

Hence, the required value of $\cot A$ is $\sqrt{3}$.

Thus (a) is correct option.

6. If $\sin \theta = \frac{a}{b}$, then $\cos \theta$ is equal to

(a) $\frac{b}{\sqrt{b^2 - a^2}}$ (b) $\frac{b}{a}$
 (c) $\frac{\sqrt{b^2 - a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2 - a^2}}$

Ans :

We have $\sin \theta = \frac{a}{b} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\text{Base} = \sqrt{b^2 - a^2}$$

So, $\cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{b^2 - a^2}}{b}$

Thus (c) is correct option.

7. If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to

(a) $\cos \beta$ (b) $\cos 2\beta$
 (c) $\sin \alpha$ (d) $\sin 2\alpha$

Ans :

Given, $\cos(\alpha + \beta) = 0 = \cos 90^\circ$ $[\cos 90^\circ = 0]$

$$\alpha + \beta = 90^\circ$$

$$\alpha = 90^\circ - \beta$$

Now, $\sin(\alpha - \beta) = \sin(90^\circ - \beta - \beta)$ h 22

$$= \sin(90^\circ - 2\beta)$$

$$= \cos 2\beta$$

Thus (b) is correct option.

8. If $\cos 9\alpha = \sin \alpha$ and $9\alpha < 90^\circ$, then the value of $\tan 5\alpha$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$
 (c) 1 (d) 0

Ans :

We have $\cos 9\alpha = \sin \alpha$ where $9\alpha < 90^\circ$

$$\sin(90^\circ - 9\alpha) = \sin \alpha$$

$$90^\circ - 9\alpha = \alpha$$

$$10\alpha = 90^\circ \Rightarrow \alpha = 9^\circ$$

$$\tan 5\alpha = \tan(5 \times 9^\circ)$$

$$= \tan 45^\circ = 1 \quad [\tan 45^\circ = 1]$$

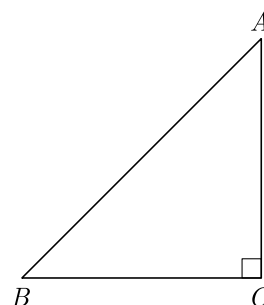
Thus (c) is correct option.

9. If ΔABC is right angled at C , then the value of $\cos(A + B)$ is

(a) 0 (b) 1
 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Ans :

We know that in ΔABC ,



$$\angle A + \angle B + \angle C = 180^\circ$$

But right angled at C i.e., $\angle C = 90^\circ$, thus

$$\angle A + \angle B + 90^\circ = 180^\circ$$

$$A + B = 90^\circ$$

$$\cos(A + B) = \cos 90^\circ = 0$$

Thus (a) is correct option.

10. If $\sin \alpha = \frac{1}{2}$ and $\cos \beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is

(a) 0° (b) 30°
 (c) 60° (d) 90°

Ans :

Given, $\sin \alpha = \frac{1}{2} = \sin 30^\circ \Rightarrow \alpha = 30^\circ$

and $\cos \beta = \frac{1}{2} = \cos 60^\circ \Rightarrow \beta = 60^\circ$

$$\alpha + \beta = 30^\circ + 60^\circ = 90^\circ$$

Thus (d) is correct option.

11. If $4 \tan \theta = 3$, then $\left(\frac{4 \sin \theta - \cos \theta}{4 \sin \theta + \cos \theta}\right)$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
 (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Ans :

Given, $4 \tan \theta = 3$
 $\tan \theta = \frac{3}{4}$... (i)

$$\frac{4 \sin \theta - \cos \theta}{4 \sin \theta + \cos \theta} = \frac{4 \frac{\sin \theta}{\cos \theta} - 1}{4 \frac{\sin \theta}{\cos \theta} + 1} = \frac{4 \tan \theta - 1}{4 \tan \theta + 1}$$

$$= \frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} = \frac{3 - 1}{3 + 1} = \frac{2}{4} = \frac{1}{2}$$

Thus (c) is correct option.

12. If $\sin \theta - \cos \theta = 0$, then the value of $(\sin^4 \theta + \cos^4 \theta)$ is

- (a) 1 (b) $\frac{3}{4}$
 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

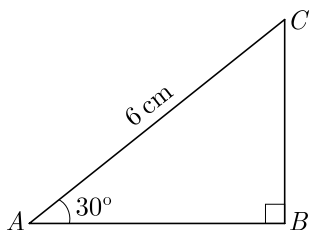
Ans :

Given, $\sin \theta - \cos \theta = 0$
 $\sin \theta = \cos \theta$
 $\sin \theta = \sin(90^\circ - \theta)$
 $\theta = 90^\circ - \theta \Rightarrow \theta = 45^\circ$

Now, $\sin^4 \theta + \cos^4 \theta = \sin^4 45^\circ + \cos^4 45^\circ$
 $= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Thus (c) is correct option.

13. In the adjoining figure, the length of BC is



- (a) $2\sqrt{3}$ cm (b) $3\sqrt{3}$ cm
 (c) $4\sqrt{3}$ cm (d) 3 cm

Ans :

In ΔABC , $\sin 30^\circ = \frac{BC}{AC}$

$$\frac{1}{2} = \frac{BC}{6}$$

$$BC = 3 \text{ cm}$$

Thus (d) is correct option.

14. If $x = p \sec \theta$ and $y = q \tan \theta$, then

- (a) $x^2 - y^2 = p^2 q^2$ (b) $x^2 q^2 - y^2 p^2 = pq$
 (c) $x^2 q^2 - y^2 p^2 = \frac{1}{p^2 q^2}$ (d) $x^2 q^2 - y^2 p^2 = p^2 q^2$

Ans :

We know, $\sec^2 \theta - \tan^2 \theta = 1$

Substituting $\sec \theta = \frac{x}{p}$ and $\tan \theta = \frac{y}{q}$ in above equation we have

$$\left(\frac{x}{p}\right)^2 - \left(\frac{y}{q}\right)^2 = 1$$

$$x^2 q^2 - y^2 p^2 = p^2 q^2$$

Thus (d) is correct option.

15. If $b \tan \theta = a$, the value of $\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta}$ is

- (a) $\frac{a-b}{a^2+b^2}$ (b) $\frac{a+b}{a^2+b^2}$
 (c) $\frac{a^2+b^2}{a^2-b^2}$ (d) $\frac{a^2-b^2}{a^2+b^2}$

Ans :

We have $\tan \theta = \frac{a}{b}$

$$\frac{a \sin \theta - b \cos \theta}{a \sin \theta + b \cos \theta} = \frac{a \frac{\sin \theta}{\cos \theta} - b}{a \frac{\sin \theta}{\cos \theta} + b} = \frac{a \tan \theta - b}{a \tan \theta + b}$$

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

Thus (d) is correct option.

16. $(\cos^4 A - \sin^4 A)$ is equal to

- (a) $1 - 2 \cos^2 A$ (b) $2 \sin^2 A - 1$
 (c) $\sin^2 A - \cos^2 A$ (d) $2 \cos^2 A - 1$

Ans :

$$\cos^4 A - \sin^4 A = (\cos^2 A)^2 - (\sin^2 A)^2$$

$$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$$

$$= (\cos^2 A - \sin^2 A)(1)$$

$$= \cos^2 A - (1 - \cos^2 A)$$

$$= 2 \cos^2 A - 1$$

Thus (d) is correct option.

17. If $\sec 5A = \operatorname{cosec}(A + 30^\circ)$, where $5A$ is an acute angle, then the value of A is

- (a) 15° (b) 5°
 (c) 20° (d) 10°

Ans :

22. Assertion : The value of $\sin \theta = \frac{4}{3}$ is not possible.
Reason : Hypotenuse is the largest side in any right angled triangle.

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- (c) Assertion (A) is true but reason (R) is false.
- (d) Assertion (A) is false but reason (R) is true.

Ans :

$$\sin \theta = \frac{P}{H} = \frac{4}{3}$$

Here, perpendicular is greater than the hypotenuse which is not possible in any right triangle. Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). Thus (a) is correct option.

23. Assertion : $\sin^2 67^\circ + \cos^2 67^\circ = 1$
Reason : For any value of θ , $\sin^2 \theta + \cos^2 \theta = 1$

- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- (c) Assertion (A) is true but reason (R) is false.
- (d) Assertion (A) is false but reason (R) is true.

Ans :

We have $\sin^2 \theta + \cos^2 \theta = 1$
 $\sin^2 67^\circ + \cos^2 67^\circ = 1$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). Thus (a) is correct option.

2. Triangle in which we study trigonometric ratios is called

Ans :
 Right Triangle

3. Cosine of 90° is

Ans :
 Zero

4. Sum of of sine and cosine of angle is one.

Ans :
 Square

5. Reciprocal of $\sin \theta$ is

Ans :
 $\operatorname{cosec} \theta$

6. The value of $\sin A$ or $\cos A$ never exceeds

Ans :
 1

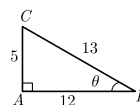
7. sine of $(90^\circ - \theta)$ is

Ans :
 $\cos \theta$

8. If $\sin \theta = \frac{5}{13}$, then the value of $\tan \theta$ is

Ans : [Board 2020 OD Basic]

From $\sin \theta = \frac{5}{13}$ we can draw the figure as given below.



Now, $\tan \theta = \frac{AC}{BC} = \frac{5}{12}$

9. The value of the $(\tan^2 60^\circ + \sin^2 45^\circ)$ is

Ans : [Board 2020 OD Basic]

$$\begin{aligned} \tan^2 60^\circ + \sin^2 45^\circ &= (\sqrt{3})^2 + \left(\frac{1}{\sqrt{2}}\right)^2 \\ &= 3 + \frac{1}{2} = \frac{7}{2} \end{aligned}$$

10. If $\cot \theta = \frac{12}{5}$, then the value of $\sin \theta$ is

Ans : [Board 2020 Delhi Basic]

Given, $\cot \theta = \frac{12}{5} \Rightarrow \tan \theta = \frac{5}{12}$

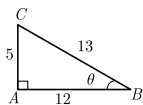
1. FILL IN THE BLANK

1. Maximum value for sine of any angle is

Ans :

1

From $\tan \theta = \frac{5}{12}$ we can draw the figure as given below.



So, $\sin \theta = \frac{AC}{CB} = \frac{5}{13}$

11. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, $A > B$, then the value of A is

Ans : [Board 2020 Delhi Basic]

We have $\tan(A + B) = \sqrt{3}$
 $= \tan 60^\circ$

Hence, $A + B = 60^\circ$
 ...(1)

Again, $\tan(A - B) = \frac{1}{\sqrt{3}}$
 $= \tan 30^\circ$
 $A - B = 30^\circ$... (2)

Adding equation (1) and (2) we get

$$2A = 90^\circ \Rightarrow A = 45^\circ$$

12. The value of $\left(\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}\right) = \dots\dots\dots$

Ans : [Board 2020 Delhi Standard]

$$\begin{aligned} \sin^2 \theta + \frac{1}{1 + \tan^2 \theta} &= \sin^2 \theta + \frac{1}{\sec^2 \theta} \\ &= \sin^2 \theta + \cos^2 \theta = 1 \end{aligned}$$

13. The value of $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta) = \dots\dots\dots$

Ans : [Board 2020 Delhi Standard]

$$\begin{aligned} (1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta) &= \sec^2 \theta (1 - \sin^2 \theta) \\ &= \sec^2 \theta \times \cos^2 \theta \\ &= \frac{1}{\cos^2 \theta} \times \cos^2 \theta = 1 \end{aligned}$$

VERY SHORT ANSWER QUESTIONS

14. Prove that

$$(1 + \tan A - \sec A) \times (1 + \tan A + \sec A) = 2 \tan A$$

Ans : [Board 2020 Delhi Basic]

$$\begin{aligned} \text{LHS} &= (1 + \tan A - \sec A) \times (1 + \tan A + \sec A) \\ &= (1 + \tan A)^2 - \sec^2 A \\ &= 1 + \tan^2 A + 2 \tan A - \sec^2 A \\ &= \sec^2 A + 2 \tan A - \sec^2 A \\ &= 2 \tan A = \text{RHS} \end{aligned}$$

15. If $\tan A = \cot B$, then find the value of $(A + B)$.

Ans : [Board 2020 OD Standard]

$$\begin{aligned} \text{We have } \tan A &= \cot B \\ \tan A &= \tan(90^\circ - B) \\ A &= 90^\circ - B \end{aligned}$$

Thus $A + B = 90^\circ$

16. If $x = 3 \sin \theta + 4 \cos \theta$ and $y = 3 \cos \theta - 4 \sin \theta$ then prove that $x^2 + y^2 = 25$.

Ans : [Board 2020 OD Basic]

$$\begin{aligned} \text{We have } x &= 3 \sin \theta + 4 \cos \theta \\ \text{and } y &= 3 \cos \theta - 4 \sin \theta \\ x^2 + y^2 &= (3 \sin \theta + 4 \cos \theta)^2 + (3 \cos \theta - 4 \sin \theta)^2 \\ &= (9 \sin^2 \theta + 16 \cos^2 \theta + 24 \sin \theta \cos \theta) + \\ &\quad + (9 \cos^2 \theta + 16 \sin^2 \theta - 24 \sin \theta \cos \theta) \\ &= 9(\sin^2 \theta + \cos^2 \theta) + 16(\sin^2 \theta + \cos^2 \theta) \\ &= 9 + 16 = 25 \end{aligned}$$

17. Evaluate $\sin^2 60^\circ - 2 \tan 45^\circ - \cos^2 30^\circ$

Ans : [Board 2019 OD]

$$\begin{aligned} \sin^2 60^\circ - 2 \tan 45^\circ - \cos^2 30^\circ &= \left(\frac{\sqrt{3}}{2}\right)^2 - 2(1) - \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= \frac{3}{4} - 2 - \frac{3}{4} = -2 \end{aligned}$$

18. If $\sin \theta + \sin^2 \theta = 1$ then prove that $\cos^2 \theta + \cos^4 \theta = 1$.

Ans : [Board 2020 OD Basic]

We have $\sin \theta + \sin^2 \theta = 1$

$$\sin \theta + (1 - \cos^2 \theta) = 1$$

$$\sin \theta - \cos^2 \theta = 0$$

$$\sin \theta = \cos^2 \theta$$

Squaring both sides, we get

$$\sin^2 \theta = \cos^4 \theta$$

$$1 - \cos^2 \theta = \cos^4 \theta$$

$$\cos^4 \theta + \cos^2 \theta = 1 \quad \text{Hence Proved}$$

19. In a triangle ABC , write $\cos\left(\frac{B+C}{2}\right)$ in terms of angle A .

Ans : [Board Term-1 2016]

In a triangle $A + B + C = 180^\circ$

$$B + C = 180^\circ - A$$

Thus $\cos\left(\frac{B+C}{2}\right) = \cos\left[\frac{180^\circ - A}{2}\right]$

$$= \cos\left[90 - \frac{A}{2}\right]$$

$$= \sin \frac{A}{2}$$

20. If $\sec \theta \cdot \sin \theta = 0$, then find the value of θ .

Ans : [Board Term-1 2016]

We have $\sec \theta \cdot \sin \theta = 0$

$$\frac{1}{\cos \theta} \cdot \sin \theta = 0$$

$$\frac{\sin \theta}{\cos \theta} = 0$$

$$\tan \theta = 0 = \tan 0^\circ$$

Thus $\theta = 0^\circ$

21. If $\tan 2A = \cot(A + 60^\circ)$, find the value of A where $2A$ is an acute angle.

Ans : [Board Term-1 2016]

We have $\tan 2A = \cot(A + 60^\circ)$

$$\cot(90^\circ - 2A) = \cot(A + 60^\circ)$$

$$90^\circ - 2A = A + 60^\circ \quad \text{h104}$$

$$3A = 30^\circ \Rightarrow A = 10^\circ$$

22. If $\tan(3x + 30^\circ) = 1$ then find the value of x .

Ans : [Board Term-1 2015]

We have $\tan(3x + 30^\circ) = 1 = \tan 45^\circ$

$$3x + 30^\circ = 45^\circ$$

$$x = 5^\circ$$

23. What happens to value of $\cos \theta$ when θ increases from 0° to 90° .

Ans : [Board Term-1 2015]

$\cos \theta$ decreases from 1 to 0.

24. If A and B are acute angles and $\sin A = \cos B$, ^{h108} then find the value of $A + B$.

Ans : [Board Term-1 2016]

We have $\sin A = \cos B$

$$\sin A = \sin(90^\circ - B)$$

$$A = 90^\circ - B$$

$$A + B = 90^\circ$$

25. If $\cos A = \frac{2}{5}$, find the value of $4 + 4 \tan^2 A$.

Ans : [Board SQP 2018]

$$4 + 4 \tan^2 A = 4(1 + \tan^2 A)$$

$$4 \sec^2 A = \frac{4}{\cos^2 A} = \frac{4}{\left(\frac{2}{5}\right)^2} = 4 \times \frac{25}{4} = 25$$

26. If $k + 1 = \sec^2 \theta (1 + \sin \theta)(1 - \sin \theta)$, then find the value of k .

Ans : [Board Term-1 2015]

We have $k + 1 = \sec^2 \theta (1 + \sin \theta)(1 - \sin \theta)$

$$= \sec^2 \theta (1 - \sin^2 \theta)$$

$$= \sec^2 \theta \cdot \cos^2 \theta$$

$$= \sec^2 \theta \times \frac{1}{\sec^2 \theta} \quad \text{h1}$$

$$k + 1 = 1 \Rightarrow k = 1 - 1 = 0$$

Thus $k = 0$

27. Find the value of $\sin^2 41^\circ + \sin^2 49^\circ$

Ans : [Board Term-1 2012, NCERT]

We have

$$\sin^2 41 + \sin^2 49 = \sin^2(90^\circ - 49^\circ) + \sin^2 49^\circ$$

$$= \cos^2 49 + \sin^2 49^\circ$$

$$= 1$$

TWO MARKS QUESTIONS

28. Prove that $1 + \frac{\cot^2 \alpha}{1 + \operatorname{cosec} \alpha} = \operatorname{cosec} \alpha$

Ans : [Board 2020 OD Standard]

$$1 + \frac{\cot^2 \alpha}{1 + \operatorname{cosec} \alpha} = 1 + \frac{\operatorname{cosec}^2 \alpha - 1}{1 + \operatorname{cosec} \alpha}$$

$$= 1 + \frac{(1 + \operatorname{cosec} \alpha)(\operatorname{cosec} \alpha - 1)}{1 + \operatorname{cosec} \alpha}$$

$$= 1 + \operatorname{cosec} \alpha - 1$$

$$= \operatorname{cosec} \alpha \quad \text{Hence Proved}$$

29. Prove that : $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$.

Ans : [Board 2018]

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)} \quad \text{h275}$$

$$= \tan A \frac{[1 - 2(1 - \cos^2 A)]}{(2\cos^2 A - 1)}$$

$$= \tan A \frac{[1 - 2 + 2\cos^2 A]}{(2\cos^2 A - 1)}$$

$$= \tan A \frac{(2\cos^2 A - 1)}{(2\cos^2 A - 1)}$$

$$= \tan A \quad \text{Hence Proved}$$

30. Show that $\tan^4 \theta + \tan^2 \theta = \sec^4 \theta - \sec^2 \theta$

Ans : [Board 2020 OD Standard]

$$\tan^4 \theta + \tan^2 \theta = \tan^2 \theta(1 + \tan^2 \theta)$$

$$= \tan^2 \theta \times \sec^2 \theta$$

$$= (\sec^2 \theta - 1)\sec^2 \theta$$

$$= \sec^4 \theta - \sec^2 \theta \quad \text{Hence Proved}$$

31. Prove that $\sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = \sec \theta - \tan \theta$.

Ans :

$$\text{LHS} = \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = \sqrt{\frac{(1 - \sin \theta)(1 - \sin \theta)}{(1 + \sin \theta)(1 - \sin \theta)}}$$

$$= \sqrt{\frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta}} = \sqrt{\frac{(1 - \sin \theta)^2}{\cos^2 \theta}}$$

$$= \frac{1 - \sin \theta}{\cos \theta} = \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}$$

$$= \sec \theta - \tan \theta = \text{RHS} \quad \text{Hence Proved}$$

32. Prove that : $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \cos^2 \theta - \sin^2 \theta$

Ans : [Board 2020 OD Basic]

$$\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1 - \tan^2 \theta}{\sec^2 \theta}$$

$$= \frac{1}{\sec^2 \theta} - \frac{\tan^2 \theta}{\sec^2 \theta}$$

$$= \cos^2 \theta - \frac{\sin^2 \theta}{\cos^2 \theta} \times \cos^2 \theta$$

$$= \cos^2 \theta - \sin^2 \theta \quad \text{Hence Proved}$$

33. Prove that $\frac{\tan^2 \theta}{1 + \tan^2 \theta} + \frac{\cot^2 \theta}{1 + \cot^2 \theta} = 1$.

Ans : [Board 2020 Delhi Basic]

$$\text{LHS} = \frac{\tan^2 \theta}{1 + \tan^2 \theta} + \frac{\cot^2 \theta}{1 + \cot^2 \theta}$$

$$= \frac{\tan^2 \theta}{\sec^2 \theta} + \frac{\cot^2 \theta}{\operatorname{cosec}^2 \theta}$$

$$= \frac{\frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{1}{\cos^2 \theta}} + \frac{\frac{\cos^2 \theta}{\sin^2 \theta}}{\frac{1}{\sin^2 \theta}}$$

$$= \sin^2 \theta + \cos^2 \theta = 1 = \text{RHS}$$

34. Prove that : $\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} = 2\sec^2 \theta$

Ans : [Board 2020 Delhi Basic]

$$\text{LHS} = \frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta}$$

$$= \frac{(1 - \sin \theta) + (1 + \sin \theta)}{(1 + \sin \theta)(1 - \sin \theta)}$$

$$= \frac{2}{1 - \sin^2 \theta} = 2\sec^2 \theta = \text{RHS}$$

35. Prove that $\frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - 1} + \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta + 1} = 2\sec^2 \theta$.

Ans : [Board 2020 Delhi Basic]

$$\begin{aligned} \text{LHS} &= \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - 1} + \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta + 1} \\ &= \operatorname{cosec} \theta \left[\frac{1}{\operatorname{cosec} \theta - 1} + \frac{1}{\operatorname{cosec} \theta + 1} \right] \\ &= \operatorname{cosec} \theta \left[\frac{\operatorname{cosec} \theta + 1 + \operatorname{cosec} \theta - 1}{(\operatorname{cosec} \theta - 1)(\operatorname{cosec} \theta + 1)} \right] \\ &= \operatorname{cosec} \theta \left(\frac{2 \operatorname{cosec} \theta}{\operatorname{cosec}^2 \theta - 1} \right) \\ &= \frac{2 \operatorname{cosec}^2 \theta}{\operatorname{cosec}^2 \theta - 1} = \frac{2 \operatorname{cosec}^2 \theta}{\cot^2 \theta} \quad \text{h283} \\ &= \frac{2 \times \frac{1}{\sin^2 \theta}}{\frac{\cos^2 \theta}{\sin^2 \theta}} = \frac{2}{\cos^2 \theta} \\ &= 2 \sec^2 \theta = \text{RHS} \quad \text{Hence Proved} \end{aligned}$$

36. If $5 \tan \theta = 3$, then what is the value of $\left(\frac{5 \sin \theta - 3 \cos \theta}{4 \sin \theta + 3 \cos \theta} \right)$?

Ans : [Board 2020 Delhi Basic]

We have $5 \tan \theta = 3 \Rightarrow \tan \theta = \frac{3}{5}$

Dividing numerator and denominator by $\cos \theta$ we have

$$\begin{aligned} \frac{5 \sin \theta - 3 \cos \theta}{4 \sin \theta + 3 \cos \theta} &= \frac{5 \frac{\sin \theta}{\cos \theta} - 3}{4 \frac{\sin \theta}{\cos \theta} + 3} = \frac{5 \tan \theta - 3}{4 \tan \theta + 3} \\ &= \frac{5 \times \frac{3}{5} - 3}{4 \times \frac{3}{5} + 3} = \frac{3 - 3}{\frac{12}{5} + 3} = 0 \end{aligned}$$

37. Evaluate :

$$\frac{3 \tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

Ans : [Board Term-1 2016]

$$\begin{aligned} &\frac{3 \tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ} \\ &= \frac{3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2} \\ &= \frac{3 \times \frac{1}{3} + 3 + 2 - 1}{1} \\ &= 1 + 3 + 2 - 1 = 5 \end{aligned}$$

38. If $\sin(A + B) = 1$ and $\sin(A - B) = \frac{1}{2}$, $0 \leq A + B < 90^\circ$ and $A > B$, then find A and B .

Ans : [Board Term-1 2016]

We have $\sin(A + B) = 1 = \sin 90^\circ$
 $A + B = 90^\circ \quad \dots(1)$

and $\sin(A - B) = \frac{1}{2} = \sin 30^\circ$
 $A - B = 30^\circ \quad \dots(2)$

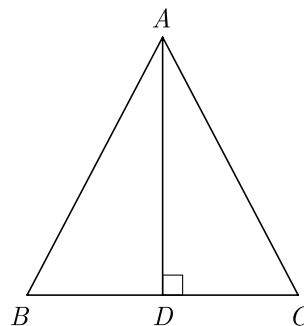
Solving eq. (1) and (2), we obtain

$$A = 60^\circ \text{ and } B = 30^\circ$$

39. Find $\operatorname{cosec} 30^\circ$ and $\cos 60^\circ$ geometrically.

Ans : [Board Term-1 2015]

Let a triangle ABC with each side equal to $2a$ as shown below.



In $\triangle ABC$, $\angle A = \angle B = \angle C = 60^\circ$

Now we draw AD perpendicular to BC , then

$$\triangle BDA \cong \triangle CDA$$

$$BD = CD$$

$$\angle BAD = \angle CAD = 30^\circ \quad \text{by CPCT}$$

$$AD = \sqrt{3}a$$

In $\triangle BDA$, $\operatorname{cosec} 30^\circ = \frac{AB}{BD} = \frac{2a}{a} = 2$

and $\cos 60^\circ = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$

40. Evaluate : $\frac{\sin 90^\circ}{\cos 45^\circ} + \frac{1}{\operatorname{cosec} 30^\circ}$

Ans : [Board Term-1 2013]

We have $\frac{\sin 90^\circ}{\cos 45^\circ} + \frac{1}{\operatorname{cosec} 30^\circ} = \frac{1}{\frac{1}{\sqrt{2}}} + \frac{1}{2}$

$$= \sqrt{2} + \frac{1}{2} = \frac{2\sqrt{2} + 1}{2}$$

41. If $\sqrt{2} \sin \theta = 1$, find the value of $\sec^2 \theta - \operatorname{cosec}^2 \theta$.

Ans : [Board Term-1 2012]

We have $\sqrt{2} \sin \theta = 1$

$$\sin \theta = \frac{1}{\sqrt{2}} = \sin 45^\circ$$

Thus $\theta = 45^\circ$

Now $\sec^2 \theta - \operatorname{cosec}^2 \theta = \sec^2 45^\circ - \operatorname{cosec}^2 45^\circ$

$$= (\sqrt{2})^2 - (\sqrt{2})^2$$

$$= 0$$

42. If $4 \cos \theta = 11 \sin \theta$, find the value of $\frac{11 \cos \theta - 7 \sin \theta}{11 \cos \theta + 7 \sin \theta}$.

Ans : [Board Term-1 2012]

We have $4 \cos \theta = 11 \sin \theta$

or, $\cos \theta = \frac{11}{4} \sin \theta$

Now $\frac{11 \cos \theta - 7 \sin \theta}{11 \cos \theta + 7 \sin \theta} = \frac{11 \times \frac{11}{4} \sin \theta - 7 \sin \theta}{11 \times \frac{11}{4} \sin \theta + 7 \sin \theta}$ h121

$$= \frac{\sin \theta (\frac{121}{4} - 7)}{\sin \theta (\frac{121}{4} + 7)}$$

$$= \frac{121 - 28}{121 + 28} = \frac{93}{149}$$

43. If $\tan(A + B) = \sqrt{3}$, $\tan(A - B) = \frac{1}{\sqrt{3}}$, $0^\circ < A + B \leq 90^\circ$, then find A and B .

Ans : [Board Term-1 2012]

We have $\tan(A + B) = \sqrt{3} = \tan 60^\circ$

$$A + B = 60^\circ \quad \dots(1)$$

Also $\tan(A - B) = \frac{1}{\sqrt{3}} = \tan 30^\circ$

$$A - B = 30^\circ \quad \dots(2)$$

Adding equations (1) and (2), we obtain,

$$2A = 90^\circ$$

$$A = \frac{90^\circ}{2} = 45^\circ$$

Substituting this value of A in equation (1), we get

$$B = 60^\circ - A = 60^\circ - 45^\circ = 15^\circ$$

Hence, $A = 45^\circ$ and $B = 15^\circ$

44. If $\cos(A - B) = \frac{\sqrt{3}}{2}$ and $\sin(A + B) = \frac{\sqrt{3}}{2}$, find $\sin A$ and B , where $(A + B)$ and $(A - B)$ are acute angles.

Ans : [Board Term-1 2012]

We have $\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30^\circ$

$$A - B = 30^\circ \quad \dots(1)$$

Also $\sin(A + B) = \frac{\sqrt{3}}{2} = \sin 60^\circ$

$$A + B = 60^\circ \quad \dots(2)$$

Adding equations (1) and (2), we obtain,

$$2A = 90^\circ$$

$$A = 45^\circ$$

Substituting this value of A in equation (1), we get $B = 15^\circ$

45. Find the value of $\cos 2\theta$, if $2 \sin \theta = \sqrt{3}$.

Ans : [Board Term-1 2012, Set-25]

We have $2 \sin \theta = \sqrt{3}$

$$\sin \theta = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

$$2\theta = 60^\circ$$

Hence, $\cos 2\theta = \cos 60^\circ = \frac{1}{2}$.

46. Find the value of $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$ is it equal to $\sin 90^\circ$ or $\cos 90^\circ$?

Ans : [Board Term-1 2016]

$$\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

It is equal to $\sin 90^\circ = 1$ but not equal to $\cos 90^\circ$ as $\cos 90^\circ = 0$.

h126

47. If $\sqrt{3} \sin \theta - \cos \theta = 0$ and $0^\circ < \theta < 90^\circ$, find the value of θ .

Ans : [Boar Term-1, 2012]

We have

$$\sqrt{3} \sin \theta - \cos \theta = 0 \text{ and } 0^\circ < \theta < 90^\circ$$

$$\sqrt{3} \sin \theta = \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} = \frac{1}{\sqrt{3}}$$

$$\tan \theta = \frac{1}{\sqrt{3}} = \tan 30^\circ \quad \left[\tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\theta = 30^\circ$$

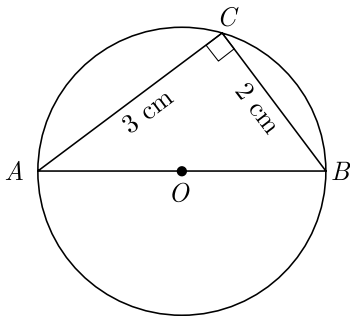
48. Evaluate : $\frac{\cos 45^\circ}{\sec 30^\circ} + \frac{1}{\sec 60^\circ}$

Ans :

[Board Term-1 2012]

$$\begin{aligned} \text{We have } \frac{\cos 45^\circ}{\sec 30^\circ} + \frac{1}{\sec 60^\circ} &= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} + \frac{1}{2} \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \\ &= \frac{\sqrt{6}}{4} + \frac{1}{2} = \frac{\sqrt{6} + 2}{4} \end{aligned}$$

49. In the given figure, AOB is a diameter of a circle with centre O , find $\tan A \tan B$.



Ans :

[Board Term-1 2012]

In ΔABC , $\angle C$ is a angle in a semi-circle, thus

$$\angle C = 90^\circ$$

$$\tan A = \frac{BC}{AC} = \frac{2}{3}$$

and

$$\tan B = \frac{AC}{BC} = \frac{3}{2}$$

$$\tan A \tan B = \frac{2}{3} \times \frac{3}{2} = 1$$

50. If $\sin \phi = \frac{1}{2}$, show that $3 \cos \phi - 4 \cos^3 \phi = 0$.

Ans :

$$\text{We have } \sin \phi = \frac{1}{2}$$

$$\phi = 30^\circ$$

Now substituting this value of θ in LHS we have

$$\begin{aligned} 3 \cos \phi - 4 \cos^3 \phi &= 3 \cos 30^\circ - 4 \cos^3 30^\circ \\ &= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3 \end{aligned}$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$$= 0$$

Hence Proved

51. Express the trigonometric ratio of $\sec A$ and $\tan A$ in terms of $\sin A$.

Ans :

[Board Term-1 2015]

$$\text{We have } \sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1 - \sin^2 A}}$$

$$\text{and } \tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$$

52. Prove that : $\frac{(\sin^4 \theta + \cos^4 \theta)}{1 - 2 \sin^2 \theta \cos^2 \theta} = 1$

Ans :

[Board Term-1 2015]

$$\begin{aligned} \frac{(\sin^4 \theta + \cos^4 \theta)}{1 - 2 \sin^2 \theta \cos^2 \theta} &= \frac{(\sin^2 \theta)^2 + (\cos^2 \theta)^2}{1 - 2 \sin^2 \theta \cos^2 \theta} \\ &= \frac{(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta}{1 - 2 \sin^2 \theta \cos^2 \theta} \\ &= \frac{1 - 2 \sin^2 \theta \cos^2 \theta}{1 - 2 \sin^2 \theta \cos^2 \theta} \\ &= 1 \end{aligned}$$

53. Prove that : $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$

Ans :

[Board Term-1 2015]

We have

$$\begin{aligned} \sec^4 \theta - \sec^2 \theta &= \sec^2 \theta (\sec^2 \theta - 1) \\ &= \sec^2 \theta (\tan^2 \theta) \\ &= (1 + \tan^2 \theta) \tan^2 \theta \\ &= \tan^2 \theta + \tan^4 \theta \end{aligned}$$

Hence Proved.

54. Find the value of θ , if,

$$\frac{\cos \theta}{1 - \sin \theta} + \frac{\cos \theta}{1 + \sin \theta} = 4; \theta \leq 90^\circ$$

Ans :

[Board Term-1 2015]

$$\text{We have } \frac{\cos \theta}{1 - \sin \theta} + \frac{\cos \theta}{1 + \sin \theta} = 4$$

$$\frac{\cos\theta(1 + \sin\theta) + \cos\theta(1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)} = 4$$

$$\frac{\cos\theta[1 + \sin\theta + 1 - \sin\theta]}{1 - \sin^2\theta} = 4$$

$$\frac{\cos\theta(2)}{\cos^2\theta} = 4$$

$$\frac{1}{\cos\theta} = 2$$

$$\cos\theta = \frac{1}{2}$$

$$\cos\theta = \cos 60^\circ$$

Thus $\theta = 60^\circ$.

55. Prove that : $-1 + \frac{\sin A \sin(90^\circ - A)}{\cot(90^\circ - A)} = -\sin^2 A$

Ans : [Board Term-1 2012]

$$-1 + \frac{\sin A \sin(90^\circ - A)}{\cot(90^\circ - A)} = -\sin^2 A$$

$$\frac{\sin A \sin(90^\circ - A)}{\cot(90^\circ - A)} = 1 - \sin^2 A$$

$$\frac{\sin A \cos A}{\tan A} = \cos^2 A$$

$$\frac{\sin A \cos A}{\frac{\sin A}{\cos A}} = \cos^2 A$$

$$\frac{\cos A}{\sin A} \sin A \cos A = \cos^2 A$$

$$\cos^2 A = \cos^2 A \quad \text{Hence Proved.}$$

56. Prove that : $\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \operatorname{cosec} A - \cot A$

Ans : [Board Term-1 2012]

$$\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \sqrt{\frac{1 - \cos A}{1 + \cos A} \times \frac{1 - \cos A}{1 - \cos A}}$$

$$= \sqrt{\frac{(1 - \cos A)^2}{(1 - \cos^2 A)}}$$

$$= \sqrt{\frac{(1 - \cos A)^2}{\sin^2 A}}$$

$$= \frac{1 - \cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$$= \operatorname{cosec} A - \cot A \quad \text{Hence Proved.}$$

57. If $\sin\theta - \cos\theta = \frac{1}{2}$, then find the value of $\sin\theta + \cos\theta$.

Ans : [Board Term-1 2013]

We have $\sin\theta - \cos\theta = \frac{1}{2}$

Squaring both sides, we get

$$(\sin\theta - \cos\theta)^2 = \left(\frac{1}{2}\right)^2$$

$$\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = \frac{1}{4}$$

$$1 - 2\sin\theta\cos\theta = \frac{1}{4}$$

$$2\sin\theta\cos\theta = 1 - \frac{1}{4} = \frac{3}{4}$$

Again, $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$$= 1 + 2\sin\theta\cos\theta$$

$$= 1 + \frac{3}{4} = \frac{7}{4}$$

Thus $\sin\theta + \cos\theta = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$

58. If θ be an acute angle and $5 \operatorname{cosec}\theta = 7$, then evaluate $\sin\theta + \cos^2\theta - 1$.

Ans : [Board Term-1 2012]

We have $5 \operatorname{cosec}\theta = 7$

$$\operatorname{cosec}\theta = \frac{7}{5}$$

$$\sin\theta = \frac{5}{7} \quad \left[\operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$

$$\sin\theta + \cos^2\theta - 1 = \sin\theta - (1 - \cos^2\theta)$$

$$= \sin\theta - \sin^2\theta \quad [\sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{5}{7} - \left(\frac{5}{7}\right)^2 = \frac{35 - 25}{49} = \frac{10}{49}$$

59. If $\sin A = \frac{\sqrt{3}}{2}$, find the value of $2 \cot^2 A - 1$.

Ans : [Board Term-1 2012]

Using $\cot^2\theta = -1 + \operatorname{cosec}^2\theta$ we have

$$2 \cot^2 A - 1 = 2(\operatorname{cosec}^2 A - 1) - 1$$

$$= \frac{2}{\sin^2 A} - 3$$

$$= \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} - 3 = \frac{8}{3} - 3 = \frac{-1}{3}$$

Thus $2 \cot^2 A - 1 = \frac{-1}{3}$

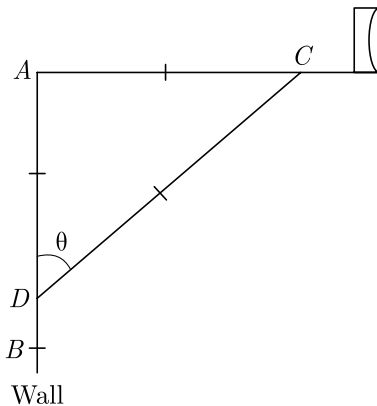
THREE MARKS QUESTIONS

60. Show that : $\frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta)\tan(30^\circ - \theta)} = 1$

Ans : [Board 2020 OD Standard]

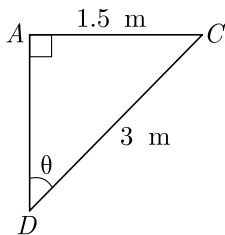
$$\begin{aligned} \text{LHS} &= \frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta)\tan(30^\circ - \theta)} \\ &= \frac{\cos^2(45^\circ + \theta) + \sin^2(90^\circ - 45^\circ + \theta)}{\tan(60^\circ + \theta)\cot(90^\circ - 30^\circ + \theta)} \\ &= \frac{\cos^2(45^\circ + \theta) + \sin^2(45^\circ + \theta)}{\tan(60^\circ + \theta)\cot(60^\circ + \theta)} \\ &= \frac{1}{1} = 1 = \text{RHS} \end{aligned}$$

61. The rod of TV disc antenna is fixed at right angles to wall AB and a rod CD is supporting the disc as shown in Figure. If AC = 1.5 m long and CD = 3 m, find (i) tan θ (ii) sec θ + cosec θ.



Ans : [Board 2020 Delhi Standard]

From the given information we draw the figure as below



In right angle triangle ΔCAD, applying Pythagoras theorem,

$$\begin{aligned} AD^2 + AC^2 &= DC^2 \\ AD^2 + (1.5)^2 &= (3)^2 \\ AD^2 &= 9 - 2.25 = 6.75 \\ AD &= \sqrt{6.75} = 2.6 \text{ m (Approx)} \end{aligned}$$

(i) $\tan \theta = \frac{AC}{AD} = \frac{1.5}{2.6} = \frac{15}{26}$

(ii) $\sec \theta + \operatorname{cosec} \theta = \frac{CD}{AD} + \frac{CD}{AC} = \frac{3}{2.6} + \frac{3}{1.5} = \frac{41}{13}$

62. Prove that : $\frac{\cot \theta + \operatorname{cosec} \theta - 1}{\cot \theta - \operatorname{cosec} \theta + 1} = \frac{1 + \cot \theta}{\sin \theta}$

Ans : [Board 2020 Delhi Standard]

$$\begin{aligned} \text{LHS} &= \frac{\cot \theta + \operatorname{cosec} \theta - 1}{\cot \theta - \operatorname{cosec} \theta + 1} \\ &= \frac{\frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta} - 1}{\frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta} + 1} \\ &= \frac{\sin \theta (\cos \theta + 1 - \sin \theta)}{\sin \theta (\cos \theta - 1 + \sin \theta)} \\ &= \frac{\sin \theta \cos \theta + \sin \theta - \sin^2 \theta}{\sin \theta (\cos \theta + \sin \theta - 1)} \\ &= \frac{\sin \theta \cos \theta + \sin \theta - (1 - \cos^2 \theta)}{\sin \theta (\cos \theta + \sin \theta - 1)} \\ &= \frac{\sin \theta (\cos \theta + 1) - (1 - \cos^2 \theta)}{\sin \theta (\cos \theta + \sin \theta - 1)} \\ &= \frac{(1 + \cos \theta) (\sin \theta - 1 + \cos \theta)}{\sin \theta (\cos \theta + \sin \theta - 1)} \\ &= \frac{1 + \cos \theta}{\sin \theta} = \text{RHS} \end{aligned}$$

63. If $\sin \theta + \cos \theta = \sqrt{2}$ prove that $\tan \theta + \cot \theta = 2$

Ans : [Board 2020 OD Standard]

We have $\sin \theta + \cos \theta = \sqrt{2}$
Squaring both the sides, we get

$$\begin{aligned} (\sin \theta + \cos \theta)^2 &= (\sqrt{2})^2 \\ \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta &= 2 \\ 1 + 2 \sin \theta \cos \theta &= 2 \\ 2 \sin \theta \cos \theta &= 1 \\ \sin \theta \cos \theta &= \frac{1}{2} \quad \dots(1) \end{aligned}$$

Now $\tan \theta + \cot \theta = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$

$$\begin{aligned} &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \\ &= \frac{1}{\sin \theta \cos \theta} = \frac{1}{\frac{1}{2}} = 2 = \text{RHS} \end{aligned}$$

64. If $\sin \theta + \cos \theta = \sqrt{3}$, then prove that $\tan \theta + \cot \theta = 1$.

Ans : [Board 2020 SQP Standard]

Given, $\sin\theta + \cos\theta = \sqrt{3}$

Squaring above equation, we have

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$

$$1 + 2\sin\theta\cos\theta = 3$$

$$2\sin\theta\cos\theta = 3 - 1 = 2$$

$$\sin\theta\cos\theta = 1$$

Now, $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta}$$

Substituting value of $\sin\theta\cos\theta$ we have

$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta} = \frac{1}{1} = 1$$

65. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that $\tan\theta = 1$ or $\frac{1}{2}$.

Ans : [Board 2020 OD Standard]

We have, $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Dividing by $\sin^2\theta$ on both sides, we get

$$\frac{1}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{3\sin\theta\cos\theta}{\sin^2\theta}$$

$$\frac{1}{\sin^2\theta} + 1 = 3\cot\theta$$

$$\operatorname{cosec}^2\theta + 1 = 3\cot\theta$$

$$1 + \cot^2\theta + 1 = 3\cot\theta$$

$$\cot^2\theta - 3\cot\theta + 2 = 0$$

$$\cot^2\theta - 2\cot\theta - \cot\theta + 2 = 0$$

$$\cot\theta(\cot\theta - 2) - 1(\cot\theta - 2) = 0$$

$$(\cot\theta - 2)(\cot\theta - 1) = 0$$

$$\cot\theta = 1 \text{ or } 2$$

$$\tan\theta = 1 \text{ or } \frac{1}{2}.$$

66. Prove that

$$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$$

Ans : [Board 2019 Delhi Standard]

$$\text{LHS} = (\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$$

$$= (\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta) +$$

h292

$$+(\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta)$$

$$= (\sin^2\theta + \cos^2\theta) + (\operatorname{cosec}^2\theta + \sec^2\theta)$$

$$+ 2\sin\theta \times \frac{1}{\sin\theta} + 2\cos\theta \times \frac{1}{\cos\theta}$$

$$= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 2 + 2$$

$$= 7 + \tan^2\theta + \cot^2\theta$$

$$= \text{RHS}$$

67. Prove that $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

Ans : [Board 2019 Delhi]

$$\text{LHS} = (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$$

$$= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right) \left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right)$$

$$= \left(\frac{\sin A + \cos A - 1}{\sin A}\right) \left(\frac{\cos A + \sin A + 1}{\cos A}\right)$$

$$= \frac{(\sin A + \cos A - 1)(\cos A + \sin A + 1)}{\sin A \cos A} \quad \text{h293}$$

$$= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cos A}$$

$$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A - 1}{\sin A \cos A}$$

$$= \frac{1 + 2\sin A \cos A - 1}{\sin A \cos A}$$

$$= 2 = \text{RHS}$$

68. Prove that $\frac{\sin A - \cos A - 1}{\sin A + \cos A - 1} = \frac{1}{\sec A - \tan A}$

Ans : [Board 2019 Delhi]

$$\text{LHS} = \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$$

$$= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \times \frac{1 + \sin A}{1 + \sin A}$$

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\sin A + \cos A - 1 + \sin^2 A + \cos A \sin A - \sin A}$$

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{-1 + \cos A + (1 - \cos^2 A) + \sin A \cos A}$$

$$= \frac{(\sin A - \cos A + 1)(1 + \sin A)}{\cos A(1 - \cos A + \sin A)}$$

$$= \frac{1 + \sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

$$= \frac{(\sec A + \tan A)}{(\sec A - \tan A)} \times (\sec A - \tan A)$$

$$= \frac{\sec^2 A - \tan^2 A}{\sec A - \tan A}$$

$$= \frac{1}{\sec A - \tan A} = \text{RHS}$$

69. Prove that: $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$

Ans : [Board 2020 Delhi Standard]

$$\text{LHS} = 2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2[(\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta)] +$$

$$- 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2(\sin^4\theta - \sin^2\theta\cos^2\theta + \cos^4\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= 2(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$$

$$= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta\cos^2\theta + 1$$

$$= -(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) + 1$$

$$= -(\sin^2\theta + \cos^2\theta)^2 + 1$$

h295

$$= -1 + 1 = 0 = \text{RHS}$$

70. Prove that $\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}$

Ans : [Board 2019 Delhi]

$$\text{LHS} = \frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - 1} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A \sin^2 A}}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{1 - \cos^2 A - \cos^2 A}$$

$$= \frac{1}{1 - 2\cos^2 A}$$

$$= \text{RHS}$$

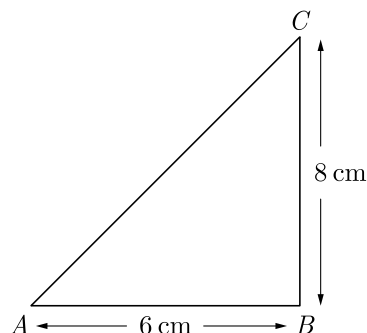
71. If in a triangle ABC right angled at B , $AB = 6$ units and $BC = 8$ units, then find the value of

$$\sin A \cos C + \cos A \sin C.$$

Ans :

[Board Term-1 2016]

As per question statement figure is shown below.



We have $AC^2 = 8^2 + 6^2 = 100$

$$AC = 10 \text{ cm}$$

Now $\sin A = \frac{BC}{AC} = \frac{8}{10};$

$$\cos A = \frac{AB}{AC} = \frac{6}{10}$$

and $\sin C = \frac{AB}{AC} = \frac{6}{10};$

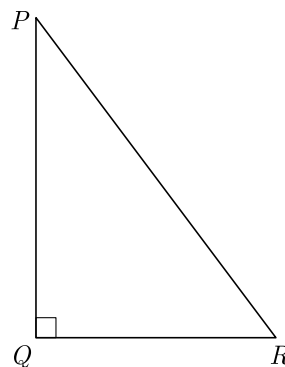
$$\cos C = \frac{BC}{AC} = \frac{8}{10}$$

$$\text{Thus } \sin A \cos C + \cos A \sin C = \frac{8}{10} \times \frac{8}{10} + \frac{6}{10} \times \frac{6}{10}$$

$$= \frac{64}{100} + \frac{36}{100}$$

$$= \frac{100}{100} = 1$$

72. In the given $\angle PQR$, right-angled at Q , $QR = 9$ cm and $PR - PQ = 1$ cm. Determine the value of $\sin R + \cos R$.



Ans :

[Board Term-1 2015]

Using Pythagoras theorem we have

$$PQ^2 + QR^2 = PR^2$$

$$PQ^2 + 9^2 = (PQ + 1)^2$$

$$PQ^2 + 81 = (PQ + 1)^2$$

$$PQ^2 + 81 = PQ^2 + 1 + 2PQ$$

$$PQ = 40$$

Since $PR - PQ = 1$, thus,

$$PR = 1 + 40 = 41$$

$$\sin R + \cos R = \frac{40}{41} + \frac{9}{41} = \frac{49}{41}$$

73. If $\cos(40^\circ + x) = \sin 30^\circ$, find the value of x .

Ans :

[Board Term-1 2015]

We have

$$\cos(40^\circ - x) = \sin 30^\circ$$

$$\cos(40^\circ + x) = \sin(90^\circ - 60^\circ)$$

$$\cos(40^\circ + x) = \cos 60^\circ$$

$$40^\circ + x = 60^\circ$$

$$x = 60^\circ - 40^\circ = 20^\circ$$

Thus $x = 20^\circ$.74. Evaluate : $\frac{5 \cos^2 60^\circ + 4 \cos^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 60^\circ}$

Ans :

[Board Term-1 2013]

$$\frac{5 \cos^2 60^\circ + 4 \cos^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 60^\circ}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{\sqrt{3}}{2}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \frac{\frac{5}{4} + 3 - 1}{\frac{1}{4} + \frac{1}{4}}$$

$$= \frac{\frac{5}{4} + 2}{\frac{1}{2}} = \frac{\frac{13}{4}}{\frac{1}{2}} = \frac{13}{2}$$

75. Verify : $\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{\sin \theta}{1 + \cos \theta}$, for $\theta = 60^\circ$

Ans :

$$\text{LHS} = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \sqrt{\frac{1 - \cos 60^\circ}{1 + \cos 60^\circ}}$$

$$= \sqrt{\frac{1 - \frac{1}{2}}{1 + \frac{1}{2}}} = \sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}} = \frac{1}{\sqrt{3}} \quad \left(\cos 60^\circ = \frac{1}{2}\right)$$

$$\text{RHS} = \frac{\sin \theta}{1 + \cos \theta} = \frac{\sin 60^\circ}{1 + \cos 60^\circ}$$

$$= \frac{\frac{\sqrt{3}}{2}}{1 + \frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{1}{\sqrt{3}}$$

$$\text{RHS} = \text{LHS}$$

Hence, relation is verified for $\theta = 60^\circ$.76. If $\tan A + \cot A = 2$, then find the value of $\tan^2 A + \cot^2 A$.

Ans :

[Board Term-1 2015]

$$\text{We have} \quad \tan A + \cot A = 2$$

Squaring both sides, we have

$$(\tan A + \cot A)^2 = (2)^2$$

$$\tan^2 A + \cot^2 A + 2 \tan A \cot A = 4$$

$$\tan^2 A + \cot^2 A + 2 \tan A \times \frac{1}{\tan A} = 4$$

$$\tan^2 A + \cot^2 A + 2 = 4$$

$$\tan^2 A + \cot^2 A = 4 - 2$$

$$\tan^2 A + \cot^2 A = 2$$

77. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, show that $\cos \theta - \sin \theta = \sqrt{2} \cos \theta$.

Ans :

[Board Term-1 2011]

$$\text{We have} \quad \cos \theta + \sin \theta = \sqrt{2} \cos \theta$$

$$\text{We have} \quad \sin \theta = \sqrt{2} \cos \theta - \cos \theta$$

$$= (\sqrt{2} - 1) \cos \theta$$

$$= \frac{(\sqrt{2} - 1)(\sqrt{2} + 1)}{(\sqrt{2} + 1)} \cos \theta$$

$$\text{Thus} \quad \sin \theta = \frac{1}{\sqrt{2} + 1} \cos \theta$$

$$(\sqrt{2} + 1) \sin \theta = \cos \theta$$

$$\sqrt{2} \sin \theta + \sin \theta = \cos \theta$$

$$\cos \theta - \sin \theta = \sqrt{2} \sin \theta \quad \text{Hence proved.}$$

78. Prove that : $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$.

Ans :

[Board Term-1 2013, 2011]

$$\text{LHS} = \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$$

$$= \frac{\cos A}{1 - \left(\frac{\sin A}{\cos A}\right)} + \frac{\sin A}{1 - \left(\frac{\cos A}{\sin A}\right)}$$

$$\begin{aligned}
 &= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A} \\
 &= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A} \\
 &= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A} \\
 &= \frac{(\cos A - \sin A)(\cos A + \sin A)}{(\cos A - \sin A)} \\
 &= \cos A + \sin A \\
 &= \sin A + \cos A \\
 &= \text{RHS}
 \end{aligned}$$

Hence proved.

We have $AC - AB = 1$

Let $AB = x$, then we have

$$AC = x + 1$$

Now $AC^2 = AB^2 + BC^2$

$$(x + 1)^2 = x^2 + 5^2$$

$$x^2 + 2x + 1 = x^2 + 25$$

$$2x = 24$$

$$x = \frac{24}{2} = 12 \text{ cm}$$

Hence, $AB = 12$ cm and $AC = 13$ cm

Now $\sin C = \frac{AB}{AC} = \frac{12}{13}$

$$\cos C = \frac{BC}{AC} = \frac{5}{13}$$

Now $\frac{1 + \sin C}{1 + \cos C} = \frac{1 + \frac{12}{13}}{1 + \frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}$

80. Prove that : $\frac{\cos A}{1 + \tan A} - \frac{\sin A}{1 + \cot A} = \cos A - \sin A$

Ans :

[Board Term-1 2016]

$$\begin{aligned}
 &\frac{\cos A}{1 + \tan A} - \frac{\sin A}{1 + \cot A} \\
 &= \frac{\cos A}{1 + \frac{\sin A}{\cos A}} - \frac{\sin A}{1 + \frac{\cos A}{\sin A}} \\
 &= \frac{\cos^2 A}{\cos A + \sin A} - \frac{\sin^2 A}{\sin A + \cos A} \\
 &= \frac{\cos^2 A - \sin^2 A}{(\sin A + \cos A)} \\
 &= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\sin A + \cos A} \\
 &= \cos A - \sin A
 \end{aligned}$$

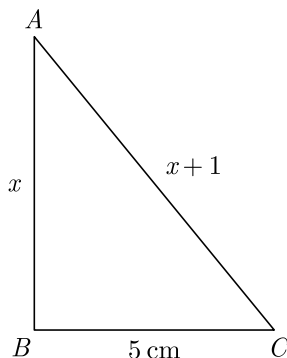
Hence Proved.

79. In ΔABC , $\angle B = 90^\circ$, $BC = 5$ cm, $AC - AB = 1$,
Evaluate : $\frac{1 + \sin C}{1 + \cos C}$.

Ans :

[Board Term-1 2011]

As per question we have drawn the figure given below.



81. If $b \cos \theta = a$, then prove that $\operatorname{cosec} \theta + \cot \theta = \sqrt{\frac{b+a}{b-a}}$.

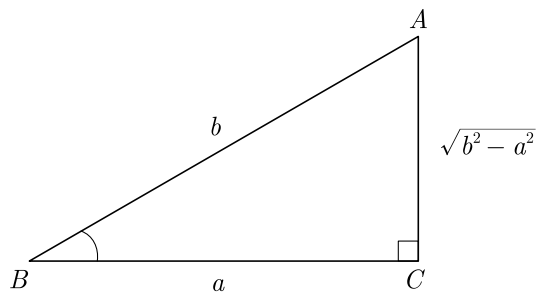
Ans :

[Board Term-1 2015]

We have $b \cos \theta = a$

or, $\cos \theta = \frac{a}{b}$

Now consider the triangle shown below.



$$AC^2 = AB^2 - BC^2$$

or, $\cos \theta = \frac{a}{b}$

$$AC = \sqrt{b^2 - a^2}$$

Now $\operatorname{cosec} \theta = \frac{b}{\sqrt{b^2 - a^2}}, \cot \theta = \frac{a}{\sqrt{b^2 - a^2}}$

$$\operatorname{cosec} \theta + \cot \theta = \frac{b+a}{\sqrt{b^2 - a^2}} = \sqrt{\frac{b+a}{b-a}}$$

82. Prove that : $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 - \cos \theta} = \tan \theta$

Ans :

[Bard Term-1 2015]

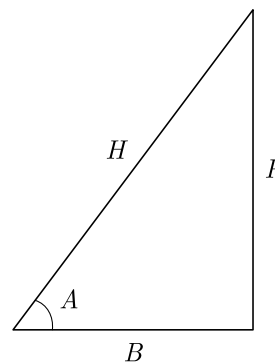
$$\begin{aligned} \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 - \cos \theta} &= \frac{\sin \theta(1 - 2 \sin^2 \theta)}{\cos \theta(2 \cos^2 \theta - 1)} \\ &= \frac{\sin \theta(\sin^2 \theta + \cos^2 \theta - 2 \sin^2 \theta)}{\cos \theta(2 \cos^2 \theta - \sin^2 \theta - \cos^2 \theta)} \\ &= \frac{\tan \theta(\cos^2 \theta - \sin^2 \theta)}{(\cos^2 \theta - \sin^2 \theta)} \\ &= \tan \theta \end{aligned}$$

83. When is an equation called 'an identity'. Prove the trigonometric identity $1 + \tan^2 A = \sec^2 A$.

Ans :

[Board Term-1 2015, NCERT]

Equations that are true no matter what value is plugged in for the variable. On simplifying an identity equation, one always get a true statement. Consider the triangle shown below.



Let $\tan A = \frac{P}{B}$ and $\sec A = \frac{H}{B}$

$$H^2 = P^2 + B^2$$

Now $1 + \tan^2 A = 1 + \left(\frac{P}{B}\right)^2 = 1 + \frac{P^2}{B^2}$

$$= \frac{B^2 + P^2}{B^2} = \frac{H^2}{B^2}$$

$$= \left(\frac{H}{B}\right)^2$$

$$= \sec^2 A$$

Hence Proved.

84. Prove that : $(\cot \theta - \operatorname{cosec} \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

Ans :

[Board Term-1 2015]

$$\cot \theta - \operatorname{cosec} \theta = \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}$$

$$(\cot \theta - \operatorname{cosec} \theta)^2 = \left(\frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right)^2$$

$$= \left(\frac{\cos \theta - 1}{\sin \theta}\right)^2$$

$$= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \quad [[\sin^2 \theta + \cos^2 \theta = 1]]$$

$$= \frac{(1 - \cos \theta)^2}{(1 - \cos^2 \theta)}$$

$$= \frac{(1 - \cos \theta)(1 - \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)}$$

$$= \frac{1 - \cos \theta}{1 + \cos \theta}$$

Hence Proved.

85. Prove that :

$$(\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta) = 1$$

Ans :

[Board Term-1 2015]

$$\text{LHS} = (\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta)$$

$$\begin{aligned} &= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) \\ &= \left(\frac{1 - \sin^2\theta}{\sin\theta}\right)\left(\frac{1 - \cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cos\theta}\right) \\ &= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \left(\frac{1}{\sin\theta \cos\theta}\right) \quad [\sin^2\theta + \cos^2\theta = 1] \\ &= \cos\theta \sin\theta \times \frac{1}{\sin\theta \cos\theta} = 1 \end{aligned}$$

86. Show that :

$$\operatorname{cosec}^2\theta - \tan^2(90^\circ - \theta) = \sin^2\theta + \sin(90^\circ - \theta)$$

Ans : [Board Term-1 2013]

$$\begin{aligned} &\operatorname{cosec}^2\theta - \tan^2(90^\circ - \theta) \\ &= \operatorname{cosec}^2\theta - \cot^2\theta \\ &= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta} \\ &= \frac{1 - \cos^2\theta}{\sin^2\theta} = \frac{\sin^2\theta}{\sin^2\theta} \\ &= 1 \\ &= \sin^2\theta + \cos^2\theta \\ &= \sin^2\theta + \sin^2(90^\circ - \theta) \end{aligned}$$

Hence Proved

87. Prove that : $\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta$

Ans : [Board Term-1 2013]

We have

$$\begin{aligned} &\frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta - 1} - \frac{\operatorname{cosec}^2\theta}{\operatorname{cosec}\theta + 1} = \operatorname{cosec}^2\theta \left[\frac{1}{\frac{1}{\sin\theta} - 1} - \frac{1}{\frac{1}{\sin\theta} + 1} \right] \\ &= \operatorname{cosec}^2\theta \left[\frac{\sin\theta}{1 - \sin\theta} - \frac{\sin\theta}{1 + \sin\theta} \right] \\ &= \frac{1}{\sin^2\theta} \sin\theta \left[\frac{(1 + \sin\theta) - (1 - \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)} \right] \\ &= \frac{1}{\sin\theta} \left[\frac{2\sin\theta}{1 - \sin^2\theta} \right] \\ &= \frac{2}{\cos^2\theta} = 2\sec^2\theta \end{aligned}$$

Hence Proved

88. Prove that :

$$\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$$

Ans : [Board Term-1 2011]

$$\begin{aligned} &\frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A} \\ &\frac{1}{\operatorname{cosec} A - \cot A} + \frac{1}{\operatorname{cosec} A + \cot A} = \frac{1}{\sin A} + \frac{1}{\sin A} \\ &\frac{1}{\operatorname{cosec} A - \cot A} + \frac{1}{\operatorname{cosec} A + \cot A} = \frac{2}{\sin A} \\ &\frac{\operatorname{cosec} A + \cot A + \operatorname{cosec} A - \cot A}{(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)} = \frac{2}{\sin A} \end{aligned}$$

$$\frac{2 \operatorname{cosec} A}{\operatorname{cosec}^2 A - \cot^2 A} = \frac{2}{\sin A}$$

$$\frac{2 \cdot \frac{1}{\sin A}}{1} = \frac{2}{\sin A}$$

$$\frac{2}{\sin A} = \frac{2}{\sin A} \quad \text{Hence Proved.}$$

89. If $\sec\theta = x + \frac{1}{4x}$ prove that $\sec\theta + \tan\theta = 2x$ or, $\frac{1}{2x}$

Ans : [Board Term-1 2011]

We have $\sec\theta = x + \frac{1}{4x}$ (1)

Squaring both side we have

$$\sec^2\theta = x^2 + 2x \cdot \frac{1}{4x} + \frac{1}{16x^2}$$

$$1 + \tan^2\theta = x^2 + \frac{1}{2} + \frac{1}{16x^2}$$

$$\tan^2\theta = x^2 + \frac{1}{2} + \frac{1}{16x^2} - 1$$

$$= x^2 - \frac{1}{2} + \frac{1}{16x^2}$$

$$= x^2 - 2x \cdot \frac{1}{4x} + \frac{1}{16x^2}$$

$$\tan^2\theta = \left(x - \frac{1}{4x}\right)^2$$

Taking square root both sides we obtain

$$\tan\theta = \pm \left(x - \frac{1}{4x}\right)$$

Now $\tan\theta = x - \frac{1}{4x}$ (2)

or $\tan\theta = -\left(x - \frac{1}{4x}\right) = -x + \frac{1}{4x}$ (3)

Adding (1) and (2) we have

$$\tan \theta + \sec \theta = 2x$$

Adding (1) and (3) we have

$$\sec \theta + \tan \theta = \frac{1}{4x} + \frac{1}{4x} = \frac{1}{2x} \quad \text{Hence proved.}$$

90. Prove that : $\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} + \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = \frac{2}{2 \sin^2 \theta - 1}$

Ans : [Board Term-1 2011]

$$\begin{aligned} \text{LHS} &= \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} + \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} \\ &= \frac{(\sin \theta - \cos \theta)^2 + (\sin \theta + \cos \theta)^2}{\sin^2 \theta - \cos^2 \theta} \\ &= \frac{(\sin^2 \theta + \cos^2 \theta) - 2 \sin \theta \cos \theta + (\sin^2 \theta + \cos^2 \theta) + 2 \sin \theta \cos \theta}{\sin^2 \theta - (1 - \sin^2 \theta)} \\ &= \frac{1 + 1}{\sin^2 \theta - 1 + \sin^2 \theta} \\ &= \frac{2}{2 \sin^2 \theta - 1} = \text{RHS} \end{aligned}$$

Hence Proved.

91. If $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$ and $x \sin \theta = y \cos \theta$, prove that $x^2 + y^2 = 1$.

Ans : [Board Term-1 2011]

We have $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$ (1)

and $x \sin \theta = y \cos \theta$

or, $x = \frac{y \cos \theta}{\sin \theta}$ (2)

Eliminating x from equation (1) and (2) we obtain,

$$\begin{aligned} \frac{y \cos \theta}{\sin \theta} \sin^3 \theta + y \cos^3 \theta &= \sin \theta \cos \theta \\ y \cos \theta \sin^2 \theta + y \cos^3 \theta &= \sin \theta \cos \theta \\ y \cos \theta [\sin^2 \theta + \cos^2 \theta] &= \sin \theta \cos \theta \\ y(\sin^2 \theta + \cos^2 \theta) &= \sin \theta \\ y &= \sin \theta \quad \dots(3) \end{aligned}$$

Substituting this value of y in equation (2) we have,

$$x = \cos \theta \quad (4)$$

Squaring and adding equation (3) and (4), we get

$$x^2 + y^2 = \cos^2 \theta + \sin^2 \theta = 1 \quad \text{Hence Proved.}$$

92. Prove that $\frac{\cos^3 \theta + \sin^3 \theta}{\cos \theta + \sin \theta} + \frac{\cos^3 \theta - \sin^3 \theta}{\cos \theta - \sin \theta} = 2$

Ans : [Board Term-1 2011]

$$X = \frac{\cos^3 \theta + \sin^3 \theta}{\cos \theta + \sin \theta}$$

$$= \frac{(\cos \theta + \sin \theta)(\cos^2 \theta + \sin^2 \theta - \sin \theta \cos \theta)}{(\cos \theta + \sin \theta)}$$

$$= (1 - \sin \theta \cos \theta)$$

$$Y = \frac{\cos^3 \theta - \sin^3 \theta}{\cos \theta - \sin \theta}$$

$$= \frac{(\cos \theta - \sin \theta)(\cos^2 \theta + \sin^2 \theta + \sin \theta \cos \theta)}{(\cos \theta - \sin \theta)}$$

$$= (1 + \sin \theta \cos \theta)$$

Now given expression

$$X + Y = \frac{\cos^3 \theta + \sin^3 \theta}{\cos \theta + \sin \theta} + \frac{\cos^3 \theta - \sin^3 \theta}{\cos \theta - \sin \theta}$$

$$= (1 - \sin \theta \cos \theta) + (1 + \sin \theta \cos \theta)$$

$$= 2 - \sin \theta \cos \theta + \sin \theta \cos \theta$$

$$= 2 = \text{RHS}$$

Hence Proved.

93. Express : $\sin A, \tan A$ and $\text{cosec } A$ in terms of $\sec A$.

Ans : [Board Term-1 2011]

(1) $\sin^2 A + \cos^2 A = 1$

$$\sin A = \sqrt{1 - \cos^2 A}$$

$$= \sqrt{1 - \frac{1}{\sec^2 A}}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

(2) $\tan A = \frac{\sin A}{\cos A} = \sin A \sec A$

$$= \frac{\sqrt{\sec^2 A - 1}}{\sec A} \times \sec A$$

$$= \sqrt{\sec^2 A - 1}$$

(3) $\text{cosec } A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$

94. If $\sin \theta + \cos \theta = \sqrt{2}$, then evaluate $\tan \theta + \cot \theta$.

Ans : [Board SQP 2018]

We have $\sin \theta + \cos \theta = \sqrt{2}$

Squaring both sides, we get

$$(\sin \theta + \cos \theta)^2 = (\sqrt{2})^2$$

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 2$$

$$1 + 2 \sin \theta \cos \theta = 2$$

$$2 \sin \theta \cos \theta - 1 = 1$$

$$\frac{1}{\sin \theta \cos \theta} = 2$$

Now,

$$\begin{aligned} \tan \theta + \cot \theta &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \\ &= \frac{1}{\cos \theta \sin \theta} = 2 \end{aligned}$$

96. If $\sec \theta = x + \frac{1}{4x}$, $x \neq 0$ find $(\sec \theta + \tan \theta)$.

Ans :

[Board 2019 Delhi]

We have $\sec \theta = x + \frac{1}{4x}$... (1)

Since, $\tan^2 \theta = \sec^2 \theta - 1$

Substituting value of $\sec \theta$ we have

$$\begin{aligned} \tan^2 \theta &= \left(x + \frac{1}{4x}\right)^2 - 1 \\ &= x^2 + \frac{2x}{4x} + \frac{1}{16x^2} - 1 \\ &= x^2 + \frac{1}{16x^2} - \frac{1}{2} \\ &= \left(x - \frac{1}{4x}\right)^2 \end{aligned}$$

$$\tan \theta = \pm \left(x - \frac{1}{4x}\right)$$

When $\sec \theta = x + \frac{1}{4x}$ and $\tan \theta = x - \frac{1}{4x}$ we have

$$\sec \theta + \tan \theta = \left(x + \frac{1}{4x}\right) + \left(x - \frac{1}{4x}\right) = 2x$$

When $\sec \theta = x + \frac{1}{4x}$ and $\tan \theta = -\left(x - \frac{1}{4x}\right)$ we have

$$\begin{aligned} \sec \theta + \tan \theta &= \left(x + \frac{1}{4x}\right) + \left\{-\left(x - \frac{1}{4x}\right)\right\} \\ &= x + \frac{1}{4x} - x + \frac{1}{4x} \\ &= \frac{2}{4x} = \frac{1}{2x} \end{aligned}$$

97. If $\sin A = \frac{3}{4}$ calculate $\sec A$.

Ans :

[Board 2019 OD]

We have $\sin A = \frac{3}{4}$

Now $\cos^2 A = 1 - \sin^2 A$

$$\cos^2 A = 1 - \left(\frac{3}{4}\right)^2 = 1 - \frac{9}{16} = \frac{7}{16}$$

$$\cos A = \frac{\sqrt{7}}{4}$$

Thus $\sec A = \frac{1}{\cos A} = \frac{4}{\sqrt{7}}$

98. Prove that: $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$

FOUR MARKS QUESTIONS

95. If $\sin \theta + \cos \theta = \sqrt{3}$, then prove that $\tan \theta + \cot \theta = 1$.

Ans :

[Board 2020 Delhi Standard]

We have $\sin \theta + \cos \theta = \sqrt{3}$

Squaring both the sides, we get

$$(\sin \theta + \cos \theta)^2 = (\sqrt{3})^2$$

$$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 3$$

$$1 + 2 \sin \theta \cos \theta = 3$$

$$2 \sin \theta \cos \theta = 3 - 1 = 2$$

$$\sin \theta \cos \theta = 1 \quad \dots(1)$$

Now

$$\begin{aligned} \tan \theta + \cot \theta &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \end{aligned}$$

or

$$\tan \theta + \cot \theta = \frac{1}{\sin \theta \cos \theta}$$

Substituting the value of $\sin \theta \cos \theta$ from equation (1) we have

$$\tan \theta + \cot \theta = \frac{1}{1} = 1$$

Hence,

$$\tan \theta + \cot \theta = 1$$

Ans :

[Board 2019 OD]

= LHS

Hence Proved

$$\begin{aligned}
\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} &= \frac{\tan \theta}{1 - \frac{1}{\tan \theta}} + \frac{\frac{1}{\tan \theta}}{1 - \tan \theta} \\
&= \frac{\tan^2 \theta}{\tan \theta - 1} + \frac{1}{\tan \theta(1 - \tan \theta)} \\
&= \frac{\tan^2 \theta}{\tan \theta - 1} - \frac{1}{\tan \theta(\tan \theta - 1)} \\
&= \frac{\tan^3 \theta - 1}{\tan \theta(\tan \theta - 1)} \\
&= \frac{(\tan \theta - 1)(\tan^2 \theta + 1 + \tan \theta)}{\tan \theta(\tan \theta - 1)} \\
&= \frac{\tan^2 \theta + 1 + \tan \theta}{\tan \theta} \\
&= \tan \theta + \cot \theta + 1 \quad \text{h301} \\
&= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} + 1 \\
&= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} + 1 \\
&= \frac{1}{\sin \theta \cos \theta} + 1 \\
&= \operatorname{cosec} \theta \sec \theta + 1 \\
&= 1 + \sec \theta \operatorname{cosec} \theta \text{ Hence Proved}
\end{aligned}$$

99. Prove that: $\frac{\sin \theta}{\cot \theta + \operatorname{cosec} \theta} = 2 + \frac{\sin \theta}{\cot \theta - \operatorname{cosec} \theta}$

Ans :

[Board 2019 OD]

$$\begin{aligned}
\text{LHS} &= \frac{\sin \theta}{\cot \theta + \operatorname{cosec} \theta} \\
&= \frac{\sin \theta}{\frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta}} = \frac{\sin^2 \theta}{\cos \theta + 1} \\
&= \frac{1 - \cos^2 \theta}{\cos \theta + 1} = \frac{(1 - \cos \theta)(1 + \cos \theta)}{\cos \theta + 1} \\
&= 1 - \cos \theta \quad \dots(1)
\end{aligned}$$

Now, RHS = $2 + \frac{\sin \theta}{\cot \theta - \operatorname{cosec} \theta}$

$$\begin{aligned}
&= 2 + \frac{\sin \theta}{\frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}} = 2 + \frac{\sin^2 \theta}{\cos \theta - 1} \\
&= 2 + \frac{1 - \cos^2 \theta}{\cos \theta - 1} = 2 - \frac{(\cos^2 \theta - 1)}{(\cos \theta - 1)} \\
&= 2 - \frac{(\cos \theta - 1)(\cos \theta + 1)}{\cos \theta - 1} \\
&= 2 - (\cos \theta + 1) = 1 - \cos \theta
\end{aligned}$$

100. Find A and B if $\sin(A + 2B) = \frac{\sqrt{3}}{2}$ and $\cos(A + 4B) = 0$, where A and B are acute angles.

Ans :

[Board 2019 OD]

We have $\sin(A + 2B) = \frac{\sqrt{3}}{2}$

$$\sin(A + 2B) = \sin 60^\circ \quad (\sin 60^\circ = \frac{\sqrt{3}}{2})$$

$$A + 2B = 60^\circ \quad \dots(1)$$

Also, given $\cos(A + 4B) = 0$

$$\cos(A + 4B) = \cos 90^\circ \quad (\cos 90^\circ = 0)$$

$$A + 4B = 90^\circ \quad \dots(2)$$

Subtracting equation (2) from equation (1) we get

$$-2B = -30^\circ \Rightarrow B = 15^\circ$$

From equation (1) we have

$$A + 2(15^\circ) = 60^\circ$$

$$A = 60^\circ - 30^\circ$$

$$= 30^\circ$$

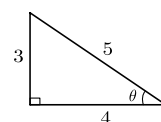
Hence angle $A = 30^\circ$ and angle $B = 15^\circ$.

101. If $4 \tan \theta = 3$, evaluate $\left(\frac{4 \sin \theta - \cos \theta + 1}{4 \sin \theta + \cos \theta - 1}\right)$

Ans :

[Board 2018]

We have $4 \tan \theta = 3 \Rightarrow \tan \theta = \frac{3}{4}$

We know very well that if $\tan \theta = \frac{3}{4}$, then

$$\sin \theta = \frac{3}{5} \text{ and } \cos \theta = \frac{4}{5}$$

Substituting above values in given expression,

$$\frac{4 \sin \theta - \cos \theta + 1}{4 \sin \theta + \cos \theta - 1} = \frac{4 \times \frac{3}{5} - \frac{4}{5} + 1}{4 \times \frac{3}{5} + \frac{4}{5} - 1} = \frac{13}{11}$$

102. Evaluate :

$$\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \tan^2 60^\circ - 2 \tan 45^\circ \cos^2 0^\circ \sin 90^\circ$$

Ans :

[Board Term-1 2015]

$$\tan^2 30^\circ \sin 30^\circ + \cos 60^\circ \sin^2 90^\circ \tan^2 60^\circ - 2 \tan 45^\circ \cos^2 0^\circ \sin 90^\circ$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{2} + \frac{1}{2} \times (1)^2 \times (\sqrt{3})^2 - 2 \times 1 \times 1^2 \times 1$$

$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times 3 - 2$$

$$= \frac{1}{6} + \frac{3}{2} - 2 = \frac{1+9-12}{6} = -\frac{2}{6} = -\frac{1}{3}$$

103. Given that

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B},$$

find the values of $\tan 75^\circ$ and $\tan 90^\circ$ by taking suitable values of A and B .

Ans : [Board Term-1 2012, NCERT]

$$\text{We have } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\begin{aligned} \text{(i)} \quad \tan 75^\circ &= \tan(45^\circ + 30^\circ) \\ &= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} \quad \text{h145} \\ &= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \\ &= \frac{(\sqrt{3} + 1)(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)} \\ &= \frac{3 + 2\sqrt{3} + 1}{(\sqrt{3})^2 - (1)^2} = \frac{4 + 2\sqrt{3}}{2} \\ &= 2 + \sqrt{3} \end{aligned}$$

Hence $\tan 75^\circ = 2 + \sqrt{3}$

$$\begin{aligned} \text{(ii)} \quad \tan 90^\circ &= \tan(60^\circ + 30^\circ) \\ &= \frac{\tan 60^\circ + \tan 30^\circ}{1 - \tan 60^\circ \tan 30^\circ} \\ &= \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{1 - \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{3 + 1}{0} \end{aligned}$$

Hence, $\tan 90^\circ = \infty$

104. Evaluate :

$$\sin^2 30^\circ \cos^2 45^\circ + 4 \tan^2 30^\circ + \frac{1}{2} \sin 90^\circ - 2 \cos^2 90^\circ + \frac{1}{24}$$

Ans : [Board Term-1 2013]

$$\begin{aligned} &\sin^2 30^\circ \cos^2 45^\circ + 4 \tan^2 30^\circ + \frac{1}{2} \sin 90^\circ - 2 \cos^2 90^\circ + \frac{1}{24} \\ &= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2(0) + \frac{1}{24} \\ &= \frac{1}{4}\left(\frac{1}{2}\right) + 4\left(\frac{1}{3}\right) + \frac{1}{2} + \frac{1}{24} = \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} \end{aligned}$$

$$= \frac{3 + 32 + 12 + 1}{24} = \frac{48}{24} = 2$$

105. Evaluate : $4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ)$

Ans : [Board Term-1 2013]

$$\begin{aligned} &4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) \\ &= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right] \\ &= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right] \\ &= 4\left(\frac{2}{16}\right) - 3\left(-\frac{1}{2}\right) = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2 \end{aligned}$$

106. If $15 \tan^2 \theta + 4 \sec^2 \theta = 23$, then find the value of $(\sec \theta + \operatorname{cosec} \theta)^2 - \sin^2 \theta$.

Ans : [Board Term-1 2012]

$$\begin{aligned} \text{We have } 15 \tan^2 \theta + 4 \sec^2 \theta &= 23 \\ 15 \tan^2 \theta + 4(\tan^2 \theta + 1) &= 23 \\ 15 \tan^2 \theta + 4 \tan^2 \theta + 4 &= 23 \\ 19 \tan^2 \theta &= 19 \\ \tan \theta &= 1 = \tan 45^\circ \end{aligned}$$

Thus $\theta = 45^\circ$

$$\begin{aligned} \text{Now, } (\sec \theta + \operatorname{cosec} \theta)^2 - \sin^2 \theta &= (\sec 45^\circ + \operatorname{cosec} 45^\circ)^2 - \sin^2 45^\circ \\ &= (\sqrt{2} + \sqrt{2})^2 - \left(\frac{1}{\sqrt{2}}\right)^2 \\ &= (2\sqrt{2})^2 - \frac{1}{2} = 8 - \frac{1}{2} = \frac{15}{2} \end{aligned}$$

107. If $\sqrt{3} \cot^2 \theta - 4 \cot \theta + \sqrt{3} = 0$, then find the value of $\cot^2 \theta + \tan^2 \theta$.

Ans : [Board Term-1 2012]

$$\text{We have } \sqrt{3} \cot^2 \theta - 4 \cot \theta + \sqrt{3} = 0$$

Let $\cot \theta = x$, then we have

$$\begin{aligned} \sqrt{3} x^2 - 4x + \sqrt{3} &= 0 \\ \sqrt{3} x^2 - 3x - x + \sqrt{3} &= 0 \\ (x - \sqrt{3})(\sqrt{3}x - 1) &= 0 \\ x &= \sqrt{3} \text{ or } \frac{1}{\sqrt{3}} \end{aligned}$$

Thus $\cot \theta = \sqrt{3}$ or $\cot \theta = \frac{1}{\sqrt{3}}$

Therefore $\theta = 30^\circ$ or $\theta = 60^\circ$

If $\theta = 30^\circ$, then

$$\begin{aligned}\cot^2 30^\circ + \tan^2 30^\circ &= (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2 \\ &= 3 + \frac{1}{3} = \frac{10}{3}\end{aligned}$$

If $\theta = 60^\circ$, then

$$\begin{aligned}\cot^2 60^\circ + \tan^2 60^\circ &= \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 \\ &= \frac{1}{3} + 3 = \frac{10}{3}.\end{aligned}$$

108. Evaluate the following :

$$\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$$

Ans :

[Board Term-1 2012]

$$\begin{aligned}\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ} &= \frac{2\left(\frac{1}{2}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2(1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} \\ &= \frac{2\left(\frac{1}{2}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2(1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} \\ &= \frac{\frac{2}{4} + 4 - 2}{\frac{1}{4} + \frac{1}{2}} = \frac{10}{3}\end{aligned}$$

109. Prove that : $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta$.

Ans :

[Board Term-1 2012]

$$\begin{aligned}\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} &= \frac{\tan \theta}{1 - \frac{1}{\tan \theta}} + \frac{\frac{1}{\tan \theta}}{1 - \tan \theta} \\ &= \frac{\tan^2 \theta}{\tan \theta - 1} + \frac{1}{(1 - \tan \theta)\tan \theta} \\ &= \frac{\tan^2 \theta}{\tan \theta - 1} - \frac{1}{(\tan \theta - 1)\tan \theta} \\ &= \frac{\tan^3 \theta - 1}{(\tan \theta - 1)\tan \theta} \\ &= \frac{(\tan \theta - 1)(\tan^2 \theta + \tan \theta + 1)}{(\tan \theta - 1)(\tan \theta)} \\ &= \frac{\tan^2 \theta + \tan \theta + 1}{\tan \theta} \\ &= \tan \theta + 1 + \cot \theta\end{aligned}$$

h152

Hence Proved.

110. In an acute angled triangle ABC if $\sin(A + B - C) = \frac{1}{2}$ and $\cos(B + C - A) = \frac{1}{\sqrt{2}}$ find $\angle A, \angle B$ and $\angle C$.

Ans :

[Board Term-1 2012]

$$\text{We have } \sin(A + B - C) = \frac{1}{2} = \sin 30^\circ$$

$$A + B - C = 30^\circ \quad \dots(1)$$

$$\text{and } \cos(B + C - A) = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

$$B + C - A = 45^\circ \quad \dots(2)$$

Adding equation (1) and (2), we get

$$2B = 75^\circ \Rightarrow B = 37.5^\circ$$

Subtracting equation (2) from equation (1) we get,

$$2(A - C) = -15^\circ$$

$$A - C = -7.5^\circ \quad \dots(3)$$

$$\text{Now } A + B + C = 180^\circ$$

$$A + C = 180^\circ - 37.5^\circ = 142.5^\circ \quad \dots(4)$$

Adding equation (3) and (4), we have

$$2A = 135^\circ \Rightarrow A = 67.5^\circ$$

$$\text{and, } C = 75^\circ$$

$$\text{Hence, } \angle A = 67.5^\circ, \angle B = 37.5^\circ, \angle C = 75^\circ$$

111. Prove that $b^2 x^2 - a^2 y^2 = a^2 b^2$, if :

(1) $x = a \sec \theta, y = b \tan \theta$, or

(2) $x = a \operatorname{cosec} \theta, y = b \cot \theta$

Ans :

[Board Term-1 2015]

(1) We have $x = a \sec \theta, y = b \tan \theta$,

$$\frac{x^2}{a^2} = \sec^2 \theta, \frac{y^2}{b^2} = \tan^2 \theta$$

$$\text{or, } \frac{x^2}{a^2} - \frac{y^2}{b^2} = \sec^2 \theta - \tan^2 \theta = 1$$

$$\text{Thus } b^2 x^2 - a^2 y^2 = a^2 b^2 \quad \text{Hence Proved}$$

(ii) We have $x = a \operatorname{cosec} \theta, y = b \cot \theta$

$$\frac{x^2}{a^2} = \operatorname{cosec}^2 \theta, \frac{y^2}{b^2} = \cot^2 \theta$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$\text{Thus } b^2 x^2 - a^2 y^2 = a^2 b^2 \quad \text{Hence Proved}$$

112. If $\operatorname{cosec} \theta - \cot \theta = \sqrt{2} \cot \theta$, then prove that $\operatorname{cosec} \theta + \cot \theta = \sqrt{2} \operatorname{cosec} \theta$.

Ans :

[Board Term-1 2015]

$$\text{We have } \operatorname{cosec} \theta - \cot \theta = \sqrt{2} \cot \theta$$

Squaring both sides we have

$$\operatorname{cosec}^2 \theta + \cot^2 \theta - 2 \operatorname{cosec} \theta \cot \theta = 2 \cot^2 \theta$$

$$\begin{aligned} \operatorname{cosec}^2\theta - \cot^2\theta &= 2 \operatorname{cosec}\theta \cot\theta \\ (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) &= 2 \operatorname{cosec}\theta \cot\theta \\ (\operatorname{cosec}\theta - \cot\theta) &= \sqrt{2} \cot\theta \\ (\operatorname{cosec}\theta + \cot\theta)\sqrt{2} \cot\theta &= 2 \operatorname{cosec}\theta \cot\theta \\ \operatorname{cosec}\theta + \cot\theta &= \sqrt{2} \operatorname{cosec}\theta \end{aligned}$$

Hence Proved.

113. Prove that :

$$\frac{\cot^3\theta \sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\tan^3\theta \cos^3\theta}{(\cos\theta + \sin\theta)^2} = \frac{\sec\theta \operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta + \sec\theta}$$

Ans : [Board Term-1 2015]

$$\begin{aligned} &\frac{\cot^3\theta \sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\tan^3\theta \cos^3\theta}{(\cos\theta + \sin\theta)^2} \\ &= \frac{\frac{\cos^3\theta}{\sin^3\theta} \times \sin^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\frac{\sin^3\theta}{\cos^3\theta} \times \cos^3\theta}{(\cos\theta + \sin\theta)^2} \\ &= \frac{\cos^3\theta}{(\cos\theta + \sin\theta)^2} + \frac{\sin^3\theta}{(\cos\theta + \sin\theta)^2} \\ &= \frac{(\cos\theta + \sin\theta)(\cos^2\theta + \sin^2\theta - \sin\theta \cos\theta)}{(\cos\theta + \sin\theta)^2} \\ &= \frac{1 - \sin\theta \cos\theta}{\cos\theta + \sin\theta} = \frac{\frac{1}{\cos\theta \sin\theta} - \frac{\sin\theta \cos\theta}{\cos\theta \sin\theta}}{\frac{\cos\theta}{\cos\theta \sin\theta} + \frac{\sin\theta}{\cos\theta \sin\theta}} \\ &= \frac{\operatorname{cosec}\theta \sec\theta - 1}{\operatorname{cosec}\theta + \sec\theta} \end{aligned}$$

Hence Proved

114. Prove that : $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2 \operatorname{cosec}\theta$.

Ans : [Board Terim-1, 2012, Set-9]

$$\begin{aligned} \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} &= \frac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{(\sec\theta + 1)(\sec\theta - 1)}} \\ &= \frac{2 \sec\theta}{\sqrt{\sec^2\theta - 1}} = \frac{2 \sec\theta}{\sqrt{\tan^2\theta}} = \frac{2 \sec\theta}{\tan\theta} \\ &= 2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} \\ &= 2 \times \frac{1}{\sin\theta} \\ &= 2 \operatorname{cosec}\theta \end{aligned}$$

Hence Proved

115. Prove that : $\frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\sec\theta + 1}{\sec\theta - 1}$.

Ans : [Board Term-1 2012]

We have
$$\frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \sin\theta}{\frac{\sin\theta}{\cos\theta} - \sin\theta}$$

$$\begin{aligned} &= \frac{\sin\theta(\frac{1}{\cos\theta} + 1)}{\sin\theta(\frac{1}{\cos\theta} - 1)} \\ &= \frac{\sec\theta + 1}{\sec\theta - 1} \end{aligned}$$

Hence Proved.

116. Prove that : $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2 \sec^2 A$

Ans : [Board Term-1 2012]

$$\begin{aligned} &\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} \\ &= \frac{\operatorname{cosec}^2 A + \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{(\operatorname{cosec} A - 1)(\operatorname{cosec} A + 1)} \\ &= \frac{2 \operatorname{cosec}^2 A}{\operatorname{cosec}^2 A - 1} = \frac{2 \operatorname{cosec}^2 A}{\cot^2 A} \\ &= \frac{\frac{2}{\sin^2 A}}{\frac{\cos^2 A}{\sin^2 A}} = \frac{2}{\sin^2 A} \times \frac{\sin^2 A}{\cos^2 A} \\ &= \frac{2}{\cos^2 A} = 2 \sec^2 A \end{aligned}$$

Hence Proved.

117. If $\operatorname{cosec}\theta + \cot\theta = p$, then prove that $\cos\theta = \frac{p^2 - 1}{p^2 + 1}$.

Ans : [Board Term-1 2016]

$$\begin{aligned} \frac{p^2 - 1}{p^2 + 1} &= \frac{(\operatorname{cosec}\theta + \cot\theta)^2 - 1}{(\operatorname{cosec}\theta + \cot\theta)^2 + 1} \\ &= \frac{\operatorname{cosec}^2\theta + \cot^2\theta + 2 \operatorname{cosec}\theta \cot\theta - 1}{\operatorname{cosec}^2\theta + \cot^2\theta + 2 \operatorname{cosec}\theta \cot\theta + 1} \\ &= \frac{1 + \cot^2\theta + \cot^2\theta + 2 \operatorname{cosec}\theta \cot\theta - 1}{\operatorname{cosec}^2\theta + \operatorname{cosec}^2\theta - 1 + 2 \operatorname{cosec}\theta \cot\theta + 1} \\ &= \frac{2 \cot\theta(\cot\theta + \operatorname{cosec}\theta)}{2 \operatorname{cosec}\theta(\operatorname{cosec}\theta + \cot\theta)} \\ &= \frac{\cos\theta}{\sin\theta} \times \sin\theta = \cos\theta \end{aligned}$$

118. If $a \cos\theta + b \sin\theta = m$ and $a \sin\theta - b \cos\theta = n$, prove that $m^2 + n^2 = a^2 + b^2$

Ans : [Board Term-1 2012]

We have

$$m^2 = a^2 \cos^2\theta + 2ab \sin\theta \cos\theta + b^2 \sin^2\theta \dots(1)$$

and, $n^2 = a^2 \sin^2 \theta - 2ab \sin \theta \cos \theta + b^2 \cos^2 \theta \dots (2)$

Adding equations (1) and (2) we get

$$\begin{aligned} m^2 + n^2 &= a^2(\cos^2 \theta + \sin^2 \theta) + b^2(\cos^2 \theta + \sin^2 \theta) \\ &= a^2(1) + b^2(1) \\ &= a^2 + b^2 \end{aligned}$$

119. Prove that : $\frac{\cos^2 \theta}{1 - \tan \theta} + \frac{\sin^3 \theta}{\sin \theta - \cos \theta} = 1 + \sin \theta \cos \theta.$

Ans : [Board Term-1 2012]

$$\begin{aligned} &\frac{\cos^2 \theta}{1 - \tan \theta} + \frac{\sin^3 \theta}{\sin \theta - \cos \theta} \\ &= \frac{\cos^2 \theta}{1 - \frac{\sin \theta}{\cos \theta}} + \frac{\sin^3 \theta}{\sin \theta - \cos \theta} \\ &= \frac{\cos^3 \theta}{\cos \theta - \sin \theta} - \frac{\sin^3 \theta}{\cos \theta - \sin \theta} \\ &= \frac{\cos^3 \theta - \sin^3 \theta}{\cos \theta - \sin \theta} \\ &= \frac{(\cos \theta - \sin \theta)(\cos^2 \theta + \sin^2 \theta + \sin \theta \cos \theta)}{(\cos \theta - \sin \theta)} \\ &= 1 + \sin \theta \cos \theta \quad \text{Hence Proved} \end{aligned}$$

120. If $\cos \theta + \sin \theta = p$ and $\sec \theta + \operatorname{cosec} \theta = q$, prove that $q(p^2 - 1) = 2p$

Ans : [Board Term-1 2012]

We have $\cos \theta + \sin \theta = p$ and $\sec \theta + \operatorname{cosec} \theta = q$

$$\begin{aligned} q(p^2 - 1) &= (\sec \theta + \operatorname{cosec} \theta)[(\cos \theta + \sin \theta)^2 - 1] \\ &= (\sec \theta + \operatorname{cosec} \theta)(\cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta - 1) \\ &= (\sec \theta + \operatorname{cosec} \theta)[1 + 2 \sin \theta \cos \theta - 1] \\ &= \left(\frac{1}{\cos \theta} + \frac{1}{\sin \theta}\right)(2 \sin \theta \cos \theta) \\ &= \left(\frac{\sin \theta + \cos \theta}{\cos \theta \sin \theta}\right) 2 \sin \theta \cos \theta \\ &= 2(\sin \theta + \cos \theta) = 2p \quad \text{Hence Proved.} \end{aligned}$$

121. If $x = r \sin A \cos C$, $y = r \sin A \sin C$ and $z = r \cos A$, then prove that $x^2 + y^2 + z^2 = r^2$

Ans : [Board Term-1 2012, Set-50]

Since, $x^2 = r^2 \sin^2 A \cos^2 C$

$$y^2 = r^2 \sin^2 A \sin^2 C$$

and $z^2 = r^2 \cos^2 A$

$$x^2 + y^2 + z^2 = r^2 \sin^2 A \cos^2 C + r^2 \sin^2 A \sin^2 C + r^2 \cos^2 A$$

$$= r^2 \sin^2 A (\cos^2 C + \sin^2 C) + r^2 \cos^2 A$$

$$= r^2 \sin^2 A + r^2 \cos^2 A$$

$$= r^2 (\sin^2 A + \cos^2 A)$$

$$= r^2$$

Hence Proved.

122. Prove that: $\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} + \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = 2 \sec \theta.$

Ans : [Board Term-1 2012]

$$\begin{aligned} &\sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}} + \sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} \\ &= \sqrt{\frac{(1 + \sin \theta)}{(1 - \sin \theta)} \times \frac{(1 + \sin \theta)}{(1 + \sin \theta)}} + \sqrt{\frac{(1 - \sin \theta)}{(1 + \sin \theta)} \times \frac{(1 - \sin \theta)}{(1 - \sin \theta)}} \\ &= \sqrt{\frac{(1 + \sin \theta)^2}{(1 - \sin^2 \theta)}} + \sqrt{\frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta}} \\ &= \sqrt{\frac{(1 + \sin \theta)^2}{\cos^2 \theta}} + \sqrt{\frac{(1 - \sin \theta)^2}{\cos^2 \theta}} \\ &= \frac{1 + \sin \theta}{\cos \theta} + \frac{1 - \sin \theta}{\cos \theta} = \frac{1 + \sin \theta + 1 - \sin \theta}{\cos \theta} \\ &= \frac{2}{\cos \theta} = 2 \sec \theta \quad \text{Hence Proved} \end{aligned}$$

123. Prove that

$$(1 - \sin \theta + \cos \theta)^2 = 2(1 + \cos \theta)(1 - \sin \theta).$$

Ans : [Board Term-1 2012]

$$\begin{aligned} &(1 - \sin \theta + \cos \theta)^2 \\ &= 1 + \sin^2 \theta + \cos^2 \theta - 2 \sin \theta - 2 \sin \theta \cos \theta + 2 \cos \theta \\ &= 1 + 1 - 2 \sin \theta - 2 \sin \theta \cos \theta + 2 \cos \theta \\ &= 2 + 2 \cos \theta - 2 \sin \theta - 2 \sin \theta \cos \theta \\ &= 2(1 + \cos \theta) - 2 \sin \theta(1 + \cos \theta) \\ &= (1 + \cos \theta)(2 - 2 \sin \theta) \\ &= 2(1 + \cos \theta)(1 - \sin \theta) \quad \text{Hence Proved} \end{aligned}$$

124. Prove that : $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \sec \theta + \tan \theta$

Ans : [Board Term-1 2012]

$$\begin{aligned} &\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} \\ &= \frac{(\tan \theta + \sec \theta) - (\sec^2 \theta - \tan^2 \theta)}{\tan \theta - \sec \theta + 1} \\ &= \frac{(\tan \theta + \sec \theta) - (\sec \theta - \tan \theta)(\sec \theta + \tan \theta)}{\tan \theta - \sec \theta + 1} \end{aligned}$$

$$= \frac{(\tan \theta + \sec \theta)(1 - \sec \theta + \tan \theta)}{\tan \theta - \sec \theta + 1}$$

$$= \tan \theta + \sec \theta$$

Hence Proved

125. Prove that :

$$(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta \cot^2 \theta$$

Ans : [Board Term-1 2012]

$$(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2$$

$$= \sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \operatorname{cosec} \theta + \cos^2 \theta + \sec^2 \theta + 2 \cos \theta \sec \theta$$

$$= (\sin^2 \theta + \cos^2 \theta) + \operatorname{cosec}^2 \theta + 2 + \sec^2 \theta + 2$$

$$= 1 + (1 + \cot^2 \theta) + 2 + (1 + \tan^2 \theta) + 2$$

$$= 7 + \tan^2 \theta + \cot^2 \theta \quad \text{Hence Proved}$$

126. If $\sin \theta = \frac{c}{\sqrt{c^2 + d^2}}$ and $d > 0$, find the value of $\cos \theta$ and $\tan \theta$.

Ans : [Board Term-1 2013]

We have $\sin \theta = \frac{c}{\sqrt{c^2 + d^2}}$

Now $\cos^2 \theta = 1 - \sin^2 \theta$

$$= 1 - \left(\frac{c}{\sqrt{c^2 + d^2}}\right)^2$$

$$= 1 - \frac{c^2}{c^2 + d^2} \quad \text{h200}$$

$$= \frac{c^2 + d^2 - c^2}{c^2 + d^2} = \frac{d^2}{c^2 + d^2}$$

Thus $\cos \theta = \frac{d}{\sqrt{c^2 + d^2}}$

Again, $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{c}{\sqrt{c^2 + d^2}}}{\frac{d}{\sqrt{c^2 + d^2}}} = \frac{c}{d}$

Thus $\tan \theta = \frac{c}{d}$

127. If $\tan \theta = \frac{1}{\sqrt{5}}$,

(1) Evaluate : $\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta}$

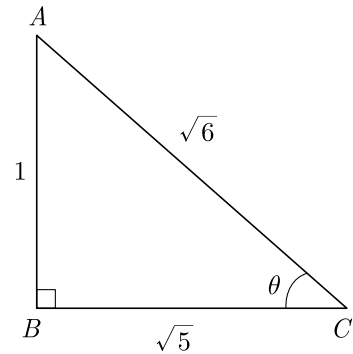
(2) Verify the identity : $\sin^2 \theta + \cos^2 \theta = 1$

Ans : [Board Term-1 2012]

We have $\tan \theta = \frac{1}{\sqrt{5}}$

We draw the triangle as shown below and write all

dimensions.



Now $\cot \theta = \frac{1}{\tan \theta} = \sqrt{5}$

$$\sin \theta = \frac{1}{\sqrt{6}}$$

$$\cos \theta = \frac{\sqrt{5}}{\sqrt{6}}$$

$$\begin{aligned} (1) \frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} &= \frac{(1 + \cot^2 \theta) - (1 + \tan^2 \theta)}{(1 + \cot^2 \theta) + (1 + \tan^2 \theta)} \\ &= \frac{\cot^2 \theta - \tan^2 \theta}{2 + \cot^2 \theta + \tan^2 \theta} \\ &= \frac{(\sqrt{5})^2 - \left(\frac{1}{\sqrt{5}}\right)^2}{2 + (\sqrt{5})^2 + \left(\frac{1}{\sqrt{5}}\right)^2} \\ &= \frac{5 - \frac{1}{5}}{2 + 5 + \frac{1}{5}} = \frac{25 - 1}{35 + 1} = \frac{24}{36} = \frac{2}{3} \end{aligned}$$

$$\begin{aligned} (2) \quad \sin^2 \theta + \cos^2 \theta &= \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2 \\ &= \frac{1}{6} + \frac{5}{6} = \frac{6}{6} \\ &= 1 \quad \text{Hence proved.} \end{aligned}$$

128. If $\sec \theta + \tan \theta = p$, show that $\sec \theta - \tan \theta = \frac{1}{p}$. Hence, find the values of $\cos \theta$ and $\sin \theta$.

Ans : [Board Term-1 2015]

We have $\sec \theta + \tan \theta = p \quad (1)$

Now $\frac{1}{p} = \frac{1}{\sec \theta + \tan \theta} \times \frac{(\sec \theta - \tan \theta)}{(\sec \theta - \tan \theta)}$

$$= \frac{\sec \theta - \tan \theta}{\sec^2 \theta - \tan^2 \theta} = \sec \theta - \tan \theta$$

$$\text{or } \frac{1}{p} = \sec \theta - \tan \theta \quad (2)$$

$$\text{Solving } \sec \theta + \tan \theta = p \text{ and } \sec \theta - \tan \theta = \frac{1}{p},$$

$$\sec \theta = \frac{1}{2} \left(p + \frac{1}{p} \right) = \frac{p^2 + 1}{2p}$$

$$\text{Thus } \cos \theta = \frac{2p}{p^2 + 1}$$

$$\text{and } \tan \theta = \frac{1}{2} \left(p - \frac{1}{p} \right) = \frac{p^2 - 1}{2p}$$

$$\text{and } \sin \theta = \tan \theta \cos \theta = \frac{p^2 - 1}{p^2 + 1}$$

$$129. \text{ Prove that : } (\operatorname{cosec} \theta + \cot \theta)^2 = \frac{\sec \theta + 1}{\sec \theta - 1}$$

Ans :

$$(\operatorname{cosec} \theta + \cot \theta)^2 = \operatorname{cosec}^2 \theta + \cot^2 \theta + 2 \operatorname{cosec} \theta \cdot \cot \theta$$

$$= \left(\frac{1}{\sin \theta} \right)^2 + \left(\frac{\cos \theta}{\sin \theta} \right)^2 + \frac{2 \times 1}{\sin \theta} \times \frac{\cos \theta}{\sin \theta}$$

$$= \frac{1}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} + \frac{2 \cos \theta}{\sin^2 \theta}$$

$$= \frac{1 + \cos^2 \theta + 2 \cos \theta}{\sin^2 \theta} = \frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta}$$

$$= \frac{(1 + \cos \theta)(1 + \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$$

$$= \frac{1 + \cos \theta}{1 - \cos \theta} = \frac{1 + \frac{1}{\sec \theta}}{1 - \frac{1}{\sec \theta}}$$

$$= \frac{\sec \theta + 1}{\sec \theta - 1}$$

Hence Prove.

130. Prove that :

$$(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = (1 + \sec A \operatorname{cosec} A)^2$$

Ans :

[Board Term-1 2012]

$$\text{LHS} = (\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2$$

$$= \left(\sin A + \frac{1}{\cos A} \right)^2 + \left(\cos A + \frac{1}{\sin A} \right)^2$$

$$= \sin^2 A + \frac{1}{\cos^2 A} + 2 \frac{\sin A}{\cos A} + \cos^2 A + \frac{1}{\sin^2 A} + 2 \frac{\cos A}{\sin A}$$

$$= \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} + 2 \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right)$$

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A} + 2 \left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A} \right) \quad \text{h206}$$

$$= 1 + \frac{1}{\sin^2 A \cos^2 A} + \frac{2}{\sin A \cos A}$$

$$= \left(1 + \frac{1}{\sin A \cos A} \right)^2$$

$$= (1 + \sec A \operatorname{cosec} A)^2$$

Hence Proved

$$131. \text{ If } (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) \\ = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$$

Prove that each of the side is equal to ± 1 .

Ans :

[Board Term-1 2012]

We have

$$(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) \quad \text{h207}$$

$$= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$$

Multiply both sides by

$$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$$

$$\text{or, } (\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) \times$$

$$(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$$

$$= (\sec A - \tan A)^2 (\sec B - \tan B)^2 (\sec C - \tan C)^2$$

$$\text{or, } (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C)$$

$$= (\sec A - \tan A)^2 (\sec A + \tan A)^2 (\sec C - \tan C)^2$$

$$\text{or, } 1 = [(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)]^2$$

$$\text{or, } (\sec A - \tan A)(\sec B - \tan B)(\sec C + \tan C) = \pm 1$$

132. If $4 \sin \theta = 3$, find the value of x if

$$\sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} + 2 \cot \theta = \frac{\sqrt{7}}{x} + \cos \theta$$

Ans :

[Board Term-1 2012]

We have

$$\sin \theta = \frac{3}{4}$$

or,

$$\sin^2 \theta = \frac{9}{16}$$

Since $\sin^2 \theta + \cos^2 = 1$, we have

$$\cos^2 \theta = 1 - \sin^2 \theta = 1 - \frac{9}{16} = \frac{7}{16}$$

$$\cos \theta = \frac{\sqrt{7}}{4}$$

and

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{3}{4}}{\frac{\sqrt{7}}{4}} = \frac{3}{\sqrt{7}}$$

$$\text{Thus } \sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} + 2 \cot \theta = \frac{\sqrt{7}}{x} + \cos \theta$$

$$\begin{aligned} \sqrt{\frac{1}{\tan^2\theta}} + 2 \times \frac{\sqrt{7}}{3} &= \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4} \\ \frac{1}{\tan\theta} + \frac{2\sqrt{7}}{3} &= \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4} \\ \frac{\sqrt{7}}{3} + \frac{2\sqrt{7}}{3} - \frac{\sqrt{7}}{4} &= \frac{\sqrt{7}}{x} \\ \frac{4\sqrt{7} - \sqrt{7}}{4} &= \frac{\sqrt{7}}{x} \\ \frac{3\sqrt{7}}{4} &= \frac{\sqrt{7}}{x} \end{aligned}$$

Thus $x = \frac{4}{3}$

133. Prove that $\sec^2\theta + \operatorname{cosec}^2\theta$ can never be less than 2.

Ans : [Board-Term 1 2011]

Let $\sec^2\theta + \operatorname{cosec}^2\theta = x$

$$1 + \tan^2\theta + 1 + \cot^2\theta = x$$

$$2 + \tan^2\theta + \cot^2\theta = x$$

$$2 + \tan^2\theta + \cot^2\theta = x$$

$$\tan^2\theta \geq 0 \text{ and } \cot^2\theta \geq 0$$

Thus $x > 2$

Thus $\sec^2\theta + \operatorname{cosec}^2\theta > 2$

Hence $\sec^2\theta + \operatorname{cosec}^2\theta$ can never be less than 2.

134.(a) Solve for ϕ , if $\tan 5\phi = 1$

(b) Solve for ϕ , if $\frac{\sin\phi}{1+\cos\phi} + \frac{1+\cos\phi}{\sin\phi} = 4$

Ans :

(a) $\tan 5\phi = 1$

$$\tan 5\phi = \tan 45^\circ$$

$$5\phi = 45^\circ$$

Thus $\phi = 9^\circ$

(b) $\frac{\sin\phi}{1+\cos\phi} + \frac{1+\cos\phi}{\sin\phi} = 4$

$$\frac{\sin^2\phi + (1+\cos\phi)^2}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{\sin^2\phi + 1 + 2\cos\phi + \cos^2\phi}{\sin\phi + \sin\phi\cos\phi} = 4$$

$$\frac{\sin^2\phi + \cos^2\phi + 1 + 2\cos\phi}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{2 + 2\cos\phi}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{2(1+\cos\phi)}{\sin\phi(1+\cos\phi)} = 4$$

$$\frac{2}{\sin\phi} = 4$$

$$\sin\phi = \frac{1}{2}$$

$$\sin\phi = \sin 30^\circ$$

Thus $\phi = 30^\circ$

135. If $\tan A + \sin A = m$ and $\tan A - \sin A = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

Ans : [Board-Term 1 2009]

We have $\tan A + \sin A = m$

and $\tan A - \sin A = n$

$$\begin{aligned} m^2 - n^2 &= (\tan A + \sin A)^2 - (\tan A - \sin A)^2 \\ &= (\tan^2 A + \sin^2 A + 2\sin A \tan A) \\ &\quad - (\tan^2 A + \sin^2 A - 2\sin A \tan A) \\ &= \tan^2 A + \sin^2 A + 2\sin A \tan A \\ &\quad - \tan^2 A - \sin^2 A + 2\sin A \tan A \\ &= 4\sin A \tan A \end{aligned}$$

$$\begin{aligned} 4\sqrt{mn} &= 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)} \\ &= 4\sqrt{\tan^2 A - \sin^2 A} \\ &= 4\sqrt{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A} \\ &= 4\sqrt{\frac{\sin^2 A - \sin^2 A \cos^2 A}{\cos^2 A}} \\ &= 4\sqrt{\frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A}} \\ &= 4\sqrt{\frac{\sin^2 A \times \sin^2 A}{\cos^2 A}} \\ &= 4\frac{\sin A \times \sin A}{\cos A} \\ &= 4\sin A \times \frac{\sin A}{\cos A} \\ &= 4\sin A \tan A \end{aligned}$$

Thus $m^2 - n^2 = 4\sqrt{mn}$ Hence Proved

136. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that

$$(m^2 + n^2)\cos^2\beta = n^2.$$

Ans :

[Board-Term 1 2010]

We have $\frac{\cos \alpha}{\cos \beta} = m$ and $\frac{\cos \alpha}{\sin \beta} = n$

$$m^2 = \frac{\cos^2 \alpha}{\cos^2 \beta} \text{ and } n^2 = \frac{\cos^2 \alpha}{\sin^2 \beta}$$

$$\begin{aligned} (m^2 + n^2)\cos^2\beta &= \left[\frac{\cos^2 \alpha}{\cos^2 \beta} + \frac{\cos^2 \alpha}{\sin^2 \beta} \right] \cos^2 \beta \\ &= \cos^2 \alpha \left[\frac{1}{\cos^2 \beta} + \frac{1}{\sin^2 \beta} \right] \cos^2 \beta \\ &= \cos^2 \alpha \frac{\sin^2 \beta + \cos^2 \beta}{\cos^2 \beta \sin^2 \beta} \cos^2 \beta \\ &= \cos^2 \alpha \left(\frac{1}{\cos^2 \beta \sin^2 \beta} \right) \cos^2 \beta \\ &= \frac{\cos^2 \alpha}{\sin^2 \beta} \\ &= n^2 \end{aligned}$$

Hence Proved.

137.If $7 \operatorname{cosec} \phi - 3 \cot \phi = 7$, prove that $7 \cot \phi - 3 \operatorname{cosec} \phi = 3$.

Ans :

We have $7 \operatorname{cosec} \phi - 3 \cot \phi = 7$

$$7 \operatorname{cosec} \phi - 7 = 3 \cot \phi$$

$$7(\operatorname{cosec} \phi - 1) = 3 \cot \phi$$

$$7(\operatorname{cosec} \phi - 1)(\operatorname{cosec} \phi + 1) = 3 \cot \phi(\operatorname{cosec} \phi + 1)$$

$$7(\operatorname{cosec}^2 \phi - 1) = 3 \cot \phi(\operatorname{cosec} \phi + 1)$$

$$7 \cot^2 \phi = \cot \phi \operatorname{cosec} \phi$$