

Circles

2016

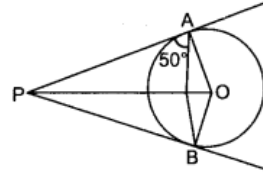
Very Short Answer Type Questions [1 Mark]

Question 1.

From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50^\circ$, then find $\angle AOB$.

Solution:

Given, $\angle PAB = 50^\circ$
 $\angle PAB + \angle OAB = 90^\circ$
 [∵ angle between radius OA and tangent PA is 90°]



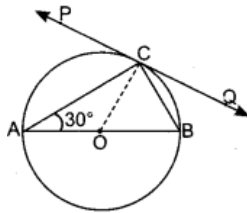
$\Rightarrow 50^\circ + \angle OAB = 90^\circ$
 $\Rightarrow \angle OAB = 90^\circ - 50^\circ = 40^\circ$

Now, $PA = PB$ [∵ tangents from an external point are same]
 $\Rightarrow \angle PBA = \angle PAB$
 $\Rightarrow \angle PBA = 50^\circ$
 $\angle PBA + \angle OBA = 90^\circ$ [∵ angle between radius OB and tangent PB is 90°]
 $\Rightarrow 50^\circ + \angle OBA = 90^\circ$
 $\Rightarrow \angle OBA = 90^\circ - 50^\circ = 40^\circ$

Now in $\triangle AOB$ we have
 $\angle AOB + \angle ABO + \angle BAO = 180^\circ$ [∵ sum of angles in triangle is 180°]
 $\Rightarrow \angle AOB + 40^\circ + 40^\circ = 180^\circ \Rightarrow \angle AOB = 180^\circ - 80^\circ = 100^\circ$

Question 2.

In given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30^\circ$, find $\angle PCA$



Solution:

Construction: Join AO.

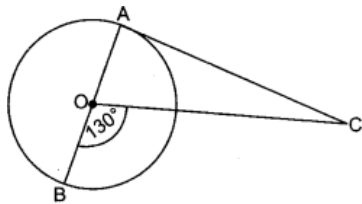
Given: PQ is tangent. AB is diameter $\angle CAB = 30^\circ$.

To Find: $\angle PCA$

Solution: In $\triangle AOC$, $AO = CO$ (∵ Equal radii)
 $\angle CAO = \angle OCA$ (∵ Angles opposite to equal sides are equal)
 or $\angle CAB = \angle OCA$
 But, $\angle CAB = 30^\circ$ So, $\angle OCA = 30^\circ$ (i)
 Since, $OC \perp PQ$ (∵ Tangent is perpendicular to radius at point of contact)
 $\Rightarrow \angle PCO = 90^\circ \Rightarrow \angle OCA + \angle PCA = 90^\circ \Rightarrow 30^\circ + \angle PCA = 90^\circ$
 $\therefore \angle PCA = 60^\circ$

Question 3.

In figure given, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130^\circ$, then find $\angle ACO$.

**Solution:**

$$\angle AOC + \angle BOC = 180^\circ$$

[\because Linear Pair Axiom]

$$\angle AOC + 130^\circ = 180^\circ$$

$$\angle AOC = 180^\circ - 130^\circ$$

$$\angle AOC = 50^\circ$$

Now,

$$\angle OAC = 90^\circ \text{ [angle between radius OA and tangent AC is } 90^\circ]$$

Now, in $\triangle AOC$,

$$\angle OAC + \angle AOC + \angle ACO = 180^\circ$$

[\because sum of angles in triangle is 180°]

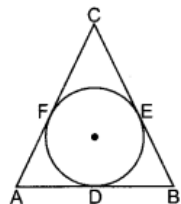
$$90^\circ + 50^\circ + \angle ACO = 180^\circ$$

$$\angle ACO = 180^\circ - 140^\circ$$

$$\angle ACO = 40^\circ$$

Short Answer Type Questions I [2Marks]**Question 4.**

In given figure, a circle is inscribed in a $\triangle ABC$, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF

**Solution:**

Given, AB = 12 cm, CA = 10 cm, BC = 8 cm

Let $AD = AF = x$ [\because Tangent drawn from external point to circle are equal]

$$\therefore DB = BE = 12 - x \text{ and } CF = CE = 10 - x$$

$$BC = BE + EC \Rightarrow 8 = 12 - x + 10 - x$$

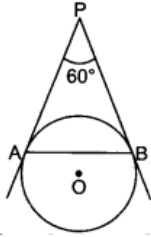
$$\Rightarrow x = 7$$

$$\therefore AD = 7 \text{ cm, } BE = 5 \text{ cm and } CF = 3 \text{ cm}$$

Question 5.

If given figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and

$\angle APB = 60^\circ$. Find the length of chord AB.



Solution:

In $\triangle APB$ we have

\Rightarrow

$$AP = BP$$

$$\angle PAB = \angle PBA$$

[\because Tangents from an external point are equally inclined to segment joining centre to point]

Let

$$\angle PAB = x,$$

then in $\triangle APB$,

$$x + x + 60^\circ = 180^\circ$$

$$2x = 180^\circ - 60^\circ = 120^\circ$$

$$x = 60^\circ$$

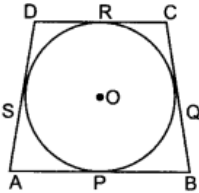
As all three angles of $\triangle APB$ are 60° . So $\triangle APB$ is an equilateral triangle.

Hence $AP = BP = AB = 5$ cm

Question 6.

In figure, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively.

Prove that $AB + CD = BC + DA$.



Solution:

We know that tangents drawn to a circle from an outer points are equal.

So,

$$AP = AS, BP = BQ,$$

$$CR = CQ \text{ and } DR = DS.$$

Now, consider

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

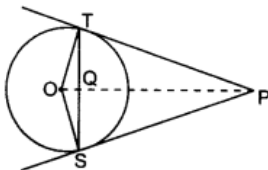
\Rightarrow

$$AB + CD = AD + BC$$

Hence proved.

Question 7.

In given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If $PO = 2r$, show that $\angle OTS = \angle OST = 30^\circ$.



Solution:

Let $\angle TOP = \theta$

In right triangle OTP we have

$$\therefore \cos \theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2} = \cos 60^\circ \Rightarrow \theta = 60^\circ$$

Hence $\angle TOS = 2 \times 60 = 120^\circ$ [$\because \angle TOP = \angle POS$ as angles opposite to equal tangent are equal]

In $\triangle OTS$, we have $OT = OS$ [\because Equal radii]

$\Rightarrow \angle OTS = \angle OST$ [\because Angle opposite to equal sides are equal]

In $\triangle OTS$,

$$\angle OTS + \angle OST + \angle TOS = 180^\circ$$

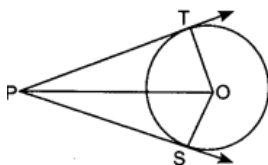
$$2\angle OST = 60^\circ$$

$$\therefore \angle OST = \angle OTS = 30^\circ$$

Hence proved.

Question 8.

In given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that $\angle SPT = 120^\circ$, Prove that $OP = 2PS$



Solution:

Let $PT = x = PS$ [\because Tangent drawn from external point to circle are equal]

$$\angle SPT = 120^\circ$$

In $\triangle OTP$ and $\triangle OSP$, $\angle OTP = \angle OSP$

[\because each equal to 90° , since tangent perpendicular r radius]

$$OT = OS \quad [\because \text{Equal radii}]$$

$$OP = OP \quad [\text{common}]$$

$\Rightarrow \triangle OSP \cong \triangle OTP$ [\because By SAS congruence rule]

$\therefore \angle TPO = \angle SPO$ [\because By CPCT]

$$\Rightarrow \angle TPO = \frac{1}{2} \angle SPT = \frac{1}{2} \times 120 = 60^\circ$$

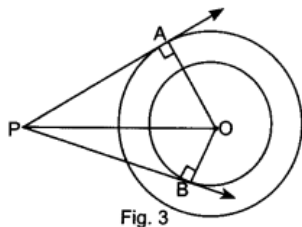
In $\triangle OTP$, $\frac{OP}{x} = \sec 60^\circ$

$$\Rightarrow \frac{OP}{x} = 2 \Rightarrow OP = 2x \Rightarrow OP = 2PS$$

Hence proved.

Question 9.

In given figure, there are two concentric circles of radii 6 cm and 4 cm with centre O. If AP is a tangent to the larger circle and BP to the smaller circle and length of AP is 8cm, find the length of BP



Solution:

$$OA = 6 \text{ cm } [\because \text{ Given radius}]$$

$$OB = 4 \text{ cm } [\because \text{ Given radius}]$$

$$AP = 8 \text{ cm}$$

In $\triangle OAP$, $OP^2 = OA^2 + AP^2 = 36 + 64 = 100$ [\because Pythagoras theorem]

$$\Rightarrow OP = 10 \text{ cm}$$

In $\triangle OBP$, $BP^2 = OP^2 - OB^2 = 100 - 16 = 84$ [\because Pythagoras theorem]

$$BP = 2\sqrt{21} \text{ cm}$$

Long Answer Type Questions [4 Marks]

Question 10.

Prove that the lengths of tangents drawn from an external point to a circle are equal

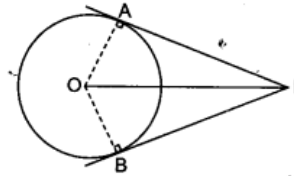
Solution:

Given: A circle $C(O, r)$, P is a point outside the circle and PA and PB are tangents to a circle.

To Prove: $PA = PB$

Construction: Draw OA, OB and OP.

Proof: Consider triangles OAP and OBP.



$$\angle OAP = \angle OBP = 90^\circ \quad \dots(i)$$

[Radius is perpendicular to the tangent at the point of contact]

$$OA = OB \text{ (radii)} \quad \dots(ii)$$

OP is common

$\dots(iii)$

$$\therefore \triangle OAP \cong \triangle OBP \text{ (RHS)}$$

[from (i), (ii) and (iii)]

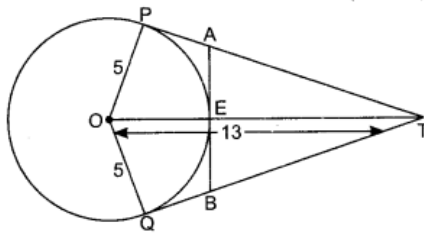
Hence,

$$AP = BP$$

(CPCT)

Question 11.

In given figure, O is the centre of a circle of radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Solution:

In $\triangle OPT$, $OP^2 + PT^2 = OT^2$ [\because Pythagoras theorem]

$$PT = \sqrt{OT^2 - OP^2}$$

$$= \sqrt{169 - 25} = 12 \text{ cm}$$

and

$$TE = OT - OE = 13 - 5 = 8 \text{ cm}$$

Let

$$PA = AE = x \quad \text{[tangent from outer point A]}$$

In $\triangle TEA$,

$$TE^2 + EA^2 = TA^2 \quad \text{[}\because \text{ Pythagoras theorem]}$$

$$(8)^2 + (x)^2 = (12 - x)^2$$

$$64 + x^2 = (12 - x)^2$$

\Rightarrow

$$64 + x^2 = 144 + x^2 - 24x$$

\Rightarrow

$$80 = 24x \Rightarrow x = 3.3 \text{ cm}$$

$$\text{Thus } AB = 2 \times 3.3 \text{ cm} = 6.6 \text{ cm}$$

[\because $AE = EB$, as AB is tangent to circle at E]

Question 12.

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

Solution:

Given: A circle $C(O, r)$ and a tangent AB at a point P .

To prove: $OP \perp AB$

Construction: Take any point Q other than P on the tangent AB .

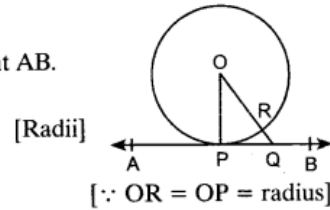
Join OQ , intersecting circle at R .

Proof: We have, $OP = OR$
 $OQ = OR + RQ$
 $\therefore OQ > OR \Rightarrow OQ > OP$

Thus, $OP < OQ$, i.e. OP is shorter than any other segment joining O to any point of AB .

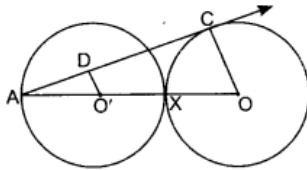
But among all line segments, joining point O to point on AB , shortest one is perpendicular from O on AB .

Hence, $OP \perp AB$



Question 13.

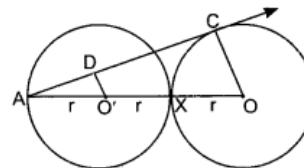
In given figure, two equal circles, with centres O and O' , touch each other at X . OO' produced meets the circle with centre O' at A . AC is tangent to the circle with centre O , at the point C . $O'D$ is perpendicular to AC . Find the value of DO'/CO .



Solution:

AC is tangent to the circle with centre O .

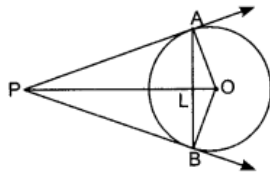
In $\triangle ADO'$ and $\triangle ACO$, $\angle ADO' = \angle ACO$ (each 90°)
 $\angle DAO' = \angle CAO$ (common)



\therefore By AA criterion, $\frac{AO'}{AO} = \frac{DO'}{CO}$ [\therefore corresponding parts of similar triangle]
 $AO = AO' + O'X + XO = r + r + r = 3r$
 $\frac{DO'}{CO} = \frac{r}{3r}$ [$\therefore AO = AO' + O'X + XO = 3AO$]
 $\Rightarrow \frac{DO'}{CO} = \frac{1}{3}$

Question 14.

In given figure, AB is a chord of a circle, with centre O , such that $AB = 16$ cm and radius of circle is 10 cm. Tangents at A and B intersect each other at P . Find the length of PA



Solution:

Let $PL = x$
 As OP is perpendicular bisector of AB . Then
 $AL = BL = 8$ cm
 In $\triangle ALO$, $OL^2 = OA^2 - AL^2 = 10^2 - 8^2 = 36 \Rightarrow OL = 6$ cm
 $AP^2 = OP^2 - OA^2$ [\because Pythagoras theorem]
 In $\triangle OAP$, $AP^2 = (x + 6)^2 - 10^2$
 $AP^2 = AL^2 + PL^2$ [\because Pythagoras theorem]
 In $\triangle ALP$, $AP^2 = x^2 + 64$
 Now, $(x + 6)^2 - 10^2 = x^2 + 64$
 $x^2 + 12x + 36 - 100 = x^2 + 64$
 $\Rightarrow 12x = 128$
 $\Rightarrow x = \frac{128}{12}$
 $= \frac{32}{3}$ cm
 From $\triangle ALP$, $AP^2 = \left(\frac{32}{3}\right)^2 + 64$
 $= \frac{1024}{9} + 64$
 $= \frac{1024 + 576}{9}$ cm
 $AP^2 = \frac{1600}{9}$ cm
 $AP = \frac{40}{3}$ cm = 13.3 cm

2015

Very Short Answer Type Questions [1 Mark]

Question 15.

In figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$. Write the measure of $\angle OAB$

Solution:

Join OB .

$\because PA$ and PB are tangents to the circle drawn from an external point P . We know that, tangent is perpendicular to radius.

$$\angle OAP = \angle OBP = 90^\circ$$

$$\text{Then, } \angle OAP + \angle APB + \angle OBP + \angle AOB = 360^\circ$$

(ASP of quadrilateral) $\angle P = 50^\circ$

$$\therefore \angle APB + \angle AOB = 180^\circ$$

$$\Rightarrow 50^\circ + \angle AOB = 180^\circ$$

$$\Rightarrow \angle AOB = 130^\circ$$

$$\text{In } \triangle OAB, \quad OA = OB \quad (\because \text{equal radii})$$

$$\Rightarrow \angle A = \angle B = x \text{ (say)} (\because \text{angles opposite to equal sides are equal})$$

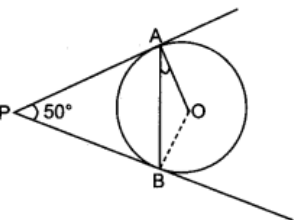
$$\angle A + \angle B + \angle AOB = 180^\circ \quad (\because \text{ASP of triangles})$$

$$\Rightarrow x + x + 130^\circ = 180^\circ$$

$$\Rightarrow 2x = 50^\circ$$

$$\Rightarrow x = 25^\circ$$

$$\therefore \angle OAB = 25^\circ$$



Question 16.

Find the relation between x and y if the points $A(x, y)$, $B(-5, 7)$ and $C(-4, 5)$ are collinear

Solution:

\therefore A, B and C are collinear. Area of triangle = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$
 So, $\text{ar}(\Delta ABC) = 0$
 $\therefore \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$
 $\frac{1}{2}[x(7 - 5) + (-5)(5 - y) + (-4)(y - 7)] = 0$
 $2x - 25 + 5y - 4y + 28 = 0$
 $\Rightarrow 2x + y + 3 = 0$. Required relation between x and y.

Question 17.

Two concentric circles of radii a and b (a > b) are given. Find the length of the chord of the larger circle which touches the smaller circle.

Solution:

AB is tangent at C to circle C(O, b)

$\therefore OC \perp AB$

$\therefore \angle OCB = 90^\circ$

$\Rightarrow AC = BC \Rightarrow AB = 2AC$

(\because perpendicular from centre to chord bisects chord)

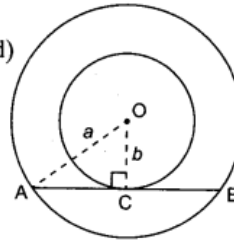
Now, in ΔOCA , $AO^2 = OC^2 + AC^2$

$\Rightarrow a^2 = b^2 + AC^2$

$\Rightarrow AC = \sqrt{a^2 - b^2}$

$\therefore AB = 2\sqrt{a^2 - b^2} = 2AC$

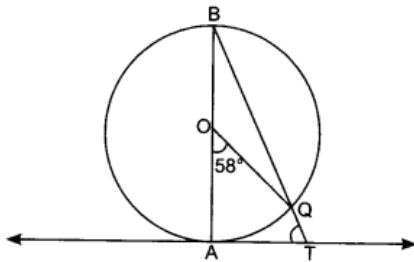
length of chord = $2\sqrt{a^2 - b^2}$



Short Answer Type Questions I [2 Marks]

Question 18.

In figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^\circ$, find $\angle ATQ$



Solution:

\because AT is a tangent and BA is a diameter.

So, $OA \perp AT$

[radius is perpendicular to the tangent at point of contact]

$\Rightarrow \angle OAT = 90^\circ$ or $\angle BAT = 90^\circ$

Arc AQ subtends an angle of 58° at the circle.

$\angle AOQ = 2\angle ABQ$

So, $\angle ABQ = 29^\circ$ [angle subtended by the arc at the centre is double the angle subtended by the same arc on the circle]

In ΔABT ,

$\angle A + \angle ABT + \angle ATB = 180^\circ$

$\Rightarrow 90^\circ + 29^\circ + \angle ATB = 180^\circ$

$\Rightarrow \angle ATB = 61^\circ$

Hence, $\angle ATQ = 61^\circ$

Question 19.

From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle.

Prove that OT is the right bisector of the line segment PQ.

Solution:

Given: TP and TQ are tangents to the circle of centre O.

To Prove: $\angle OMP = 90^\circ$ and $PM = MQ$.

Proof: \because TP and TQ are tangents at P and Q respectively.

So, $OP \perp PT$ and $OQ \perp QT$

(\because radius is perpendicular to the tangent at point of contact)

$\therefore \angle OPT = \angle OQT = 90^\circ$

In $\triangle OPT$ and $\triangle OQT$

$OP = OQ$ (radius)

$\angle P = \angle Q$ (each 90°)

$OT = OT$ (common)

$\triangle OPT \cong \triangle OQT$ (By RHS)

So,

$\angle 1 = \angle 2$ (By CPCT)

Now, In $\triangle OMP$ and $\triangle OMQ$,

$OP = OQ$ (radius)

$\angle 1 = \angle 2$ (Proved above)

$OM = OM$ (common)

$\triangle OMP \cong \triangle OMQ$ (By SAS)

So,

$PM = MQ$ and $\angle 3 = \angle 4$ (By CPCT)

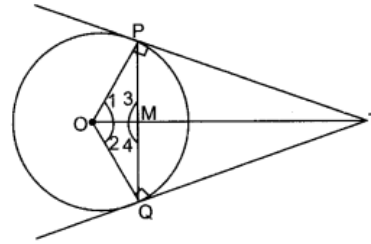
\Rightarrow

Now $\angle 3 + \angle 4 = 180^\circ$ (\because Linear Pair Axiom)

\Rightarrow

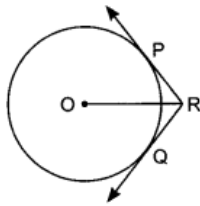
$2\angle 3 = 180^\circ \Rightarrow \angle 3 = 90^\circ \Rightarrow \angle OMP = 90^\circ$

Hence, OT is the right bisector of the line segment PQ.



Question 20.

In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120^\circ$, then prove that $OR = PR + RQ$.



Solution:

We know that, tangent is perpendicular to radius. Perpendicular from centre bisects angle.

OR bisects $\angle PRQ$

\therefore

$\angle PRO = \angle QRO = 60^\circ$

[$\because \angle PRQ = \angle ORP + \angle ORQ = 120^\circ$]

In right $\triangle OPR$ ($\because OP \perp PR$) [\because radius is perpendicular to the tangent at point of contact]

\therefore

$$\cos \angle ORP = \frac{PR}{OR} = \cos 60^\circ$$

\Rightarrow

$$OR = 2PR$$

...(i)

Similarly, in right $\triangle OQR$,

$$\frac{QR}{OR} = \frac{1}{2} = \cos 60^\circ$$

\Rightarrow

$$OR = 2QR$$

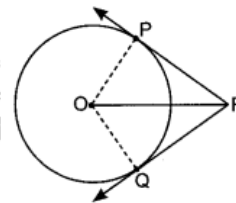
...(ii)

Adding (i) and (ii), we get

$$2OR = 2PR + 2QR$$

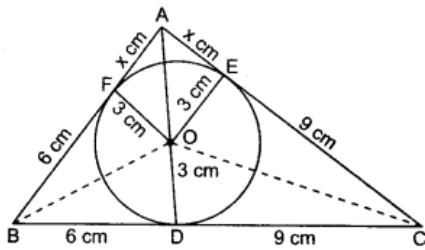
\Rightarrow

$$OR = PR + RQ$$



Question 21.

In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of $\triangle ABC$ is 54 cm^2 , then find the lengths of sides AB and AC



Solution:

Let $AF = x$ cm, $BC = (6 + 9) = 15$ cm

$\therefore AF = AE$

[tangents drawn from an external point are equal]

$\therefore AE = x$ cm

Also $BD = BF = 6$ cm

and $CD = CE = 9$ cm

$\therefore AB = (x + 6)$ cm

In $\triangle ABC$, $AC = (x + 9)$ cm

$\text{Area } \triangle ABC = \text{Area } \triangle BOC + \text{Area } \triangle COA + \text{Area } \triangle AOB$

$$\Rightarrow 54 = \frac{1}{2} BC \times OD + \frac{1}{2} AC \times OE + \frac{1}{2} AB \times OF$$

$$\Rightarrow 54 \times 2 = 15 \times 3 + (9 + x) \times 3 + (6 + x) \times 3$$

$$108 = 45 + 18 + 3x + 27 + 3x$$

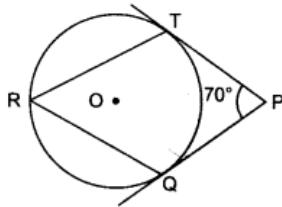
$$6x = 18 \Rightarrow x = 3$$

$$\Rightarrow AB = 6 + x = 6 + 3 = 9 \text{ cm}$$

$$AC = 9 + x = 9 + 3 = 12 \text{ cm}$$

Question 22.

In figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ = 70^\circ$, find $\angle TRQ$



Solution:

We know that tangent is perpendicular to radius. Hence,

$$\angle OTP = \angle OQP = 90^\circ$$

In quadrilateral PQOT,

$$\angle QOT + \angle OTP + \angle TPQ + \angle OQP = 360^\circ$$

$$\angle TOQ + \angle TPQ = 180^\circ$$

$$\Rightarrow \angle TOQ = 110^\circ$$

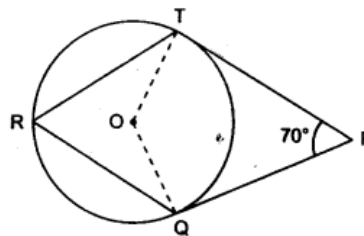
Also $\angle TOQ = 2\angle TRQ$

[angle subtended by an arc at centre of the circle is twice the angle subtended by it in alternate segment]

$$\Rightarrow 110^\circ = 2\angle TRQ$$

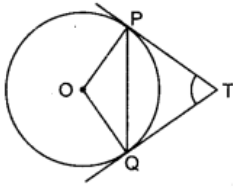
$$\Rightarrow \angle TRQ = 55^\circ$$

[\therefore ASP of quadrilateral]



Question 23.

In figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the lengths of TP and TQ



Solution:

Join OT intersecting PQ at R.

OT bisects $\angle PTQ$

$$\therefore \angle PTO = \angle QTO$$

$$\therefore \angle PTR = \angle QTR$$

$$\text{In } \triangle PTR \text{ and } \triangle QTR, \quad PT = QT$$

... (i)
[length of tangents drawn from common external point are equal]

$$RT = RT \quad [\text{common}]$$

$$\angle PTR = \angle QTR \quad [\because \text{from (i)}]$$

$$\triangle PTR \cong \triangle QTR \quad [\text{By SAS}]$$

$$\Rightarrow PR = RQ \quad [\because \text{By CPCT}]$$

\Rightarrow R is mid-point of PQ

$$\therefore OR \perp PQ$$

In right triangle ORP

$$OP^2 = PR^2 + OR^2$$

[\because Given, $OP = 5$ cm, $PQ = 8$ cm

$$\therefore PR = QR = 4 \text{ cm}]$$

$$\Rightarrow 25 = 16 + OR^2$$

$$OR = 3 \text{ cm}$$

In $\triangle ORQ$ and $\triangle OQT$

$$\angle ORQ = \angle OQT \quad (\text{Each } 90^\circ)$$

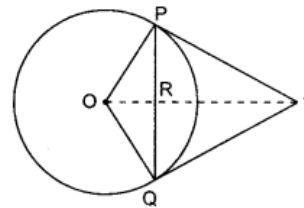
$$\angle ROQ = \angle ROQ \quad (\text{Common})$$

$$\therefore \triangle ORQ \sim \triangle OQT \quad (\text{By AA criterion})$$

$$\Rightarrow \frac{OR}{OQ} = \frac{RQ}{QT} \quad (\text{By C.P. of similar triangles})$$

$$\Rightarrow \frac{3}{5} = \frac{4}{QT} \Rightarrow QT = \frac{20}{3} \text{ cm}$$

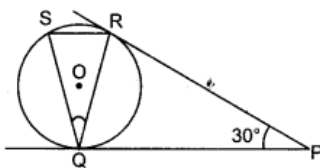
$$\text{Also } PT = QT \Rightarrow PT = \frac{20}{3} \text{ cm}$$



Long Answer Type Questions [4 Marks]

Question 24.

In figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$



Solution:

Construction: Draw a line through Q and perpendicular to PQ.

Proof: As $MQ \perp PQ$

So, MQ passes through the centre O. [If a line is perpendicular to the tangent, then it must be passed through centre of the circle]

$\Rightarrow \angle OQP = 90^\circ$
 Also $PQ = PR$
 [lengths of tangents drawn from an external point to a circle are equal]

In $\triangle PQR$, $PQ = PR$
 $\Rightarrow \angle Q = \angle R$ [\because angles opposite to equal sides are equal]
 Now, $\angle P + \angle Q + \angle R = 180^\circ$ [\because ASP of triangle]
 $\Rightarrow 30^\circ + 2\angle Q = 180^\circ$
 $\Rightarrow 2\angle Q = 150^\circ$
 $\Rightarrow \angle Q = 75^\circ$
 \therefore Given, $SR \parallel PQ$
 $\Rightarrow \angle QRS = \angle RQP$ [if 2 lines are parallel, alternate angles are equal]

$\Rightarrow \angle QRS = 75^\circ$
 Also, $\angle QSR = \angle RQP$ [\because Corresponding angles equal]
 $\Rightarrow \angle QSR = 75^\circ$

Now, In $\triangle RQS$,
 $\angle RQS + \angle RSQ + \angle SRQ = 180^\circ$ [\because ASP of triangle]
 $\Rightarrow \angle RQS + 75^\circ + 75^\circ = 180^\circ$
 $\Rightarrow \angle RQS = 180 - 150 = 30^\circ$

Alternate method:
Given: PQ and PR are tangents and $\angle RPQ = 30^\circ$
To find: $\angle RQS$
 $\therefore SR \parallel PQ$
 $\Rightarrow \angle 1 = \angle P$
 $\Rightarrow \angle 1 = 30^\circ$
 \therefore PR is tangent at P and $\angle 1 = 30^\circ$
 So, $\angle RQS = \angle 1$ [By alternate segment theorem]
 $\Rightarrow \angle RQS = 30^\circ$

Question 25.

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

Solution:

Refer to Ans. 12

Question 26.

Prove that the lengths of the tangents drawn from an external point to a circle are equal

Solution:

Refer to Ans. 10.

Question 27.

Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

Solution:

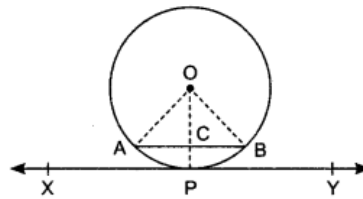
Given: APB is arc of the circle $C(O, r)$, P is mid-point of arc APB and XY is tangent to the circle at P .

To Prove: $AB \parallel XY$

Join OA and OB

Here

$$\begin{aligned} \angle AOP &= \angle BOP \\ &\text{(angle subtended by equal arc)} \\ OA &= OB && (\because \text{equal radii}) \\ OC &= OC && (\text{common}) \end{aligned}$$



$$\begin{aligned} \therefore \Delta ACO &\cong \Delta BCO && (\text{SAS}) \\ \text{So, } AC &= BC && (\text{CPCT}) \end{aligned}$$

$$\Rightarrow OC \perp AB$$

$$\therefore \angle OCB = \angle OCA = 90^\circ \quad (\text{line joining mid-point of chord with centre of the circle is perpendicular to the chord})$$

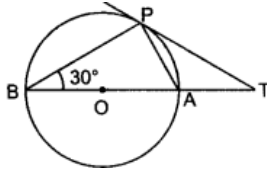
$$\text{Also } \angle OPY = 90^\circ$$

$$\Rightarrow \angle OCB = \angle OPY \quad (\text{these are corresponding angles})$$

$$\therefore AB \parallel XY \quad (\because \text{if corresponding angles are equal, 2 lines are parallel})$$

Question 28.

In figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If $\angle PBT = 30^\circ$, prove that $BA : AT = 2 : 1$



Solution:

We have, $\angle BPA = 90^\circ$ [\because PT is tangent to circle, tangent perpendicular radius]

In ΔBPA , $\angle ABP + \angle BPA + \angle PAB = 180^\circ$ [\because ASP of triangle]

$$\Rightarrow 30^\circ + 90^\circ + \angle PAB = 180^\circ$$

$$[\because \angle PBT = \angle ABP = 30^\circ]$$

$$\Rightarrow \angle PAB = 60^\circ$$

$$\text{Also } \angle POA = 2\angle PBA$$

[\because Angle subtended by an arc at centre is twice angle subtended by arc on circle]

$$\Rightarrow \angle POA = 2 \times 30^\circ = 60^\circ$$

$$\therefore \angle PAO = \angle POA$$

$$\Rightarrow OP = AP \quad (\text{sides opposite to equal angles}) \dots (i)$$

$$\text{In } \Delta OPT, \angle OPT = 90^\circ \quad (\text{radius is perpendicular to tangent})$$

$$\angle POT = 60^\circ$$

$$\therefore \angle PTO = 30^\circ \quad [\text{angle sum property of a triangle}]$$

$$\text{Also, } \angle APT + \angle ATP = \angle PAO \quad (\text{exterior angle property})$$

$$\therefore \angle APT + 30^\circ = 60^\circ \Rightarrow \angle APT = 30^\circ$$

$$\angle PTA = \angle APT \quad (\because \text{from above})$$

$$\therefore AP = AT \quad (\text{sides opposite to equal angles}) \dots (ii)$$

From (i) and (ii)

$$\Rightarrow AT = OP = \text{radius of the circle} = r \quad [\because AP \text{ is radius of circle}]$$

$$\text{Now } AB = 2r$$

$$\Rightarrow AB = 2AT \Rightarrow \frac{AB}{AT} = 2 \Rightarrow AB : AT = 2 : 1$$

2014

Short Answer Type Questions I [2 Marks]

Question 29.

Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.

Solution:

Given: PQ and RS are two parallel tangents to a circle at B and A respectively. O is the centre of the circle. AOB is a line segment.

To prove: AB passes through O.

Construction: Join OA and OB.

Proof: As we know, OB is perpendicular to PQ.

[Tangent is perpendicular to radius at the point of contact.]

Now, given, $PQ \parallel RS \Rightarrow BO$ (Produced to RS) is perpendicular to RS.(i)

[A line perpendicular to one of the two parallel lines is perpendicular to other line also]

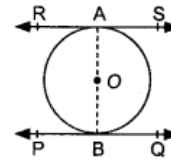
Also, OA is perpendicular to RS [\because Tangent perpendicular to radius] ... (ii)

From (i) and (ii), OA and OB must coincide as only one line can be drawn perpendicular from a point outside the line to the line.

\therefore AOB is straight line.

\therefore A, O, B are collinear.

\Rightarrow AB Passes through O, the centre of the circle.



Question 30.

If from an external point P of a circle with centre O, two tangents PQ and PR are drawn, such that $\angle QPR = 120^\circ$, prove that $2PQ = PO$.

Solution:

Given: PQ and PR are tangents from point P to circle with centre O.

Also, $\angle QPR = 120^\circ$

To Prove: $2PQ = OP$

Construction: Join OQ, OP and OR

Proof: In triangles OQP and ORP,

$$OQ = OR = r \text{ (say) } [\because \text{equal radii}]$$

$$OP = OP \text{ (common)}$$

$$PQ = PR$$

[The lengths of the tangents drawn from an external point to a circle are equal]

$$\therefore \triangle OQP \cong \triangle ORP \text{ (by SSS)}$$

$$\therefore \angle OPQ = \angle OPR$$

(By CPCT)

Now, given $\angle QPR = 120^\circ$

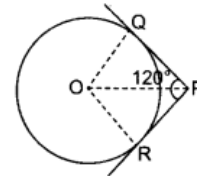
$$\Rightarrow \angle OPQ + \angle OPR = 120^\circ$$

$$\Rightarrow 2\angle OPQ = 120^\circ$$

$$\Rightarrow \angle OPQ = 60^\circ = \angle OPR$$

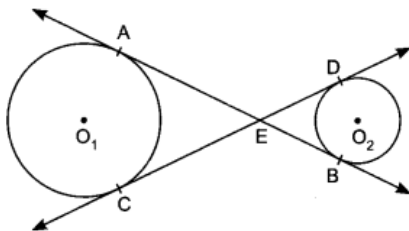
Now, In $\triangle OQP$, $\angle Q = 90^\circ$ [\because Tangent perpendicular to radius]

Then, $\frac{PQ}{OP} = \cos 60^\circ = \frac{1}{2} \Rightarrow OP = 2PQ$



Question 31.

In figure, common tangents AB and CD to the two circles with Centres O_1 and O_2 intersect at E. Prove that $AB = CD$.



Solution:

In the given figure, AB and CD are common tangents to the two given circles with centres O_1 and O_2 respectively.

We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal in length.

$$\therefore AE = EC \text{ and } EB = ED$$

$$\Rightarrow AE + EB = CE + ED$$

$$\Rightarrow AB = CD.$$

Question 32.

The incircle of an isosceles triangle ABC, in which $AB = AC$, touches the sides BC, CA and AB at D, E and F respectively. Prove that $BD = DC$

Solution:

We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal in length.

$$\therefore AF = AE, BF = BD \text{ and } CD = CE \quad \dots(i)$$

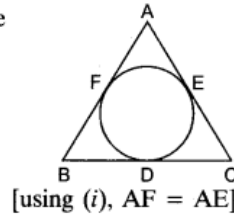
Given $AB = AC$

$$\Rightarrow AF + FB = AE + EC$$

$$\Rightarrow FB = EC$$

$$\Rightarrow BD = CD$$

[using (i), $BF = BD$ and $CD = CE$]



Question 33.

In figure, XP and XQ are two tangents to the circle with centre O, drawn from an external point X. ARB is another tangent, touching the circle at R. Prove that $XA + AR = XB + BR$.

Solution:

Lengths of the tangents drawn from a point outside the circle to the circle are equal.

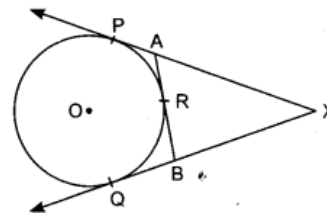
$$\therefore XP = XQ, AP = AR \text{ and } BR = BQ \quad \dots(i)$$

Now, $XP = XQ$ [\because equal tangents]

$$\Rightarrow XA + AP = XB + BQ$$

$$\Rightarrow XA + AR = XB + BR \quad [\text{using (i)}]$$

Hence proved.



Question 34.

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solution:

AB is diameter of a circle with centre O and l_1, l_2 are the tangents to the circle at A and B.

We know that radius is perpendicular to the tangent at the point of contact or diameter is perpendicular to the tangent at the point of contact.

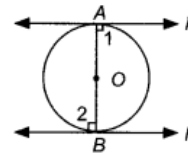
$$\therefore \angle 1 = 90^\circ \text{ and } \angle 2 = 90^\circ \quad [\because \text{See from figure}]$$

$$\Rightarrow \angle 1 = \angle 2$$

But these are alternate angles.

$$\therefore l_1 \text{ is parallel to } l_2.$$

[\because If alternate angles are equal, so 2 lines are parallel]



Long Answer Type Questions [4 Marks]

Question 35.

Prove that the length of the tangents drawn from an external point to a circle are equal.

Solution:

Refer to Ans. 10.

Question 36.

Prove that a parallelogram circumscribing a circle is a rhombus

Solution:

Given: ABCD is parallelogram circumscribing a circle.

To prove: ABCD is a rhombus

Proof: We have,

$$DR = DS \quad \dots(i)$$

[Lengths of tangents drawn from an external point to a circle are equal]

Also,

$$AP = AS \quad \dots(ii)$$

$$BP = BQ \quad \dots(iii)$$

$$CR = CQ \quad \dots(iv)$$

Adding (i), (ii), (iii) and (iv),

$$(DR + CR) + (AP + BP) = (DS + AS) + (BQ + CQ)$$

$$\Rightarrow CD + AB = AD + BC$$

\Rightarrow

$$2AB = 2AD \quad [\because \text{In parallelogram, opposite sides are equal}$$

$$AB = CD \text{ and } AD = BC]$$

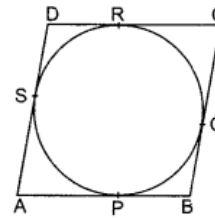
\Rightarrow

$$AB = AD$$

\therefore

$$AB = AD = BC = CD$$

Hence, ABCD is a rhombus as all sides are equal in rhombus.



Question 37.

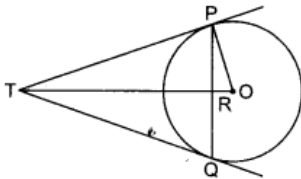
Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

Solution:

Refer to Ans. 12.

Question 38.

In figure, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T. Find the length of TP.



Solution:

Given: PQ is chord of length 16 cm, TP and TQ are the tangents to a circle with centre O, radius 10 cm.

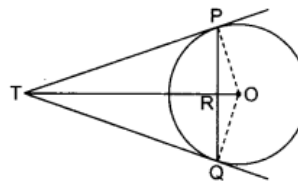
To find: TP.

Solution: Join OP and OQ.

In triangles OTP and OTQ,

OT is common

$$OP = OQ \quad (\text{radii})$$



$$TP = TQ$$

[length of the tangents drawn from a point outside the circle to the circle are equal]

$$\begin{aligned} \therefore \quad \triangle OPT &\cong \triangle OQT && \text{(SSS congruence rule)} \\ \therefore \quad \angle POT &= \angle QOT && \dots(i) \text{ (By CPCT)} \end{aligned}$$

Consider, triangles OPR and OQR

$$\begin{aligned} OP &= OQ && \text{(radii)} \\ \text{OR is common} \quad \angle POR &= \angle QOR && \text{[from (i)]} \\ \therefore \quad \triangle OPR &\cong \triangle OQR && \text{(SAS congruence rule)} \end{aligned}$$

$$\begin{aligned} \text{So,} \quad PR &= RQ = \frac{1}{2} \times 16 = 8 \text{ cm} && \dots(ii) \text{ (By CPCT)} \\ \angle ORP &= \angle ORQ = 90^\circ && \dots(iii) \text{ (By CPCT)} \end{aligned}$$

In right-angled triangle TRP,

$$TR^2 = TP^2 - (8)^2 = TP^2 - 64 \quad \dots(iv) \text{ [From (iii)]}$$

$$\text{Also, in } \triangle TOP, \quad OT^2 = TP^2 + (10)^2 = TP^2 + PO^2 \quad (\because \text{Pythagoras theorem})$$

$$(TR + OR)^2 = TP^2 + 100$$

$$(TR + 6)^2 = TP^2 + 100$$

$$[\because OR = \sqrt{100 - 64} = 6]$$

$$TR^2 + 12TR + 36 = TP^2 + 100$$

$$TP^2 - 64 + 12TR + 36 = TP^2 + 100 \quad \text{[From (iv)]}$$

$$12TR = 128 \Rightarrow TR = \frac{32}{3} \text{ cm}$$

$$\text{From (iv),} \quad \left(\frac{32}{3}\right)^2 = TP^2 - 64$$

$$\Rightarrow TP^2 = \frac{1024}{9} + 64 = \frac{1024 + 576}{9} = \frac{1600}{9} \Rightarrow TP = \frac{40}{3} \text{ cm.}$$

Question 39.

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:

Given: ABCD is a quadrilateral, circumscribing a circle with centre O and touches the quadrilateral at P, Q, R and S respectively.

To Prove: (i) $\angle AOB + \angle COD = 180^\circ$

(ii) $\angle BOC + \angle AOD = 180^\circ$

Construction: Join OP, OQ, OR and OS.

Proof: Consider, triangles APO and ASO,

$$AP = AS$$

$$OS = OP \text{ (radii)}$$

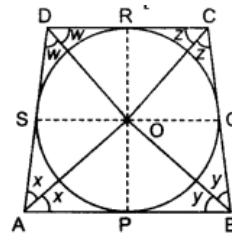
OA is common

$$\therefore \quad \triangle APO \cong \triangle ASO \quad \text{(SSS congruency rule)}$$

$$\therefore \quad \angle OAP = \angle OAS = x \text{ (say)} \quad \text{(CPCT)}$$

$$\text{Similarly,} \quad \angle OBP = \angle OBQ = y \text{ (say)}$$

$$\angle OCQ = \angle OCR = z \text{ (say)}$$



[Lengths of the tangents drawn from a point outside the circle to the circle are equal]

and $\angle ODR = \angle ODS = w$ (say)

We have, $\angle DAB + \angle ABC + \angle BCD + \angle CDA = 360^\circ$

[\because Angle sum property of quadrilateral]

$$\Rightarrow 2x + 2y + 2z + 2w = 360^\circ$$

$$\Rightarrow x + y + z + w = 180^\circ \quad \dots(i)$$

Consider, $\angle AOB + \angle COD = [180^\circ - x - y] + [180^\circ - w - z]$

[Sum of angles of a triangle is 180°]

$$= 360^\circ - (x + y + z + w)$$

$$= 360^\circ - 180^\circ$$

[using (i)]

$\therefore \angle AOB + \angle COD = 180^\circ$

Again consider, $\angle BOC + \angle AOD$

$$= [180^\circ - y - z] + [180^\circ - x - w]$$

[Sum of angles of a triangle is 180°]

$$= 360^\circ - (x + y + z + w)$$

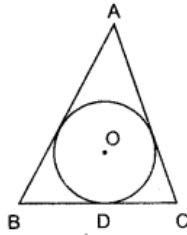
$$= 360^\circ - 180^\circ = 180^\circ$$

[using (i)]

Hence proved.

Question 40.

In figure, a triangle ABC is drawn to circumscribe a circle of radius 4 cm, such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.



Solution:

We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal in length.

$$\therefore AF = AE = x, \text{ (say)}$$

$$BF = BD = 8 \text{ cm}$$

$$CE = CD = 6 \text{ cm}$$

$$\therefore AB = (x + 8) \text{ cm} = a \text{ (say)}$$

$$AC = (x + 6) \text{ cm} = b \text{ (say)}$$

$$\text{and } c \text{ (say)} = BC = 14 \text{ cm} = (8 + 6) \text{ cm}$$

Now, perimeter of triangle using Heron's formula

$$2S = x + 8 + x + 6 + 14$$

$$= 28 + 2x$$

$$\Rightarrow S = 14 + x$$

\therefore Area of ΔABC using Heron's formula,

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

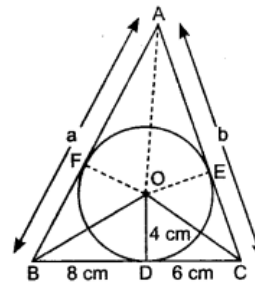
$$= \sqrt{(14+x)(14+x-x-8)(14+x-x-6)(14+x-14)}$$

$$= \sqrt{(14+x) \times 6 \times 8 \times x} \quad \dots(i)$$

Also, area of $\Delta ABC = \text{ar}(\triangle BOC) + \text{ar}(\triangle BOA) + \text{ar}(\triangle AOC)$

$$= \frac{1}{2} \times 14 \times 4 + \frac{1}{2} \times (8+x) \times 4 + \frac{1}{2} \times (6+x) \times 4$$

$$= 28 + 16 + 2x + 12 + 2x = 56 + 4x \quad \dots(ii)$$



From (i) and (ii), we get

$$\sqrt{48x(14+x)} = 56 + 4x$$

Squaring both sides, we get

$$48x(14+x) = (56+4x)^2$$

$$\Rightarrow 48x(14+x) = 16(14+x)^2$$

$$\Rightarrow 3x(14+x) - (14+x)^2 = 0$$

$$\Rightarrow (14+x)(3x-14-x) = 0$$

$$\Rightarrow (14+x)(2x-14) = 0$$

$$\Rightarrow 14+x = 0 \text{ or } 2x-14 = 0$$

$$\Rightarrow x = -14 \text{ (rejected) or } x = 7$$

$$\therefore AB = (7+8) \text{ cm} = 15 \text{ cm,}$$

$$\text{and } AC = (7+6) \text{ cm} = 13 \text{ cm.}$$

Question 41.

Prove that the lengths of the tangents drawn from an external point to a circle are equal.

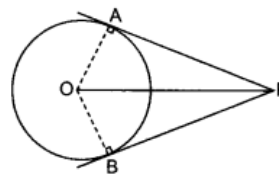
Solution:

Given: A circle $C(O, r)$. P is a point outside the circle and PA and PB are tangents to a circle.

To Prove: PA = PB

Construct: Draw OA, OB and OP.

Proof: Consider triangle OAP and OBP.



$$\angle OAP = \angle OBP = 90^\circ \quad \dots(i)$$

[Radius is perpendicular to the tangent at the point of contact]

$$OA = OB$$

(radii) ... (ii)

OP is common

... (iii)

$$\therefore \triangle OAP \cong \triangle OBP$$

(RHS) [from (i), (ii), (iii)]

$$\Rightarrow AP = BP$$

(CPCT)

Question 42.

A quadrilateral is drawn to circumscribe a circle. Prove that the sums of opposite sides are equal

Solution:

Given: A quadrilateral ABCD which circumscribes a circle.

Let it touches the circle at P, Q, R and S as shown in figure.

To Prove: AB + CD = AD + BC

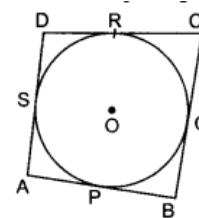
Proof: We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal.

$$\therefore AP = AS; BP = BQ; CQ = CR \text{ and } DR = DS \quad \dots(i)$$

$$\text{Consider, } AB + CD = AP + PB + CR + RD$$

$$= AS + BQ + CQ + DS$$

$$= (AS + DS) + (BQ + CQ) = AD + BC$$



* [using (i)]

2013

Short Answer Type Questions I [2 Marks]

Question 43.

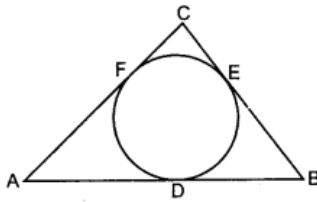
Prove that the parallelogram circumscribing a circle is a rhombus

Solution:

Refer to Ans. 36.

Question 44.

In the given figure, a circle inscribed in $\triangle ABC$ touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the lengths of AD, BE and CF



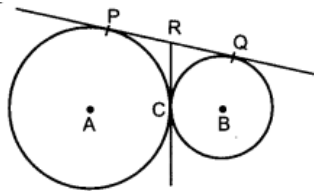
Solution:

Given, $AB = 12$ cm, $BC = 8$ cm, $AC = 10$ cm

Let	$AD = x$ cm	
\therefore	$BD = AB - AD = (12 - x)$ cm	
\therefore	$AD = AF$	[tangents from point A]
\therefore	$AF = x$ cm	
Now,	$CF = AC - AF = (10 - x)$ cm	
Also,	$CE = CF \Rightarrow CE = (10 - x)$ cm	
And	$BD = BE$	[\because tangents from B]
\Rightarrow	$BE = (12 - x)$ cm	[From (i)]
Now,	$BC = CE + BE$	
\Rightarrow	$8 = (10 - x) + (12 - x)$	
\Rightarrow	$8 = 22 - 2x \Rightarrow 2x = 14$	
\Rightarrow	$x = 7$ cm	
\Rightarrow	$AD = 7$ cm	
\Rightarrow	$BE = 12 - x = 12 - 7 = 5$ cm	
and	$CF = 10 - x = 10 - 7 = 3$ cm	

Question 45.

In the given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.



Solution:

PR and RC are tangents to circle with centre A.

$\therefore PR = RC$ [tangent from common point R] ... (i)

Similarly RQ and RC are tangents to circle with centre B

$\therefore RQ = RC$... (ii)

From (i) and (ii), $PR = RQ$

$\therefore CR$ bisects PQ .

Question 46.

In the given figure, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$

Solution:

Quadrilateral ABCD circumscribing a circle.

$\therefore AP = AS$ [tangents drawn from common external point to a circle are equal in length.]

$$BP = BQ$$

$$DR = DS$$

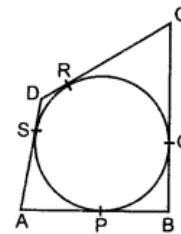
$$CR = CQ$$

On adding,

$$AP + BP + DR + CR = AS + BQ + DS + CQ$$

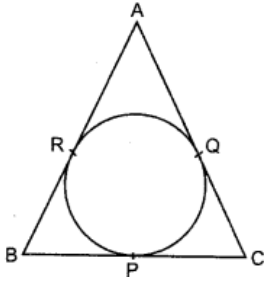
$$(AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ)$$

$$AB + CD = AD + BC$$



Question 47.

In the given figure, a circle inscribed in $\triangle ABC$, touches its sides BC, CA and AB at the points P, Q and R respectively. If $AB = AC$, then prove that $BP = CP$.

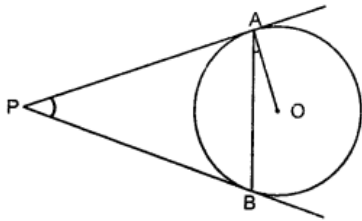


Solution:

$AB = AC$
 $\therefore AR + BR = AQ + CQ$
 $AR + BR = AR + CQ$
 [AQ = AR, equal tangents]
 $\Rightarrow BR = CQ$
 Now,
 $BR = BP$ [Length of equal tangents]
 $CQ = CP$
 $\Rightarrow BP = CP$

Question 48.

In the given figure, two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that $\angle APB = 2 \angle OAB$



Solution:

Construction: Join OP and OB.

Proof: Now, $OA \perp AP$ [Radius is perpendicular to tangent at the point of contact]

$\Rightarrow \angle OAP = 90^\circ$

Similarly, $OB \perp BP$

$\Rightarrow \angle OBP = 90^\circ$

In quadrilateral OAPB

$$\angle OAP + \angle OBP = 90^\circ + 90^\circ = 180^\circ$$

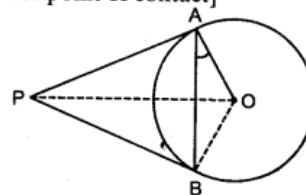
\therefore Quadrilateral OAPB is a cyclic quadrilateral.

$\Rightarrow \angle OAB = \angle OPB$ [Angles in same segment]

Also, $\angle OPB = \angle OPA$ [OP bisects $\angle APB$]

$\Rightarrow \angle OPB + \angle OPA = 2\angle OPB$

$\Rightarrow \angle APB = 2\angle OPB$



...(i)

[From (i)]

...(ii)

From (i) and (ii)

$$\angle APB = 2\angle OAB$$

$$[\because \angle OAB = \angle OPB]$$

Long Answer Type Questions [4 Marks]

Question 49.

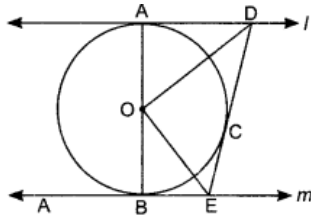
Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

Solution:

Refer to ANS.12

Question 50.

In the given figure l, m are two parallel tangents to the circle with center O , touching the circle at A and B respectively. Another tangent at C intersect the line l at D and m at E . prove that $\angle DOE = 90^\circ$



Solution:

Given: Line $l \parallel m$ and both are tangents to a circle at points A and B respectively.

To prove: $\angle DOE = 90^\circ$

Construction: Join OC

Proof: In $\triangle ADO$ and $\triangle CDO$,

$AD = DC$ [tangents from an external point are equal]

$OD = OD$ [common]

$\angle OAD = \angle OCD = 90^\circ$ [radius is perpendicular to tangent]

$\triangle ADO \cong \triangle CDO$ [By RHS]

$\therefore \angle 1 = \angle 2$... (i) [By CPCT]

Similarly, in $\triangle BOE$ and $\triangle COE$,

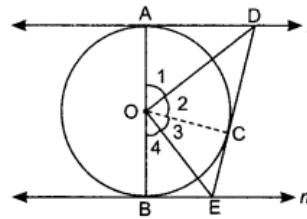
$\angle 3 = \angle 4$... (ii)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$ [angles on a straight line]

$\Rightarrow 2\angle 2 + 2\angle 3 = 180^\circ$ [$\because \angle 1 = \angle 2$ and $\angle 3 = \angle 4$]

$\Rightarrow 2(\angle 2 + \angle 3) = 180^\circ$

$\Rightarrow \angle 2 + \angle 3 = 90^\circ \Rightarrow \angle DOE = 90^\circ$



Question 51.

Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution:

Given: PA and PB are tangents to a circle with centre O .

To Prove: $PA = PB$

Construction: Join OP , OA , OB .

Proof: In $\triangle AOP$ and $\triangle BOP$

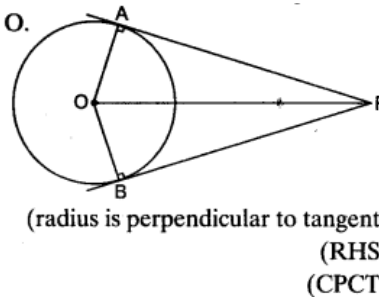
$OP = OP$ (common)

$OA = OB$ (radii of circle)

$\angle OAP = \angle OBP = 90^\circ$

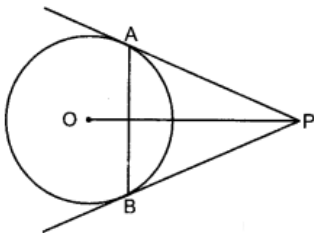
$\triangle AOP \cong \triangle BOP$

$\therefore PA = PB$



Question 52.

In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre O . Prove that OP is the right bisector of line segment AB .



Solution:

Join OA and OB.

In $\triangle PAO$ and $\triangle PBO$

$$OA = OB$$

[Radii]

$$OP = OP$$

[Common]

and

$$AP = BP$$

[Tangents from P]

\therefore

$$\triangle PAO \cong \triangle PBO$$

(SSS congruence rule)

\Rightarrow

$$\angle 1 = \angle 2$$

In $\triangle APC$ and $\triangle BPC$

$$\angle 1 = \angle 2 \text{ [Proved]}$$

$$AP = BP$$

$$PC = PC$$

and

$$\triangle APC \cong \triangle BPC$$

[SAS congruence rule]

\Rightarrow

$$AC = BC$$

[CPCT]

and

$$\angle ACP = \angle BCP$$

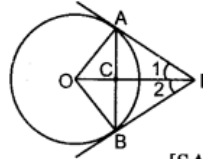
Also,

$$\angle ACP + \angle BCP = 180^\circ$$

\Rightarrow

$$\angle ACP = \angle BCP = 90^\circ$$

\therefore OP is the right bisector of AB.



Question 53.

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

Solution:

Given: A circle with centre O, line l is tangent to the circle at A.

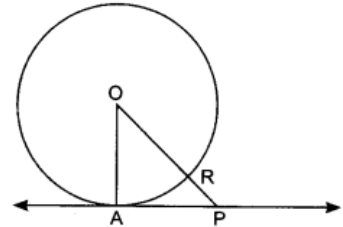
To Prove: Radius OA is perpendicular to the tangent at A.

Construction: Take a point P, other than A, on tangent l . Join OP, meeting the circle at R.

Proof: We know that tangent to the circle touches, the circle at one point and all other points on the tangent lie in the exterior of a circle.

$$\therefore OP > OR \text{ (radius of circle)}$$

$$\Rightarrow OP > OA \text{ (}\because OR = OA, \text{ radius of circle)}$$



$$\Rightarrow OA < OP$$

$$\Rightarrow OA \text{ is the smallest segment, from O to a point on the tangent.}$$

We know that smallest line segment from a point outside the circle to the line is perpendicular segment.

Hence, $OA \perp$ tangent l .

\Rightarrow tangent at any point of a circle is perpendicular to the radius through the point of contact.

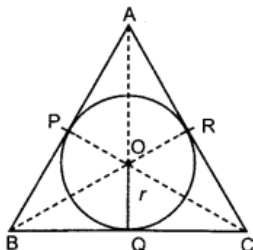
Question 54.

In the given figure, the sides AB, BC and CA of $\triangle ABC$ touch a circle with centre O and radius r at P, Q and R respectively.

Prove that:

- $AB + CQ = AC + BQ$

- Area ($\triangle ABC$) = $1/2$ (perimeter of $\triangle ABC$) $\times r$



Solution:

- (i) We have, $AP = AR$ [Tangents from A] ...*(i)*
Similarly, $BP = BQ$ [Tangents from B] ...*(ii)*
 $CR = CQ$ [Tangents from C] ...*(iii)*

Now, we have

$$\therefore AP = AR$$

$$\Rightarrow (AB - BP) = (AC - CR)$$

$$\Rightarrow AB + CR = AC + BP$$

$$\Rightarrow AB + CQ = AC + BQ$$

[Using eq. *(ii)* and *(iii)*]

(ii) $\text{Area}(\Delta ABC) = \text{Area}(\Delta ABO + \Delta OBC + \Delta OAC)$

$$= \frac{1}{2}(AB + BC + AC) \times r \quad [\because \text{Area}(\Delta) = \frac{1}{2} \times \text{base} \times \text{height}]$$

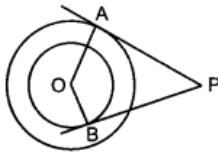
$$= \frac{1}{2}(\text{perimeter of } \Delta ABC) \times r$$

2012

Short Answer Type Questions I [2 Marks]

Question 55.

Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radii 8 cm and 5 cm respectively, as shown in Fig. If AP = 15 cm, then find the length of BP



Solution:

Join OP.

In ΔPAO and ΔPBO

$$\angle PAO = 90^\circ, \angle PBO = 90^\circ$$

(\because tangent is perpendicular to radius at the point of contact)

In right angled ΔPAO

$$PA^2 + OA^2 = OP^2 \quad [\because \text{Pythagoras theorem}]$$

$$15^2 + 8^2 = OP^2$$

$$225 + 64 = OP^2$$

$$OP^2 = 289$$

$$OP = \sqrt{289} = 17 \text{ cm}$$

Now, In right angled ΔPBO

$$PB^2 + BO^2 = PO^2$$

[\because Pythagoras theorem]

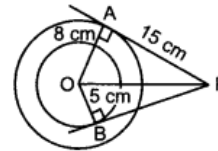
$$PB^2 + 5^2 = (17)^2$$

$$PB^2 + 25 = 289$$

$$PB^2 = 289 - 25$$

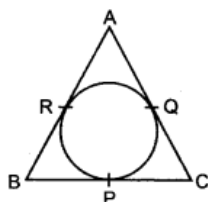
$$PB^2 = 264$$

$$PB = \sqrt{264} = 2\sqrt{66} \text{ cm.}$$



Question 56.

In figure, an isosceles triangle ABC, with $AB = AC$, circumscribes a circle. Prove that the point of contact P bisects the base BC



Solution:

Let the centre of circle be O.
Join OR, OQ, OB, OP, OC.

$$\angle 1 = \angle 2 = \angle 3 = \angle 4 = 90^\circ$$

(\because Radius is perpendicular to tangent at the point of contact)

In $\triangle ORB$ and $\triangle OQC$

$$OR = OQ \quad (\text{Radii of same circle})$$

$$\angle 1 = \angle 2 \quad (\text{each } 90^\circ)$$

$$RB = QC \quad (\because AB = AC \text{ and } AR = AQ)$$

(So, $AB - AR = AC - AQ$)

By SAS congruence rule,

$$\triangle ORB \cong \triangle OQC$$

\therefore

$$OB = OC$$

In $\triangle OPB$ and $\triangle OPC$

$$OP = OP \quad (\text{common})$$

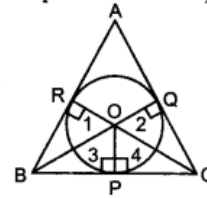
$$\angle 3 = \angle 4 \quad (\text{each } 90^\circ)$$

$$OB = OC \quad (\text{Proved above})$$

By RHS congruence rule,

$$\triangle OPB \cong \triangle OPC$$

$$BP = PC$$



(By CPCT)

(common)

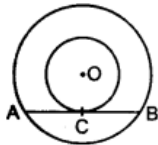
(each 90°)

(Proved above)

Hence, P bisects the base BC.

Question 57.

In figure, the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that $AC = CB$.



Solution:

Given: Two concentric circles with centre O.

AB is chord of bigger circle which touches the smaller circle to C.

To Prove: $AC = CB$

Construction: Join OA, OC, OB

Proof: In $\triangle OCA$ and $\triangle OCB$

$$OC = OC$$

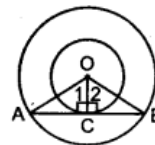
$$\angle 1 = \angle 2$$

$$OA = OB$$

(common)

(Radius perpendicular tangent)

(radii of same circle).



By RHS congruence rule,

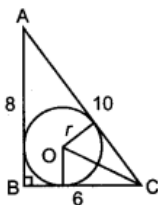
$$\triangle OCA \cong \triangle OCB$$

\therefore

$$AC = BC$$

Question 58.

In figure, a right triangle ABC, circumscribes a circle of radius r. If AB and BC are of lengths 8 cm and 6 cm respectively, find the value of r.



Solution:

$\therefore \triangle ABC$ is right angle \triangle , right $\angle d$ at B.

So, By Pythagoras theorem

$$AC^2 = AB^2 + BC^2 = 8^2 + 6^2 = 100$$

$$AC = 10 \text{ cm}$$

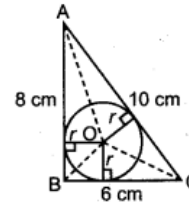
So, $\text{ar}(\triangle ABC) = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$

Also, $\text{ar}(\triangle ABC) = \text{ar}(\triangle OBC) + \text{ar}(\triangle OAC) + \text{ar}(\triangle OAB)$

$$\Rightarrow 24 = \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r + \frac{1}{2} \times 8 \times r$$

$$\Rightarrow 24 = 3r + 5r + 4r \Rightarrow 12r = 24$$

$$\Rightarrow r = 2 \text{ cm}$$



Question 59.

Prove that the tangents drawn at the ends of a diameter of a circle are parallel

Solution:

AB is the diameter.

R_1T_1 and R_2T_2 are the tangents at point A and B respectively.

Now, $OB \perp R_2T_2$
[radius perpendicular the tangent at point of contact]

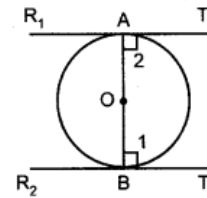
$$\Rightarrow \angle 1 = 90^\circ$$

Also, $OA \perp R_1T_1$
[radius perpendicular the tangent at point of contact]

$$\Rightarrow \angle 2 = 90^\circ$$

Now, $\angle 1 + \angle 2 = 90^\circ + 90^\circ = 180^\circ$

$$\Rightarrow R_1T_1 \parallel R_2T_2 \quad [\because \text{if interior angles on same side is supplementary, 2 lines are parallel}]$$



Question 60.

The incircle of an isosceles triangle ABC, with $AB = AC$, touches the sides AB, BC and CA at D, E and F respectively. Prove that E bisects BC

Solution:

We have, $AB = AC$ [Given] ...(i)
 $AD = AF$

[tangents drawn from an external point are equal] ...(ii)

On subtracting eq (ii) from eq (i), we get

$$AB - AD = AC - AF \Rightarrow BD = CF \text{ ... (iii)}$$

\therefore Also, $BD = BE$...(iv)

\therefore from (iii) and (iv), we have

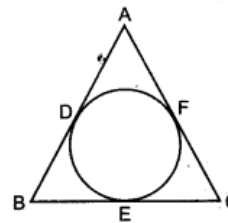
$$BE = CF \text{ ... (v)}$$

Also, $CF = CE$...(vi)

\therefore from (v) and (vi)

$$BE = CE$$

\therefore E bisects BC.

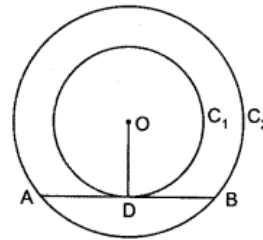


Question 61.

Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact

Solution:

Given: Let O be the centre of two concentric circles C_1 and C_2 .
Let AB be the chord of larger circle C_2 which is a tangent to the smaller circle C_1 at D.



To prove: Now we have to prove that the chord AB is bisected at D that is $AD = BD$.

Construction: Join OD.

Proof: Now since OD is the radius of the circle C_1 and AB is the tangent to the circle C_1 at D.

So, $OD \perp AB$ [radius of the circle is perpendicular to tangent at any point of contact]

Since AB is the chord of the circle C_2 and $OD \perp AB$.

$\therefore AD = DB$ [perpendicular drawn from the centre to the chord always bisects the chord]

Short Answer Type Questions II [3 Marks]

Question 62.

Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:

Given: A circle with centre O.

ABCD is a parallelogram circumscribing the circle, touching it at P, Q, R, S

To Prove: ABCD is a Rhombus.

Proof: $AR = AS$

[\because tangents from an external point are equal] ...*(i)*

$RB = BQ$

...*(ii)*

$DP = DS$

...*(iii)*

$PC = CQ$

...*(iv)*

Consider $AB + DC = AR + RB + DP + PC$
 $= AS + BQ + DS + QC$

[By *(i)*, *(ii)*, *(iii)*, *(iv)*]

Now, $AB + DC = AD + BC$

$AB + AB = AD + AD$

[\because ABCD is a parallelogram, so opposite sides are equal, i.e. $AB = CD$. $AD = BC$]

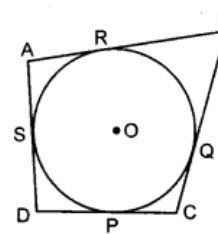
$2AB = 2AD$

$AB = AD$

So, $AB = BC = CD = AD$

ABCD is a parallelogram with all sides equal.

Hence, ABCD is a Rhombus.



Question 63.

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:

Given: ABCD is a quadrilateral circumscribing the circle with centre O touching it at P, Q, R, S.

To Prove: $\angle AOB + \angle DOC = 180^\circ$
 $\angle AOD + \angle BOC = 180^\circ$

Construction: Join AO, PO, BO, QO, CO, RO, DO, SO,

Proof: In $\triangle AOS$ and $\triangle AOP$

$$\begin{aligned} AO &= AO && \text{(common)} \\ AS &= AP && \text{(tangents from external point)} \\ OS &= OP && \text{(radii of same circle)} \end{aligned}$$

By SSS congruence

$$\begin{aligned} \triangle AOS &\cong \triangle AOP \\ \angle 1 &= \angle 2 && \text{(By CPCT) ... (i)} \end{aligned}$$

Similarly, $\angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$... (ii)

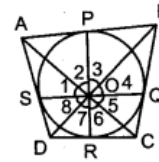
Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$ [\because ASP of quadrilateral]

$$\angle 2 + \angle 2 + \angle 3 + \angle 3 + \angle 6 + \angle 6 + \angle 7 + \angle 7 = 360^\circ$$

$$2[\angle 2 + \angle 3 + \angle 6 + \angle 7] = 360^\circ$$

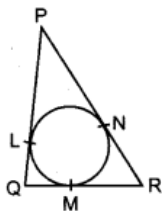
$$\angle AOB + \angle COD = 180^\circ$$

Similarly, $\angle AOD + \angle BOC = 180^\circ$



Question 64.

In figure, a circle is inscribed in a triangle PQR with PQ = 10 cm, QR = 8 cm and PR = 12 cm. Find the lengths QM, RN and PL.



Solution:

We know that the tangents drawn from an external point to a circle are equal.

Therefore

Let

$$QM = x = QL$$

$$MR = y = RN$$

and

$$PL = z = PN$$

Now

$$PQ = 10 \text{ cm, } QR = 8 \text{ cm, } PR = 12 \text{ cm}$$

$$\Rightarrow x + y = 8, y + z = 12, z + x = 10$$

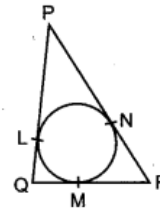
$$\Rightarrow 2x + 2y + 2z = 8 + 12 + 10 = 30$$

$$\Rightarrow x + y + z = 15 \Rightarrow 8 + z = 15 \Rightarrow z = 7$$

$$\Rightarrow x + 12 = 15 \Rightarrow x = 3$$

$$\Rightarrow y + 10 = 15 \Rightarrow y = 5$$

Hence, QM = 3 cm, RN = 5 cm and PL = 7 cm.

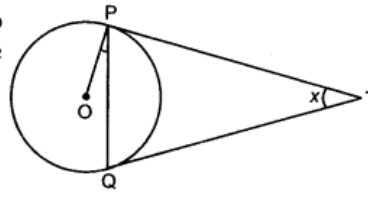


Question 65.

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$

Solution:

Given: A circle with centre O. External point T and two tangents TP and TQ to the circle, where P, Q are the points of contact.



To Prove: $\angle PTQ = 2\angle OPQ$

Proof: Let $\angle PTQ = x$

In $\triangle PTQ$, $PT = PQ$ [The lengths of tangents drawn from an external point to a circle are equal]

$\angle TPQ = \angle TQP$ [angles opposite to equal sides are equal]

$\angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - x) = 90^\circ - \frac{x}{2}$ [\because ASP of triangle]

$\angle OPT = 90^\circ$ [The tangent at any point of a circle is perpendicular to the radius through the point of contact]

\therefore From figure, $\angle OPQ = \angle OPT - \angle TPQ$

$$= 90^\circ - \left(90^\circ - \frac{x}{2}\right) = 90^\circ - 90^\circ + \frac{x}{2}$$

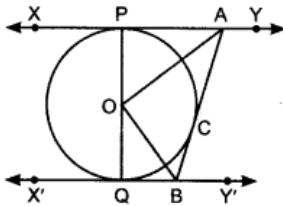
$$\angle OPQ = \frac{1}{2}\angle PTQ$$

This gives $\angle PTQ = 2\angle OPQ$

Hence, Proved.

Question 66.

In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X'Y' at B. Prove that $\angle AOB = 90^\circ$.



Solution:

Given: XY and X'Y' are two parallel tangents to circle with centre O. Tangent AB with point of contact C intersects XY at A and X'Y' at B.

To Prove: $\angle AOB = 90^\circ$

Construction: Join OC.

Proof: In $\triangle OPA$ and $\triangle OCA$,

$OP = OC$ (Radii of the same circle)

$AP = AC$ (Tangents from point A)

$AO = AO$ (common side)

$\triangle OPA \cong \triangle OCA$

Therefore, $P \rightarrow e, A \rightarrow A, O \rightarrow o,$

$\angle POA = \angle COA$

...(i) (CPCT)

Similarly, we prove: $\triangle OQB \cong \triangle OCB$

Then: $\angle QOB = \angle COB$

...(ii) (CPCT)

Since, POQ is the diameter of the circle, it is a straight line.

$\therefore \angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$

from equation (i) and (ii),

$$2\angle COA + 2\angle COB = 180^\circ$$

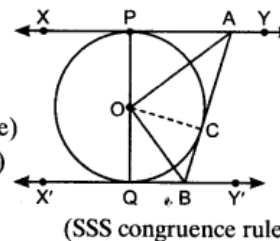
$$2(\angle COA + \angle COB) = 180^\circ$$

$$\angle COA + \angle COB = \frac{180^\circ}{2}$$

$$\angle COA + \angle COB = 90^\circ$$

$$\angle AOB = 90^\circ$$

Hence, Proved.



Long Answer Type Questions [4 Marks]

Question 67.

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

Solution:

Refer to Ans. 12.

Question 68.

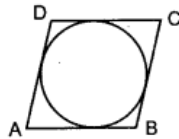
A quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$.

Solution:

Refer to Ans. 46.

Question 69.

Prove that the lengths of tangents drawn from an external point to a circle are equal. Using it, prove: quadrilateral ABCD is drawn to circumscribe a circle. Such that $AB + CD = AD + BC$



Solution:

Given: A circle with centre O. Through the external point A, tangents AP and AQ are drawn.

To prove: $AP = AQ$

Construction: Join OA, OP and OQ

Proof: In $\triangle OAP$ and $\triangle OAQ$,

$$OP = OQ$$

$$OA = OA$$

$$\angle OPA = \angle OQA = 90^\circ$$

$$\therefore \triangle OAP \cong \triangle OAQ$$

$$\Rightarrow AP = AQ$$

Hence proved.

Second Part:

In the given figure, $AE = AH$...*(i)*

[Tangents drawn from an external point are equal]

$BE = BF$...*(ii)*

[Tangents drawn from an external point are equal]

$DG = DH$...*(iii)*

[Tangents drawn from an external point are equal]

$CG = CF$...*(iv)* [Tangents drawn from an external point are equal]

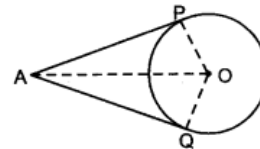
Adding equation *(i)*, *(ii)*, *(iii)* and *(iv)*, we get

$$AE + BE + DG + CG = AH + BF + DH + CF$$

$$\Rightarrow (AE + BE) + (DG + CG) = (AH + DH) + (BF + CF)$$

$$\Rightarrow AB + CD = AD + BC.$$

Hence proved.



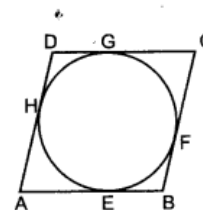
[Radii of the same circle]

[Common]

[radius is perpendicular to the tangent at point of contact]

[By RHS]

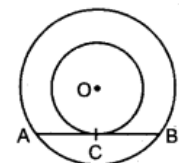
[CPCT]



Question 70.

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Solution:



Short Answer Type Questions I [2 Marks]

Question 71.

Two concentric circles are of radii 7 cm and r cm respectively, where $r > 7$. A chord of the larger circle, of length 48 cm, touches the smaller circle. Find the value of r .

Solution:

Given:

$$OP = 7 \text{ cm}; OA = r \text{ cm}$$

$$AB = 48 \text{ cm}$$

Now, $OP \perp AB$

(as radius makes an angle of 90° with the tangent at point of contact)

Also, $AP = PB$

(perpendicular drawn from centre to the chord bisects the chord)

So,

$$AP = 24 \text{ cm}$$

In $\triangle OPA$,

$$\angle P = 90^\circ$$

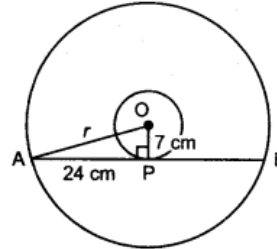
By Pythagoras theorem in $\triangle OPA$,

$$OA^2 = AP^2 + OP^2$$

$$r^2 = 24^2 + 7^2 = 576 + 49 = 625$$

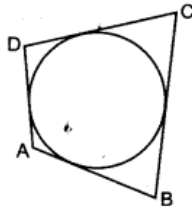
\Rightarrow

$$r = 25 \text{ cm}$$



Question 72.

In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are $AB = 6$ cm, $BC = 9$ cm and $CD = 8$ cm. Find the length of side AD.



Solution:

If a circle touches all the four sides of quadrilateral ABCD, then we know that

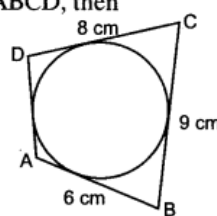
$$AD + BC = AB + CD$$

\therefore

$$AD + 9 = 6 + 8$$

\Rightarrow

$$AD = 5 \text{ cm}$$



Question 73.

If d_1, d_2 ($d_2 > d_1$) be the diameters of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, prove that $d_2^2 = c^2 + d_1^2$.

Solution:

\therefore Diameter of bigger circle = d_2

So, Radius of bigger circle = $\frac{1}{2}d_2 = OB$

and Diameter of smaller circle = d_1

So, Radius of smaller circle = $\frac{1}{2}d_1 = OA$

$$AB = \frac{c}{2}$$

In right $\triangle OAB$,

$$\angle A = 90^\circ$$

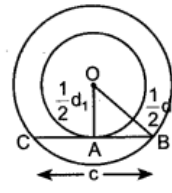
[\because radius is perpendicular the tangent at point of contact]

By pythagoras theorem

$$OB^2 = AB^2 + OA^2$$

$$\Rightarrow \left(\frac{1}{2}d_2\right)^2 = \left(\frac{1}{2}c\right)^2 + \left(\frac{1}{2}d_1\right)^2 \Rightarrow \frac{1}{4}d_2^2 = \frac{1}{4}c^2 + \frac{1}{4}d_1^2$$

$$\Rightarrow d_2^2 = c^2 + d_1^2$$



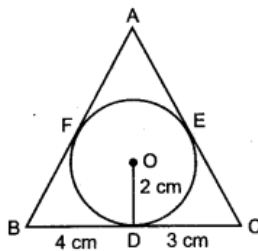
[\because Diameter of CB = 'C']

Hence proved.

Short Answer Type Questions II [3 Marks]

Question 74.

In figure, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D are the lengths 4 cm and 3 cm respectively. If area of $\triangle ABC = 21 \text{ cm}^2$, then find the lengths of sides AB and AC.



Solution:

Let $AE = AF = y$ (say)

[Tangents drawn from an external point are equal]

$$\text{ar } \triangle BOC = \frac{1}{2} \times 7 \times 2 = 7 \text{ cm}^2 = b \text{ (say)}$$

$$\text{ar } \triangle AOB = \frac{1}{2} \times (4 + y) \times 2 = (4 + y) \text{ cm}^2 = a \text{ (say)}$$

$$\text{ar } \triangle AOC = \frac{1}{2} \times (3 + y) \times 2 = (3 + y) \text{ cm}^2 = c \text{ (say)}$$

Now,

$$\text{ar } \triangle ABC = \text{ar } \triangle AOB + \text{ar } \triangle BOC + \text{ar } \triangle AOC$$

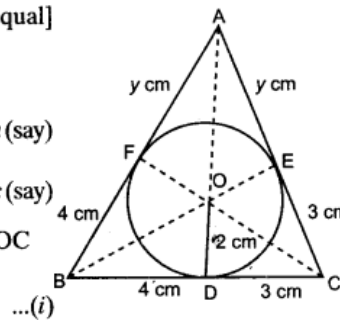
$$= 4 + y + 7 + 3 + y$$

$$\text{ar } \triangle ABC = 14 + 2y$$

For $\triangle ABC$,

Semi-perimeter,

$$s = \frac{a+b+c}{2} = \frac{4+y+7+3+y}{2} = \frac{14+2y}{2} = 7+y$$



$$\begin{aligned} \therefore \text{ar } \Delta ABC &= \sqrt{s(s-a)(s-b)(s-c)} \quad [\because \text{By Heron's formula}] \\ &= \sqrt{(7+y)(7+y-4-y)[7+y-7](7+y-3-y)} \\ &= \sqrt{(7+y) \times 3 \times y \times 4} \\ \text{ar } \Delta ABC &= 2\sqrt{3y(7+y)} \quad \dots(ii) \end{aligned}$$

From (i) and (ii)

$$\Rightarrow 2\sqrt{3y(7+y)} = 14 + 2y$$

$$\Rightarrow \sqrt{3y(7+y)} = 7 + y$$

Squaring both sides, we get

$$\Rightarrow 3y(7+y) = (7+y)^2 \Rightarrow 21y + 3y^2 = 49 + y^2 + 14y$$

$$\Rightarrow 2y^2 + 7y - 49 = 0 \Rightarrow 2y^2 + 14y - 7y - 49 = 0$$

$$\Rightarrow 2y(y+7) - 7(y+7) = 0 \Rightarrow (2y-7)(y+7) = 0$$

$$\Rightarrow y = \frac{7}{2}, y = -7$$

[Rejected]

Hence, length of side AB = 4 + 3.5 = 7.5 cm and AC = 3 + 3.5 = 6.5 cm.

Long Answer Type Questions [4 Marks]

Question 75.

Prove that the lengths of tangents drawn from an external point to a circle are equal.

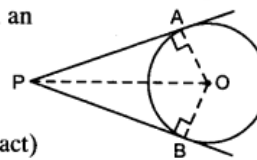
Solution:

Given: PA and PB are two tangents to a given circle drawn from an external point P.

To prove: PA = PB

Proof: OA ⊥ PA and OB ⊥ PB

(radius perpendicular to tangent at point of contact)



Join OP.

Now, In ΔOAP and ΔOBP,	OA = OB	(radii)
	∠A = ∠B	(each 90°)
	OP = OP	(common)
So,	ΔOAP ≅ ΔOBP	(By RHS)
So,	PA = PB	(By CPCT)

Question 76.

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

Solution:

Refer to Ans. 12.

Question 77.

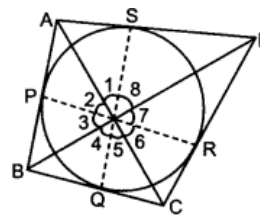
Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:

The given quadrilateral ABCD is circumscribing the circle having its centre at O.

The sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively.

Join OA, OB, OC, OD; OP, OQ, OR, OS.



We observe that OA bisects $\angle POS$ [\because By CPCT, applied to ΔPOA and ΔSOA]
 $\Rightarrow \angle 1 = \angle 2$... (i)
 similarly $\angle 3 = \angle 4$... (ii)
 $\angle 5 = \angle 6$... (iii)
 and $\angle 7 = \angle 8$... (iv)
 Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$ [\because ASP of quadrilateral]
 $\Rightarrow 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360^\circ$
 $\Rightarrow (\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180^\circ \Rightarrow \angle AOD + \angle BOC = 180^\circ$
 Similarly, $\angle AOB + \angle COD = 180^\circ$
 Hence, opposite sides of the quadrilateral ABCD subtend supplementary angles at the centre

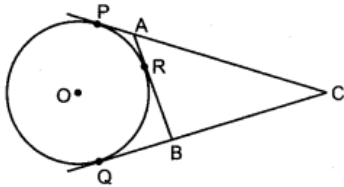
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Very Short Answer Type Questions [1 Mark]

Question 78.

In figure, CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm, find the length of BC.

Solution:



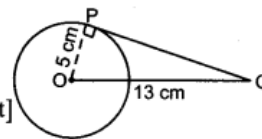
In the given figure, $CP = CQ$
 [tangents drawn from an external point are equal]
 So, $CP = CQ = 11$ cm
 Also, $BR = BQ$
 [tangents drawn from an external point are equal]
 So, $BR = BQ = 4$ cm
 \therefore Now, $BC = CQ - BQ = (11 - 4)$ cm = 7 cm

Question 79.

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 13 cm. Find the length PQ.

Solution:

Given, $OP = 5$ cm [radius]
 $OQ = 13$ cm
 Now, In ΔOPQ , $\angle P = 90^\circ$ [radius is perpendicular to tangent at point of to contact]



$(OQ)^2 = (OP)^2 + (PQ)^2$
 $\therefore PQ = \sqrt{(13)^2 - (5)^2} = 12$ cm [By pythagoras theorem]

Short Answer Type Questions I [2 Marks]

Question 80.

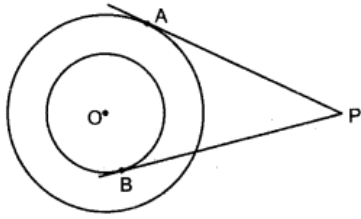
Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above prove the following: In Fig., PA and PB are tangents from an external point P, to a circle with centre O. LN touches the circle at M. Prove that $PL + LM = PN + MN$.

Solution:

Refer to Ans. 10 and 33.

Question 81.

In figure, there are two concentric circles, with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.



Solution:

Construction: Join OA, OB and OP.

AP = 12 cm, OA = 5 cm, OB = 3 cm

In $\triangle AOP$, $\angle A = 90^\circ$ [radius is perpendicular to the tangent at point of contact]

$\triangle BOP$, $\angle B = 90^\circ$ [radius is perpendicular to the tangent at point of contact]

So, $OP^2 = OA^2 + AP^2$
and $OP^2 = OB^2 + BP^2$

Using Pythagoras theorem for $\triangle AOP$ and $\triangle BOP$.

$$\therefore OA^2 + AP^2 = OB^2 + BP^2$$

$$5^2 + 12^2 = 3^2 + BP^2 \Rightarrow 25 + 144 = 9 + BP^2 \Rightarrow 169 - 9 = BP^2$$

$$\Rightarrow BP = \sqrt{160} \text{ cm} = 12.65 \text{ cm}$$

