5}{7}=\dfrac{y-2}{-5}=\dfrac{z-0}{1}$ ...(ii)

Since (i) and (ii) are parallel lines.

$\therefore\ \dfrac{a}{7}=\dfrac{b}{-5}=\dfrac{c}{1}$

From (i), we get

$\dfrac{x-1}{7}=\dfrac{y-2}{-5}=\dfrac{z+1}{1}$ is the cartesian equation of line.

Also, the vector equation of line is $(\hat{i}+2\hat{j}-\hat{k})+\lambda(7\hat{i}-5\hat{j}+\hat{k})$.

37. The given lines are perpendicular, if

$a_1a_2+b_1b_2+c_1c_2=0$

The given lines are as

$l_1:\dfrac{x-0}{1}=\dfrac{y-0}{-1}=\dfrac{z-0}{\frac{1}{k}}\ ;\ l_2:\dfrac{x-2}{1}=\dfrac{y+\frac{1}{2}}{\frac{1}{2}}=\dfrac{z-1}{-1}$

$\because\ l_1$ is perpendicular to l_2

here, a, b, c, are 1, −1, $1/k$ and a_2, b_2, c_2 are 1, 1/2, −1 respectively

$\therefore\ 1(1)+(-1)\left(\dfrac{1}{2}\right)+\left(\dfrac{1}{k}\right)(-1)=0$

$\Rightarrow\ 1-\dfrac{1}{2}-\dfrac{1}{k}=0 \Rightarrow \dfrac{1}{2}=\dfrac{1}{k} \Rightarrow k=2$

Commonly Made Mistake

- Remember two lines $\dfrac{x-x_1}{a_1}=\dfrac{y-y_1}{b_1}=\dfrac{z-z_1}{c_1}$ and $\dfrac{x-x_2}{a_2}=\dfrac{y-y_2}{b_2}=\dfrac{z-z_2}{c_2}$ are perpendicular to each other if $a_1a_2+b_1b_2+c_1c_2=0$ and parallel if $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}=\dfrac{c_1}{c_2}$.

38. Vector equation of the line passing through (1, 2, −1) and parallel to the line

$5x-25=14-7y=35z$

i.e., $\dfrac{x-5}{1/5}=\dfrac{y-2}{-1/7}=\dfrac{z}{1/35}$ or $\dfrac{x-5}{7}=\dfrac{y-2}{-5}=\dfrac{z-0}{1}$

is $\vec{r}=(\hat{i}+2\hat{j}-\hat{k})+\lambda(7\hat{i}-5\hat{j}+\hat{k})$

39. We have point $P(5, 7, 3)$ and equation of line as

$\dfrac{x-15}{3}=\dfrac{y-29}{8}=\dfrac{z-5}{-5}=k$ (say)

Any point on this line is given by

$Q(3k+15, 8k+29, -5k+5)$

Direction ratio of line PQ are

$\langle 3k+15-5, 8k+29-7, -5k+5-3\rangle$

i.e., $\langle 3k+10, 8k+22, -5k+2\rangle$

As, PQ is perpendicular to given line

$\therefore\ 3(3k+10)+8(8k+22)-5(-5k+2)=0$

$\Rightarrow\ 98k+196=0 \Rightarrow k=-2$

$\therefore$ Foot of perpendicular drawn from given point $P(5, 7, 3)$ on the given line is

$(-6+15, -16+29, 10+5)$ *i.e.*, (9, 13, 15)

40. Let M be the foot of the perpendicular drawn from point $P(0, 2, 3)$ to the line

$\dfrac{x+3}{5}=\dfrac{y-1}{2}=\dfrac{z+4}{3}=\lambda$ (say) ... (i)

$\therefore$ Any point on line (i) is $(5\lambda-3, 2\lambda+1, 3\lambda-4)$

So, coordinates of $M\equiv(5\lambda-3, 2\lambda+1, 3\lambda-4)$ ... (ii)

Now, direction ratios of PM are $\langle 5\lambda-3-0, 2\lambda+1-2, 3\lambda-4-3\rangle$

i.e., $\langle 5\lambda-3, 2\lambda-1, 3\lambda-7\rangle$

Since, PM is perpendicular to line (i).

$\therefore\ 5(5\lambda-3)+2(2\lambda-1)+3(3\lambda-7)=0$

$\Rightarrow\ 25\lambda-15+4\lambda-2+9\lambda-21=0$

$\Rightarrow\ 38\lambda-38=0 \Rightarrow \lambda=1$

So, coordinates of M are (2, 3, −1).

41. The given lines are

$l_1:\dfrac{x-1}{-3}=\dfrac{y-2}{\lambda/7}=\dfrac{z-3}{2}$ and $l_2:\dfrac{x-1}{-3\lambda/7}=\dfrac{y-5}{1}=\dfrac{z-6}{-5}$

Now, $l_1 \perp l_2$ [Given]

$\therefore\ (-3)\left(-\dfrac{3\lambda}{7}\right)+\dfrac{\lambda}{7}-10=0$

$\Rightarrow\ \dfrac{9\lambda}{7}+\dfrac{\lambda}{7}-10=0 \Rightarrow \dfrac{10\lambda}{7}=10 \Rightarrow \lambda=7$

Since for $\lambda = 7$, given lines are at right angle.

$\therefore$ Lines are intersecting.

42. The given lines are

$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$

and $\vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

Equation of any line through (1, 2, −4) with d.r's l, m, n is

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + p(l\hat{i} + m\hat{j} + n\hat{k})$ ...(i)

Since, the required line is perpendicular to both the given lines.

$\therefore$ $3l - 16m + 7n = 0$ and $3l + 8m - 5n = 0$

$\Rightarrow \frac{l}{80-56} = \frac{m}{21+15} = \frac{n}{24+48} \Rightarrow \frac{l}{2} = \frac{m}{3} = \frac{n}{6}$

$\therefore$ From (i), the required line is

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + p(2\hat{i} + 3\hat{j} + 6\hat{k})$

Here, the position vector of passing point is $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$ and parallel vector is $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$.

$\therefore$ Cartesian equation of line is given by

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$

43. Let the equation of line passing through (2, 1, 3) and perpendicular to the lines

$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$ be

$\frac{x-2}{l} = \frac{y-1}{m} = \frac{z-3}{n}$ ...(i)

$\therefore$ $l \cdot 1 + m \cdot 2 + n \cdot 3 = 0$ and $l \cdot (-3) + m \cdot 2 + n \cdot 5 = 0$

$\Rightarrow \frac{l}{10-6} = \frac{m}{-9-5} = \frac{n}{2+6} \Rightarrow \frac{l}{2} = \frac{m}{-7} = \frac{n}{4}$.

$\therefore$ The equation of the required line is

$\frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$.

Also its vector equation is

$\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$.

44. The given lines are

$l_1: \frac{x-1}{-3} = \frac{y-2}{p/7} = \frac{z-3}{2}$

$l_2: \frac{x-1}{-3p/7} = \frac{y-5}{1} = \frac{z-6}{-5}$

$\because$ l_1 is perpendicular to l_2.

$\therefore$ $(-3)\left(\frac{-3p}{7}\right) + \frac{p}{7} \cdot 1 + 2(-5) = 0$

$\Rightarrow \frac{9p}{7} + \frac{p}{7} = 10 \Rightarrow \frac{10p}{7} = 10 \Rightarrow p = 7$

Now, equation of the line passing through (3, 2, −4) and parallel to l_1 is

$\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$.

45. The given lines are

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

Equation of any line through (2, −1, 3) with d.r's l, m, n is

$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + p(l\hat{i} + m\hat{j} + n\hat{k})$ ...(i)

Since, the required line is perpendicular to both the given lines.

$\therefore$ $2l - 2m + n = 0$ and $l + 2m + 2n = 0$

$\Rightarrow \frac{l}{-4-2} = \frac{m}{1-4} = \frac{n}{4+2} \Rightarrow \frac{l}{2} = \frac{m}{1} = \frac{n}{-2}$

$\therefore$ From (i), the required line is

$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + p(2\hat{i} + \hat{j} - 2\hat{k})$.

46. The given line is $\frac{x+2}{2} = \frac{2y-7}{6} = \frac{5-z}{6}$

$\Rightarrow \frac{x+2}{2} = \frac{y-\frac{7}{2}}{3} = \frac{z-5}{-6}$ ...(i)

Its d.r's are 2, 3, −6

$\because \sqrt{2^2 + 3^2 + (-6)^2} = 7$

$\therefore$ Its d.c's are $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$

Equation of a line through (−1, 2, 3) and parallel to (i) is

$\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-3}{-6} = \lambda$ (say)

$\therefore$ Vector equation of a line passing through (−1, 2, 3) and parallel to (i) is given by

$\vec{r} = (-\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - 6\hat{k})$

Answer Tips

- If a, b, c are d.r.'s of a line, then d.c.'s of the line are $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

47. The given lines are

$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} = \lambda$ ...(i) $\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} = \mu$ ...(ii)

Let P be the general point on line (i), then

$x = 3\lambda + 1$, $y = 2\lambda - 1$ and $z = 5\lambda + 1$

$\therefore$ $P \equiv (3\lambda + 1, 2\lambda - 1, 5\lambda + 1)$

Let Q be the general point on line (ii), then

$x = 4\mu - 2$, $y = 3\mu + 1$ and $z = -2\mu - 1$

$\therefore$ $Q \equiv (4\mu - 2, 3\mu + 1, -2\mu - 1)$

Let the given lines intersects.

So P and Q coincide for some particular values of λ and μ.

$\therefore$ $3\lambda + 1 = 4\mu - 2 \Rightarrow 3\lambda - 4\mu = -3$ ...(iii)

$2\lambda - 1 = 3\mu + 1 \Rightarrow 2\lambda - 3\mu = 2$ ...(iv)

and $5\lambda + 1 = -2\mu - 1 \Rightarrow 5\lambda + 2\mu = -2$ ...(v)

Solving equation (iii) and (iv), we get

$\lambda = -17$ and $\mu = -12$

But $\lambda = -17$ and $\mu = -12$ do not satisfy the equation (v).

It means our assumption is wrong hence the given lines do not intersect.

48. Given line, $2x = 3y = -z$ can be written as

$\frac{x}{1/2} = \frac{y}{1/3} = \frac{z}{-1}$ ...(i)

The direction ratios of line (i) are $<\frac{1}{2}, \frac{1}{3}, -1>$

and the line, $6x = -y = -4z$ can be written as

$$\frac{x}{1/6} = \frac{y}{-1} = \frac{z}{-1/4} \quad ...(ii)$$

The direction ratios of line (ii) are $<\frac{1}{6}, -1, -\frac{1}{4}>$

It is known that if two lines are perpendicular then the dot product of the direction ratios of the two lines is equal to 0.

Product of direction ratios $= \frac{1}{2} \times \frac{1}{6} + \frac{1}{3} \times (-1) + (-1) \times \left(-\frac{1}{4}\right)$

$= \frac{1}{12} - \frac{1}{3} + \frac{1}{4} = 0$

So, angle between the lines is 90°.

49. Let the equation of line passing through (1, 2, –4) and perpendicular to the lines

$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

be $\frac{x-1}{l} = \frac{y-2}{m} = \frac{z+4}{n} \quad ...(i)$

$\therefore \quad l(3) + m(-16) + n(7) = 0$ and $l(3) + m(8) + n(-5) = 0$

$\Rightarrow \frac{l}{80-56} = \frac{m}{21+15} = \frac{n}{24+48}$

$\Rightarrow \frac{l}{24} = \frac{m}{36} = \frac{n}{72} \Rightarrow \frac{l}{2} = \frac{m}{3} = \frac{n}{6}$

$\therefore$ The equation of the required line is

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

and its vector equation is

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

50. The line of shortest distance between two skew lines is perpendicular to both the lines.

51. For the given lines, $\frac{l_1}{l_2} = \frac{1}{1} = 1$; $\frac{m_1}{m_2} = \frac{2}{2} = 1$; $\frac{n_1}{n_2} = \frac{3}{3} = 1$

Since, $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$, therefore the given lines are parallel.

Let $x = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$ (say)

Any point on this line is $P(\lambda, 2\lambda + 1, 3\lambda + 2)$

If it is the foot of the perpendicular on the line, then

$1(\lambda) + 2(2\lambda + 1) + 3(3\lambda + 2) = 0$

$\Rightarrow \quad \lambda = -\frac{4}{7}$

$\therefore \quad P(\lambda, 2\lambda+1, 3\lambda+2) = P\left(-\frac{4}{7}, \frac{3}{7}, \frac{2}{7}\right)$

Similarly $x + 1 = \frac{y+2}{2} = \frac{z-1}{3} = \mu$ (say)

Any point on this line is $Q(\mu - 1, 2\mu - 2, 3\mu + 1)$.

$\Rightarrow \quad 1(\mu - 1) + 2(2\mu - 2) + 3(3\mu + 1) = 0$

$\Rightarrow \quad \mu = \frac{1}{7}$

$\therefore \quad Q(\mu - 1, 2\mu - 2, 3\mu + 1) = \left(-\frac{6}{7}, -\frac{12}{7}, \frac{10}{7}\right)$

$PQ = \sqrt{\left(\frac{-6}{7} + \frac{4}{7}\right)^2 + \left(\frac{-12}{7} - \frac{3}{7}\right)^2 + \left(\frac{10}{7} - \frac{2}{7}\right)^2} = \frac{\sqrt{293}}{7}$ units

52. Comparing the gives lines with $\vec{r} = \vec{a} + s\vec{b}, \vec{r} = \vec{c} + t\vec{b}$

$\vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$; $\vec{b} = \hat{i} + \hat{j} - \hat{k}$; $\vec{a}_2 = \hat{i} - 2\hat{j} + \hat{k}$

Distance between two given lines $= \left|\frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|}\right|$

$= \left|\frac{(\hat{i} + \hat{j} - \hat{k}) \times (-\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{1+1+1}}\right| = \frac{1}{\sqrt{3}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & -3 & 2 \end{vmatrix}$

$= \frac{1}{\sqrt{3}}\left|\hat{i}(2-3) + \hat{j}(-2+1) + \hat{k}(-3+1)\right| = \frac{1}{\sqrt{3}}|-\hat{i} - \hat{j} - 2\hat{k}|$

$= \frac{\sqrt{1+1+4}}{\sqrt{3}} = \sqrt{2}$ units

53. For the given lines,

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$

$$\Delta = \left|\frac{\begin{vmatrix} 4-1 & 1-2 & 0-3 \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}}{\sqrt{470}}\right| = \left|\frac{\begin{vmatrix} 3 & -1 & -3 \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}}{\sqrt{470}}\right|$$

$\left[\because \sqrt{(-5)^2 + (18)^2 + (11)^2} = \sqrt{470}\right]$

$\Delta = \left|\frac{3(3-8) + 1(2-20)) - 3(4-15)}{\sqrt{470}}\right|$

$\Delta = \left|\frac{-15-18+33}{\sqrt{470}}\right|$

$\Delta = 0$

Since, $\Delta = 0$, therefore, the gives lines are not skew lines.

Concept Applied

- Shortest distance between two lines

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is gives by

$$\Delta = \left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2 + (c_1a_2-c_2a_1)^2 + (a_1b_2-a_2b_1)^2}}\right|$$

54. (i) We have, $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k}) \quad ...(i)$

and $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k}) \quad ...(ii)$

Here, $\vec{a}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}$, $\vec{a}_2 = 3\hat{i} + 3\hat{j}$;

$\vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b}_2 = 2\hat{i} + \hat{j} + \hat{k}$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = \hat{i}(2+1) - \hat{j}(1+2) + \hat{k}(1-4) = 3\hat{i} - 3\hat{j} - 3\hat{k}$$

Now, $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j})\cdot(3\hat{i} - 3\hat{j} - 3\hat{k}) = 9 - 9 = 0$

Hence, shortest distance between the given lines is 0.

(ii) Equation of line (i) : $x = \lambda, y = 2\lambda, z = -\lambda$

Equation of line (ii) : $x = 3 + 2\mu, y = 3 + \mu, z = \mu$

So, $\lambda = 3 + 2\mu$ ...(iii)

$2\lambda = 3 + \mu$ ...(iv)

$-\lambda = \mu$ ...(v)

Substitute $\mu = -\lambda$ is (iii), we get

$\lambda = 3 - 2\lambda$

$\Rightarrow 3\lambda = 3$

$\Rightarrow \lambda = 1$

$\Rightarrow \mu = -1$

So, the two lines intersect at point (1, 2, – 1).

Since, the point (1, 2, –1) satisfies both the equation of lines, therefore point of intersection of given lines is (1, 2, –1).

So, the motorcycles may collide at point (1, 2, –1).

55. We have, $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ ...(i)

$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$ ...(ii)

Comparing with lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$, we get

$\vec{a}_1 = (4\hat{i} - \hat{j})$; $\vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}$; $\vec{b}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$

Now, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = \hat{i}(-10+12) - \hat{j}(-5+6) + \hat{k}(4-4) = 2\hat{i} - \hat{j}$

and $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$

$\vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k}$

$\therefore$ Shortest distance, $d = \left|\dfrac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$= \left|\dfrac{(2\hat{i} - \hat{j})\cdot(-3\hat{i} + 2\hat{k})}{\sqrt{5}}\right| = \left|\dfrac{-6+0+0}{\sqrt{5}}\right| = \dfrac{6}{\sqrt{5}}$

56. The given lines are

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and

$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

On comparing, we get

$\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}, \vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$; $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}, \vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$= \hat{i}(-2-1) - \hat{j}(2-2) + k(1+2) = -3\hat{i} + 0\cdot\hat{j} + 3\hat{k}$

$\Rightarrow |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 3^2} = 3\sqrt{2}$

$\Rightarrow (\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = 1(-3) - 3(0) - 2(3) = -9.$

$\therefore d = \left|\dfrac{-9}{3\sqrt{2}}\right| = \dfrac{3}{\sqrt{2}} = \dfrac{3}{2}\sqrt{2}$ units

Concept Applied

- S.D. between the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$, and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is given by $d = \left|\dfrac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

57. The given lines are

$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and

$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

S.D. between the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is given by

$d = \left|\dfrac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

On comparing, we get

$\vec{a}_1 = 2\hat{i} - 5\hat{j} + \hat{k}, \vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$; $\vec{a}_2 = 7\hat{i} - 6\hat{k}, \vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = 5\hat{i} + 5\hat{j} - 7\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 6 \\ 1 & 2 & 2 \end{vmatrix} = \hat{i}(4-12) - \hat{j}(6-6) + \hat{k}(6-2) = -8\hat{i} + 4\hat{k}$$

$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-8)^2 + 4^2} = 4\sqrt{5}$

Hence, $d = \left|\dfrac{(5\hat{i} + 5\hat{j} - 7\hat{k})\cdot(-8\hat{i} + 4\hat{k})}{4\sqrt{5}}\right|$

$= \dfrac{|5(-8) - 7(4)|}{4\sqrt{5}} = \dfrac{68}{4\sqrt{5}} = \dfrac{17\sqrt{5}}{5}$ units

58. Let $l_1 : \dfrac{x+1}{7} = \dfrac{y+1}{-6} = \dfrac{z+1}{1}$

$\Rightarrow \dfrac{x-(-1)}{7} = \dfrac{y-(-1)}{-6} = \dfrac{z-(-1)}{1}$ and $l_2 : \dfrac{x-3}{1} = \dfrac{y-5}{-2} = \dfrac{z-7}{1}$

$\therefore$ Vector equation of lines are

$\vec{r} = -\hat{i} - \hat{j} - \hat{k} + \lambda(7\hat{i} - 6\hat{j} + \hat{k})$ and $\vec{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \mu(\hat{i} - 2\hat{j} + \hat{k})$

We get $\vec{a}_1 = -\hat{i} - \hat{j} - \hat{k}, \vec{b}_1 = 7\hat{i} - 6\hat{j} + \hat{k}$

and $\vec{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}, \vec{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$

So, $\vec{a}_2 - \vec{a}_1 = (3\hat{i} + 5\hat{j} + 7\hat{k}) - (-\hat{i} - \hat{j} - \hat{k}) = 4\hat{i} + 6\hat{j} + 8\hat{k}$

And, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -4\hat{i} - 6\hat{j} - 8\hat{k}$

Shortest distance between two skew lines is,

$d = \left|\dfrac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$

$\Rightarrow d = \left|\dfrac{(-4\hat{i} - 6\hat{j} - 8\hat{k}).(4\hat{i} + 6\hat{j} + 8\hat{k})}{\sqrt{(-4)^2 + (-6)^2 + (-8)^2}}\right|$

$\Rightarrow d = \left|\dfrac{-16 - 36 - 64}{\sqrt{116}}\right| \Rightarrow d = 2\sqrt{29}$ units

59. We have, $\vec{r}=\hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k})$

$\therefore\ \vec{a}_1=\hat{i}+\hat{j}, \vec{b}_1=2\hat{i}-\hat{j}+\hat{k}$

Also, $\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

$\therefore\ \vec{a}_2=2\hat{i}+\hat{j}-\hat{k}, \vec{b}_2=3\hat{i}-5\hat{j}+2\hat{k}$

So, $\vec{a}_2-\vec{a}_1=\hat{i}-\hat{k}$

And, $\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -5 & 2\end{vmatrix}=\hat{i}(-2+5)-\hat{j}(4-3)+\hat{k}(-10+3)=3\hat{i}-\hat{j}-7\hat{k}$

Shortest distance between two skew lines is,

$$d=\left|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

$$\Rightarrow\ d=\left|\frac{(3\hat{i}-\hat{j}-7\hat{k})\cdot(\hat{i}-\hat{k})}{\sqrt{3^2+(-1)^2+(-7)^2}}\right|\ \Rightarrow\ d=\left|\frac{3+7}{\sqrt{59}}\right|=\frac{10}{\sqrt{59}}\text{units.}$$

60. Given lines are

$l_1:\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k});$

$l_2:\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(4\hat{i}+6\hat{j}+12\hat{k})$

$\therefore$ We have $\vec{a}_1=\hat{i}+2\hat{j}-4\hat{k}, \vec{b}_1=2\hat{i}+3\hat{j}+6\hat{k}$

and $\vec{a}_2=3\hat{i}+3\hat{j}-5\hat{k}, \vec{b}_2=4\hat{i}+6\hat{j}+12\hat{k}$

So, $\vec{a}_2-\vec{a}_1=2\hat{i}+\hat{j}-\hat{k}$

Also, $\vec{b}_2=4\hat{i}+6\hat{j}+12\hat{k}=2\vec{b}_1 \Rightarrow \vec{b}_1||\vec{b}_2$

Hence l_1 and l_2 are parallel lines.

Shortest distance between two parallel lines is,

$$d=\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|$$

$$\Rightarrow\ d=\left|\frac{(2\hat{i}+3\hat{j}+6\hat{k})\times(2\hat{i}+\hat{j}-\hat{k})}{\sqrt{2^2+3^2+6^2}}\right|\Rightarrow\ d=\left|\frac{-9\hat{i}+14\hat{j}-4\hat{k}}{7}\right|$$

$$\Rightarrow\ d=\frac{\sqrt{(-9)^2+14^2+(-4)^2}}{7}=\frac{\sqrt{293}}{7}\text{units.}$$

61. Here, the lines are

$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ and

$\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$

Here, $\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k}, \vec{b}_1=\hat{i}-3\hat{j}+2\hat{k}$ and

$\vec{a}_2=4\hat{i}+5\hat{j}+6\hat{k}, \vec{b}_2=2\hat{i}+3\hat{j}+\hat{k}$

The shortest distance between the lines is given by

$$d=\left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

$\vec{a}_2-\vec{a}_1=3\hat{i}+3\hat{j}+3\hat{k}$

$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 2\\ 2 & 3 & 1\end{vmatrix}=\hat{i}(-3-6)-\hat{j}(1-4)+\hat{k}(3+6)=-9\hat{i}+3\hat{j}+9\hat{k}$

$\Rightarrow\ |\vec{b}_1\times\vec{b}_2|=\sqrt{(-9)^2+3^2+9^2}=3\sqrt{19}$

Also, $(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)=(3\hat{i}+3\hat{j}+3\hat{k})\cdot(-9\hat{i}+3\hat{j}+9\hat{k})$

$= 3\times(-9)+3\times3+3\times9=9$

$\therefore\ d=\left|\frac{9}{3\sqrt{19}}\right|=\frac{3}{\sqrt{19}}$ unit.

62. Vector equation of a line passing through (2, 3, 2) and parallel to the line

$\vec{r}=(-2\hat{i}+3\hat{j})+\lambda(2\hat{i}-3\hat{j}+6\hat{k})$ is given by

$\vec{r}=(2\hat{i}+3\hat{j}+2\hat{k})+\mu(2\hat{i}-3\hat{j}+6\hat{k})$

Now, $\vec{a}_1=-2\hat{i}+3\hat{j}, \vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$

and $\vec{a}_2=2\hat{i}+3\hat{j}+2\hat{k}$

Distance between given parallel lines

$$=\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|=\left|\frac{(2\hat{i}-3\hat{j}+6\hat{k})\times(4\hat{i}+0\hat{j}+2\hat{k})}{|\sqrt{4+9+36}|}\right|$$

$$=\frac{1}{7}|\hat{i}(-6)+20\hat{j}+12\hat{k}|$$

$$=\frac{1}{7}\sqrt{(-6)+(20)^2+(12)^2}=\frac{\sqrt{580}}{7}\text{ units}$$

CBSE Sample Questions

1. **(b) :** The line through the points (0, 5, –2) and (3, –1, 2) is

$$\frac{x}{3-0}=\frac{y-5}{-1-5}=\frac{z+2}{2+2}$$

or $\quad \frac{x}{3}=\frac{y-5}{-6}=\frac{z+2}{4}$

Any point on the line is $P(3k, -6k+5, 4k-2)$, where k is an arbitrary scalar.

$\because\quad 3k=6$

$\Rightarrow\quad k=2$

The z-coordinate of the point P will be $4\times2-2=6$. **(1)**

2. The equations of the line are $6x-12=3y+9=2z-2$, which, when written in standard symmetric form, will be

$\frac{x-2}{1/6}=\frac{y-(-3)}{1/3}=\frac{z-1}{1/2}$ **(1/2)**

Since, lines are parallel, we have $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ **(1/2)**

Hence, the required direction ratios are

$\left(\frac{1}{6},\frac{1}{3},\frac{1}{2}\right)$ or (1, 2, 3) **(1/2)**

and the required direction cosines are $\left(\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}\right)$

(1/2)

3. The given line can be written as $\frac{x-3}{1}=\frac{y-\frac{1}{2}}{1}=\frac{z}{4}$ **(1)**

$\therefore$ Direction ratios of the given line are $<1, 1, 4>$. **(1/2)**

Thus, direction cosines are $\langle\frac{1}{3\sqrt{2}},\frac{1}{3\sqrt{2}},\frac{4}{3\sqrt{2}}\rangle$. **(1/2)**

4. (a): The equation of the x-axis may be written as $\vec{r} = t\hat{i}$. Hence, the acute angle θ between the given line and the x-axis is given by

$$\cos\theta = \frac{|1\times1+(-1)\times0+0\times0|}{\sqrt{1^2+(-1)^2+0^2}\times\sqrt{1^2+0^2+0^2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$$

Hence, both A and R are true and R is the correct explanation of A. **(1)**

5. Let $\vec{a}_1 = \hat{i}+\hat{j}-\hat{k}, \vec{a}_2 = \hat{i}+\hat{j}+2\hat{k}$ and $\vec{b} = 2\hat{i}+\hat{j}+\hat{k}$

Here, the lines are parallel.

$\therefore$ Shortest distance between lines $= \frac{|(\vec{a}_2-\vec{a}_1)\times\vec{b}|}{|\vec{b}|}$

$$= \frac{|(3\hat{k})\times(2\hat{i}+\hat{j}+\hat{k})|}{\sqrt{4+1+1}} \quad \textbf{(1½)}$$

Now, $(3\hat{k})\times(2\hat{i}+\hat{j}+\hat{k}) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & 3\\ 2 & 1 & 1\end{vmatrix} = -3\hat{i}+6\hat{j}$ **(1)**

$\Rightarrow |(3\hat{k})\times(2\hat{i}+\hat{j}+\hat{k})| = \sqrt{9+36} = 3\sqrt{5}$

Hence, the required shortest distance $= \frac{3\sqrt{5}}{\sqrt{6}}$ units. **(1/2)**

6. The given lines are non-parallel lines. There is a unique line-segment AB. A lying on one and B on the other, which is at right angles to both the lines, AB is the shortest distance between the lines. Hence, the shortest possible distance between the insects = AB

The position vector of A lying on the line

$\vec{r} = 6\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$

is $(6+\lambda)\hat{i}+(2-2\lambda)\hat{j}+(2+2\lambda)\hat{k}$ for some λ. **(1)**

The position vector of B lying on the line

$\vec{r} = -4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$

is $(-4+3\mu)\hat{i}+(-2\mu)\hat{j}+(-1-2\mu)\hat{k}$ for some μ. **(1)**

$AB = (-10+3\mu-\lambda)\hat{i}+(-2\mu-2+2\lambda)\hat{j}+(-3-2\mu-2\lambda)\hat{k}$

Since, AB is perpendicular to both the lines

$(-10+3\mu-\lambda)+(-2\mu-2+2\lambda)(-2)+(-3-2\mu-2\lambda)2 = 0,$ **(1)**

i.e., $\mu - 3\lambda = 4$ ...(i)

and $(-10+3\mu-\lambda)3+(-2\mu-2+2\lambda)(-2)+(-3-2\mu-2\lambda)(-2) = 0$

i.e., $17\mu - 3\lambda = 20$ ...(ii)

Solving (i) and (ii) for λ and μ, we get $\mu = 1, \lambda = -1$. **(1)**

The position vector of the points, at which they should be so that the distance between them is the shortest, are $5\hat{i}+4\hat{j}$ and $-\hat{i}-2\hat{j}-3\hat{k}$

$AB = -6\hat{i}-6\hat{j}-3\hat{k}$

$\therefore$ The shortest distance $= |AB| = \sqrt{6^2+6^2+3^2} = 9$ **(1)**

7. Eliminating t between the equations, we obtain the equations of the path $\frac{x}{2} = \frac{y}{-4} = \frac{z}{4}$, which are the equations of the line passing through the origin having direction ratios $< 2, -4, 4 >$. This line is the path of the rocket. **(1)**

When t = 10 seconds, the rocket will be at the point (20, −40, 40). **(1)**

Hence, the required distance from the origin at 10 seconds $= \sqrt{20^2+40^2+40^2}$ km $= 20\times3$ km $= 60$ km **(1)**

The distance of the point (20, −40, 40) from the given line

$$= \frac{|(\vec{a}_2-\vec{a}_1)\times\vec{b}|}{|\vec{b}|} = \frac{|-30\hat{j}\times(10\hat{i}-20\hat{j}+10\hat{k})|}{|10\hat{i}-20\hat{j}+10\hat{k}|}\text{km} \quad \textbf{(1)}$$

$$= \frac{|-300\hat{i}+300\hat{k}|}{|10\hat{i}-20\hat{j}+10\hat{k}|}\text{km} = \frac{300\sqrt{2}}{10\sqrt{6}}\text{km} = 10\sqrt{3}\text{ km} \quad \textbf{(1)}$$

8. We have, $\vec{a}_1 = 3\hat{i}+2\hat{j}-4\hat{k}$, $\vec{b}_1 = \hat{i}+2\hat{j}+2\hat{k}$

$\vec{a}_2 = 5\hat{i}-2\hat{j}$, $\vec{b}_2 = 3\hat{i}+2\hat{j}+6\hat{k}$

$\vec{a}_2-\vec{a}_1 = 2\hat{i}-4\hat{j}+4\hat{k}$ **(1)**

$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 2\\ 3 & 2 & 6\end{vmatrix} = \hat{i}(12-4)-\hat{j}(6-6)+\hat{k}(2-6)$ **(1)**

$= 8\hat{i}-4\hat{k}$

And $(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1) = 16-16 = 0$ **(1)**

$\therefore$ The lines are intersecting and the shortest distance between the lines is 0.

Now for point of intersection, consider

$3\hat{i}+2\hat{j}-4\hat{k}+\lambda(\hat{i}+2\hat{j}+2\hat{k}) = 5\hat{i}-2\hat{j}+\mu(3\hat{i}+2\hat{j}+6\hat{k})$

$\Rightarrow$ $3+\lambda = 5+3\mu$ ... (i)

$2+2\lambda = -2+2\mu$ ... (ii)

$-4+2\lambda = 6\mu$ ... (iii) **(1)**

Solving (i) and (ii) we get $\mu = -2$ and $\lambda = -4$.

These values satisfy equation (iii) also.

Now, substituting the value of μ in equation of line, we get

$\vec{r} = 5\hat{i}-2\hat{j}+(-2)(3\hat{i}+2\hat{j}+6\hat{k}) = -\hat{i}-6\hat{j}-12\hat{k}$

$\therefore$ Point of intersection is (−1, −6, −12). **(1)**