

## Introduction To Trigonometry

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(2025)

1. Which of the following statements is false ? (1 Mark) (2025)

- (A)  $\tan 45^\circ \cot 45^\circ$
- (B)  $\sin 90^\circ = \tan 45^\circ$
- (C)  $\sin 30^\circ = \cos 30^\circ$
- (D)  $\sin 45^\circ = \cos 45^\circ$

2.

The value of  $\left( \tan^2 A - \frac{1}{\cos^2 A} \right)$  is : (1 Mark) (2025)

- (A) more than 1
- (B) 1
- (C) 0
- (D) -1

3. (a) Find the values of A and B ( $0 \leq A < 90^\circ, 0 \leq B < 90^\circ$ ), if (2 Mark) (2025)

$$\tan(A + B) = 1 \text{ and } \tan(A - B) = \frac{1}{\sqrt{3}}.$$

4. Prove that  $\tan 45^\circ = 1$  geometrically. (2 Mark) (2025)

5. Prove that : (3 Mark) (2025)

$$\frac{1 + \cot^2 A}{1 + \tan^2 A} = \left( \frac{1 - \cot A}{1 - \tan A} \right)^2$$

6. If  $\theta$  is an acute angle and  $7 + 4 \sin \theta = 9$ , then the value of  $\theta$  is : (1 Mark) (2025)

- (A)  $90^\circ$
- (B)  $30^\circ$
- (C)  $45^\circ$
- (D)  $60^\circ$

7.

The value of  $\tan^2 \theta - \left( \frac{1}{\cos \theta} \times \sec \theta \right)$  is :  
(1 Mark) (2025)

- (A) 1
- (B) 0
- (C) 1
- (D) 2

8. (a) If  $x \cos 60^\circ + y \cos 0^\circ + \sin 30^\circ \cot 45^\circ = 5$ , then find the value of  $x + 2y$ .  
(2 Mark) (2025)

9.

Evaluate :  $\frac{\tan^2 60^\circ}{\sin^2 60^\circ + \cos^2 30^\circ}$   
(2 Mark) (2025)

10.

Prove that :  $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$   
(3 Mark) (2025)

11.

Prove that :  $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{2 \sin^2 A - 1}$   
(3 Mark) (2025)

## Answers

1. (C)  $\sin 30^\circ = \cos 30^\circ$

2. (D) - 1

3.  $A + B = 45^\circ$

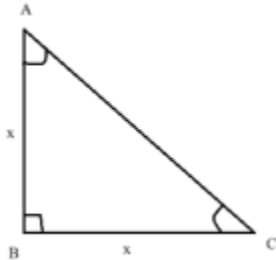
$A - B = 30^\circ$

Solving and getting  $A = 37.5^\circ$  and  $B = 7.5^\circ$

4. Consider an isosceles right  $\triangle ABC$

Using angle sum property  $\angle A = \angle C = 45^\circ$

$$\text{Clearly, } \tan 45^\circ = \frac{AB}{BC} = \frac{x}{x} = 1$$



5.

$$\begin{aligned} \text{LHS} &= \frac{1 + \frac{\cos^2 A}{\sin^2 A}}{1 + \frac{\sin^2 A}{\cos^2 A}} = \frac{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}}{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}} = \frac{\cos^2 A}{\sin^2 A} \\ &= \frac{\cos^2 A}{\sin^2 A} \left( \frac{\sin A - \cos A}{\cos A - \sin A} \right)^2 = \left( \frac{\frac{\sin A - \cos A}{\sin A}}{\frac{\cos A - \sin A}{\cos A}} \right)^2 \\ &= \left( \frac{1 - \cot A}{1 - \tan A} \right)^2 = \text{RHS} \end{aligned}$$

6. (B)  $30^\circ$

7. (C)  $-1$

8.

$$\begin{aligned} x \left( \frac{1}{2} \right) + y(1) + \frac{1}{2} - 1 &= 5 \\ \Rightarrow x + 2y &= 11 \end{aligned}$$

9.

$$\begin{aligned} &\frac{(\sqrt{3})^2}{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \\ &= 2 \end{aligned}$$

10.

$$\begin{aligned}\text{LHS} &= \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} \\ &= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\sin \theta}{\cos \theta}} \\ &= \frac{\sin^2 \theta}{\cos \theta (\sin \theta - \cos \theta)} - \frac{\cos^2 \theta}{\sin \theta (\sin \theta - \cos \theta)} \\ &= \frac{1}{(\sin \theta - \cos \theta)} \left[ \frac{\sin^3 \theta - \cos^3 \theta}{\sin \theta \cos \theta} \right] \\ &= \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta)}{(\sin \theta - \cos \theta) \sin \theta \cos \theta} \\ &= \frac{(1 + \sin \theta \cos \theta)}{\sin \theta \cos \theta} \\ &= 1 + \sec \theta \operatorname{cosec} \theta = \text{RHS}\end{aligned}$$

11.

$$\begin{aligned}\text{LHS} &= \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} \\ &= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)} \\ &= \frac{\sin^2 A + \cos^2 A + 2 \sin A \cos A + \sin^2 A + \cos^2 A - 2 \sin A \cos A}{\sin^2 A - \cos^2 A} \\ &= \frac{1 + 1}{\sin^2 A - (1 - \sin^2 A)} \\ &= \frac{2}{2 \sin^2 A - 1} = \text{RHS}\end{aligned}$$

(2024)

1.

If  $\sin \theta = \frac{1}{3}$ , then  $\sec \theta$  is equal to :

- (a)  $\frac{2\sqrt{2}}{3}$       (b)  $\frac{3}{2\sqrt{2}}$       (c) 3      (d)  $\frac{1}{\sqrt{3}}$       (2024)

Answer. (b)  $\frac{3}{2\sqrt{2}}$

2. For what value of  $\theta$ ,  $\sin^2\theta + \sin\theta + \cos^2\theta$  is equal to 2 ? (2024)

- (a)  $45^\circ$   
(b)  $0^\circ$   
(c)  $90^\circ$   
(d)  $30^\circ$

Answer. (c)  $90^\circ$

3. In a  $\Delta ABC$ ,  $\angle A = 90^\circ$ . If  $\tan C = \sqrt{3}$ , then find the value of  $\sin B + \cos C - \cos^2 B$ . (2024)

Answer.

$$\tan C = \sqrt{3} \Rightarrow \angle C = 60^\circ \Rightarrow \angle B = 30^\circ$$

$$\therefore \sin B + \cos C - \cos^2 B = \frac{1}{2} + \frac{1}{2} - \frac{3}{4} = \frac{1}{4}$$

4.

Prove that :  $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$  (2024)

Answer.

$$\begin{aligned} \text{LHS} &= \frac{\sec A - 1 + \sec A + 1}{\sqrt{\sec^2 A - 1}} \\ &= \frac{2 \sec A}{\tan A} \\ &= 2 \operatorname{cosec} A = \text{RHS} \end{aligned}$$

5. If  $\sec \theta = m$ , then the value of  $\sec \theta + \tan \theta$  is: (2024)

- (a)  $1 - \frac{1}{m}$       (b)  $m^2 - 1$       (c)  $\frac{1}{m}$       (d)  $-m$

Answer.

(c)  $\frac{1}{m}$

6.

If  $\cos(\alpha + \beta) = 0$ , then value of  $\cos\left(\frac{\alpha + \beta}{2}\right)$  is equal to :

- (a)  $\frac{1}{\sqrt{2}}$       (b)  $\frac{1}{2}$       (c) 0      (d)  $\sqrt{2}$  (2024)

Answer.

(a)  $\frac{1}{\sqrt{2}}$

7. (A) Evaluate:  $2\sqrt{2} \cos 45^\circ \sin 30^\circ + 2\sqrt{3} \cos 30^\circ$  (2024)

Answer.

$$\begin{aligned} &2\sqrt{2} \times \frac{1}{\sqrt{2}} \times \frac{1}{2} + 2\sqrt{3} \times \frac{\sqrt{3}}{2} \\ &= 4 \end{aligned}$$

OR

$\sin(A + B) = \sin A \cos B + \cos A \sin B$  (2024)

Answer.

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = 1$$

$\therefore$  LHS = RHS

8.

Prove that :  $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$  (2024)

Answer.

$$\begin{aligned} \text{LHS} &= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{(\sin \theta - \cos \theta)}{\sin \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{(\cos \theta - \sin \theta)}{\cos \theta}} \\ &= \frac{1}{(\sin \theta - \cos \theta)} \left[ \frac{\sin^2 \theta}{\cos \theta} - \frac{\cos^2 \theta}{\sin \theta} \right] \\ &= \frac{1}{(\sin \theta - \cos \theta)} \times \frac{(\sin \theta - \cos \theta)(\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)}{\sin \theta \cos \theta} \\ &= \frac{1}{\sin \theta \cos \theta} + 1 \\ &= 1 + \sec \theta \operatorname{cosec} \theta = \text{RHS} \end{aligned}$$

# Introduction to Trigonometry

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## Previous Years' CBSE Board Questions

### 8.2 Trigonometric Ratios

#### MCQ

1.

If  $2 \tan A = 3$ , then the value of  $\frac{4\sin A + 3\cos A}{4\sin A - 3\cos A}$  is

(a)  $\frac{7}{\sqrt{13}}$

(b)  $\frac{1}{\sqrt{13}}$

(c) 3

(d) does not exist (2023)

2.

Given that  $\cos \theta = \frac{\sqrt{3}}{2}$ , then the value of

$\frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta}$  is

(a) -1

(b) 1

(c)  $\frac{1}{2}$

(d)  $-\frac{1}{2}$

(Term I, 2021-22)

3.

$\frac{1}{\operatorname{cosec} \theta (1 - \cot \theta)} + \frac{1}{\sec \theta (1 - \tan \theta)}$  is equal to

(a) 0

(b) 1

(c)  $\sin \theta + \cos \theta$

(d)  $\sin \theta - \cos \theta$

(Term I, 2021-22)

4. If  $\sin \theta = \cos \theta$ , then the value of  $\tan^2 \theta + \cot^2 \theta$  is

- (a) 2
- (b) 4
- (c) 1
- (d)  $10/3$  (2020C)

5.

If  $\tan \theta + \cot \theta = \frac{4\sqrt{3}}{3}$ , then find the value of  $\tan^2 \theta + \cot^2 \theta$ . (2021C)

### SA I (2 marks)

6. Given  $15 \cot A = 8$ , then find the values of  $\sin A$  and  $\sec A$ . (2020C)

7.

If  $3 \cot A = 4$ , prove that  $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ .  
(Board Term I, 2015)

### SA II (3 marks)

8.

Given  $\sin A = \frac{3}{5}$ , find the other trigonometric ratios of the angle  $A$ . (Board Term I, 2016)

9.

If  $3 \tan A = 4$  check whether

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A \text{ or not.}$$

(Board Term I, 2017)

## 8.3 Trigonometric Ratios of Some Specific Angles

### MCQ

10.

$\left[ \frac{5}{8} \sec^2 60^\circ - \tan^2 60^\circ + \cos^2 45^\circ \right]$  is equal to


- (a)  $\frac{-5}{3}$
  - (b)  $\frac{-1}{2}$
  - (c) 0
  - (d)  $\frac{-1}{4}$
- (2023)

11.

Given that  $\sin\alpha = \frac{\sqrt{3}}{2}$  and  $\tan\beta = \frac{1}{\sqrt{3}}$ , then the value

of  $\cos(\alpha - \beta)$  is

- (a)  $\frac{\sqrt{3}}{2}$       (b)  $\frac{1}{2}$       (c) 0      (d)  $\frac{1}{\sqrt{2}}$

(Term I, 2021-22) 

12. The value of  $\theta$  for which  $2 \sin 2\theta = 1$ , is

- (a)  $15^\circ$   
(b)  $30^\circ$   
(c)  $45^\circ$   
(d)  $60^\circ$  (Term I, 2021-22)

**VSA (1 mark)**

13. Evaluate:

$$2 \sec 30^\circ \times \tan 60^\circ \text{ (2020)}$$

14. Write the value of  $\sin^2 30^\circ + \cos^2 60^\circ$ . (2020)

15.


Evaluate:

$$\frac{2 \tan 45^\circ \times \cos 60^\circ}{\sin 30^\circ} \quad (2020)$$

16. If  $\sin x + \cos y = 1$ ;  $x = 30^\circ$  and  $y$  is an acute angle, find the value of  $y$ . (A/2019)

17.

If  $\sin \alpha = \frac{1}{2}$ , then find the value of  $3 \sin \alpha - 4 \sin^3 \alpha$ .

(Board Term I, 2017) 

**SAI (2 marks)**

18. Evaluate  $2 \sec 2\theta + 3 \operatorname{cosec} 2\theta - 2 \sin \theta \cos \theta$  if  $\theta = 45^\circ$  (2023)

19. If  $\sin \theta \cos \theta = 0$ , then find the value of  $\sin^2 \theta + \cos^2 \theta$ . (2023)

20.

Evaluate:  $\frac{5}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cot^2 45^\circ + 2 \sin^2 90^\circ$   
(2023)

21. If  $\theta$  is an acute angle and  $\sin \theta = \cos \theta$ , find the value of  $\tan^2 \theta + \cot^2 \theta - 2$ .  
(2023)

22. Take  $A = 60^\circ$  and  $B = 30^\circ$ . Write the values of  $\cos A + \cos B$  and  $\cos(A + B)$ .

Is  $\cos(A + B) = \cos A + \cos B$ ? (Board Term 1, 2017)

23. Find  $\operatorname{cosec} 30^\circ$  and  $\cos 60^\circ$  geometrically. (Board Term 1, 2017)

24.

$$\sin(A + B) = 1 \text{ \& \; } \sin(A - B) = \frac{1}{2},$$

$0 \leq A + B = 90^\circ$  &  $A > B$ , then find  $A$  &  $B$ .

(Board Term I, 2017)

**LA (4/5/6 marks)**

25. If  $\theta = 30^\circ$ , verify the following:

(i)  $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$

(ii)  $\sin 3\theta = 3\sin \theta - 4\sin^3 \theta$  (Board Term 1, 2017)

26. Find trigonometric ratios of  $30^\circ$  &  $45^\circ$  in all values of T.R. (Board Term 1, 2017)

27. If  $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$  and  $\cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

Find the value of (i)  $\sin 75^\circ$  (ii)  $\cos 15^\circ$  (Board Term 1, 2016)

## 8.4 Trigonometric Identities

**MCQ**

28.  $(\sec^2 \theta - 1)(\operatorname{cosec}^2 \theta - 1)$  is equal to

(a) -1

(b) 1

(c) 0

(d) 2 (2023)

29. Which of the following is true for all values of  $\theta(0^\circ \leq \theta \leq 90^\circ)$ ?

- (a)  $\cos^2\theta - \sin^2\theta = 1$       (b)  $\operatorname{cosec}^2\theta - \sec^2\theta = 1$   
(c)  $\sec^2\theta - \tan^2\theta = 1$       (d)  $\cot^2\theta - \tan^2\theta = 1$   
(2023)

30.

Given that  $\sin\theta = \frac{p}{q}$ ,  $\tan\theta$  is equal to

- (a)  $\frac{p}{\sqrt{p^2 - q^2}}$       (b)  $\frac{q}{\sqrt{p^2 - q^2}}$   
(c)  $\frac{p}{\sqrt{q^2 - p^2}}$       (d)  $\frac{q}{\sqrt{q^2 - p^2}}$   
(Term I, 2021-22)

31.

The simplest form of  $\sqrt{(1 - \cos^2\theta)(1 + \tan^2\theta)}$  is

- (a)  $\cos\theta$     (b)  $\sin\theta$     (c)  $\cot\theta$     (d)  $\tan\theta$   
(Term I, 2021-22)

32. If  $\sin^2\theta + \sin\theta = 1$ , then the value of  $\cos^2\theta + \cos^4\theta$  is

- (a) -1  
(b) 1  
(c) 0  
(d) 2 (Term I, 2021-22)

33. The distance between the points  $(a\cos\theta + b\sin\theta, 0)$  and  $(0, a\sin\theta - b\cos\theta)$ , is

- (a)  $a^2 + b^2$       (b)  $a^2 - b^2$   
(c)  $\sqrt{a^2 + b^2}$       (d)  $\sqrt{a^2 - b^2}$  (2020)

34. If  $3 \sin A = 1$ , then find the value of  $\sec A$ . (2021 C)

35.

Show that:  $\frac{1 + \cot^2\theta}{1 + \tan^2\theta} = \cot^2\theta$  (2021C)

36.  $5 \tan^2 \theta - 5 \sec^2 \theta = \underline{\hspace{2cm}}$  (2020 C)

37. Simplest form of  $(1 - \cos^2 A)(1 + \cot^2 A)$  is  $\underline{\hspace{2cm}}$  (2020)

38.

Simplest form of  $\frac{1 + \tan^2 A}{1 + \cot^2 A}$  is  $\underline{\hspace{2cm}}$ . (2020)

39.

The value of  $\left( \sin^2 \theta + \frac{1}{1 + \tan^2 \theta} \right) = \underline{\hspace{2cm}}$ . (2020)

40. The value of  $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta)$  (2020)

41. If  $\operatorname{cosec}^2 \theta (1 + \cos \theta)(1 - \cos \theta) = k$ , then find the value of  $k$ . (2019 C)

42. If  $\sec \theta + \tan \theta = x$ , find the value of  $\sec \theta - \tan \theta$ . (Board Term 1, 2017)

43. Find the value of  $(\sec^2 \theta - 1) \cdot \cot^2 \theta$  (Board Term 1, 2017)

44. Write the expression in simplest form:

$\sec^2 \theta - \frac{1}{\operatorname{cosec}^2 \theta - 1}$ . (Board Term I, 2016)

**SAI (2 marks)**

45. If  $\sin \theta + \operatorname{cosec} \theta = \sqrt{3}$ , then find the value of  $\sin \theta \operatorname{cosec} \theta$ . (2023)

46.

If  $\sin \alpha = \frac{1}{\sqrt{2}}$  and  $\cot \beta = \sqrt{3}$ , then find the value of  $\operatorname{cosec} \alpha + \operatorname{cosec} \beta$ . (2023)

47. If  $x = p \sec \theta + q \tan \theta$  and  $y = p \tan \theta + q \sec \theta$ , then prove that  $x^2 - y^2 = p^2 - q^2$ . (Board Term I, 2017)

48.

Prove that :

$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \tan^2 A$ . (Board Term I, 2016)

49.

Prove that:  $\sqrt{\frac{1+\cos A}{1-\cos A}} = \operatorname{cosec} A + \cot A.$   
(Board Term I, 2015)

SA II (3 marks)

50. Prove that:

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$$

(2023, 2018, Board Term I, 2016)

51. Prove that  $\sec A (1 - \sin A) (\sec A + \tan A) = 1.$  (2023)

52. Prove that

$$(\operatorname{cosec} A - \sin A) (\sec A - \cos A) = \frac{1}{\cot A + \tan A}.$$

(NCERT, 2023)

53. Show that  $\sin^6 A + 3 \sin^2 A \cos^2 A = 1 - \cos^6 A$  (2021 C)

54.

Prove that  $\frac{1 + \sec \theta - \tan \theta}{1 + \sec \theta + \tan \theta} = \frac{1 - \sin \theta}{\cos \theta}$  (2020 C)

55.

Show that  $\frac{1 + \tan A}{2 \sin A} + \frac{1 + \cot A}{2 \cos A} = \operatorname{cosec} A + \sec A$   
(2020 C)

56.

Prove that:  $\frac{2\cos^3 \theta - \cos \theta}{\sin \theta - 2\sin^3 \theta} = \cot \theta$  (2020)

57. Prove that:

$$(\sin^4 \theta - \cos^4 \theta + 1) \operatorname{cosec}^2 \theta = 2$$
 (2020)

58.

Prove that:  $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$  (2020)

59. If  $\sin\theta + \csc\theta = \sqrt{3}$ , then prove that  $\tan\theta + \cot\theta = 1$ . (2020)

60.

Prove that  $1 + \frac{\cot^2\theta}{1 + \csc\theta} = \csc\theta$  (2019C)

61. Prove that  $(\sin\theta + \csc\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$ . (Delhi 2019, Board Term I, 2015)

62. Prove that

$(1 + \cot A - \csc A)(1 + \tan A + \sec A) = 2$ . (Delhi 2019)

63.

Prove that :

$$\frac{\tan\theta}{1 - \tan\theta} - \frac{\cot\theta}{1 - \cot\theta} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} \quad (\text{AI 2019})$$

64. If  $\csc\theta + \sin\theta = \sqrt{2} \csc\theta$ , show that  $\csc\theta - \sin\theta = \sqrt{2} \sin\theta$ . (AI 2019)

65.

If  $4 \tan\theta = 3$ , evaluate  $\left( \frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} \right)$  (2018)

66.

Prove that :  $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$   
(Board Term I, 2017, 2015)

67.

If  $\tan A = \frac{1}{2}$ , find the value of

$$\frac{\cos A}{\sin A} + \frac{\sin A}{1 + \cos A}. \quad (\text{Board Term I, 2017})$$

68.

Prove that :

$$\frac{\csc A - \sin A}{\csc A + \sin A} = \frac{\sec^2 A - \tan^2 A}{\sec^2 A + \tan^2 A} \quad (\text{Board Term I, 2017})$$

69.

If  $\sin\theta = \frac{12}{13}$ ,  $0^\circ < \theta < 90^\circ$ , find the value of

$$\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta \cdot \cos\theta} \times \frac{1}{\tan^2\theta}. \quad (\text{Board Term I, 2017})$$

70. Prove that :  $\sin 2\theta - \tan\theta + \cos 2\theta \cdot \cot\theta + 2\sin\theta \cdot \cos\theta = \tan\theta + \cot\theta$ . (Board Term 1, 2017)

71.

Prove the identity :

$$\frac{1}{\operatorname{cosec}\theta + \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta - \cot\theta} \quad (\text{Board Term I, 2017})$$

LA (4/5/6 marks)

72. If  $1 + \sin^2\theta = 3\sin\theta \cos\theta$  then prove that  $\tan\theta = 1$

$$\text{or } \tan\theta = \frac{1}{2} \quad (2019)$$

73.

Prove that

$$\frac{\tan^2 A}{\tan^2 A - 1} + \frac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A} = \frac{1}{1 - 2\cos^2 A}. \quad (\text{Delhi 2019})$$

74. Express  $\sin A$ ,  $\cos A$ ,  $\operatorname{cosec} A$  and  $\sec A$  in terms of  $\cot A$ . (Board Term 1, 2017)

75. If  $\sin A + \sin^3 A = \cos 2A$ , prove that  $\cos A - 4\cos^4 A + 8\cos^2 A = 4$  (Board Term 1, 2017)

76. Prove that  $(\cot A + \sec B)^2 - (\tan A - \operatorname{cosec} B)^2 = 2(\cot A \cdot \sec B + \tan A - \operatorname{cosec} B)$  (Board Term I, 2017)

7.

If  $\sec A - \tan A = x$ , show that  $\frac{x^2 + 1}{x^2 - 1} = -\operatorname{cosec} A$ .

(Board Term I, 2017)

78.

$$\begin{aligned} \text{Prove that: } & \frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A + \cot A} + \frac{\operatorname{cosec} A + \cot A}{\operatorname{cosec} A - \cot A} \\ & = 2(2\operatorname{cosec}^2 A - 1) = 2 \left( \frac{1 + \cos^2 A}{1 - \cos^2 A} \right) \quad (\text{Board Term I, 2017}) \end{aligned}$$

79.

If  $m = \cos A - \sin A$  and  $n = \cos A + \sin A$ , then show that

$$\frac{m}{n} - \frac{n}{m} = -\frac{4\sin A \cos A}{\cos^2 A - \sin^2 A} = -\frac{4}{\cot A - \tan A} \quad (\text{Board Term I, 2017}) \quad \text{Ap}$$

80.

$$\begin{aligned} \text{Prove that: } & \frac{\sec^3 \theta}{\sec^2 \theta - 1} + \frac{\operatorname{cosec}^3 \theta}{\operatorname{cosec}^2 \theta - 1} \\ & = \sec \theta \operatorname{cosec} \theta (\sec \theta + \operatorname{cosec} \theta) \quad (\text{Board Term I, 2017}) \end{aligned}$$

81.

$$\begin{aligned} \text{Prove that: } & (\tan \theta + \sec \theta - 1) \cdot (\tan \theta + 1 + \sec \theta) = \frac{2\sin \theta}{1 - \sin \theta} \\ & \quad (\text{Board Term I, 2016}) \end{aligned}$$

82.

$$\begin{aligned} \text{Prove that: } & \sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta} = (\tan \theta + \cot \theta) \\ & \quad (\text{Board Term I, 2016}) \end{aligned}$$

83. If  $\tan \theta = m$  and  $\csc \theta = n$ ; prove that:

$$m^2 - n^2 = 4\sqrt{mn}. \quad (\text{Board Term 1, 2015})$$

## CBSE Sample Questions

### 8.2 Trigonometric Ratios

#### MCQ

1.

If  $5 \tan \beta = 4$ , then  $\frac{5 \sin \beta - 2 \cos \beta}{5 \sin \beta + 2 \cos \beta} =$

- (a)  $\frac{1}{3}$     (b)  $\frac{2}{5}$     (c)  $\frac{3}{5}$     (d) 6  
(2022-23)

2.

If  $4 \tan \beta = 3$ , then  $\frac{4 \sin \beta - 3 \cos \beta}{4 \sin \beta + 3 \cos \beta} =$

- (a) 0    (b)  $\frac{1}{3}$     (c)  $\frac{2}{3}$     (d)  $\frac{3}{4}$

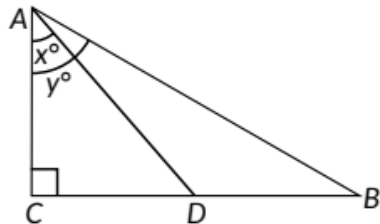
(Term I, 2021-22)

3. If  $\tan a + \cot a = 2$ , then  $\tan^{20} a + \cot^{20} a =$


- (a) 0  
(b) 2  
(c) 20  
(d) 220 (Term I, 2021-22)

4.

In the given figure,  $D$  is the mid-point of  $BC$ , then the value of  $\frac{\cot y^\circ}{\cot x^\circ}$  is



- (a) 2    (b)  $\frac{1}{2}$     (c)  $\frac{1}{3}$     (d)  $\frac{1}{4}$

(Term I, 2021-22) 

SAI (2 marks)

5. If  $\tan A = 3/4$ , find the value of  $1/\sin A + 1/\cos A$ . (2020-21)

### 8.3 Trigonometric Ratios of Some Specific Angles

#### MCQ

6. If  $x \tan 60^\circ \cos 60^\circ = \sin 60^\circ \cot 60^\circ$ , then  $x =$

- (a)  $\cos 30^\circ$
- (b)  $\tan 30^\circ$
- (c)  $\sin 30^\circ$
- (d)  $\cot 30^\circ$  (2022-23)

7. In  $\triangle ABC$  right angled at  $B$ , if  $\tan A = \sqrt{3}$ , then  $\cos A \cos C - \sin A \sin C =$

- (a)  $-1$
  - (b)  $0$
  - (c)  $1$
  - (d)  $\frac{\sqrt{3}}{2}$
- (Term I, 2021-22)

8. If the angles of  $\triangle ABC$  are in the ratio  $1:1:2$ , respectively (the largest angle being angle  $C$ ), then

the value of  $\frac{\sec A}{\operatorname{cosec} B} - \frac{\tan A}{\cot B}$  is

- (a)  $0$
  - (b)  $\frac{1}{2}$
  - (c)  $1$
  - (d)  $\frac{\sqrt{3}}{2}$
- (Term I, 2021-22)

#### VSA (1 mark)

9.  $\sin A + \cos B = 1$ ,  $A = 30^\circ$  and  $B$  is an acute angle, then find the value of  $B$ . (2020-21)

#### SAI (2 marks)

10.

If  $\sin(A + B) = 1$  and  $\cos(A - B) = \frac{\sqrt{3}}{2}$ ,  $0^\circ < A + B \leq 90^\circ$  and  $A > B$ , then find the measures of angles  $A$  and  $B$ . (2022-23)

11.

Find an acute angle  $\theta$  when  $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$ .  
(2022-23)

12. If  $\sqrt{3} \sin \theta - \cos \theta = 0$  and  $0^\circ < \theta < 90^\circ$ , find the value of  $\theta$ . (2020-21)

## 8.4 Trigonometric Identities

### MCQ

13. If  $\sin \theta + \csc \theta = \sqrt{2}$ , then  $\tan \theta + \cot \theta =$

- (a) 1
- (b) 2
- (c) 3
- (d) 4 (2022-23)

14. If  $2\sin^2 \beta - \cos^2 \beta = 2$ , then  $\beta$  is

- (a)  $0^\circ$
- (b)  $90^\circ$
- (c)  $45^\circ$
- (d)  $30^\circ$  (Term I, 2021-22)

15. If  $1 + \sin^2 \alpha = 3 \sin \alpha \cos \alpha$ , then values of  $\cot \alpha$  are

- (a) -1, 1
- (b) 0, 1
- (c) 1, 2
- (d) -1, -1 (Term I, 2021-22)

### VSA (1 mark)

16. If  $x = 2 \sin 2\theta$  and  $y = 2 \cos^2 \theta + 1$ , then find  $x + y$ . (2020-21)

### SA II (3 marks)

17.

Prove that :

$$\frac{\tan^3 \theta}{1 + \tan^2 \theta} + \frac{\cot^3 \theta}{1 + \cot^2 \theta} = \sec \theta \operatorname{cosec} \theta - 2 \sin \theta \cos \theta$$

(2022-23)

## SOLUTIONS

### Previous Years' CBSE Board Questions

1.

(c): We have,  $2 \tan A = 3$

$$\Rightarrow \tan A = \frac{3}{2} = \frac{P}{B}$$

Let  $P = 3k$  and  $B = 2k$

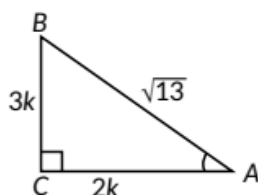
$$AB = \sqrt{2^2 + 3^2}$$

(By Pythagoras theorem)

$$\Rightarrow H = \sqrt{13}$$

$$\therefore \sin A = \frac{P}{H} = \frac{3}{\sqrt{13}}, \cos A = \frac{B}{H} = \frac{2}{\sqrt{13}}$$

$$\text{Now, } \frac{4 \sin A + 3 \cos A}{4 \sin A - 3 \cos A} = \frac{4 \left( \frac{3}{\sqrt{13}} \right) + 3 \left( \frac{2}{\sqrt{13}} \right)}{4 \left( \frac{3}{\sqrt{13}} \right) - 3 \left( \frac{2}{\sqrt{13}} \right)} = 3$$



2.

(c): Given,  $\cos \theta = \frac{\sqrt{3}}{2} = \frac{B}{H}$

Let  $B = \sqrt{3}k$  and  $H = 2k$

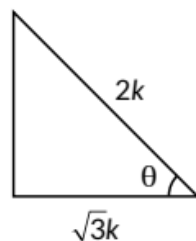
$$\therefore P = \sqrt{(2k)^2 - (\sqrt{3}k)^2}$$

[By Pythagoras Theorem]

$$\Rightarrow P = \sqrt{k^2} = k$$

$$\therefore \operatorname{cosec} \theta = \frac{H}{P} = \frac{2k}{k} = 2 \quad \sec \theta = \frac{H}{B} = \frac{2k}{\sqrt{3}k} = \frac{2}{\sqrt{3}}$$

$$\therefore \frac{\operatorname{cosec}^2 \theta - \sec^2 \theta}{\operatorname{cosec}^2 \theta + \sec^2 \theta} = \frac{(2)^2 - \left( \frac{2}{\sqrt{3}} \right)^2}{(2)^2 + \left( \frac{2}{\sqrt{3}} \right)^2} = \frac{4 - \frac{4}{3}}{4 + \frac{4}{3}} = \frac{8}{16} = \frac{1}{2}$$



3.

(c): We have,  $\frac{1}{\operatorname{cosec}\theta(1-\cot\theta)} + \frac{1}{\sec\theta(1-\tan\theta)}$   
 $= \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}$

$$\left[ \because \frac{1}{\operatorname{cosec}\theta} = \sin\theta, \frac{1}{\sec\theta} = \cos\theta, \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta} \right]$$

$$= \frac{\sin^2\theta}{\sin\theta - \cos\theta} + \frac{\cos^2\theta}{\cos\theta - \sin\theta} = \frac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta} = \sin\theta + \cos\theta$$

4.

(a): We have  $\sin\theta = \cos\theta$

or  $\frac{\sin\theta}{\cos\theta} = 1$

$$\Rightarrow \tan\theta = 1 \text{ and } \cot\theta = 1$$

$$\therefore \tan^2\theta + \cot^2\theta = 1^2 + 1^2 = 2$$

$$\left[ \because \cot\theta = \frac{1}{\tan\theta} \right]$$

Hence, A option is correct.

5.

$$\text{We have } \tan\theta + \cot\theta = \frac{4\sqrt{3}}{3} \quad \dots(i)$$

On squaring both sides of equation (i), we get

$$\tan^2\theta + \cot^2\theta + 2\tan\theta \cdot \cot\theta = \frac{16 \times 3}{9}$$

$$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta \cdot \frac{1}{\tan\theta} = \frac{16}{3}$$

$$\Rightarrow \tan^2\theta + \cot^2\theta = \frac{16}{3} - 2 = \frac{10}{3}$$

6.

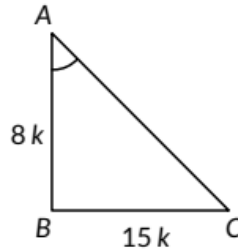
In right angle  $\triangle ABC$ , we have

$$15 \cot A = 8$$

$$\Rightarrow \cot A = \frac{8}{15}$$

$$\text{Since, } \cot A = \frac{AB}{BC}$$

$$\therefore \frac{AB}{BC} = \frac{8}{15}$$



Let  $AB = 8k$  and  $BC = 15k$

By using Pythagoras theorem, we get

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow (8k)^2 + (15k)^2 = 64k^2 + 225k^2 = 289k^2 = (17k)^2$$

$$\Rightarrow AC = \sqrt{(17k)^2} = 17k$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17} \text{ and } \cos A = \frac{AB}{AC} = \frac{8k}{17k} = \frac{8}{17}$$

$$\text{So, } \sec A = \frac{1}{\cos A} = \frac{17}{8}$$

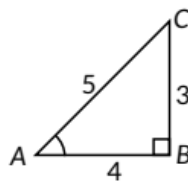
7.

$$\text{Given, } 3 \cot A = 4 \Rightarrow \cot A = \frac{4}{3} \Rightarrow \tan A = \frac{3}{4}$$

$$\text{In } \triangle ABC, AC^2 = AB^2 + BC^2 = 16 + 9 = 25 \Rightarrow AC = 5$$

$$\text{Now, L.H.S.} = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$



$$\text{R.H.S.} = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

8.

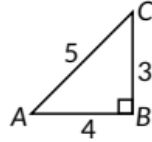
$$\text{We have, } \sin A = \frac{3}{5} = \frac{P}{H}$$

In right angled  $\triangle ABC$ , by Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow 5^2 = AB^2 + (3)^2$$

$$\Rightarrow AB^2 = 16 \Rightarrow AB = 4$$



$$\therefore \cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{4}{5}, \tan A = \frac{P}{B} = \frac{3}{4}, \operatorname{cosec} A = \frac{5}{3},$$

$$\sec A = \frac{5}{4} \text{ and } \cot A = \frac{4}{3}$$

9.

We have,  $3 \tan A = 4$

$$\Rightarrow \tan A = \frac{4}{3} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\therefore \text{Hypotenuse} = \sqrt{(4)^2 + (3)^2} = 5$$

$$\therefore \sin A = \frac{4}{5} \text{ and } \cos A = \frac{3}{5}$$

$$\text{Now, L.H.S.} = \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{4}{3}\right)^2}{1 + \left(\frac{4}{3}\right)^2} = \frac{1 - \frac{16}{9}}{1 + \frac{16}{9}} = -\frac{7}{25}$$

$$\text{and R.H.S.} = \cos^2 A - \sin^2 A$$

$$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

10.

$$= \frac{5}{8} \times (2)^2 - (\sqrt{3})^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{5}{8} \times 4 - 3 + \frac{1}{2} = 0$$

11.

(a): Given,  $\sin\alpha = \frac{\sqrt{3}}{2}$

$\Rightarrow \alpha = 60^\circ$   $\left( \because \sin 60^\circ = \frac{\sqrt{3}}{2} \right)$

and  $\tan\beta = \frac{1}{\sqrt{3}}$

$\Rightarrow \beta = 30^\circ$   $\left( \because \tan 30^\circ = \frac{1}{\sqrt{3}} \right)$

Now,  $\cos(\alpha - \beta) = \cos(60^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

12.

(a): Given,  $2 \sin 2\theta = 1 \Rightarrow \sin 2\theta = 1/2$

$\Rightarrow 2\theta = 30^\circ$   $\left( \because \sin 30^\circ = \frac{1}{2} \right)$

$\Rightarrow \theta = 15^\circ$

13.

We have,  $2 \sec 30^\circ \times \tan 60^\circ$

$$= 2 \times \frac{2}{\sqrt{3}} \times \sqrt{3} = 4$$

14.

We have,  $\sin^2 30^\circ + \cos^2 60^\circ$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$$

15.

$$\text{We have, } \frac{2 \tan 45^\circ \times \cos 60^\circ}{\sin 30^\circ} = \frac{2 \times 1 \times \frac{1}{2}}{\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

16.

$$\text{Given, } \sin x + \cos y = 1$$

$$\Rightarrow \sin 30^\circ + \cos y = 1 \quad [\text{Given, } x = 30^\circ]$$

$$\Rightarrow \frac{1}{2} + \cos y = 1 \Rightarrow \cos y = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \cos y = \cos 60^\circ \Rightarrow y = 60^\circ$$

17.

$$\text{We have, } \sin \alpha = \frac{1}{2}$$

$$\text{Now, } 3\sin \alpha - 4\sin^3 \alpha = 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3 = \frac{3}{2} - \frac{4}{8} = \frac{3}{2} - \frac{1}{2} = 1$$

18.

$$\begin{aligned} \text{Put } \theta = 45^\circ \text{ in } 2\sec^2 \theta + 3\operatorname{cosec}^2 \theta - 2\sin \theta \cos \theta \\ = 2\sec^2(45^\circ) + 3\operatorname{cosec}^2(45^\circ) - 2\sin(45^\circ)\cos(45^\circ) \\ = 2(\sqrt{2})^2 + 3(\sqrt{2})^2 - 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 4 + 6 - 1 = 9 \end{aligned}$$

19.

$$\text{Given, } \sin \theta - \cos \theta = 0$$

$$\Rightarrow \sin \theta = \cos \theta \Rightarrow \frac{\sin \theta}{\cos \theta} = 1$$

$$\Rightarrow \tan \theta = 1$$

$$\left[ \because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\Rightarrow \tan \theta = \tan 45^\circ$$

$$[\because \tan 45^\circ = 1]$$

$$\Rightarrow \theta = 45^\circ$$

$$\therefore \sin^4 \theta + \cos^4 \theta = \sin^4(45^\circ) + \cos^4(45^\circ)$$

$$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = 2\left(\frac{1}{\sqrt{2}}\right)^4 = 2\left(\frac{1}{4}\right) = \frac{1}{2}$$

20.

$$\begin{aligned} \frac{5}{\cot^2 30^\circ} + \frac{1}{\sin^2 60^\circ} - \cot^2 45^\circ + 2\sin^2 90^\circ \\ = \frac{5}{(\sqrt{3})^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - (1)^2 + 2(1)^2 = \frac{5}{3} + \frac{4}{3} - 1 + 2 = 4 \end{aligned}$$

21.

Given,  $\sin\theta = \cos\theta$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = 1 \Rightarrow \tan\theta = 1$$

$$\Rightarrow \tan\theta = \tan\frac{\pi}{4} \quad [\theta \text{ is acute}]$$

$$\therefore \theta = \frac{\pi}{4}$$

So,  $\tan^2\theta + \cot^2\theta - 2$

$$= \tan^2\left(\frac{\pi}{4}\right) + \cot^2\left(\frac{\pi}{4}\right) - 2 = 1 + 1 - 2 = 0$$

22.

Given that,  $A = 60^\circ, B = 30^\circ$

$$\therefore \cos A = \cos 60^\circ = \frac{1}{2}; \cos B = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\text{Now, } \cos A + \cos B = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1 + \sqrt{3}}{2}$$

and  $\cos(A + B) = \cos(60^\circ + 30^\circ) = \cos 90^\circ = 0$

$$\therefore \cos(A + B) \neq \cos A + \cos B.$$

23.

Consider an equilateral triangle  $ABC$  with each side of length  $2a$ , and  $\angle A = \angle B = \angle C = 60^\circ$

$$\Rightarrow AB = BC = CA = 2a$$

Now, draw  $AD \perp BC$

Now, in  $\triangle ADB$  and  $\triangle ADC$

$\angle ADB = \angle ADC$  (each  $90^\circ$ )

$$AB = AC$$

$$AD = AD \quad (\text{common})$$

$$\therefore \triangle ADB \cong \triangle ADC \quad (\text{By R.H.S.})$$

$$\therefore BD = DC \text{ and } \angle BAD = \angle CAD$$

(By C.P.C.T.)

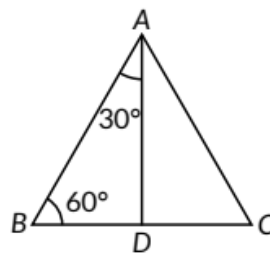
$$\therefore BD = DC = a \text{ and } \angle BAD = 30^\circ$$

In  $\triangle ADB$ ,  $AB = 2a, BD = a, \angle DAB = 30^\circ$

$$\therefore \operatorname{cosec} 30^\circ = \frac{AB}{BD} = \frac{2a}{a} = 2$$

Again in  $\triangle ADB$ , we have  $\angle ABD = 60^\circ$

$$\therefore \cos 60^\circ = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$$



24.

We have,  $\sin(A + B) = 1$

$$\Rightarrow \sin(A + B) = \sin 90^\circ \Rightarrow A + B = 90^\circ \quad \dots(i)$$

$$\text{Also, } \sin(A - B) = \frac{1}{2}$$

$$\Rightarrow \sin(A - B) = \sin 30^\circ \Rightarrow A - B = 30^\circ \quad \dots(ii)$$

Adding (i) and (ii), we get

$$\Rightarrow (A + B) + (A - B) = 120^\circ \Rightarrow 2A = 120^\circ \Rightarrow A = 60^\circ$$

$$\text{From (i), we have } 60^\circ + B = 90^\circ \Rightarrow B = 30^\circ$$

25.

Given,  $\theta = 30^\circ$

$$(i) \quad \cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

Put  $\theta = 30^\circ$ , we get

$$\text{R.H.S.} = 4 \cos^3 30^\circ - 3 \cos 30^\circ$$

$$= 4 \left( \frac{\sqrt{3}}{2} \right)^3 - 3 \left( \frac{\sqrt{3}}{2} \right) = \frac{4 \times 3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0 = \cos 90^\circ$$

$$= \cos(3 \times 30^\circ) = \cos 3\theta = \text{L.H.S.}$$

$$(ii) \quad \text{R.H.S.} = 3 \sin \theta - 4 \sin^3 \theta$$

$$= 3 \sin 30^\circ - 4 \sin^3 30^\circ = 3 \times \frac{1}{2} - 4 \times \frac{1}{8} = \frac{3}{2} - \frac{1}{2} = 1$$

$$\text{L.H.S.} = \sin 3\theta = \sin(3 \times 30^\circ) = \sin 90^\circ = 1 \therefore \text{L.H.S.} = \text{R.H.S.}$$

26.

We know that,  $\sin 30^\circ = \frac{1}{2}$ ;  $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$$\tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}; \cot 30^\circ = \frac{1}{\tan 30^\circ} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

$$\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}; \operatorname{cosec} 30^\circ = \frac{1}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}; \cos 45^\circ = \frac{1}{\sqrt{2}}; \tan 45^\circ = \frac{\sin 45^\circ}{\cos 45^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

$$\cot 45^\circ = \frac{1}{\tan 45^\circ} = 1; \sec 45^\circ = \frac{1}{\cos 45^\circ} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\operatorname{cosec} 45^\circ = \frac{1}{\sin 45^\circ} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

27.

Given,  $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

(i) Putting  $A = 45^\circ, B = 30^\circ$ , we get,

$\sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$$\Rightarrow \sin 75^\circ = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

(ii)  $\cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$$\Rightarrow \cos 15^\circ = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

28.

(b): We have,  $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1)$

$$= (\tan^2\theta)(\cot^2\theta)$$

( $\because \sec^2\theta - 1 = \tan^2\theta, \operatorname{cosec}^2\theta - 1 = \cot^2\theta$ )

$$= \tan^2\theta \times \frac{1}{\tan^2\theta} = 1 \quad \left( \because \cot\theta = \frac{1}{\tan\theta} \right)$$

29. (c):  $\sec^2\theta - \tan^2\theta = 1$

30.

(c): Given,  $\sin\theta = \frac{p}{q}$

$$\therefore \cos\theta = \sqrt{1 - \left(\frac{p}{q}\right)^2} \quad \left[ \because \cos\theta = \sqrt{1 - \sin^2\theta} \right]$$

$$\Rightarrow \cos\theta = \frac{\sqrt{q^2 - p^2}}{q}$$

$$\text{Now, } \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{p}{q}}{\frac{\sqrt{q^2 - p^2}}{q}} = \frac{p}{\sqrt{q^2 - p^2}}$$

31.

(d):  $\sqrt{(1 - \cos^2\theta)(1 + \tan^2\theta)} = \sqrt{\sin^2\theta \cdot \sec^2\theta}$

$$\left[ \because 1 + \tan^2\theta = \sec^2\theta \right]$$

$$= \sqrt{\frac{\sin^2\theta}{\cos^2\theta}}$$

$$\left[ \because \sec^2\theta = \frac{1}{\cos^2\theta} \right]$$

$$= \sqrt{\tan^2\theta} = \tan\theta$$

$$\left[ \because \tan\theta = \frac{\sin\theta}{\cos\theta} \right]$$

32. (b): Given,  $\sin^2\theta + \sin\theta = 1$  ... (i)

$$\Rightarrow \sin\theta = 1 - \sin^2\theta \Rightarrow \sin\theta = \cos^2\theta \text{ ... (ii)}$$

$$\therefore \cos^2\theta + \cos^2\theta$$

$$= \sin\theta + \sin^2\theta \text{ [From (ii)]}$$

$$= 1 \text{ [From (i)]}$$

33. (c): Let A( $a\cos\theta + b\sin\theta, 0$ ) and B( $0, a\sin\theta - b\cos\theta$ ) Using distance formula, we have

$$AB = \sqrt{(a\cos\theta + b\sin\theta - 0)^2 + (0 - a\sin\theta + b\cos\theta)^2}$$

$$= \sqrt{a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta}$$

$$= \sqrt{a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)}$$

$$= \sqrt{a^2 + b^2} \quad (\because \sin^2\theta + \cos^2\theta = 1)$$

34.

We have  $3 \sin A = 1$

$$\therefore \sin A = \frac{1}{3}$$

Now by using  $\cos^2 A = 1 - \sin^2 A$ , we get

$$\cos^2 A = 1 - \frac{1}{9} = \frac{8}{9} \Rightarrow \cos A = \frac{2\sqrt{2}}{3}$$

$$\therefore \sec A = \frac{1}{\cos A} = \frac{1}{\frac{2\sqrt{2}}{3}} = \frac{3}{2\sqrt{2}}$$

35.

We have L.H.S.

$$\frac{1 + \cot^2 \theta}{1 + \tan^2 \theta} = \frac{\operatorname{cosec}^2 \theta}{\sec^2 \theta}$$

[By using  $1 + \tan^2 \theta = \sec^2 \theta$  and  $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$ ]

$$\Rightarrow \frac{1/\sin^2 \theta}{1/\cos^2 \theta} = \frac{\cos^2 \theta}{\sin^2 \theta} = \cot^2 \theta = \text{R.H.S.}$$

$$\text{Hence, } \frac{1 + \cot^2 \theta}{1 + \tan^2 \theta} = \cot^2 \theta$$

36. We have,  $5(\tan^2 0 - \sec^2 0)$

$$= 5(-1) = -5$$

{By using  $1 + \tan^2 0 = \sec^2 0 \Rightarrow \tan^2 0 - \sec^2 0 = -1$ }

37.

$$\begin{aligned} & (1 - \cos^2 A)(1 + \cot^2 A) \\ &= (1 - \cos^2 A) \left( 1 + \frac{\cos^2 A}{\sin^2 A} \right) \quad \left( \because \cot A = \frac{\cos A}{\sin A} \right) \\ &= (1 - \cos^2 A) \left( \frac{\sin^2 A + \cos^2 A}{\sin^2 A} \right) \\ &= \frac{\sin^2 A}{\sin^2 A} \quad (\because \sin^2 A + \cos^2 A = 1) = 1 \end{aligned}$$

38.

We have,

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

39.

$$\begin{aligned} \text{We have, } & \sin^2 \theta + \frac{1}{1 + \tan^2 \theta} \\ &= \sin^2 \theta + \frac{1}{\sec^2 \theta} \quad [\because 1 + \tan^2 \theta = \sec^2 \theta] \\ &= \sin^2 \theta + \cos^2 \theta \quad [\because \sec \theta = 1/\cos \theta] \\ &= 1 \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \end{aligned}$$

40.

$$\begin{aligned} \text{We have, } & (1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta) \\ &= \sec^2 \theta (1 - \sin^2 \theta) = \sec^2 \theta \cos^2 \theta = \frac{1}{\cos^2 \theta} \times \cos^2 \theta \\ &= 1 \quad \left[ \sec \theta = \frac{1}{\cos \theta} \right] \end{aligned}$$

41.

$$\begin{aligned} \text{We have } & \operatorname{cosec}^2 \theta (1 + \cos \theta)(1 - \cos \theta) = k \\ & \quad [\because (a + b)(a - b) = (a^2 - b^2)] \\ \Rightarrow & \operatorname{cosec}^2 \theta (1 - \cos^2 \theta) = k \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \\ \Rightarrow & \operatorname{cosec}^2 \theta (\sin^2 \theta) = k \\ \Rightarrow & \frac{1}{\sin^2 \theta} \cdot \sin^2 \theta = k \\ \Rightarrow & k = 1 \end{aligned}$$

42.

Given,  $\sec \theta + \tan \theta = x$

Now, we know that,  $1 = \sec^2 \theta - \tan^2 \theta$

$$\Rightarrow 1 = (\sec \theta + \tan \theta)(\sec \theta - \tan \theta)$$

$$\Rightarrow 1 = x(\sec \theta - \tan \theta)$$

$$\Rightarrow \frac{1}{x} = \sec \theta - \tan \theta \therefore \sec \theta - \tan \theta = \frac{1}{x}$$

43.

We have,  $(\sec^2 \theta - 1) \times \cot^2 \theta$

$$= \left( \frac{1}{\cos^2 \theta} - 1 \right) \cdot \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1 - \cos^2 \theta}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} = 1$$

44.

We have,  $\sec^2 \theta - \frac{1}{\operatorname{cosec}^2 \theta - 1}$

$$= \frac{1}{\cos^2 \theta} - \frac{1}{\frac{1}{\sin^2 \theta} - 1} = \frac{1}{\cos^2 \theta} - \frac{1}{\frac{1 - \sin^2 \theta}{\sin^2 \theta}}$$

$$= \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1 - \sin^2 \theta}{\cos^2 \theta} \quad [\because \cos^2 \theta + \sin^2 \theta = 1]$$

$$= \frac{\cos^2 \theta}{\cos^2 \theta} = 1, \text{ which is the simplest form.}$$

45. (a) Given,  $\sin \theta + \cos \theta = \sqrt{3}$

Squaring both sides, we get  $(\sin \theta + \cos \theta)^2 = 3$

$$= \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 3$$

$$= 2 \sin \theta \cos \theta = 3 - 1 \quad (\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$= 2 \sin \theta \cos \theta = 2$$

$$= \sin \theta \cos \theta = 1$$

46.

$$\text{Given, } \sin\alpha = \frac{1}{\sqrt{2}} \text{ and } \cot\beta = \sqrt{3}$$

$$\text{We know that, } \operatorname{cosec}\alpha = \frac{1}{\sin\alpha} = \sqrt{2}$$

$$\text{Also, } 1 + \cot^2\beta = \operatorname{cosec}^2\beta$$

$$\Rightarrow \operatorname{cosec}^2\beta = 4$$

$$\Rightarrow \operatorname{cosec}\beta = 2$$

$$\text{Now, } \operatorname{cosec}\alpha + \operatorname{cosec}\beta = \sqrt{2} + 2$$

47.

$$\text{We have, } x = p \sec\theta + q \tan\theta \text{ and } y = p \tan\theta + q \sec\theta$$

$$\text{Now, L.H.S.} = x^2 - y^2$$

$$= (p \sec\theta + q \tan\theta)^2 - (p \tan\theta + q \sec\theta)^2$$

$$= (p^2 \sec^2\theta + q^2 \tan^2\theta + 2pq \sec\theta \tan\theta)$$

$$- (p^2 \tan^2\theta + q^2 \sec^2\theta + 2pq \tan\theta \sec\theta)$$

$$= p^2 \sec^2\theta + q^2 \tan^2\theta - p^2 \tan^2\theta - q^2 \sec^2\theta$$

$$= p^2(\sec^2\theta - \tan^2\theta) - q^2(\sec^2\theta - \tan^2\theta)$$

$$= p^2 - q^2 \quad [\because \sec^2\theta - \tan^2\theta = 1]$$

$$= \text{R.H.S.}$$

48.

$$\text{We have, L.H.S.} = \frac{1 + \tan^2 A}{1 + \cot^2 A}$$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A = \text{R.H.S.}$$

49.

$$\begin{aligned}\text{L.H.S.} &= \sqrt{\frac{1+\cos A}{1-\cos A}} = \sqrt{\frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}} \\ &= \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}} = \sqrt{\left(\frac{1+\cos A}{\sin A}\right)^2} = \frac{1+\cos A}{\sin A} \\ &= \frac{1}{\sin A} + \frac{\cos A}{\sin A} = \operatorname{cosec} A + \cot A = \text{R.H.S.}\end{aligned}$$

50.

$$\begin{aligned}\text{We have, L.H.S.} &= \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} \\ &= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)} = \frac{\sin A(1 - 2(1 - \cos^2 A))}{\cos A(2\cos^2 A - 1)} \\ &= \frac{\sin A(2\cos^2 A - 1)}{\cos A(2\cos^2 A - 1)} = \tan A = \text{R.H.S.}\end{aligned}$$

51.

$$\begin{aligned}\text{L.H.S.} &= \sec A (1 - \sin A)(\sec A + \tan A) \\ &= \frac{(1 - \sin A)(1 + \sin A)}{\cos^2 A} \\ &= \frac{1 - \sin^2 A}{\cos^2 A} && (\because (a - b)(a + b) = a^2 - b^2) \\ &= \frac{\cos^2 A}{\cos^2 A} && (\because 1 - \sin^2 A = \cos^2 A) \\ &= 1 = \text{R.H.S.}\end{aligned}$$

52.

$$\begin{aligned}
 \text{L.H.S.} &= (\operatorname{cosec} A - \sin A)(\sec A - \cos A) \\
 &= \left( \frac{1}{\sin A} - \sin A \right) \left( \frac{1}{\cos A} - \cos A \right) \\
 &= \left( \frac{1 - \sin^2 A}{\sin A} \right) \left( \frac{1 - \cos^2 A}{\cos A} \right) = \frac{\cos^2 A \times \sin^2 A}{\sin A \cos A} \\
 &\quad [\because 1 - \sin^2 A = \cos^2 A \text{ and } 1 - \cos^2 A = \sin^2 A] \\
 &= \frac{\sin A \cdot \cos A}{1} = \frac{\sin A \cdot \cos A}{\sin^2 A + \cos^2 A} \quad [\because 1 = \sin^2 A + \cos^2 A] \\
 &= \frac{\frac{\sin A \cos A}{\sin A \cos A}}{\frac{\sin^2 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A}} \quad [\text{Dividing numerator and denominator by } \sin A \cos A] \\
 &= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\tan A + \cot A} = \text{R.H.S.}
 \end{aligned}$$

53.

We have  $\sin^6 A + 3 \sin^2 A \cos^2 A = 1 - \cos^6 A$   
 Rewriting and arranging the given equation as  
 $\sin^6 A + \cos^6 A = 1 - 3 \sin^2 A \cos^2 A$  ... (i)  
 Now taking L.H.S. of equation (i), we get  
 $\sin^6 A + \cos^6 A = (\sin^2 A)^3 + (\cos^2 A)^3$   
 {By using  $(a + b)^3 = a^3 + b^3 + 3ab(a + b) \Rightarrow a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ , here  $a = \sin^2 A$  and  $b = \cos^2 A$ }  
 $\therefore \sin^6 A + \cos^6 A = (\sin^2 A + \cos^2 A)^3 - 3 \sin^2 A \cos^2 A$   
 $(\sin^2 A + \cos^2 A)$   
 $= 1^2 - 3 \sin^2 A \cos^2 A = \text{R.H.S.} \quad [\because \sin^2 A + \cos^2 A = 1]$   
 $\Rightarrow \text{L.H.S.} = \text{R.H.S.}$   
 Hence proved.

54.

$$\text{We have, } \frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{1-\sin\theta}{\cos\theta} \quad \dots(i)$$

On taking L.H.S. of equation (i), we get

$$\begin{aligned} \frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} &= \frac{(\sec^2\theta-\tan^2\theta)+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} \\ & \quad [\because 1+\tan^2\theta = \sec^2\theta] \\ &= \frac{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)+(\sec\theta-\tan\theta)}{1+\tan\theta+\sec\theta} \\ & \quad [\because (a-b)(a+b) = a^2 - b^2] \\ &= \frac{(\sec\theta-\tan\theta)[(\sec\theta+\tan\theta+1)]}{[\sec\theta+\tan\theta+1]} = \sec\theta - \tan\theta \\ &= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \frac{1-\sin\theta}{\cos\theta} = \text{R.H.S.} \end{aligned}$$

$$\text{So, } \frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{1-\sin\theta}{\cos\theta}$$

Hence proved.

55.

$$\text{We have, } \frac{1+\tan A}{2\sin A} + \frac{1+\cot A}{2\cos A} = \operatorname{cosec} A + \sec A \quad \dots(i)$$

On taking L.H.S. of equation (i), we get

$$\begin{aligned} &\Rightarrow \frac{1+\frac{\sin A}{\cos A}}{2\sin A} + \frac{1+\frac{\cos A}{\sin A}}{2\cos A} \\ &\Rightarrow \frac{\cos A + \sin A}{2\sin A \cos A} + \frac{\sin A + \cos A}{2\sin A \cos A} \\ &= \frac{\cos A + \sin A + \sin A + \cos A}{2\sin A \cos A} = \frac{2[\cos A + \sin A]}{2\sin A \cos A} \\ &= \frac{\sin A + \cos A}{\sin A \cos A} = \frac{\sin A}{\sin A \cos A} + \frac{\cos A}{\sin A \cos A} \\ &= \frac{1}{\cos A} + \frac{1}{\sin A} = \sec A + \operatorname{cosec} A = \text{R.H.S.} \end{aligned}$$

Hence,  $\frac{1+\tan A}{2\sin A} + \frac{1+\cot A}{2\cos A} = \operatorname{cosec} A + \sec A$  proved.

56.

$$\begin{aligned} \text{L.H.S.} &= \frac{2\cos^3 \theta - \cos \theta}{\sin \theta - 2\sin^3 \theta} \\ &= \frac{\cos \theta (2\cos^2 \theta - 1)}{\sin \theta (1 - 2\sin^2 \theta)} = \frac{\cot \theta (2(1 - \sin^2 \theta) - 1)}{(1 - 2\sin^2 \theta)} \\ &\qquad\qquad\qquad (\because \sin^2 \theta + \cos^2 \theta = 1) \\ &= \frac{\cot \theta (2 - 2\sin^2 \theta - 1)}{(1 - 2\sin^2 \theta)} = \frac{\cot \theta (1 - 2\sin^2 \theta)}{(1 - 2\sin^2 \theta)} = \cot \theta = \text{R.H.S.} \end{aligned}$$

57.

We know that  $\sin^2\theta + \cos^2\theta = 1$

Squaring both sides, we get

$$(\sin^2\theta + \cos^2\theta)^2 = 1$$

$$\Rightarrow \sin^4\theta + \cos^4\theta + 2\sin^2\theta \cos^2\theta = 1$$

$$\Rightarrow \sin^4\theta + \cos^4\theta + 2\sin^2\theta(1 - \sin^2\theta) = 1$$

$$\Rightarrow \sin^4\theta + \cos^4\theta + 2\sin^2\theta - 2\sin^4\theta = 1$$

$$\Rightarrow \cos^4\theta - \sin^4\theta + 2\sin^2\theta = 1$$

$$\Rightarrow \sin^4\theta - \cos^4\theta - 2\sin^2\theta = -1$$

$$\Rightarrow \sin^4\theta - \cos^4\theta + 1 = 2\sin^2\theta$$

$$\Rightarrow (\sin^4\theta - \cos^4\theta + 1) \operatorname{cosec}^2\theta = 2$$

58.

$$\begin{aligned} \text{L.H.S.} &= \sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}} \\ &= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} = \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \sqrt{\left(\frac{1+\sin A}{\cos A}\right)^2} \\ &= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A = \text{R.H.S.} \end{aligned}$$

59.

Given,  $\sin\theta + \cos\theta = \sqrt{3}$

Squaring both sides, we get  $(\sin\theta + \cos\theta)^2 = 3$

$$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta \cos\theta = 3$$

$$\Rightarrow 2\sin\theta \cos\theta = 3 - 1 \Rightarrow 2\sin\theta \cos\theta = 2$$

$$\Rightarrow \sin\theta \cos\theta = 1$$

...(i)

$$\text{L.H.S.} = \tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cos\theta} = \frac{1}{\sin\theta \cos\theta} = \frac{1}{1}$$

[Using (i)]

$$= 1 = \text{R.H.S.}$$

60.

$$\text{We have, } 1 + \frac{\cot^2 \theta}{1 + \operatorname{cosec} \theta} = \operatorname{cosec} \theta$$

On taking L.H.S. of given equation, we have

$$\begin{aligned} 1 + \frac{\cot^2 \theta}{1 + \operatorname{cosec} \theta} &= 1 + \frac{\cos^2 \theta / \sin^2 \theta}{1 + \frac{1}{\sin \theta}} = 1 + \frac{\frac{\cos^2 \theta}{\sin^2 \theta}}{\frac{\sin \theta + 1}{\sin \theta}} \\ &= 1 + \frac{\cos^2 \theta}{\sin^2 \theta + \sin \theta} \\ &= \frac{\sin^2 \theta + \sin \theta + \cos^2 \theta}{\sin \theta (\sin \theta + 1)} = \frac{(\sin^2 \theta + \cos^2 \theta) + \sin \theta}{\sin \theta (\sin \theta + 1)} \\ &= \frac{(1 + \sin \theta)}{\sin \theta (\sin \theta + 1)} = \frac{1}{\sin \theta} \quad [\because \sin^2 \theta + \cos^2 \theta = 1] \end{aligned}$$

$$= \operatorname{cosec} \theta = \text{R.H.S.}$$

$$\Rightarrow \text{L.H.S.} = \text{R.H.S.}$$

Hence proved.

61.

We have,

$$\begin{aligned} \text{L.H.S.} &= (\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 \\ &= (\sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \operatorname{cosec} \theta) + \\ &\quad (\cos^2 \theta + \sec^2 \theta + 2 \cos \theta \sec \theta) \\ &= (\sin^2 \theta + \operatorname{cosec}^2 \theta + 2) + (\cos^2 \theta + \sec^2 \theta + 2) \\ &= (\sin^2 \theta + \cos^2 \theta) + \operatorname{cosec}^2 \theta + \sec^2 \theta + 4 \\ &= 1 + (1 + \cot^2 \theta) + (1 + \tan^2 \theta) + 4 \\ &= 1 + 1 + \cot^2 \theta + 1 + \tan^2 \theta + 4 = 7 + \tan^2 \theta + \cot^2 \theta = \text{R.H.S.} \end{aligned}$$

62.

$$\begin{aligned}\text{L.H.S.} &= (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) \\ &= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right) \left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right) \\ &= \left(\frac{(\sin A + \cos A - 1)(\sin A + \cos A + 1)}{\sin A \cos A}\right) = \frac{(\sin A + \cos A)^2 - 1^2}{\sin A \cos A} \\ &= \frac{\sin^2 A + \cos^2 A + 2 \sin A \cos A - 1}{\sin A \cos A} \\ &= \frac{1 + 2 \sin A \cos A - 1}{\sin A \cos A} \quad [\because \sin^2 A + \cos^2 A = 1] \\ &= 2 = \text{R.H.S.}\end{aligned}$$

63.

$$\begin{aligned}\text{L.H.S.} &= \frac{\tan \theta}{1 - \tan \theta} - \frac{\cot \theta}{1 - \cot \theta} \\ &= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\sin \theta}{\cos \theta}} - \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\cos \theta}{\sin \theta}} = \frac{\sin \theta}{\cos \theta - \sin \theta} + \frac{\cos \theta}{\cos \theta - \sin \theta} \\ &= \frac{\sin \theta + \cos \theta}{\cos \theta - \sin \theta} = \text{R.H.S.}\end{aligned}$$

64.

$$\begin{aligned}\text{Given, } \cos \theta + \sin \theta &= \sqrt{2} \cos \theta \\ \Rightarrow \sin \theta &= (\sqrt{2} - 1) \cos \theta \\ \text{Multiplying both sides by } (\sqrt{2} + 1), &\text{ we get}\end{aligned}$$

65.

$$\text{Given, } 4 \tan \theta = 3$$

$$\Rightarrow \tan \theta = \frac{3}{4} \text{ and } \sec \theta = \sqrt{1 + \tan^2 \theta}$$

$$\Rightarrow \sec \theta = \sqrt{1 + \left(\frac{3}{4}\right)^2} = \sqrt{\frac{16+9}{16}} = \frac{5}{4}$$

$$\text{We have, } \frac{4 \sin \theta - \cos \theta + 1}{4 \sin \theta + \cos \theta - 1} = \frac{4 \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\cos \theta} + \frac{1}{\cos \theta}}{4 \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\cos \theta} - \frac{1}{\cos \theta}}$$

$$= \frac{4 \tan \theta - 1 + \sec \theta}{4 \tan \theta + 1 - \sec \theta}$$

$$\left[ \because \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \sec \theta = \frac{1}{\cos \theta} \right]$$

$$= \frac{4\left(\frac{3}{4}\right) - 1 + \frac{5}{4}}{4\left(\frac{3}{4}\right) + 1 - \frac{5}{4}} = \frac{\frac{12}{4} - 1 + \frac{5}{4}}{\frac{12}{4} + 1 - \frac{5}{4}} = \frac{\frac{12-4+5}{4}}{\frac{12+4-5}{4}} = \frac{13}{11}$$

66.

$$\text{We have, L.H.S.} = \frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1}$$

$$= \frac{\frac{\sin \theta - \cos \theta + 1}{\cos \theta}}{\frac{\sin \theta + \cos \theta - 1}{\cos \theta}} = \frac{\tan \theta - 1 + \sec \theta}{\tan \theta + 1 - \sec \theta}$$

$$= \frac{\tan \theta + \sec \theta - 1}{\tan \theta + 1 - \sec \theta} = \frac{\tan \theta + \sec \theta - [\sec^2 \theta - \tan^2 \theta]}{\tan \theta + 1 - \sec \theta}$$

$$= \frac{\tan \theta + \sec \theta - [(\sec \theta + \tan \theta)(\sec \theta - \tan \theta)]}{\tan \theta + 1 - \sec \theta}$$

$$= \frac{(\tan \theta + \sec \theta)(1 - \sec \theta + \tan \theta)}{1 - \sec \theta + \tan \theta}$$

$$= (\tan \theta + \sec \theta) = (\tan \theta + \sec \theta) \times \frac{\tan \theta - \sec \theta}{\tan \theta - \sec \theta}$$

$$= \frac{\tan^2 \theta - \sec^2 \theta}{\tan \theta - \sec \theta} = \frac{-1}{\tan \theta - \sec \theta} = \frac{1}{\sec \theta - \tan \theta} = \text{R.H.S.}$$

67.

$$\text{Given, } \tan A = \frac{1}{2}$$

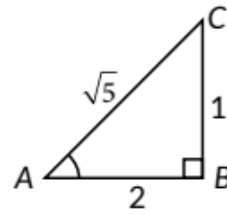
Consider a right angled  $\triangle ABC$ ,  
By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = 4 + 1 \Rightarrow AC = \sqrt{5}$$

Now,

$$\begin{aligned} \frac{\cos A}{\sin A} + \frac{\sin A}{1 + \cos A} &= \frac{\cos A(1 + \cos A) + \sin^2 A}{\sin A(1 + \cos A)} \\ &= \frac{\cos A + (\cos^2 A + \sin^2 A)}{\sin A(1 + \cos A)} = \frac{\cos A + 1}{\sin A(1 + \cos A)} \\ &= \frac{1}{\sin A} = \operatorname{cosec} A = \sqrt{5}. \end{aligned}$$



68.

We have,

$$\begin{aligned} \text{L.H.S.} &= \frac{\operatorname{cosec} A - \sin A}{\operatorname{cosec} A + \sin A} = \frac{\frac{1}{\sin A} - \sin A}{\frac{1}{\sin A} + \sin A} \\ &= \frac{1 - \sin^2 A}{1 + \sin^2 A} = \frac{\frac{1 - \sin^2 A}{\cos^2 A}}{\frac{1 + \sin^2 A}{\cos^2 A}} = \frac{\sec^2 A - \tan^2 A}{\sec^2 A + \tan^2 A} = \text{R.H.S.} \end{aligned}$$

69.

$$\text{Given, } \sin \theta = \frac{12}{13} \Rightarrow \cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$= \sqrt{1 - \left(\frac{12}{13}\right)^2} = \sqrt{\frac{169 - 144}{169}} = \frac{5}{13}$$

$$\therefore \cos \theta = \frac{5}{13} \text{ and } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{12}{5}$$

$$\text{Now, } \frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta} = \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2 \left(\frac{12}{13}\right) \left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

$$= \frac{\frac{144}{169} - \frac{25}{169}}{\frac{120}{169}} \times \frac{1}{\frac{144}{25}} = \frac{119}{120} \times \frac{25}{144} = \frac{595}{3456}$$

70.

We have,

$$\text{L.H.S.} = \sin^2 \theta \tan \theta + \cos^2 \theta \cot \theta + 2 \sin \theta \cos \theta$$

$$= \sin^2 \theta \frac{\sin \theta}{\cos \theta} + \cos^2 \theta \frac{\cos \theta}{\sin \theta} + 2 \sin \theta \cos \theta$$

$$= \frac{\sin^3 \theta}{\cos \theta} + \frac{\cos^3 \theta}{\sin \theta} + 2 \sin \theta \cos \theta = \frac{\sin^4 \theta + \cos^4 \theta}{\cos \theta \sin \theta} + 2 \sin \theta \cos \theta$$

$$= \frac{\sin^4 \theta + \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{(\sin^2 \theta + \cos^2 \theta)^2}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} \quad \dots(\text{i})$$

$$\text{Now, R.H.S.} = \tan \theta + \cot \theta$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{\sin \theta \cos \theta} \quad \dots(\text{ii})$$

From (i) and (ii), L.H.S. = R.H.S.

71.

$$\begin{aligned}\text{We have, L.H.S.} &= \frac{1}{\operatorname{cosec}\theta + \cot\theta} - \frac{1}{\sin\theta} \\ &= \frac{1}{\operatorname{cosec}\theta + \cot\theta} \times \frac{\operatorname{cosec}\theta - \cot\theta}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} \\ &= \frac{\operatorname{cosec}\theta - \cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta} - \frac{1}{\sin\theta} = \operatorname{cosec}\theta - \cot\theta - \operatorname{cosec}\theta = -\cot\theta \\ \text{Now, R.H.S.} &= \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta - \cot\theta} \\ &= \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta - \cot\theta} \times \frac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}\theta + \cot\theta} \\ &= \operatorname{cosec}\theta - \frac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta} = \operatorname{cosec}\theta - \operatorname{cosec}\theta - \cot\theta \\ &= -\cot\theta \quad [\because \operatorname{cosec}^2\theta - \cot^2\theta = 1] \\ \therefore \text{L.H.S.} &= \text{R.H.S.}\end{aligned}$$

72.

$$\begin{aligned}\text{We have } 1 + \sin^2\theta &= 3 \sin\theta \cos\theta \\ \sin^2\theta + \cos^2\theta + \sin^2\theta &= 3 \sin\theta \cos\theta \\ 2 \sin^2\theta + \cos^2\theta &= 3 \sin\theta \cos\theta \quad \dots(i) \\ \text{On dividing equation (i) each term by } \cos^2\theta, &\text{ we get} \\ 2 \tan^2\theta + 1 &= 3 \tan\theta \\ 2 \tan^2\theta - 2 \tan\theta - \tan\theta + 1 &= 0 \\ 2 \tan\theta (\tan\theta - 1) - 1 (\tan\theta - 1) &= 0 \\ \Rightarrow (\tan\theta - 1)(2 \tan\theta - 1) &= 0 \\ \Rightarrow \text{If } \tan\theta - 1 = 0 \Rightarrow \tan\theta &= 1 \\ \Rightarrow \text{If } 2 \tan\theta - 1 = 0 \Rightarrow \tan\theta &= \frac{1}{2}\end{aligned}$$

73.

We have,

$$\begin{aligned} \text{L.H.S.} &= \frac{\tan^2 A}{\tan^2 A - 1} + \frac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A} \\ &= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - 1} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}} \\ &= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A} = \frac{\sin^2 A + \cos^2 A}{\sin^2 A - \cos^2 A} \\ &= \frac{1}{\sin^2 A - \cos^2 A} \quad [\because \sin^2 A + \cos^2 A = 1] \\ &= \frac{1}{1 - \cos^2 A - \cos^2 A} = \frac{1}{1 - 2\cos^2 A} = \text{R.H.S.} \end{aligned}$$

74.

We know that  $\operatorname{cosec}^2 A = 1 + \cot^2 A$

$$\Rightarrow \operatorname{cosec} A = \sqrt{1 + \cot^2 A} \Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

$$\text{Now, } \sec^2 A = 1 + \tan^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A} \text{ and } \cos A = \frac{\cot A}{\sqrt{\cot^2 A + 1}}$$

75.

We have,  $\sin A + \sin^3 A = \cos^2 A$

$$\Rightarrow \sin A(1 + \sin^2 A) = \cos^2 A$$

Squaring both sides, we get  $\sin^2 A(1 + \sin^2 A)^2 = \cos^4 A$

$$\Rightarrow (1 - \cos^2 A)(1 + (1 - \cos^2 A))^2 = \cos^4 A$$

$$\Rightarrow (1 - \cos^2 A)(2 - \cos^2 A)^2 = \cos^4 A$$

$$\Rightarrow (1 - \cos^2 A)(4 + \cos^4 A - 4\cos^2 A) = \cos^4 A$$

$$\Rightarrow 4 + \cos^4 A - 4\cos^2 A - 4\cos^2 A - \cos^6 A$$

$$+ 4\cos^4 A = \cos^4 A$$

$$\Rightarrow \cos^6 A - 4\cos^4 A + 8\cos^2 A = 4$$

76.

We have,

$$\begin{aligned}
 \text{L.H.S.} &= (\cot A + \sec B)^2 - (\tan B - \operatorname{cosec} A)^2 \\
 &= \left( \frac{\cos A}{\sin A} + \frac{1}{\cos B} \right)^2 - \left( \frac{\sin B}{\cos B} - \frac{1}{\sin A} \right)^2 \\
 &= \frac{\cos^2 A}{\sin^2 A} + \frac{1}{\cos^2 B} + \frac{2\cos A}{\sin A \cos B} - \frac{\sin^2 B}{\cos^2 B} - \frac{1}{\sin^2 A} + \frac{2\sin B}{\cos B \sin A} \\
 &= \left( -\frac{1}{\sin^2 A} + \frac{\cos^2 A}{\sin^2 A} \right) + \left( -\frac{\sin^2 B}{\cos^2 B} + \frac{1}{\cos^2 B} \right) \\
 &\quad + 2 \left( \frac{\cos A}{\sin A \cos B} + \frac{\sin B}{\cos B \sin A} \right) \\
 &= -\left( \frac{1 - \cos^2 A}{\sin^2 A} \right) + \left( \frac{1 - \sin^2 B}{\cos^2 B} \right) + 2(\cot A \sec B + \tan B \operatorname{cosec} A) \\
 &= -1 + 1 + 2(\cot A \sec B + \tan B \operatorname{cosec} A) \\
 &= 2(\cot A \sec B + \tan B \operatorname{cosec} A) = \text{R.H.S.}
 \end{aligned}$$

77.

We have,  $x = \sec A - \tan A$

$$\Rightarrow x = \frac{1}{\cos A} - \frac{\sin A}{\cos A} = \frac{1 - \sin A}{\cos A} \quad \dots(i)$$

$$\text{Now, L.H.S.} = \frac{x^2 + 1}{x^2 - 1} = \frac{\frac{(1 - \sin A)^2}{\cos^2 A} + 1}{\frac{(1 - \sin A)^2}{\cos^2 A} - 1} \quad [\text{Using (i)}]$$

$$\begin{aligned}
 &= \frac{\frac{1 + \sin^2 A - 2\sin A + \cos^2 A}{\cos^2 A}}{\frac{1 + \sin^2 A - 2\sin A - \cos^2 A}{\cos^2 A}} = \frac{1 + (\sin^2 A + \cos^2 A) - 2\sin A}{(1 - \cos^2 A) + \sin^2 A - 2\sin A} \\
 &= \frac{1 + 1 - 2\sin A}{\sin^2 A + \sin^2 A - 2\sin A} = \frac{2(1 - \sin A)}{2\sin^2 A - 2\sin A} \\
 &= \frac{2(1 - \sin A)}{-2\sin A(1 - \sin A)} = -\frac{1}{\sin A} = -\operatorname{cosec} A = \text{R.H.S.}
 \end{aligned}$$

78.

$$\begin{aligned}
 & \text{We have, } \frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A + \cot A} + \frac{\operatorname{cosec} A + \cot A}{\operatorname{cosec} A - \cot A} \\
 &= \frac{\frac{1}{\sin A} - \frac{\cos A}{\sin A}}{\frac{1}{\sin A} + \frac{\cos A}{\sin A}} + \frac{\frac{1}{\sin A} + \frac{\cos A}{\sin A}}{\frac{1}{\sin A} - \frac{\cos A}{\sin A}} = \frac{\frac{1 - \cos A}{\sin A}}{\frac{1 + \cos A}{\sin A}} + \frac{\frac{1 + \cos A}{\sin A}}{\frac{1 - \cos A}{\sin A}} \\
 &= \frac{1 - \cos A}{1 + \cos A} + \frac{1 + \cos A}{1 - \cos A} = \frac{(1 - \cos A)^2 + (1 + \cos A)^2}{(1 + \cos A)(1 - \cos A)} \\
 &= \frac{1 + \cos^2 A - 2\cos A + 1 + \cos^2 A + 2\cos A}{1 - \cos^2 A} \\
 &= \frac{2 + 2\cos^2 A}{1 - \cos^2 A} = \frac{2(1 + \cos^2 A)}{1 - \cos^2 A} \quad \dots(i)
 \end{aligned}$$

Also,  $2(2\operatorname{cosec}^2 A - 1)$

$$= 2\left(\frac{2}{\sin^2 A} - 1\right) = 2\left(\frac{2 - \sin^2 A}{1 - \cos^2 A}\right)$$

$$= 2\left(\frac{2 - (1 - \cos^2 A)}{1 - \cos^2 A}\right) = 2\left(\frac{1 + \cos^2 A}{1 - \cos^2 A}\right) \quad \dots(ii)$$

From (i) and (ii), we get  $\frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A + \cot A} + \frac{\operatorname{cosec} A + \cot A}{\operatorname{cosec} A - \cot A}$

$$= 2(2\operatorname{cosec}^2 A - 1) = 2\left(\frac{1 + \cos^2 A}{1 - \cos^2 A}\right)$$

79.

Given,  $m = \cos A - \sin A$ ,  $n = \cos A + \sin A$

$$\begin{aligned} \text{Now, } \frac{m}{n} - \frac{n}{m} &= \frac{m^2 - n^2}{mn} = \frac{(\cos A - \sin A)^2 - (\cos A + \sin A)^2}{(\cos A - \sin A)(\cos A + \sin A)} \\ &= \frac{\cos^2 A + \sin^2 A - 2\cos A \sin A - \cos^2 A - \sin^2 A - 2\sin A \cos A}{\cos^2 A - \sin^2 A} \\ &= \frac{-4\sin A \cos A}{\cos^2 A - \sin^2 A} \quad \dots(i) \end{aligned}$$

Divide numerator and denominator by  $\sin A \cos A$ , we get,

$$\frac{\frac{-4}{\cos^2 A} - \frac{\sin^2 A}{\sin A \cos A}}{\frac{\sin^2 A}{\sin A \cos A}} = \frac{-4}{\cot A - \tan A} \quad \dots(ii)$$

$$\text{From (i) and (ii), we get } \frac{m}{n} - \frac{n}{m} = \frac{-4\sin A \cos A}{\cos^2 A - \sin^2 A} = \frac{-4}{\cot A - \tan A}$$

80.

$$\text{We have, L.H.S.} = \frac{\sec^3 \theta}{\sec^2 \theta - 1} + \frac{\operatorname{cosec}^3 \theta}{\operatorname{cosec}^2 \theta - 1}$$

$$= \frac{\frac{\sec^3 \theta}{\sec^2 \theta}}{\frac{\sec^2 \theta - 1}{\sec^2 \theta}} + \frac{\frac{\operatorname{cosec}^3 \theta}{\operatorname{cosec}^2 \theta}}{\frac{\operatorname{cosec}^2 \theta - 1}{\operatorname{cosec}^2 \theta}} = \frac{\sec \theta}{1 - \cos^2 \theta} + \frac{\operatorname{cosec} \theta}{1 - \sin^2 \theta}$$

$$= \frac{\sec \theta}{\sin^2 \theta} + \frac{\operatorname{cosec} \theta}{\cos^2 \theta} = \sec \theta \operatorname{cosec}^2 \theta + \operatorname{cosec} \theta \sec^2 \theta$$

$$= \sec \theta \operatorname{cosec} \theta (\sec \theta + \operatorname{cosec} \theta) = \text{R.H.S.}$$

81.

We have,

$$\begin{aligned} \text{L.H.S.} &= (\tan \theta + \sec \theta - 1)(\tan \theta + 1 + \sec \theta) \\ &= \left( \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} - 1 \right) \left( \frac{\sin \theta}{\cos \theta} + 1 + \frac{1}{\cos \theta} \right) \\ &= \left( \frac{\sin \theta + 1 - \cos \theta}{\cos \theta} \right) \left( \frac{\sin \theta + \cos \theta + 1}{\cos \theta} \right) \\ &= \frac{\sin^2 \theta + \sin \theta \cos \theta + \sin \theta + \sin \theta + \cos \theta + 1}{\cos^2 \theta} \\ &\quad - \frac{\sin \theta \cos \theta - \cos^2 \theta - \cos \theta}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta + 2\sin \theta + 1 - \cos^2 \theta}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta + 2\sin \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{2\sin^2 \theta + 2\sin \theta}{\cos^2 \theta} \\ &= \frac{2\sin \theta(1 + \sin \theta)}{1 - \sin^2 \theta} = \frac{2\sin \theta(1 + \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)} = \frac{2\sin \theta}{1 - \sin \theta} = \text{R.H.S.} \end{aligned}$$

82.

$$\begin{aligned} \text{We have, L.H.S.} &= \sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta} \\ &= \sqrt{(1 + \tan^2 \theta) + (1 + \cot^2 \theta)} = \sqrt{\tan^2 \theta + \cot^2 \theta + 2} \\ &= \sqrt{\tan^2 \theta + \cot^2 \theta + 2\tan \theta \cot \theta} \quad [ \because \tan \theta \cot \theta = 1 ] \\ &= \sqrt{(\tan \theta + \cot \theta)^2} = (\tan \theta + \cot \theta) = \text{R.H.S.} \end{aligned}$$

83.

$$\text{We have, } \tan \theta + \sin \theta = m \quad \dots(i)$$

$$\text{and } \tan \theta - \sin \theta = n \quad \dots(ii)$$

Squaring (i) and (ii) and then subtracting, we get

$$m^2 - n^2 = \tan^2 \theta + \sin^2 \theta + 2 \tan \theta \sin \theta \\ - \tan^2 \theta - \sin^2 \theta + 2 \tan \theta \sin \theta$$

$$\Rightarrow m^2 - n^2 = 4 \tan \theta \sin \theta \quad \dots(iii)$$

Multiplying (i) and (ii), we get  $\tan^2 \theta - \sin^2 \theta = mn$

$$\Rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta = mn \quad \left[ \because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\Rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} (1 - \cos^2 \theta) = mn$$

$$\Rightarrow \tan^2 \theta \sin^2 \theta = mn \Rightarrow \tan \theta \sin \theta = \sqrt{mn}$$

Using above value in (iii), we get  $m^2 - n^2 = 4\sqrt{mn}$

### CBSE Sample Questions

1.

$$(a): \text{ Given, } \tan \beta = \frac{4}{5}$$

$$\text{So, } \frac{5 \sin \beta - 2 \cos \beta}{5 \sin \beta + 2 \cos \beta} = \frac{5 \tan \beta - 2}{5 \tan \beta + 2} = \frac{5 \times \frac{4}{5} - 2}{5 \times \frac{4}{5} + 2} = \frac{1}{3} \quad (1)$$

2.

$$(a): \text{ We have, } \frac{4 \sin \beta - 3 \cos \beta}{4 \sin \beta + 3 \cos \beta}$$

Dividing both numerator and denominator by  $\cos \beta$ , we get

$$\frac{4 \tan \beta - 3}{4 \tan \beta + 3} = \frac{3 - 3}{3 + 3} = 0 \quad \left( \because \tan \beta = \frac{3}{4} \text{ (given)} \right) \quad (1)$$

3.

**(b):** We have,  $\tan\alpha + \cot\alpha = 2$

$$\Rightarrow \tan\alpha + \frac{1}{\tan\alpha} = 2$$

$$\Rightarrow \tan^2\alpha - 2\tan\alpha + 1 = 0 \Rightarrow (\tan\alpha - 1)^2 = 0$$

$$\Rightarrow \tan\alpha = 1 \quad \dots(i)$$

$$\therefore \tan^{20}\alpha + \cot^{20}\alpha = (\tan\alpha)^{20} + \left(\frac{1}{\tan\alpha}\right)^{20} = 1 + \left(\frac{1}{1}\right)^{20} = 2 \quad (1)$$

4.

**(b):**  $\because \angle C = 90^\circ$ ,

$$\therefore \frac{\cot y^\circ}{\cot x^\circ} = \frac{AC/BC}{AC/CD} = \frac{CD}{BC} = \frac{CD}{2CD}$$

[ $\because D$  is the midpoint of  $BC \Rightarrow BD = CD$ ]

$$= \frac{1}{2} \quad (1)$$

5.

Given,  $\tan A = \frac{3}{4} = \frac{3k}{4k}$  (say)

$$\Rightarrow P = 3k, B = 4k$$

$$\Rightarrow H = \sqrt{9k^2 + 16k^2} = 5k \quad (1/2)$$

$$\therefore \sin A = \frac{3k}{5k} = \frac{3}{5}, \cos A = \frac{4k}{5k} = \frac{4}{5} \quad (1/2)$$

$$\text{So, } \frac{1}{\sin A} + \frac{1}{\cos A} = \frac{1}{3/5} + \frac{1}{4/5} \quad (1/2)$$

$$= \frac{5}{3} + \frac{5}{4}$$

$$= \frac{20+15}{12} = \frac{35}{12} \quad (1/2)$$

6.

**(b):** Given,  $x \tan 60^\circ \cos 60^\circ = \sin 60^\circ \cot 60^\circ$

$$\Rightarrow x \times \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = \frac{1}{\sqrt{3}} = \tan 30^\circ \quad (1)$$

7.

**(b):** We know that,  $\tan A = \sqrt{3} \Rightarrow \angle A = 60^\circ$ .

Also,  $\angle B = 90^\circ$

$$\therefore \angle A + \angle B + \angle C = 180^\circ \Rightarrow \angle C = 30^\circ$$

$$\text{So, } \cos A \cos C - \sin A \sin C = \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = 0 \quad (1)$$

8.

**(a):** Let measure of  $\angle A$ ,  $\angle B$  and  $\angle C$  be  $x$ ,  $x$  and  $2x$  respectively.

$$\therefore x + x + 2x = 180^\circ \Rightarrow x = 45^\circ$$

So,  $\angle A$ ,  $\angle B$  and  $\angle C$  are  $45^\circ$ ,  $45^\circ$  and  $90^\circ$  respectively.

$$\begin{aligned} \therefore \frac{\sec A}{\operatorname{cosec} B} - \frac{\tan A}{\cot B} &= \frac{\sec 45^\circ}{\operatorname{cosec} 45^\circ} - \frac{\tan 45^\circ}{\cot 45^\circ} \\ &= \frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{1} = 1 - 1 = 0 \end{aligned} \quad (1)$$

9.

Given,  $\sin A + \cos B = 1$

$$\Rightarrow \sin 30^\circ + \cos B = 1 \quad [ \because A = 30^\circ \text{ (Given)} ]$$

$$\Rightarrow \frac{1}{2} + \cos B = 1 \quad (1/2)$$

$$\Rightarrow \cos B = \frac{1}{2} = \cos 60^\circ \Rightarrow B = 60^\circ \quad (1/2)$$

10.

$$\begin{aligned} &\text{We have given, } \sin(A + B) = 1 \Rightarrow \sin(A + B) = \sin 90^\circ, \\ \Rightarrow &A + B = 90 \quad \dots(i) \quad (1/2) \end{aligned}$$

$$\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30^\circ \Rightarrow A - B = 30^\circ \quad \dots(ii) \quad (1/2)$$

$$\text{From (i) \& (ii) } \angle A = 60^\circ \text{ and } \angle B = 30^\circ \quad (1)$$

11.

$$\text{We have, } \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$$

Dividing the numerator and denominator of LHS by  $\cos\theta$ , we get

$$\frac{1 - \tan\theta}{1 + \tan\theta} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \quad (1)$$

Which on simplification (or comparison) gives  $\tan\theta = \sqrt{3}$

$$\text{We know that } \tan 60^\circ = \sqrt{3} \therefore \theta = 60^\circ \quad (1)$$

12.

$$\text{Given, } \sqrt{3} \sin\theta = \cos\theta \quad (1/2)$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}} \quad (1/2)$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} = \tan 30^\circ \quad (1/2)$$

$$\Rightarrow \theta = 30^\circ \quad (1/2)$$

13.

$$(b): \sin\theta + \cos\theta = \sqrt{2}$$

Squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = 2$$

$$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta \cos\theta = 2 \Rightarrow 1 + 2\sin\theta \cos\theta = 2 \quad [\because \sin^2\theta + \cos^2\theta]$$

$$\Rightarrow \sin\theta \cdot \cos\theta = \frac{1}{2} \quad \dots(i)$$

$$\text{Now, } \tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$$

$$= \frac{1}{\sin\theta \cdot \cos\theta} = 2 \quad [\text{From (i)}] \quad (1)$$

14.

(b): We have,  $2\sin^2\beta - \cos^2\beta = 2$

$$\Rightarrow 2\sin^2\beta - (1 - \sin^2\beta) = 2$$

$$\Rightarrow 3\sin^2\beta = 3 \Rightarrow \sin^2\beta = 1 \Rightarrow \beta = 90^\circ \quad (1)$$

15.

(c): We have,  $1 + \sin^2\alpha = 3\sin\alpha \cos\alpha$

$$\Rightarrow \sin^2\alpha + \cos^2\alpha + \sin^2\alpha = 3\sin\alpha \cos\alpha$$

$$(\because \sin^2\alpha + \cos^2\alpha = 1)$$

$$\Rightarrow 2\sin^2\alpha - 3\sin\alpha \cos\alpha + \cos^2\alpha = 0$$

$$\Rightarrow 2\sin^2\alpha - 2\sin\alpha \cos\alpha - \sin\alpha \cos\alpha + \cos^2\alpha = 0$$

$$\Rightarrow 2\sin\alpha = \cos\alpha \text{ or } \sin\alpha = \cos\alpha$$

$$\Rightarrow \cot\alpha = 2 \text{ or } \cot\alpha = 1 \quad (1)$$

16.

$$\text{Consider, } x + y = 2\sin^2\theta + 2\cos^2\theta + 1 \quad (1/2)$$

$$[\because x = 2\sin^2\theta, y = 2\cos^2\theta + 1 \text{ (Given)}]$$

$$= 2(\sin^2\theta + \cos^2\theta) + 1$$

$$= 2 + 1 = 3 \quad [\because \sin^2\theta + \cos^2\theta = 1] \quad (1/2)$$

17.

$$\begin{aligned} \text{L.H.S.} &= \frac{\tan^3 \theta}{1+\tan^2 \theta} + \frac{\cot^3 \theta}{1+\cot^2 \theta} \\ &= \frac{\sin^3 \theta / \cos^3 \theta}{1+\sin^2 \theta / \cos^2 \theta} + \frac{\cos^3 \theta / \sin^3 \theta}{1+\cos^2 \theta / \sin^2 \theta} \\ &\quad \left( \because \tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta} \right) \quad (1/2) \end{aligned}$$

$$= \frac{\sin^3 \theta / \cos^3 \theta}{(\cos^2 \theta + \sin^2 \theta) / \cos^2 \theta} + \frac{\cos^3 \theta / \sin^3 \theta}{(\sin^2 \theta + \cos^2 \theta) / \sin^2 \theta} \quad (1/2)$$

$$= \frac{\sin^3 \theta}{\cos \theta} + \frac{\cos^3 \theta}{\sin \theta} = \frac{\sin^4 \theta + \cos^4 \theta}{\cos \theta \sin \theta} \quad (1/2)$$

$$= \frac{(\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta \cos^2 \theta}{\cos \theta \sin \theta} \quad (1/2)$$

$$= \frac{1 - 2\sin^2 \theta \cos^2 \theta}{\cos \theta \sin \theta} \quad (\because \sin^2 \theta + \cos^2 \theta = 1)$$

$$= \frac{1}{\cos \theta \sin \theta} - \frac{2\sin^2 \theta \cos^2 \theta}{\cos \theta \sin \theta}$$

$$= \sec \theta \operatorname{cosec} \theta - 2\sin \theta \cos \theta = \text{R.H.S.} \quad (1)$$