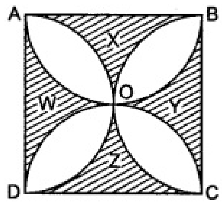
Areas Related to Circles

2016

Short Answer Type Questions II [3 Marks]

Question 1.

In figure, ABCD is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region.



Solution:

Area of the square ABCD = 14 x 14 = 196 cm^2

Area of semicircle AOB=1/2 x πr^2

=1/2×22/7x7x7

Similarly, area of semicircle DOC = 77 cm^2

Hence, the area of shaded region (Part W and Part Y) = Area of square -Area of two semicircles AOB and COD

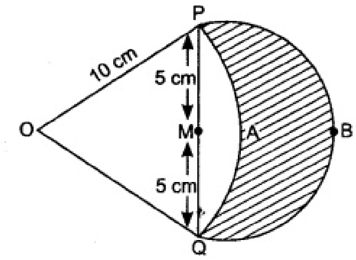
= 196 – 154 = 42 cm^2

Therefore, area of four shaded parts (i.e. X, Y, W, Z) = (2 x 42) cm^2 = 84 cm^2

Question 2.

In figure, are shown two arcs PAQ and PBQ. Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semicircle drawn on PQ as diameter with centre M. If OP = PQ = 10 cm, show that area of shaded region is

$25(\sqrt{3}- \pi /6)cm^2$



Solution:

OP = OQ = 10 cm

PQ = 10 cm

So, ΔOPQ is an equilateral triangle

∠POQ = 60°

Area of segment PAQM = Area of sector OPAQ – Area of ΔOPQ

$=60/360 \times \pi \times 10 \times 10-\sqrt{3}/4 \times 10 \times 10$

$=(100\pi/6-100\sqrt{3}/4)cm^2$

Area of semicircle $=1/2 \times \pi \times 5 \times 5 = 25/2\pi\ cm^2$

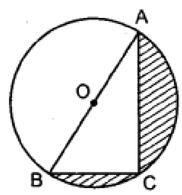
Area of the shaded region $=25/2\pi-(100\pi/6-100\sqrt{3}/4)=25\pi/2-50\pi/3+25\sqrt{3}$

$=75\pi-100\pi/6+25\sqrt{3}=25\sqrt{3}-25\pi/6$

$=25(\sqrt{3}-\pi/6)cm^2$

Question 3.

In figure, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region.



Solution:

Here, $BC^2 = AB^2-AC^2$

= 169-144 = 25 BC = 5

Area of shaded region = Area of semicircle – Area of right triangle ABC

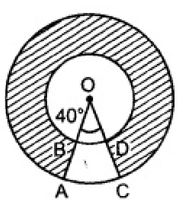
$=1/2 \times \pi r^2-1/2AC \times BC$

$=1/2 \times 3.14(13/2)^2-1/2 \times 12 \times 5$

$=66.33 – 30 = 36.33\ cm^2$

Question 4.

In figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where∠AOC = 40°.



Solution:

Area of shaded region$=360°-\theta/360° \times \pi(R^2-r^2)$

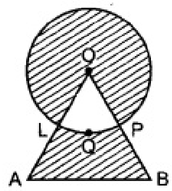
$=320°/360° \times \pi[(14)^2-(7)^2]$

$=8/9 \times 22/7(196-49)=8/9 \times 22/7 \times 147$

$=1232/3=410.67\ cm^2$

Question 5.

Find the area of shaded region in figure, where a circle of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm



Solution:

Area of $\Delta OAB = \sqrt{3}/4(\text{side})^2 = \sqrt{3}/4 \times (12)^2$

$= 36\sqrt{3} = 36 \times 1.73$

$= 62.28\ cm^2$

area of circle with center O $= \pi r^2 = 3.14 \times (6)^2$

$= 3.14 \times 36 = 113.04\ cm^2$

area of sector(OLQP) $= \pi r^2 \times \theta/360° = 3.14 \times 6^2 \times 60°/360°$

$= 3.14 \times 36 \times 1/6 = 18.84\ cm^2$

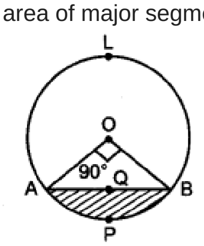
area of shaded region=area of ΔOAB+area of circle-2 area of sector OLQP

$= (62.28 + 113.04 - 2 \times 18.84) cm^2$

$= 137.64 cm^2$

Question 6.

In figure, is a chord AB of a circle, with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment AQBP. Hence, find the area of major segment ALBQA



$\left(\because \frac{90°}{360°} = \frac{1}{4}\right)$

Solution:

Area of minor segment APBQ $= \theta/360° \times \pi r^2 - r^2 \sin 45° \cos 45°$

$= 3.14 \times 100/4 - 100 \times 1/\sqrt{2} \times 1/\sqrt{2}$

$= (78.5 - 50) cm^2 = 28.5\ cm^2$

Area of major segment ALBQA $= \pi r^2$-area of minor segment

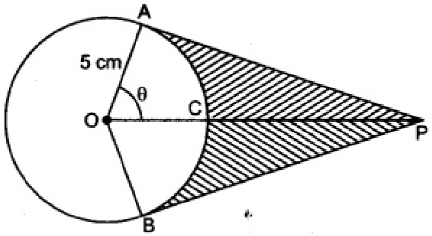
$= 3.14 \times (10)^2 - 28.5$

$= (314 - 28.5) cm^2 = 285.5\ cm^2$

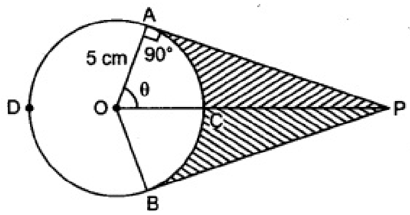
Long Answer Type Questions [4 Marks]

Question 7.

An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt, the elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley.Also find the shaded area, (use $\pi = 3.14$ and $\sqrt{3} = 1.73$)



Solution:



Given: AO = 5 cm and OP = 10 cm

In right ΔAOP,

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$= \frac{AO}{OP} = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60°$$

$$\angle AOB = \theta' = 2 \times 60 = 120°$$

$$\text{Length of ADB} = \frac{360° - \theta'}{360°} \times 2\pi r \qquad (\because Q' = 120°)$$

$$= \frac{240}{360} \times 2 \times 3.14 \times 5$$

$$= \frac{2}{3} \times 10 \times 3.14 = 20.93 \text{ cm}$$

Hence, length of belt in contact = 20.93 cm

Now, in right ΔOAP, we have

$$\tan\theta = \frac{AP}{AO}$$

$$\tan 60° = \frac{AP}{5} \quad \Rightarrow \quad AP = 5\sqrt{3} \text{ cm}$$

$$\text{Area of } (\Delta OAP + \Delta OBP) = \frac{1}{2} \times AO \times AP + \frac{1}{2} \times OB \times PB$$

$$= \frac{1}{2} \times 5 \times 5\sqrt{3} + \frac{1}{2} \times 5 \times 5\sqrt{3} \quad (\because AP = BP \text{ and } OA = OB)$$

$$= 25\sqrt{3} \text{ cm}^2 = 25 \times 1.73 \text{ cm}^2$$

$$= 43.25 \text{ cm}^2$$

$$\text{Area of sector OACB} = \frac{\theta'}{360} \times \pi r^2$$

$$= \frac{120}{360} \times 3.14 \times 5 \times 5$$

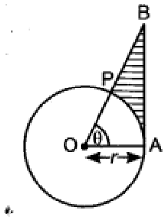
$$= \frac{1}{3} \times 3.14 \times 25 = 26.16 \text{ cm}^2$$

$$\text{Shaded Area} = \text{Area of } (\Delta OAP + \Delta OBP) - \text{Area of OACB}$$

$$= 43.25 - 26.16 = 17.09 \text{ cm}^2$$

Question 8.

In figure, is shown a sector OAP of a circle with centre O, containing Z0. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is r[tanθ+secθ+πθ/180-1]



Solution:

$$\text{Length arc } \widehat{AP} = \frac{\theta}{360} \times 2\pi r = \frac{\pi r\theta}{180}$$

$$\frac{AB}{r} = \tan\theta \Rightarrow AB = r\tan\theta$$

$$\frac{OB}{r} = \sec\theta \Rightarrow OB = r\sec\theta$$

$$PB = OB - r = r\sec\theta - r$$

$$\text{Perimeter of shaded region} = AB + PB + \widehat{AP}$$

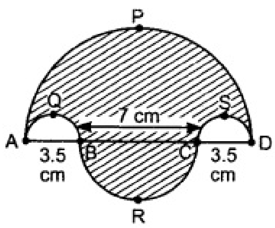
$$= r\tan\theta + r\sec\theta - r + \frac{\pi r\theta}{180}$$

$$= r\left[\tan\theta + \sec\theta - 1 + \frac{\pi\theta}{180}\right]$$

Question 9.

Find the area of the shaded region in figure, where APD, AQB, BRC and CSD are semi-circles of diameter

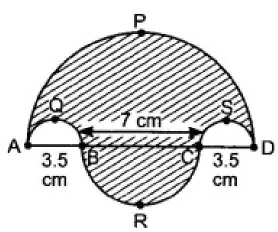
14 cm, 3.5 cm, 7 cm and 3.5 cm respectively



Solution:

Area of shaded region

= Area of semicircle APD + Area of semicircle BRC – 2 x Area of semicircle AQB



$$= \frac{1}{2}\pi(7)^2 + \frac{1}{2}\pi\left(\frac{7}{2}\right)^2 - 2 \times \frac{1}{2}\pi\left(\frac{7}{4}\right)^2 = \frac{\pi}{2}\left[49 + \frac{49}{4} - \frac{49}{8}\right]$$

$$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{392 + 98 - 49}{8}\right) = \frac{11}{7} \times \frac{441}{8}$$

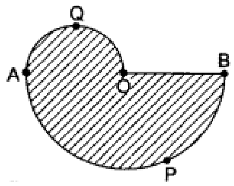
$$= \frac{693}{8}\text{ cm}^2 = 86.625\text{ cm}^2$$

2015

Short Answer Type Questions II [3 Marks

Question 10.

In figure, APB and AQO are semicircle, and AO = OB. If the perimeter of the figure is 40 cm, find the area of the shaded region.



Solution:

Let r be the radius of the semicircle APB, i.e. OB = OA = r, then r/2 is the radius of the semicircle AQO.

Given: Perimeter of the figure is 40 cm.

$\therefore$ Length of arc APB + length of arc AQO + OB = 40

$$\Rightarrow \quad \pi r + \pi \frac{r}{2} + r = 40$$

$$\frac{22}{7} \times r + \frac{22}{7} \times \frac{r}{2} + r = 40 \Rightarrow \frac{44r + 22r + 14r}{14} = 40$$

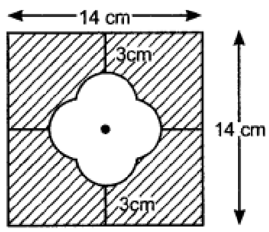
$$\Rightarrow \quad \frac{80}{14} r = 40 \Rightarrow r = \frac{40 \times 14}{80} = 7 \text{ cm}$$

Now, area of shaded portion = area of semicircle APB + area of semicircle AQO

$$= \frac{1}{2}\pi r^2 + \frac{1}{2}\pi\left(\frac{r}{2}\right)^2$$

Question 11.

In figure, find the area of the shaded region



Solution:

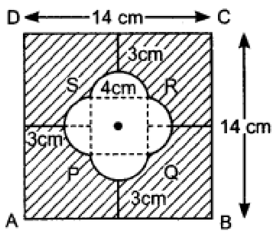
Shaded area = area of square ABCD – area of square PQRS – area of 4 semicircles

$$= 14^2 - 4^2 - 4 \times \frac{1}{2}\pi \times 2^2$$

$$= 196 - 16 - 8 \times 3.14$$

$$= 180 - 25.12$$

$$= 154.88 \text{ cm}^2$$



Question 12.

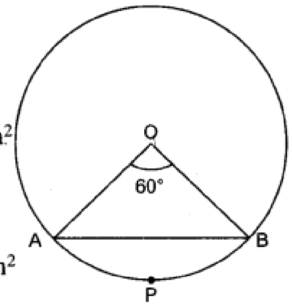
Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60°. Also find the area of the corresponding major segment

Solution:

In ΔAOB, $\angle AOB = 60°$

Also $AO = BO$

$\therefore$ ΔAOB is an equilateral triangle.



$$\text{Area of equilateral } \Delta AOB = \frac{\sqrt{3}}{4} \times 14 \times 14 = 49\sqrt{3} \text{ cm}^2$$

$$\text{Area of sector AOBP} = \frac{60}{360} \times \pi \times 14 \times 14$$

$$= \frac{1}{6} \times \frac{22}{7} \times 14 \times 14 = \frac{308}{3} \text{ cm}^2$$

$$\text{Area of minor segment} = \left(\frac{308}{3} - 49\sqrt{3}\right) = \frac{308}{3} - 49 \times 1.732 = 17.8 \text{ cm}^2$$

$$\text{Area of circle} = \pi r^2 = \frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2$$

Area of major segment = $[616 - 17.8] = 598.2 \text{ cm}^2$

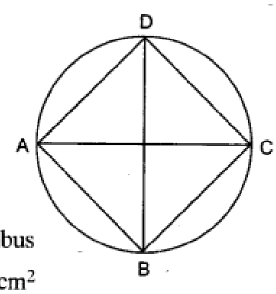
Question 13.

All the vertices of a rhombus lie on a circle.Find the area of rhombus,if the area of circle is 1256 cm^2

Solution:

Diagonal of a rhombus are perpendicular bisector of each other.

$\therefore$ Each diagonal is diameter of the circle.



Now, area of circle $= 1256 \text{ cm}^2$

$$\Rightarrow \quad \pi r^2 = 1256 \Rightarrow r^2 = \frac{1256}{\pi}$$

$$\Rightarrow \quad r^2 = \frac{1256}{3.14} = 400 \Rightarrow r = 20 \text{ cm}$$

$\therefore$ Diameter of the circle = 40 cm = Each diagonal of the rhombus

$$\text{Area of rhombus} = \frac{1}{2}(d_1 \times d_2) = \frac{1}{2} \times 40 \times 40 = 800 \text{ cm}^2$$

Question 14.

The long and short hand of a clock are 6cm and 4 cm long respectively,Find the sum of the distance travelled by their tips in 24hrs.

Solution:

Distance covered by the tip of long hand in one hour

= circumference of the circle with radius 6 cm

$= 2\pi \times 6 = 12\pi$ cm

$\therefore$ Distance travelled by long hand in 24 hours $= 24 \times 12\pi = 288\pi$ cm

Distance travelled by tip of short hand in 12 hrs

= circumference of the circle with radius 4 cm

$= 2\pi \times 4 = 8\pi$ cm

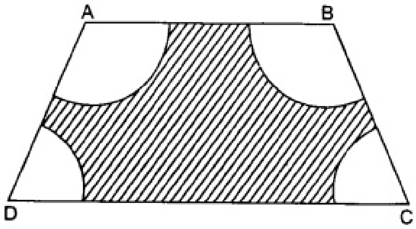
$\therefore$ Distance travelled by short hand in 24 hours $= 2 \times 8\pi = 16\pi$ cm

Total distance travelled $= 288\pi + 16\pi = 304\pi$ cm

$= 304 \times 3.14 \text{ cm} = 954.56$ cm

Question 15.

In Figure, ABCD is a trapezium with AB || DC, AB = 18 cm, DC = 32 cm and the distance between AB and DC is 14 cm. If arcs of equal radii 7 cm have been drawn, with centres A, B, C and D, then find the area of the shaded region.



Solution:

$$\text{Area of the trapezium} = \frac{1}{2} \times h(a+b)$$

$$= \frac{1}{2} \times 14 \times (18+32) = 350 \text{ cm}^2 \quad (\text{Here, } a = \text{AB}, b = \text{DC}, h = 14)$$

$$\text{Area of the four sectors} = \frac{\angle A}{360} \times \pi r^2 + \frac{\angle B}{360} \times \pi r^2 + \frac{\angle C}{360} \times \pi r^2 + \frac{\angle D}{360} \times \pi r^2$$

$$= \frac{\pi \times r^2}{360} \times (\angle A + \angle B + \angle C + \angle D)$$

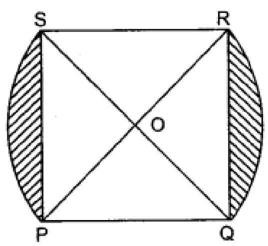
$$= \frac{\pi \times 7 \times 7}{360} \times 360 = 49\pi \text{ cm}^2 = \frac{49 \times 22}{7} = 154 \text{ cm}^2$$

$$\therefore \quad \text{Area of shaded part} = 350 - 154 = 196 \text{ cm}^2$$

Long Answer Type Questions [4 Marks]

Question 16.

In Figure, PQRS is a square lawn with side PQ = 42 metres. Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).



Solution:

$$PR^2 = PQ^2 + QR^2$$

$$\Rightarrow \quad PR^2 = (42)^2 + (42)^2$$

$$\Rightarrow \quad PR = 42\sqrt{2} \text{ m}$$

$$\Rightarrow \quad PO = \frac{42\sqrt{2}}{2} = 21\sqrt{2} \text{ m}$$

$$\text{Area of sector POS} = \frac{90°}{360°} \times \pi(21\sqrt{2})^2$$

$$\text{Area of sector POS} = \frac{90°}{360°} \times \pi(21\sqrt{2})^2 \quad (\because \text{PO} \perp \text{OS})$$

$$= \frac{1}{4} \times \frac{22}{7} \times 21 \times 21 \times 2 = 693 \text{ m}^2$$

$$\text{Area of } \Delta\text{POS} = \frac{1}{2}(\text{PO} \times \text{OS})$$

$$= \frac{1}{2} \times 21\sqrt{2} \times 21\sqrt{2} = 441 \text{ m}^2$$

$\therefore$ Area of one flower bed $= 693 - 441 = 252 \text{ m}^2$

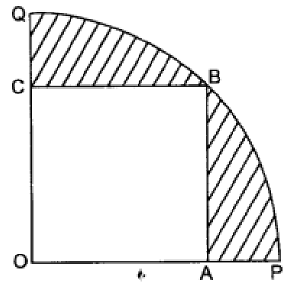
$\Rightarrow$ Area of two flower bed $= 2 \times 252 = 504 \text{ m}^2$

2014

Short Answer Type Questions I [2 Marks]

Question 17.

In figure, a square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20 cm, find the area of the shaded region



Solution:

$OA = 20 \text{ cm}$

$$OB^2 = OA^2 + AB^2 = 400 + 400 = 2 \times 400$$

$\Rightarrow \quad OB = 20\sqrt{2} \Rightarrow OB = r = 20\sqrt{2}$

$\therefore \quad \text{Shaded area} = \text{Area of quadrant} - \text{Area of square}$

$$= \frac{1}{4}\pi r^2 - (20)^2$$

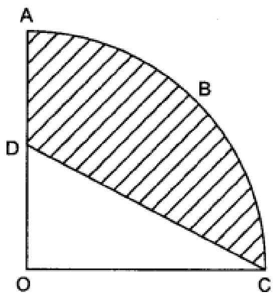
$$= \frac{1}{4} \times 3.14 \times 20\sqrt{2} \times 20\sqrt{2} - 400$$

$$= 400\left(\frac{3.14}{2} - 1\right)$$

$$= 400 \times (1.57 - 1) = 400 \times 0.57 = 228 \text{ cm}^2$$

Question 18.

In figure, OABC is a quadrant of a circle of radius 7 cm. If OD = 4 cm, find the area of the shaded region



Solution:

Area of the shaded region

= Area of the quadrant – area of the triangle DOC.

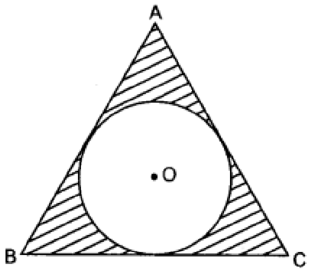
$$= \left[\frac{90°}{360°} \times \pi(7)^2 - \frac{1}{2} \times 4 \times 7\right] \text{cm}^2 = \left[\frac{1}{4} \times \frac{22}{7} \times 49 - 14\right] \text{cm}^2$$

$$= \left(\frac{77}{2} - 14\right) \text{cm}^2 = \frac{49}{2} \text{cm}^2 = 24.5 \text{ cm}^2$$

Short Answer Type Questions II [3 Marks]

Question 19.

In figure, a circle is inscribed in an equilateral triangle ABC of side 12 cm. Find the radius of inscribed circle and the area of the shaded region



Solution:

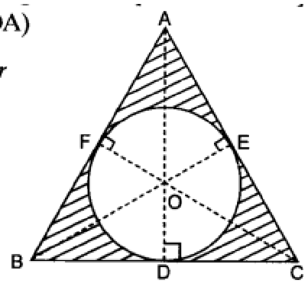
Given: ABC is an equilateral triangle of side 12 cm.
Let radius of incircle be r. Join OA, OB and OC. Also, join OD, OE and OF
Here, AB, BC and AC are the tangents for the circle.
$\therefore$ OD $\perp$ BC, OE $\perp$ AC and OF $\perp$ AB.

Now, $\text{ar}(\Delta ABC) = \text{ar}(\Delta AOB) + \text{ar}(\Delta BOC) + \text{ar}(\Delta COA)$

$\Rightarrow \frac{\sqrt{3}}{4}(12^2) = \frac{1}{2}\times 12\times r + \frac{1}{2}\times 12\times r + \frac{1}{2}\times 12\times r$

$\Rightarrow \frac{\sqrt{3}}{4}\times 12\times 12 = 6r + 6r + 6r \Rightarrow 36\sqrt{3} = 18r$

$\Rightarrow r = 2\sqrt{3} = 2\times 1.73 = 3.46$ cm

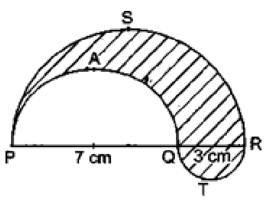


Hence, radius of incircle is 3.46 cm.
Area of shaded portion = Area of equilateral triangle – Area of circle

$= \frac{\sqrt{3}}{4}\times 12^2 - \pi(2\sqrt{3})^2 = 36\sqrt{3} - 12\pi$
$= 12(3\times 1.73 - 3.14) = 12(5.19 - 3.14)$
$= 12\times 2.05 = 24.60 \text{ cm}^2$

Question 20.

In figure, PSR, RTQ and PAQ are three semicircles of diameters 10 cm, 3 cm and 7 cm respectively. Find the perimeter of the shaded region

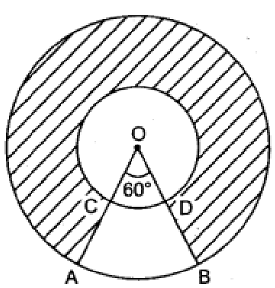


Solution:

Perimeter of shaded region = length of arc PSR + length of arc PAQ + length of arc QTR
$= 5\pi + 3.5\pi + 1.5\pi = 10\pi = 10\times 3.14 = 31.4$ cm

Question 21.

In figure, two concentric circles with centre O, have radii 21 cm and 42 cm. If ∠AOB = 60°, find the area of the shaded region



Solution:

Area of larger circle $= \pi(42)^2$

Area of smaller circle $= \pi(21)^2$

Area of CDBA = Area of sector OAB – Area of sector OCD

$$= \frac{60}{360} \times (42)^2 - \frac{60}{360} \times \pi \times (21)^2$$

$$= \frac{\pi}{6}[(42)^2 - (21)^2]$$

Area of the shaded region = Area of larger circle – Area smaller circle – Area of CDBA

$$= \pi(42)^2 - \pi(21)^2 - \frac{\pi}{6}[(42)^2 - 21)^2]$$

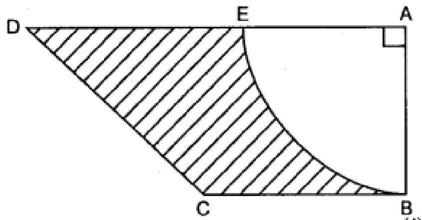
$$= \pi[1764 - 441] - \frac{\pi}{6} \times 1323$$

$$= 1323\left[\pi - \frac{\pi}{6}\right] = 1323 \times \frac{5\pi}{6}$$

$$= 1323 \times \frac{22}{7} \times \frac{5}{6} = 3465 \text{ cm}^2$$

Question 22.

In figure, ABCD is a trapezium of area 24.5 sq. cm. In it, AD ¦¦ BC, ∠DAB = 90°, AD = 10 cm and BC = 4 cm. If ABE is a quadrant of a circle, find the area of the shaded region



...(*i*)

Solution:

Area of the trapezium = 24.5 sq. cm

AD = 10 cm

BC = 4 cm

Let AB = h cm

∴ Area of the trapezium $= \frac{1}{2}(AD + BC) \times AB$

$\Rightarrow 24.5 = \frac{1}{2}(10 + 4) \times h$

$\Rightarrow h = \frac{24.5}{7} = 3.5$ cm

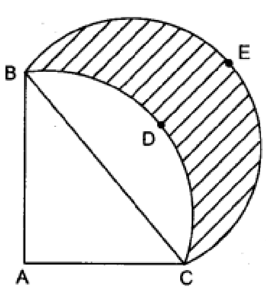
Now, area of quadrant ABE $= \frac{90°}{360°} \times \pi (3.5)^2$ sq. cm

$= \frac{1}{4} \times \frac{22}{7} \times (3.5)^2$ sq. cm = 9.625 sq. cm

∴ Area of shaded region = 24.5 – 9.625 = 14.875 sq. cm

Question 23.

In figure, ABDC is a quadrant of a circle of radius 28 cm and a semicircle BEC is drawn with BC as diameter. Find B the area of the shaded region.



Solution:

AC = 28 cm

$BC^2 = AC^2 + AB^2 = 28^2 + 28^2 \Rightarrow BC = \sqrt{28^2 + 28^2} = 28\sqrt{2}$ cm

$\therefore$ Diameter of semicircle $= 28\sqrt{2}$ cm

$\Rightarrow$ Radius of semicircle $= 14\sqrt{2}$ cm

$\therefore$ Shaded region = Area of semicircle – Area of segment BCD

= Area of semicircle – [Area of sector ABDC – Area of ΔABC]

$= \frac{1}{2}\pi(14\sqrt{2})^2 - \frac{90°}{360°} \times \pi(28)^2 + \frac{1}{2} \times 28 \times 28$

$= \frac{1}{2} \times \frac{22}{7} \times 196 \times 2 - \frac{1}{4} \times \frac{22}{7} \times 28 \times 28 + 14 \times 28$

$= 22 \times 28 - 22 \times 28 + 14 \times 28 = 392 \text{ cm}^2$

2013

Short Answer Type Questions I [2 Marks]

Question 24.

Two circular pieces of equal radii and maximum area,touching each other are cut out from a rectangular cardboard of dimensions 14 cm x 7 cm. Find the area of the remaining cardboard

Solution:

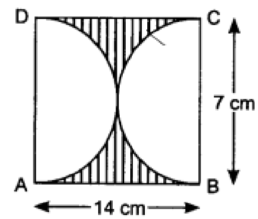
Area of remaining cardboard (i.e. area of shaded portion)

= Area of rectangle ABCD – Area of two semicircles

$= 14 \times 7 - \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

$= 98 - \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 98 - 38.5 = 59.5 \text{ cm}^2$



Question 25.

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Solution:

Angle swept by minute hand in 5 min $= \frac{360° \times 5}{60°} = 30°$

Length of minute hand (radius of circle) = 14 cm

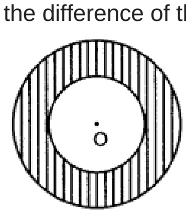
Area swept by minute hand in 5 min $= \frac{\pi r^2 \theta}{360°}$

$= \frac{22 \times 14 \times 14 \times 30°}{7 \times 360°}$

$= \frac{154}{3} = 51.33 \text{ cm}^2$

Question 26.

In the given figure,the area of the shaded region between two concentric circles is 286 cm2.If the difference of the radii of the two circles is 7 cm, find the sum of their radii.



Solution:

Let radius of outer circle is R_1 and radius of inner circle is R_2

According to question,

$$\pi R_1^2 - \pi R_2^2 = 286 \Rightarrow \pi(R_1^2 - R_2^2) = 286$$

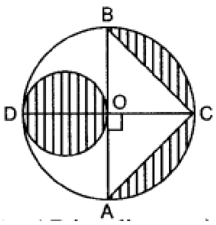
$$\Rightarrow \frac{22}{7} \times (R_1 - R_2)(R_1 + R_2) = 286 \Rightarrow \frac{22}{7} \times 7 \times (R_1 + R_2) = 286$$

$$\Rightarrow R_1 + R_2 = 13 \text{ cm}$$

Short Answer Type Questions II [3 Marks]

Question 27.

In the given figure AB and CD are two diameters of a circle with centre O,Which are perpendicular to each other.OB is the diameter of small circle.If OA=7cm,find area of shaded region



Solution:

Radius of big circle = OA = 7 cm

$$\therefore \quad AB = 14 \text{ cm}$$

$$\angle BCA = 90°$$

$$\text{Area of } \Delta BCA = \frac{1}{2} \times AB \times OC = \frac{1}{2} \times 14 \times 7 = 49 \text{ cm}^2$$

$$\text{Area of semicircle BCA} = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2$$

$$\text{Now, area of circle with OD as diameter} = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{ cm}^2$$

$$\text{Area of shaded portion} = \text{Area of semicircle BCA} - \text{Area of } \Delta BCA + \text{Area of small circle}$$

$$= 77 - 49 + \frac{77}{2} = 28 + \frac{77}{2} = \frac{133}{2} \text{ cm}^2 = 66.5 \text{ cm}^2$$

Question 28.

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find

1. the length of the arc
2. area of the sector formed by the arc.

Solution:

Radius of circle, $r = 21$ cm

Central angle subtended by an arc = 60°

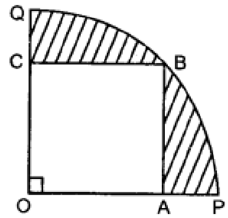
(*i*) $\text{Length of arc} = \frac{2\pi r\theta}{360°} = 2 \times \frac{22}{7} \times 21 \times \frac{60°}{360°} = 22 \text{ cm}$

(*ii*) $\text{Area of sector formed by an arc} = \frac{\pi r^2\theta}{360°}$

$$= \frac{22}{7} \times 21 \times 21 \times \frac{60°}{360°} = 231 \text{ cm}^2$$

Question 29.

In figure, a square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 21 cm, find the area of the shaded region



Solution:

OABC is a square.

$\therefore$ In ΔOBA, by Pythagoras Theorem,

$$OB^2 = OA^2 + AB^2$$
$$= (21)^2 + (21)^2 = 2 \times (21)^2$$
$$\Rightarrow \quad OB = 21\sqrt{2}$$

$\therefore$ Radius of quadrant, $r = OB = 21\sqrt{2}$ cm

Area of shaded region = area of quadrant – area of square

Area of shaded region = area of quadrant – area of square

$$= \frac{\pi r^2 \theta}{360^\circ} - (\text{side})^2$$

$$= \frac{22}{7} \times \frac{(21\sqrt{2})^2 \times 90^\circ}{360^\circ} - (21)^2$$

$$= \frac{22}{7} \times \frac{21 \times 21 \times 2}{4} - 441$$

$$= 693 - 441 = 252 \text{ cm}^2$$

Question 30.

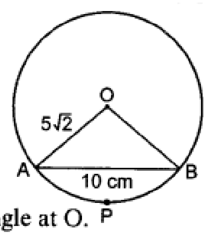
A chord of length 10 cm divides a circle of radius 5√2 cm in two segments. Find the area of the minor segment

Solution:

Consider, chord AB divides circle in two segments.

In ΔAOB,

$$AB^2 = OA^2 + OB^2$$
$$(10)^2 = (5\sqrt{2})^2 + (5\sqrt{2})^2$$
$$= 25 \times 2 + 25 \times 2$$
$$100 = 50 + 50$$
$$100 = 100$$



Hence, by converse of Pythagoras Theorem, ΔAOB is right-angled triangle at O.

$\therefore \quad \angle AOB = 90^\circ$

Area of minor segment = area of sector OAPB – area of ΔOAB

$$= \frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} \cdot OA \cdot OB$$

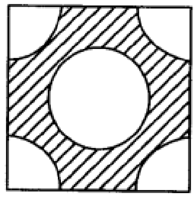
$$= \frac{3.14 \times (5\sqrt{2})^2 \times 90^\circ}{360^\circ} - \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$$

$$= \frac{314 \times 50 \times 1}{100 \times 4} - 25$$

$$= \frac{157}{4} - 25 = \frac{157 - 100}{4} = \frac{57}{4} \text{ cm}^2 = 14.25 \text{ cm}^2$$

Question 31.

In the given figure, from each corner of a square of side 4 cm, a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut. Find the area of the shaded region.



Solution:

$$\text{Area of square} = (\text{side})^2 = (4)^2 = 16 \text{ cm}^2$$

$$\text{Area of 4 quadrants} = 4 \times \frac{1}{4}\pi r^2 = \pi r^2$$

$$= 3.14 \times 1^2 = 3.14 \text{ cm}^2$$

$$\text{Area of circle with diameter 2 cm} = \pi r^2 = 3.14 \times 1^2 = 3.14 \text{ cm}^2$$

$$\text{Area of shaded part} = \text{area of square} - (\text{area of 4 quadrants} + \text{area of circle})$$

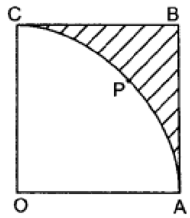
$$= 16 - (3.14 + 3.14) = 16 - 6.28 = 9.72 \text{ cm}^2$$

2012

Short Answer Type Questions I [2 Marks]

Question 32.

In figure, OABC is a square of side 7cm. If OAPC is a quadrant of a circle with centre O, then find the area of the shaded region



Solution:

$$\text{Area of square OABC} = (\text{side})^2 = (7)^2 = 49 \text{ cm}^2$$

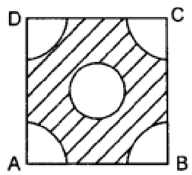
$$\text{Area of quadrant OAPC} = \frac{\pi r^2}{4} = \frac{22}{7} \times \frac{1}{4} \times 7 \times 7 = \frac{77}{2} = 38.5 \text{ cm}^2$$

$$\text{Area of shaded region} = \text{Area of square} - \text{Area of quadrant}$$

$$= 49 - 38.5 = 10.5 \text{ cm}^2$$

Question 33.

In figure, ABCD is a square of side 4 cm. A quadrant of a circle of radius 1 cm is drawn at each vertex of the square and a circle of diameter 2 cm is also drawn. Find the area of the shaded region



Solution:

Refer to Ans 31.

Question 34.

From a rectangular sheet of paper ABCD with AB = 40 cm and AD = 28 cm, semicircular portion with BC as diameter is cut off. Find the area of the remaining paper.

Solution:

$$\text{Radius of semicircle, } r = \frac{28}{2} = 14 \text{ cm}$$

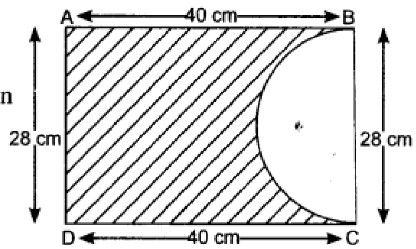
Area of remaining portion = area of shaded portion

= Area of rectangle ABCD – area of semicircle

$$= 40 \times 28 - \frac{1}{2} \times \pi \times 14 \times 14$$

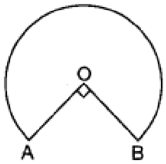
$$= 1120 - \frac{1}{2} \times \frac{22}{7} \times 196$$

$$= 1120 - 308 = 812 \text{ cm}^2$$



Question 35.

In the given figure, the shape of the top of a table is that a sector of a circle with centre O and ∠AOB = 90°. If AO = OB = 42 cm,then find the perimeter of the top of the table



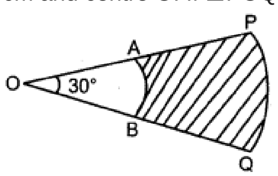
Solution:

$$\text{Perimeter} = \text{length of major arc} + 2r$$
$$= \frac{270°}{360°} \times 2 \times \pi r + 2r = \frac{3}{2} \times \frac{22}{7} \times 42 + 2 \times 42$$
$$= 198 + 84 = 282 \text{ cm}$$

Short Answer Type Questions II [3 Marks]

Question 36.

In figure, PQ and AB are respectively the arcs of two concentric circles of radii 7 cm and 3.5 cm and centre O. If ∠POQ = 30°, then find the area of the shaded region.



Solution:

Radius of bigger circle = R = 7 cm

Radius of smaller circle = r = 3.5 cm

$\theta = 30°$

Area of shaded region = Area of sector OPQ – Area of sector OAB

$$= \frac{\theta}{360°} \times \pi R^2 - \frac{\theta}{360°} \times \pi r^2$$
$$= \frac{\theta}{360°}\pi(R^2 - r^2)$$
$$= \frac{30°}{360°} \times \frac{22}{7}[7^2 - (3.5)^2]$$
$$= \frac{22}{84} \times (49 - 12.25) = \frac{22}{84} \times 36.75 = 9.625 \text{ cm}^2$$

Question 37.

In figure, find the area of the shaded region, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

Solution:

Area of the shaded region = Area of square – 2(Area of semicircle)

$$= 14 \times 14 - 2\left[\frac{1}{2} \times \pi \times (7)^2\right]$$
$$= 196 - \frac{22}{7} \times 7 \times 7$$
$$= 196 - 154 = 42 \text{ cm}^2$$

Question 38.

In the given figure, O is the centre of the circle with AC = 24 cm, AB = 7 cm and ∠BOD = 90°. Find the area of the shaded region.

Solution:

In ΔCAB,

$\angle CAB = 90°$ (Angle in a semicircle)

$\therefore \quad BC^2 = AC^2 + AB^2$ (By Pythagoras theorem)

$\Rightarrow \quad BC^2 = (24)^2 + (7)^2$

$\Rightarrow \quad BC^2 = 625 \Rightarrow BC = 25$ cm

$\Rightarrow$ Diameter of the circle $= 25$ cm

$\Rightarrow$ Radius $= \frac{25}{2}$ cm

Area of $\Delta ACB = \frac{1}{2} \times AB \times AC = \frac{1}{2} \times 7 \times 24 = 84\ cm^2$

$\because \quad \angle BOD = 90°$

$\therefore \quad \angle COD = 90°$

Area of quadrant COD $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times 3.14 \times \frac{25}{2} \times \frac{25}{2}\ cm^2 = \frac{1962.5}{16}\ cm^2$

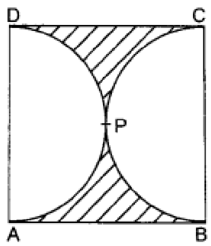
Area of shaded part = area of circle – area of ΔACB – area of quadrant COD

$= 3.14 \times \frac{25}{2} \times \frac{25}{2} - 84 - \frac{1962.5}{16}$

$= \frac{1962.5}{4} - \frac{1962.5}{16} - 84 = \frac{5887.5}{16} - 84 = 283.968\ cm^2$

Question 39.

Find the area of the shaded region in Figure, if ABCD is a square of side 28 cm and APD and BPC are semicircles.



Solution:

Side of square ABCD is 28 cm.

Radius of the semicircular APD and BPC is 14 cm.

Area of the shaded region = Area of square ABCD – Area of semicircles APD and BPC

$= (\text{side})^2 - 2 \times \frac{\pi r^2}{2}$

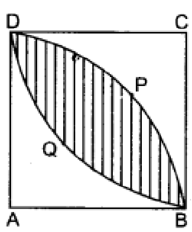
$= (\text{side})^2 - \pi r^2$

$= (28)^2 - \frac{22}{7} \times 14 \times 14$

$= (28)^2 - 22 \times 2 \times 14 = 784 - 616 = 168$

Question 40.

In figure, ABCD is a square of side 7 cm. DPBA and DQBC are quadrants of circles, each of radius 7 cm. Find the area of the shaded region.

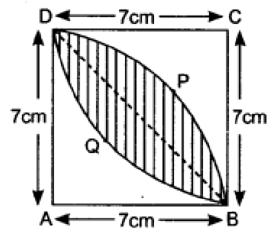


Solution:

side of square=7cm

radius of circle=7cm

Shaded area is the common region between two sector DQBC and DPBA



$$\text{Area of sector} = \frac{\theta}{360°} \times \pi r^2 = \frac{90°}{360°} \times \frac{22}{7} \times (7)^2 = \frac{1}{2} \times 11 \times 7 = \frac{77}{2}\ \text{cm}^2$$

$$\text{Areas of a triangle} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$\text{Area of } \Delta DCB = \frac{1}{2} \times 7 \times 7 = \frac{49}{2}\ \text{cm}^2$$

Area of the shaded region = 2 × (Area of sector DQBC – area of ΔDBC)

$$= 2 \times \left(\frac{77}{2} - \frac{49}{2}\right) = 2 \times \left(\frac{77-49}{2}\right) = 28\ \text{cm}^2$$

Question 41.

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 10 minutes

Solution:

In 1 hour, the minute hand rotates 360°.

In 10 minutes, minute hand will rotate $= \frac{360°}{60} \times 10 = 60°$

Therefore, the area swept by the minute hand in 10 minute will be the area of a sector of 60° in a circle of 14 cm radius.

$$\text{Area of sector of angle } \theta = \frac{\theta}{360°} \times \pi r^2$$

$$\text{Area of sector of angle } 60° = \frac{60°}{360°} \times \frac{22}{7} \times 14 \times 14 = \frac{308}{3} = 102.67\ \text{cm}^2$$

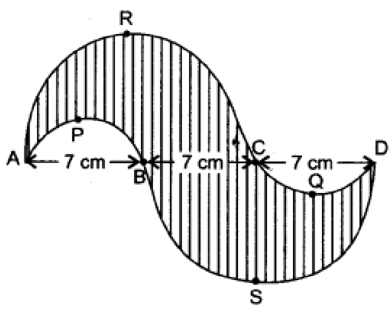
∴ The area swept by the minute hand in 10 minutes is 102.67 cm^2

2011

Short Answer Type Questions I [2 Marks]

Question 42.

In figure APB and CQD are semicircles of diameter 7 cm each, while ARC and BSD are semicircles of diameter 14 cm each. Find the perimeter of the shaded region



Solution:

$$\text{Perimeter of shaded region} = \text{Perimeter of semicircles} = ARC + APB + BSD + CQD = \pi[r_1 + r_2 + r_3 + r_4]$$

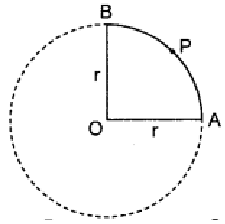
$$= \frac{22}{7}\left[7 + \frac{7}{2} + 7 + \frac{7}{2}\right] = \frac{22}{7} \times 21 = 66\ \text{cm}$$

Question 43.

Find the area of a quadrant of a circle, where the circumference of circle is 44cm

Solution:

Let r be the radius of the circle.



Given: Circumference $= 44$ cm

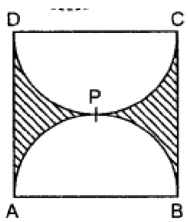
$\Rightarrow$ $2\pi r = 44$

$\Rightarrow$ $2 \times \frac{22}{7} \times r = 44 \Rightarrow r = \frac{44 \times 7}{22 \times 2} = 7$ cm

Now, area of quadrant OAPB $= \frac{\pi r^2}{4} = \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = 38.5$ cm^2

Question 44.

Find the perimeter of the shaded region in figure, if ABCD is a square of side 14 cm and APB and CPD are semicircles

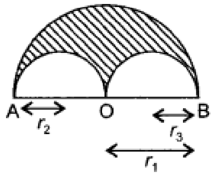


Solution:

$$\text{Perimeter of shaded region} = AD + BC + \text{length of DPC} + \text{length of APB}$$
$$= 14 + 14 + \pi r + \pi r$$
$$= 28 + 2 \times \frac{22}{7} \times \frac{14}{2} = 72 \text{ cm}$$

Question 45.

In given figure, a semicircle is drawn with O as centre and AB as diameter. Semicircles are drawn with AO and OB as diameters.If AB = 28 m, find the perimeter of the shaded region



Solution:

Diameter, AB $= 28$ m

$$\text{Radius } (r_1) = \frac{28}{2} = 14 \text{ m}$$

Diameter, AO $= 14$ m

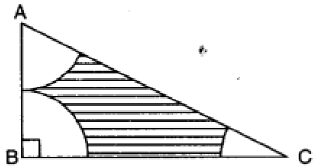
$$\text{Radius } (r_2) = \frac{14}{2} = 7 \text{ m}$$

$\therefore$ Radius $(r_3) = 7$ m

$$\text{Perimeter of the shaded region} = \pi r_1 + \pi r_2 + \pi r_3$$
$$= \pi[r_1 + r_2 + r_3]$$
$$= \frac{22}{7}[14 + 7 + 7] = \frac{22}{7} \times 28 = 88 \text{ m}$$

Question 46.

In given figure, ABC is a triangle right-angled atB, with AB = 14 cm and BC = 24 cm. With thevertices A, B and C as centres, arcs are drawn each of radius 7 cm. Find the area of the shaded region.



Solution:

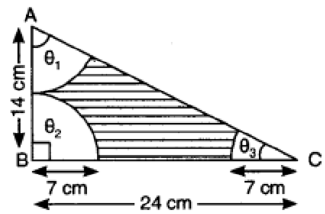
Area of shaded region = area of ΔABC – area of 3 sectors

$$= \frac{1}{2} \times 24 \times 14 - \frac{\pi r^2}{360°}[\theta_1 + \theta_2 + \theta_3]$$

$$= 12 \times 14 - \frac{22}{7} \times \frac{7 \times 7}{360°} \times 180°$$

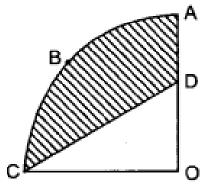
$[\therefore \theta_1 + \theta_2 + \theta_3 = 180°$, angle sum property$]$

$$= 168 - 77 = 91 \text{ cm}^2$$

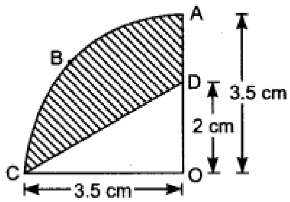


Question 47.

In the given figure, OABC is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the shaded region



Solution:



OC = OA = 3.5 cm

OD = 2 cm

Area of shaded portion

$$= \text{Area of quadrant OABC} - ar(\Delta COD)$$

$$= \frac{1}{4}\pi r^2 - \frac{1}{2}(OC \times OD)$$

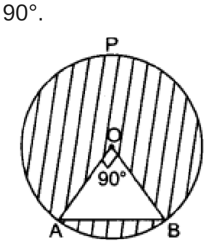
$$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 - \frac{1}{2} \times 3.5 \times 2$$

$$= 9.625 - 3.5 = 6.125 \text{ cm}^2$$

Short Answer Type Questions II [3 Marks]

Question 48.

Find the area of the major segment APB in figure of a circle of radius 35 cm and ∠AOB = 90°.



Solution:

Radius of circle = 35 cm

$\angle AOB = 90°$

$$\text{Area of sector OAB} = \frac{\pi r^2 \theta}{360°}$$

$$= \frac{22}{7} \times 35 \times 35 \times \frac{90°}{360°} = \frac{1925}{2} \text{ cm}^2$$

$$\text{Area of minor segment} = \text{area of sector OAB} - \text{area of } \Delta OAB$$
$$= \frac{1925}{2} - \frac{1}{2} \times 35 \times 35 = \frac{1925}{2} - \frac{1225}{2} = \frac{700}{2} = 350 \text{ cm}^2$$
$$\text{Area of major segment APB} = \text{area of circle} - \text{area of minor segment}$$
$$= \frac{22}{7} \times 35 \times 35 - 350 = 3500 \text{ cm}^2$$

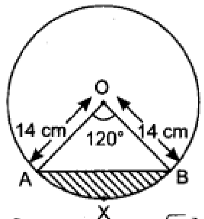
Question 49.

A chord of a circle of radius 14 cm subtends an angle of 120° at the centre. Find the area of the corresponding minor segment of the circle.

Solution:

Area of shaded portion

$$= \text{Area of sector OAXB} - \text{area of } \Delta OAB$$
$$= \frac{\pi r^2 \theta}{360°} - \frac{1}{2} r^2 \sin\theta$$
$$= \frac{22}{7} \times 14 \times 14 \times \frac{120}{360} - \frac{1}{2} \times 14 \times 14 \times \sin 120°$$
$$= \frac{616}{3} - 7 \times 14 \times \frac{\sqrt{3}}{2}$$
$$= 205.33 - 7 \times 7 \times 1.73$$
$$= 205.33 - 84.77 = 120.56 \text{ cm}^2$$



$$\left[\text{as } \sin 120° = \frac{\sqrt{3}}{2}\right]$$

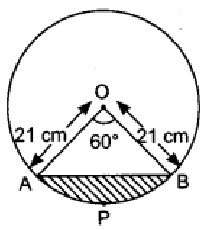
Question 50.

A chord of a circle of radius 21 cm subtends an angle of 60° at the centre. Find the area of the corresponding minor segment of the circle

Solution:

Area of shaded portion

$$= \text{Area of sector OAPB} - \text{area of } \Delta OAB$$
$$= \frac{\pi r^2 \theta}{360°} - \frac{1}{2} r^2 \sin\theta$$
$$= \frac{22}{7} \times 21 \times 21 \times \frac{60}{360} - \frac{1}{2} \times 21 \times 21 \times \sin 60°$$
$$= 22 \times 3 \times 21 \times \frac{1}{6} - \frac{1}{2} \times 21 \times 21 \times \frac{\sqrt{3}}{2}$$
$$= 11 \times 21 - \frac{441 \times 1.73}{4} = 231 - 190.73 = 40.27 \text{ cm}^2$$



Question 51.

Area of a sector of a circle of radius 14 cm is 154 cm². Find the length of the corresponding arc of the sector

Solution:

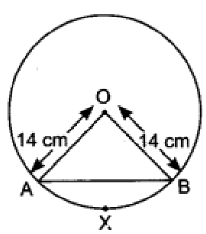
Given: Area of sector OAXB $= 154 \text{ cm}^2$

$\Rightarrow$ $$\frac{\pi r^2 \theta}{360°} = 154$$

$\Rightarrow$ $$\frac{22}{7} \times \frac{14 \times 14 \times \theta}{360} = 154$$

$\Rightarrow$ $$\theta = \frac{154 \times 7 \times 360}{22 \times 14 \times 14} \Rightarrow \theta = 90°$$

Now, length of arc AXB $$= \frac{2\pi r\theta}{360°} = 2 \times \frac{22}{7} \times 14 \times \frac{90°}{360°} = 22 \text{ cm}$$

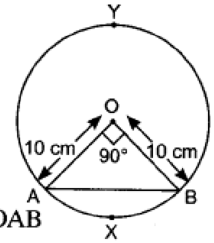


Question 52.

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding minor segment and hence find the area of the major segment

Solution:

$$\text{Area of circle} = \pi r^2 = \frac{22}{7} \times 10 \times 10 = \frac{2200}{7}\ \text{cm}^2$$



$$\text{Area of sector OAXB} = \frac{\pi r^2 \theta}{360^\circ} = \frac{22}{7} \times \frac{10 \times 10 \times 90^\circ}{360^\circ} = \frac{550}{7}\ \text{cm}^2$$

$$\text{Area of } \Delta\text{OAB} = \frac{1}{2} r^2 = \frac{1}{2} \times 10 \times 10 = 50\ \text{cm}^2$$

Now, area of minor segment AXB = area of sector OAXB – area of ΔOAB

$$= \frac{550}{7} - 50 = \frac{550 - 350}{7} = \frac{200}{7}\ \text{cm}^2$$

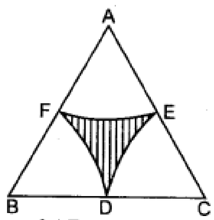
Area of major segment AYB = area of circle – area of minor segment AXB

$$= \frac{2200}{7} - \frac{200}{7} = \frac{2000}{7}\ \text{cm}^2$$

Long Answer Type Questions [4 Marks]

Question 53.

In figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle ABC of side 14 cm as centres to intersect the sides BC, CA and AB at their respective mid-point D, E and E Find the area of the shaded region



Solution:

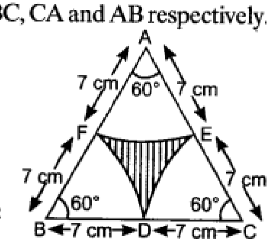
ABC is an equilateral Δ of side 14 cm. D, E, F are the mid-points of BC, CA and AB respectively.

Now, shaded area = ar(ΔABC) – areas of 3 sectors

$$= \frac{\sqrt{3}}{4}(\text{side})^2 - 3 \times \frac{\pi r^2 \theta}{360^\circ}$$

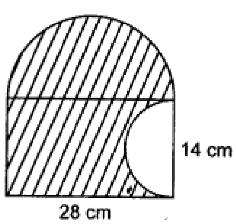
$$= \frac{\sqrt{3}}{4} \times 14 \times 14 - 3 \times \frac{22}{7} \times 7 \times 7 \times \frac{60}{360}$$

$$= \frac{1.73 \times 14 \times 14}{4} - 77 = 84.77 - 77 = 7.77\ \text{cm}^2$$



Question 54.

The length and breadth of a rectangular piece of paper are 28 cm and 14 cm respectively. A semi-circular portion is cut off from the breadth's side and a semicircular portion is added on length's side, as shown in figure. Find the area of shaded region



Solution:

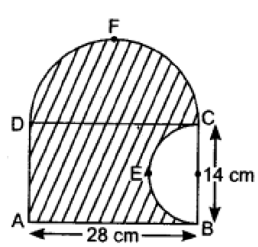
Area of shaded region

= ar (ABCD) – ar (BEC) + ar (DFC)

$$= 28 \times 14 - \frac{\pi \times 7 \times 7}{2} + \frac{\pi \times 14 \times 14}{2}$$

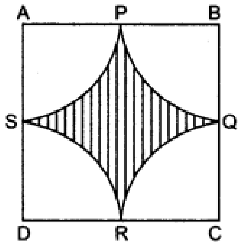
$$= 28 \times 14 - \frac{22}{7} \times \frac{7 \times 7}{2} + \frac{22}{7} \times \frac{14 \times 14}{2}$$

$$= 392 - 77 + 308 = 623\ \text{cm}^2$$



Question 55.

Find the area of shaded region in figure,where arcs drawn with centres A,B,C and D intersects at mid point P,Q,R and S of sides AB,BC,CD and DA of a square ABCD,where the length of each side of square is 14 cm



Solution:

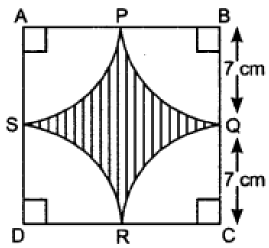
ABCD is a square of side 14 cm. P, Q, R, S are the mid-points of the sides AB, BC, CD and AD respectively.

Shaded area = area of square ABCD – area of 4 sectors

$$= 14^2 - 4 \times \frac{\pi \times r^2}{4}$$

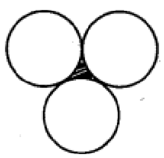
$$= 14^2 - 4 \times \frac{22}{7} \times \frac{7 \times 7}{4}$$

$$= 196 - 154 = 42 \text{ cm}^2$$



Question 56.

In the given figure, three circles each of radius 3.5 cm are drawn in such a way that each of them touches the other two. Find the area of shaded region enclosed between these three circles



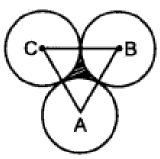
Solution:

ΔABC is an equilateral triangle each of whose side is of length = 3.5 + 3.5 = 7 cm

$\therefore \quad \angle A = \angle B = \angle C = 60°$

$$ar(\Delta ABC) = \frac{\sqrt{3}}{4}(\text{side})^2$$

$$= \frac{\sqrt{3}}{4}(7)^2 = \frac{\sqrt{3}}{4} \times 49 \text{ cm}^2$$



$$\text{Area of 3 sectors} = 3 \times \frac{60°}{360°} \times \pi r^2$$

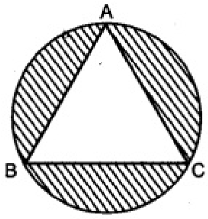
$$= 3 \times \frac{60°}{360°} \times \frac{22}{7} \times 3.5 \times 3.5 = \frac{77}{4} \text{cm}^2$$

Area of shaded region = area of ΔABC – area of 3 sectors

$$= \frac{\sqrt{3}}{4} \times 49 - \frac{77}{4} = \frac{1}{4}(49\sqrt{3} - 77) \text{ cm}^2$$

Question 57.

In given figure, an equilateral triangle has been inscribed in a circle of radius 6 cm. Find the area of the shaded region.

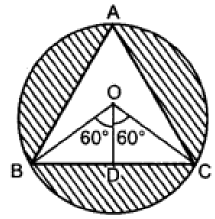


Solution:

ΔABC is equilateral,

$\therefore \quad \angle BOC = 120°$ (Angle subtended by chord at centre is double the angle subtended by the same chord at the circle)

Construction: Draw $OD \perp BC$.



So, $\angle BOD = 60°$

In ΔOBD, $\cos 60° = \frac{OD}{OB}$ and $\sin 60° = \frac{BD}{OB}$

$\Rightarrow \quad \frac{1}{2} = \frac{OD}{6}$ and $\frac{\sqrt{3}}{2} = \frac{BD}{6}$

$\Rightarrow \quad \frac{6}{2} = OD$ and $\frac{6\sqrt{3}}{2} = BD$

$\Rightarrow \quad OD = 3$ and $BD = 3\sqrt{3}$

$BC = 2BD = 2 \times 3\sqrt{3} = 6\sqrt{3}$

Area of the shaded region = area of circle – area of ΔABC

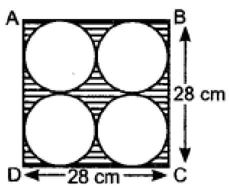
$= \pi(6)^2 - \frac{\sqrt{3}}{4}(6\sqrt{3})^2 = 3.14 \times 6 \times 6 - \frac{\sqrt{3}}{4} \times 6\sqrt{3} \times 6\sqrt{3}$

$= 113.04 - 27\sqrt{3}$ cm²

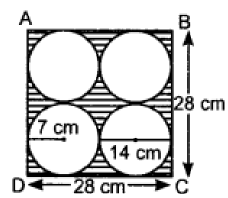
$= 113.04 - 27(1.73) = 113.04 - 46.71 = 66.33$ cm²

Question 58.

Find the area of the shaded region in given figure, where ABCD is a square of side 28 cm



Solution:



Shaded area = Area of square – Area of 4 circles

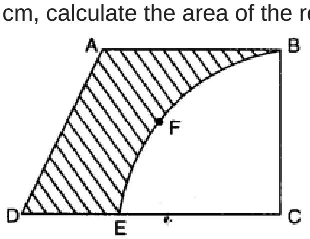
$= 28^2 - 4 \times \pi r^2$

$= (28 \times 28) - 4 \times \frac{22}{7} \times 7^2$

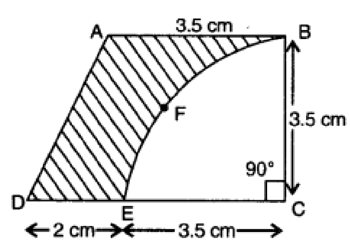
$= 784 - 616 = 168$ cm²

Question 59.

From a thin metallic piece, in the shape of a trapezium ABCD in which AB ¦¦ CD and ∠BCD = 90°, a quarter circle BFEC is removed (See figure). Given AB = BC = 3.5 cm and DE = 2 cm, calculate the area of the remaining (shaded) part of the metal sheet.



Solution:



$AB = 3.5$ cm

$DC = 2 + 3.5 = 5.5$ cm

Area of shaded region = area of trapezium ABCD – area of quarter circle

$= \frac{1}{2}$ [sum of || lines] $\times h - \frac{1}{4}\pi r^2$

$= \frac{1}{2}[3.5 + 5.5] \times 3.5 - \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5$

$= \frac{1}{2} \times 9 \times 3.5 - \frac{11 \times 0.5 \times 3.5}{2}$

$= \frac{315}{20} - \frac{1925}{200} = \frac{3150 - 1925}{200} = \frac{1225}{200}$

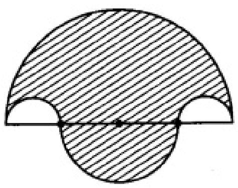
$= \frac{245}{40} = \frac{49}{8} = 6.125 \text{ cm}^2$

2010

Short Answer Type Questions II [3 Marks]

Question 60.

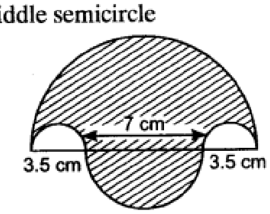
In figure, the boundary of shaded region consists of four semicircular arcs, two smallest being equal. If diameter of the largest is 14 cm and that of the smallest is 3.5 cm, calculate the area of the shaded region.



Solution:

Area of shaded region

= area of big semicircle – area of 2 small semicircles + area of middle semicircle



$= \frac{1}{2}\pi \times (7)^2 - 2 \times \frac{1}{2} \times \pi \times \left(\frac{3.5}{2}\right)^2 + \frac{1}{2}\pi \times \left(\frac{7}{2}\right)^2$

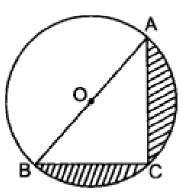
$= \frac{49}{2}\pi - \left(\frac{7}{4}\right)^2\pi + \frac{1}{2}\left(\frac{49}{4}\right)\pi = \left(\frac{49}{2} - \frac{49}{16} + \frac{49}{8}\right)\pi$

$= \left(\frac{1}{2} - \frac{1}{16} + \frac{1}{8}\right) \times 49 \times \frac{22}{7} = \left(\frac{8 - 1 + 2}{16}\right) \times 7 \times 22$

$= \frac{9}{16} \times 7 \times 22 = 86.625 \text{ cm}^2.$

Question 61.

Find the area of the shaded region in figure, if AC = 24 cm, BC = 10 cm and O is the centre of the circle

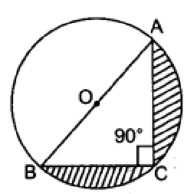


Solution:

Here, AB is diameter, AC = 24 cm, BC = 10 cm

and $\angle ACB = 90°$ [Angle in a semicircle is 90°]

$\therefore$ $AB^2 = AC^2 + BC^2$ [By Pythagoras theorem]



$\Rightarrow$ $AB = \sqrt{(24)^2 + (10)^2}$ cm

$= \sqrt{576 + 100}$ cm

$= \sqrt{676}$ cm $= 26$ cm

$\Rightarrow$ $OB = OA = \frac{AB}{2} = 13$ cm

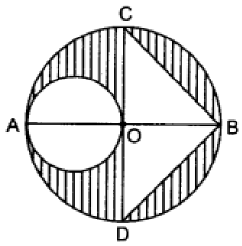
$\therefore$ Area of shaded region = Area of semicircle – Area of ΔACB

$= \left[\frac{1}{2}\pi(13)^2 - \frac{1}{2} \times 24 \times 10\right] \text{cm}^2$

$$= \left[\frac{1}{2} \times 3.14 \times 169 - 120\right] cm^2$$
$$= [265.33 - 120]\ cm^2$$
$$= 145.33\ cm^2$$

Question 62.

In figure, AB and CD are two perpendicular diameters of a circle with centre O. If OA = 7 cm, find the area of the shaded region



Solution:

$\angle CBD = 90°$ (angle in a semicircle is 90°)

Radius of big circle = OA = OB = OC = OD = 7 cm

$\therefore$ Diameter of big circle = AB = CD = 14 cm

Area of big circle $= \pi(7)^2 = 49\pi$

Area of circle with diameter AO $= \pi\left(\frac{7}{2}\right)^2 = \frac{49}{4}\pi$

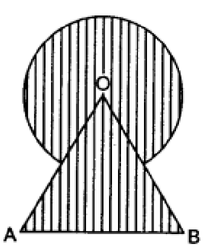
Area of $\Delta BCD = \frac{1}{2} \times CD \times OB = \frac{1}{2} \times 14 \times 7 = 49$

Area of shaded region = Area of big circle – Area of circle with diameter AO – Area of ΔBCD

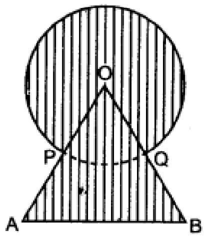
$$= 49\pi - \frac{49}{4}\pi - 49 = 49\left[\pi - \frac{\pi}{4} - 1\right] = 49\left[\frac{22}{7} - \frac{22}{28} - 1\right]$$
$$= \frac{49}{28}[88 - 22 - 28] = \frac{49 \times 38}{28} = 66.5\ cm^2$$

Question 63.

Find the area of the shaded region in figure, where a circular arc of radius 7 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm, as centre



Solution:



Radius of circular arc = 7 cm

Side of equilateral triangle = 12 cm

∴ Area of shaded portion

= Area of circle + area of equilateral Δ – area of sector OPQ

$$= \left[\left\{\pi(7)^2 - \frac{60}{360} \times \pi(7)^2\right\} + \frac{\sqrt{3}}{4}(12)^2\right]$$

$$= \left[\frac{5}{6} \times \frac{22}{7} \times 49 + \frac{\sqrt{3}}{4} \times 144\right]$$

$= (128.33 + 62.35)\ cm^2$

$= 190.68\ cm^2$