# CHAPTER 12

# AREAS RELATED TO CIRCLES

## ONE MARK QUESTIONS

### Multiple Choice Questions

**1.** The area of a circular ring formed by two concentric circles whose radii are 5.7 cm and 4.3 cm respectively is (Take $\pi = 3.1416$)

(a) 44 sq. cm. (b) 66 sq. cm.

(c) 22 sq. cm. (d) 33 sq. cm.

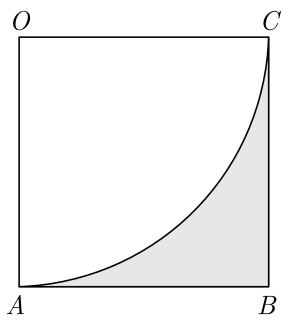
**Ans :**

Let the radii of the outer and inner circles be $r_1$ and $r_2$ respectively, we have

$$\text{Area} = \pi r_1^2 - \pi r_2^2 = \pi(r_1^2 - r_2^2)$$
$$= \pi(r_1 - r_2)(r_1 + r_2)$$
$$= \frac{22}{7} \times (5.7 - 4.3)(5.7 + 4.3)$$
$$= \frac{22}{7} \times 1.4 \times 10 \text{ sq. cm}$$
$$= 44 \text{ sq cm}$$

Thus (a) is correct option.

**2.** In the adjoining figure, $OABC$ is a square of side 7 cm. $OAC$ is a quadrant of a circle with $O$ as centre. The area of the shaded region is




(a) $10.5 \text{ cm}^2$ (b) $38.5 \text{ cm}^2$

(c) $49 \text{ cm}^2$ (d) $11.5 \text{ cm}^2$

**Ans :**

$$\text{Required area} = \left(r^2 - \frac{1}{4} \times \frac{22}{7} \times r^2\right) \text{cm}^2$$
$$= \left(7^2 - \frac{1}{4} \times \frac{22}{7} \times 7^2\right) \text{cm}^2$$
$$= (49 - 38.5)\text{cm}^2$$

Thus (a) is correct option.

**3.** A sector is cut from a circular sheet of radius 100 cm, the angle of the sector being 240°. If another circle of the area same as the sector is formed, then radius of the new circle is

(a) 79.5 cm (b) 81.5 cm

(c) 83.4 cm (d) 88.5 cm

**Ans :**

$$\text{Area of sector} = \frac{240^\circ}{360^\circ} \times \pi(100)^2 = 20933 \text{ cm}^2$$

Let $r$ be the radius of the new circle, then

$$20933 = \pi r^2$$
$$r = \sqrt{\frac{20933}{\pi}} = 81.6 \text{ cm}$$

Thus (b) is correct option.



**4.** If a circular grass lawn of 35 m in radius has a path 7 m wide running around it on the outside, then the area of the path is

(a) $1450 \text{ m}^2$ (b) $1576 \text{ m}^2$

(c) $1694 \text{ m}^2$ (d) $3368 \text{ m}^2$

**Ans :**

Radius of outer concentric circle,

$$= (35 + 7) \text{ m} = 42 \text{ m}.$$

$$\text{Area of path} = \pi(42^2 - 35^2) \text{ m}^2$$
$$= \frac{22}{7}(42^2 - 35^2) \text{ m}^2$$

Thus (c) is correct option.

**5.** If the area of a semi-circular field is 15400 sq m, then perimeter of the field is

(a) $160\sqrt{2}$ m (b) $260\sqrt{2}$ m

(c) $360\sqrt{2}$ m (d) $460\sqrt{2}$ m

**Ans :**

Let the radius of the field be $r$.

Then, $$\frac{\pi r^2}{2} = 15400$$

$$\frac{1}{2} \times \frac{22}{7} \times r^2 = 15400$$

$$r^2 = 15400 \times 2 \times \frac{7}{22} = 9800$$

$$r = 70\sqrt{2}\,\text{m}$$

Thus, perimeter of the field

$$\pi r + 2r = \frac{22}{7} \times 70\sqrt{2} + 2 \times 70\sqrt{2}$$

$$= 220\sqrt{2} + 140\sqrt{2}$$

$$= \sqrt{2}(220 + 140)$$

$$= 360\sqrt{2}\,\text{m}$$

Thus (c) is correct option.

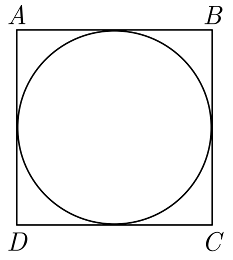
**6.** The area of the circle that can be inscribed in a square of side 6 cm is

(a) $36\pi$ cm$^2$ (b) $18\pi$ cm$^2$

(c) $12\pi$ cm$^2$ (d) $9\pi$ cm$^2$

**Ans :**

Given, side of square $= 6$ cm




Diameter of circle is equal to the side of square.

Diameter of a circle, $d = = 6$ cm

Radius of a circle, $r = \frac{d}{2} = \frac{6}{2} = 3$ cm

Area of circle, $\pi r^2 = \pi 3^2 = 9\pi$ cm$^2$

Thus (d) is correct option.

**7.** The sum of the areas of two circle, which touch each other externally, is $153\pi$. If the sum of their radii is 15, then the ratio of the larger to the smaller radius is

(a) $4:1$ (b) $2:1$

(c) $3:1$ (d) None of these

**Ans :**

Let the radii of the two circles be $r_1$ and $r_2$, then

$$r_1 + r_2 = 15 \quad ...(1)$$

and $$\pi r_1^2 + \pi r_2^2 = 153\pi \quad ...(2)$$

$$r_1^2 + r_2^2 = 153$$

$$r_1^2 + (15 - r_1)^2 = 153$$

$$r_1^2 + 225 - 30r_1 + r_1^2 = 153$$

$$2r_1^2 - 30r_1 + 72 = 0$$

$$r_1^2 - 15r_1 + 36 = 0$$

Solving, we get $r_1 = 12$ and $r_2 = 3$.

Thus required ratio is 12 : 3 or 4 : 1.

Thus (a) is correct option.

**8.** A race track is in the form of a ring whose inner and outer circumference are 437 m and 503 m respectively. The area of the track is

(a) 66 sq. cm. (b) 4935 sq. cm.

(c) 9870 sq. cm (d) None of these

**Ans :**

We have $$2\pi r_1 = 503 \Rightarrow r_1 = \frac{503}{2\pi}$$

and $$2\pi r_2 = 437 \Rightarrow r_2 = \frac{437}{2\pi}$$

Area of ring

$$\pi(r_1^2 - r_2^2) = \pi(r_1 + r_2)(r_1 - r_2)$$

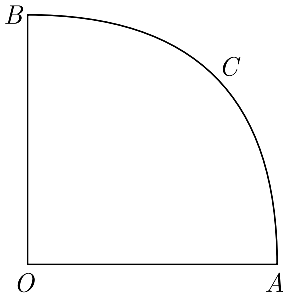
$$= \pi\left(\frac{503 + 437}{2\pi}\right)\left(\frac{503 - 437}{2\pi}\right)$$

$$= \frac{940}{2}\left(\frac{66}{2\pi}\right) = \frac{940}{2} \times \frac{66}{2} \times \frac{7}{22}$$

$$= 235 \times 21 = 4935\,\text{sq. cm.}$$

Thus (b) is correct option.

**9.** In the given figure, $OACB$ is a quadrant of a circle of radius 7 cm. The perimeter of the quadrant is




(a) 11 cm (b) 18 cm
(c) 25 cm (d) 36 cm

**Ans :**

$$\text{Perimeter} = \frac{1}{4} \times 2\pi r + 2r$$

$$= \left(\frac{1}{2} \times \frac{22}{7} \times 7 + 2 \times 7\right) \text{cm}$$

$$= 25 \text{ cm}$$

Thus (c) is correct option.

**10.** If the circumference of a circle increases from $4\pi$ to $8\pi$, then its area is

(a) halved (b) doubled
(c) tripled (d) quadrupled

**Ans :**

$$2\pi r = 4\pi \Rightarrow r = 2$$

$$\text{Area} = \pi(2)^2 = 4\pi$$

When, $2\pi r = 8\pi \Rightarrow r = 4$

$$\text{Area} = 16\pi$$

Thus area is quadrupled.

Thus (d) is correct option.

**11.** If the radius of a circle is diminished by 10%, then its area is diminished by

(a) 10% (b) 19%
(c) 36% (d) 20%

**Ans :**

Let $r$ be the radius of circle, then area $= \pi r^2$
When $r$ is diminished by 10%

Then, $$\text{area} = \pi\left(r - \frac{r}{10}\right)^2 = \pi r^2\left(\frac{81}{100}\right)$$

Thus area is diminished by

$$\left(1 - \frac{81}{100}\right)\% = 19\%$$

Thus (b) is correct option.

**12.** If the perimeter of a semi-circular protractor is 36 cm, then its diameter is

(a) 10 cm (b) 14 cm
(c) 12 cm (d) 16 cm

**Ans :**

$$\text{Perimeter} = \frac{2\pi r}{2} + 2r = \pi r + 2r$$

$$(\pi + 2)r = 36$$

$$\left(\frac{36}{7}\right) - r = 36 \Rightarrow r = 7 \text{ cm}$$

Hence, diameter $2r = 7 \times 2 = 14$ cm

Thus (b) is correct option.

**13.** The area of a circular path of uniform width $d$ surrounding a circular region of radius $r$ is

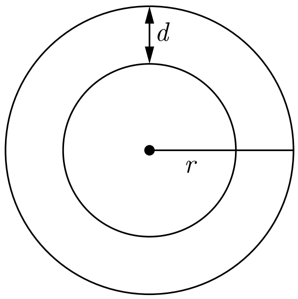
(a) $\pi d(2r + d)$ (b) $\pi(2r + d\ r$
(c) $\pi(d + r)r$ (d) $\pi(d + r)d$

**Ans :**

$$\text{Required area} = \pi\left[(r + d)^2 - r^2\right]$$

$$= \pi\left[r^2 + d^2 + 2rd - r^2\right]$$

$$= \pi\left[d^2 + 2rd\right] = \pi d\left[d + 2r\right]$$



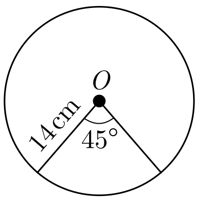

Thus (a) is correct option.

**14.** In a circle of radius 14 cm, an arc subtends an angle of 45° at the centre, then the area of the sector is

(a) $71 \text{ cm}^2$ (b) $76 \text{ cm}^2$
(c) $77 \text{ cm}^2$ (d) $154 \text{ cm}^2$

**Ans :**

Given, $r = 14$ cm and $\theta = 45°$




$$\text{Area of sector } = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{45^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14$$

$$= \frac{1}{8} \times 22 \times 2 \times 14 = 77 \text{ cm}^2$$

Thus (c) is correct option.

**15.** If the sum of the areas of two circles with radii $R_1$ and $R_2$ is equal to the area of a circle of radius $R$, then

(a) $R_1 + R_2 = R$ (b) $R_1^2 + R_2^2 = R^2$

(c) $R_1 + R_2 < R$ (d) $R_1^2 + R_2^2 < R^2$

**Ans :**

According to the given condition,

$$\text{Area of circle} = \text{Area of first circle} + \text{Area of second circle}$$

$$\pi R^2 = \pi R_1^2 + \pi R_2^2$$

$$R^2 = R_1^2 + R_2^2$$

Thus (b) is correct option.

**16.** If the sum of the circumferences of two circles with radii $R_1$ and $R_2$ is equal to the circumference of a circle of radius $R$, then

(a) $R_1 + R_2 = R$ (b) $R_1 + R_2 > R$

(c) $R_1 + R_2 > R$ (d) $R_1 + R_2 < R$

**Ans :**

According to the given condition,

$$2\pi R = 2\pi R_1 + 2\pi R_2$$

$$R = R_1 + R_2$$

Thus (a) is correct option.

**17.** If the circumference of a circle and the perimeter of a square are equal, then

(a) Area of the circle = Area of the square

(b) Area of the circle > Area of the square

(c) Area of the circle < Area of the square

(d) Nothing definite can be said about the relation between the areas of the circle and square

**Ans :**

Let $r$ and $a$ be the radius of circle and side of square respectively.

$$2\pi r = 4a$$

$$\frac{22}{7} r = 2a$$

$$11 r = 7a$$

$$r = \frac{7a}{11} \quad ...(1)$$

Now, area of circle, $A_1 = \pi r^2$

From equation (1), we get

$$A_1 = \pi\left(\frac{7a}{11}\right)^2 = \frac{22}{7} \times \frac{49a^2}{121}$$

$$A_1 = \frac{14a^2}{11} \quad ...(2)$$

and area of square, $A_2 = (a)^2 \quad ...(3)$

From equations (2) and (3),

$$A_1 = \frac{14}{11} A_2$$

$$A_1 > A_2$$

Hence, Area of the circle >Area of the square.

Thus (b) is correct option.

**18.** If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

(a) 22:7 (b) 14:11

(c) 7:22 (d) 11:14

**Ans :**

Let radius of circle be $r$ and side of a square be $a$.

According to the given condition,

$$\text{Perimeter of a circle} = \text{Perimeter of a square}$$

$$2\pi r = 4a$$

$$a = \frac{\pi r}{2} \quad ...(1)$$

Now, $\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{(a)^2} = \frac{\pi r^2}{(\pi \frac{r}{2})^2}$ [from Eq. (1)]

$$= \frac{\pi r^2}{\pi^2 \frac{r^2}{4}} = \frac{4}{\pi} = \frac{4}{\frac{22}{7}}$$

$$= \frac{28}{22} = \frac{14}{11}$$

Hence, the required ratio is 14:11.

Thus (b) is correct option.

**19.** It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be

(a) 10 m (b) 15 m

(c) 20 m (d) 24 m

**Ans :**

We have $\pi R^2 = \pi r_1^2 + \pi r_2^2$

$$R^2 = r_1^2 + r_2^2$$

$$= \left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = \left(\frac{16}{2}\right)^2 + \left(\frac{12}{2}\right)^2$$

$$= (8)^2 + (6)^2 = 100$$

$$R = \sqrt{100} = 10$$

Thus (a) is correct option.

**20.** The area of the square that can be inscribed in a circle of radius 8 cm is

(a) 256 $cm^2$ (b) 128 $cm^2$

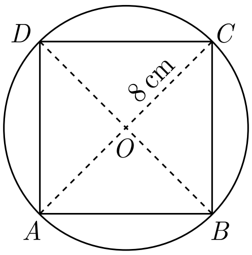
(c) $64\sqrt{2}$ $cm^2$ (d) 64 $cm^2$

**Ans :**

Radius of circle, $OC = 8$ cm,

Diameter of the circle, $AC = 2 \times OC$

$$= 2 \times 8 = 16 \text{ cm}$$




which is equal to the diagonal of a square.

Let side of square be $x$.

In right angled $\Delta ABC$,

$$AC^2 = AB^2 + BC^2$$

$$(16)^2 = x^2 + x^2$$

$$256 = 2x^2$$

$$x^2 = 128$$

Area of square, $x^2 = 128 \text{ cm}^2$

**Alternate Method :**

Radius of circle, $r = 8$ cm

Diameter of circle, $d = 2r = 2 \times 8 = 16$ cm

Since, square inscribed in circle.

Diagonal of square = Diameter of circle

$$\text{Now, Area of square} = \frac{(\text{Diagonal})^2}{2} = \frac{(16)^2}{2} = \frac{256}{2}$$

$$= 128 \text{ cm}^2$$

Thus (b) is correct option.

**21.** The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is

(a) 56 cm (b) 42 cm

(c) 28 cm (d) 16 cm

**Ans :**

We have $2\pi r = 2\pi r_1 + 2\pi r_2$

$$2\pi r = \pi d_1 + \pi d_2$$

$$2r = d_1 + d_2 = 36 + 20$$

$$2r = 56 \Rightarrow r = 28 \text{ cm}$$

Thus (c) is correct option.

**22.** The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is

(a) 31 cm (b) 25 cm

(c) 62 cm (d) 50 cm

**Ans :**

We have $\pi R^2 = \pi r_1^2 + \pi r_2^2$

$$R^2 = r_1^2 + r_2^2$$

$$= 24^2 + 7^2 = 625$$

$$R = \sqrt{625} = 25 \text{ cm}$$

Diameter of a circle

$$2R = 2 \times 25 = 50 \text{ cm}$$

Thus (d) is correct option.

**23.** **Assertion :** In a circle of radius 6 cm, the angle of a sector 60°. Then the area of the sector is $18\frac{6}{7}\,\text{cm}^2$.

**Reason :** Area of the circle with radius $r$ is $\pi r^2$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

**Ans :**

$$\text{Area of the sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{132}{7} = 18\frac{6}{7}\text{cm}^2.$$

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

**24.** **Assertion :** If the circumference of a circle is 176 cm, then its radius is 28 cm.

**Reason :** Circumference $= 2\pi \times$ radius

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

**Ans :**

We have $C = 2 \times \frac{22}{7} \times r = 176$

$$r = \frac{176 \times 7}{2 \times 22} = 28\,\text{cm}$$

Both assertion and reason are correct. Also Reason is the correct explanation of the assertion.

Thus (a) is correct option.

**25.** **Assertion :** If the outer and inner diameter of a circular path is 10 m and 6 m then area of the path is $16\pi\, m^2$.

**Reason :** If $R$ and $r$ be the radius of outer and inner circular path, then area of path is $\pi(R^2 - r^2)$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

**Ans :**

$$\text{Area of the path} = \pi\left[\left(\frac{10}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right.$$

$$= \pi(25 - 9) = 16\pi$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

**26.** **Assertion :** If a wire of length 22 cm is bent in the shape of a circle, then area of the circle so formed is $40\,\text{cm}^2$.

**Reason :** Circumference of the circle = length of the wire.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

**Ans :**

We have $2\pi r = 22$

$$r = 3.5\,\text{cm}$$

$$\text{Area of the circle} = \frac{22}{7} \times 3.5 \times 3.5$$

$= 38.5\,\text{cm}^2$

Assertion is not correct, but reason is true.

Thus (d) is correct option.

## Fill in the Blank Questions

**27.** The boundary of a sector consists of an arc of the circle and the two .........

**Ans :**

radii

**28.** Angle formed by two radii at the centre is known as ..........

**Ans :**

central angle

**29.** Concentric circles are circles having same...........

**Ans :**

centre

**30.** The area of a circle is the measurement of the region enclosed by its ..........

**Ans :**

boundary

**31.** Segment is the region enclosed between chord and ..........

**Ans :**

arc

**32.** Pie ($\pi$) is the ratio between circumference and .......... of the circle.

**Ans :**

diameter

**33.** The region enclosed by an arc and a chord is called the ......... of the circle.

**Ans :**

segment

**34.** Perimeter of a semi circle ..........

**Ans :**

$(\pi r + d)$ units

**35.** Circumference of a circle is ..........

**Ans :**

$2\pi r$

**36.** Area of a circle is ..........

**Ans :**

$\pi r^2$

**37.** Measure of angle in a semi circle is ..........

**Ans :**

$90^\circ$

**38.** Length of an arc of a sector of a circle with radius $r$ and angle with degree measure $\theta$ is ..........

**Ans :**

$\frac{\theta}{360} \times 2\pi r$

**39.** A sector of a circle is called a ......... sector if the minor arc of the circle is a part of its boundary.

**Ans :**

minor

## Very Short Answer Questions

**40.** The radii of two circles are 19 cm and 9 cm respectively. Find the radius of a circle of a circle which has circumference equal to sum of their circumferences.

**Ans :** [Board 2020 Delhi Basic]

Radius of 1st circle $r_1 = 9$ cm

Radius of 2nd circle $r_2 = 19$ cm

Let $r$ the radius of required circle. According to question, circumference of required circle is sum of circumference of two circles.

$$2\pi r = 2\pi r_1 + 2\pi r_2$$

$$2\pi r = 2\pi(r_1 + r_2)$$

$$r = r_1 + r_2 = 9 + 19 = 28 \text{ cm.}$$

Hence, radius of required circle is 28 cm

**41.** The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes.

**Ans :** [Board 2020 Delhi Standard]

Angle subtended in 1 minutes $= 6^\circ$

Angle subtended in 35 minutes $= 6^\circ \times 35 = 210^\circ$

Area of the face of the clock by the minute hand, i.e. area of sector,

$$\frac{\pi r^2\theta}{360^\circ} = \frac{22}{7} \times \frac{12 \times 12 \times 210^\circ}{360^\circ}$$

$$= \frac{22}{7} \times \frac{12 \times 12 \times 7 \times 30^\circ}{12 \times 30^\circ}$$

$$= 22 \times 12 = 264 \text{ cm}^2$$

**42.** The radius of a circle is 17.5 cm. find the area of the sector of the circle enclosed by two radii and an arc 44 cm in length.

**Ans :** [Board 2020 OD Basic]

Given, arc length $= 44$ cm

Radius of circle, $r = 17.5$ cm

So, area of sector $= \frac{\text{arc length}}{2\pi r} \times \pi r^2$

$$= \frac{\text{arc length} \times r}{2} = \frac{44 \times 17.5}{2}$$

$$= 22 \times 17.5 = 385 \text{ sq. cm.}$$

**43.** Find the area of the sector of a circle of radius 6 cm whose central angle is $30°$. (Take $\pi = 3.14$)

**Ans :** [Board 2020 OD Standard]

Radius, $r = 6$ cm

Central angle, $\theta = 30°$

Area of the sector,

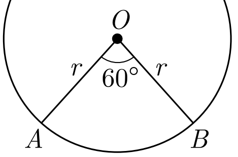
$$\frac{\pi r^2 \theta}{360°} = \frac{3.14 \times 6 \times 6 \times 30°}{360°}$$

$$= 9.42 \text{ cm}^2$$

**44.** What is the perimeter of the sector with radius 10.5 cm and sector angle $60^o$.

**Ans :** [Board Term-2 2012]

As per question the digram is shown below.




Perimeter of the sector,

$$p = 2r + \frac{2\pi r\theta}{360^o}$$

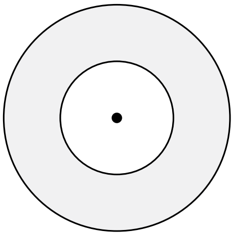
$$= 10.5 \times 2 + 2 \times \frac{22}{7} \times \frac{10.5 \times 60}{360}$$

$$= 21 + 11 = 32 \text{ cm}$$

**45.** If the circumferences of two concentric circles forming a ring are 88 cm and 66 cm respectively. Find the width of the ring.

**Ans :** [Board Term-2 Delhi 2013]

As per question statement figure is shown below.



Circumference of the outer circle, $2\pi r_1 = 88$ cm

$$r_1 = \frac{88 \times 7}{2 \times 22} = 14 \text{ cm}$$

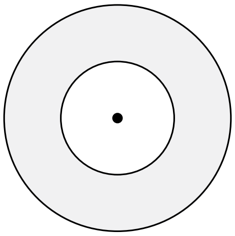
Circumference of the outer circle, $2\pi r_2 = 66$ cm

$$r_2 = \frac{66 \times 7}{2 \times 22} = \frac{21}{2} \text{ cm} = 10.5 \text{ cm}$$

Width of the ring,

$$r_1 - r_2 = 14 - 10.5 \text{ cm} = 3.5 \text{ cm}$$

**46.** Two coins of diameter 2 cm and 4 cm respectively are kept one over the other as shown in the figure, find the area of the shaded ring shaped region in square cm.



**Ans :** [Board Term-2 2012]

$$\text{Area of circle} = \pi r^2$$

$$\text{Area of the shaded region} = \pi(2)^2 - \pi(1)^2$$

$$= 4\pi - \pi = 3\pi \text{ sq cm}$$

**47.** The diameter of two circle with centre $A$ and $B$ are 16 cm and 30 cm respectively. If area of another circle with centre $C$ is equal to the sum of areas of these two circles, then find the circumference of the circle with centre $C$.

**Ans :** [Board Term-2 2012]

Let the radius of circle with centre $C$ be $r$.
According to question we have,

$$\pi(8)^2 + \pi(15)^2 = \pi r^2$$

$$64\pi + 225\pi = \pi r^2$$

$$289\pi = \pi r^2$$

$$r^2 = 289 \text{ or } R = 17 \text{ cm}$$

Circumference of circle

$$2\pi r = 2\pi \times 17$$

$$= 34\pi \text{ cm}$$

**48.** The diameter of a wheel is 1.26 m. What the distance covered in 500 revolutions.

**Ans :** [Board Term-2 2012]

Distance covered in 1 revolution is equal to circumference of wheel and that is $\pi d$.

Distance covered in 500 revolutions

$$= 500 \times \pi \times 1.26$$

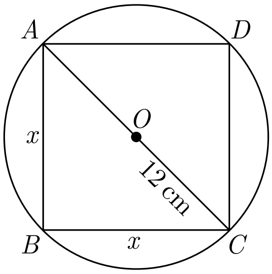
$$= 500 \times \frac{22}{7} \times 1.26$$

$$= 1980 \text{ m.} = 1.98 \text{ km}$$

**49.** What is the area of the largest square that can be inscribed in a circle of radius 12 cm.?

**Ans :** [Board Term-2 2012]

As per question the digram is shown below.




$$\text{Radius of the circle} = 12 \text{ cm}$$

$$\text{Diameter of circle} = 24 \text{ cm}$$

$$\text{Diagonal of square} = 24 \text{ cm}$$

Let the side of square be $x$.
From Pythagoras theorem we have

$$x^2 + x^2 = (24)^2$$

$$2x^2 = 24 \times 24$$

$$x^2 = \frac{24 \times 24}{2} = 288$$

Thus area of square,

$$x^2 = 288 \text{ cm}^2$$

**50.** What is the name of a line which intersects a circle at two distinct points?

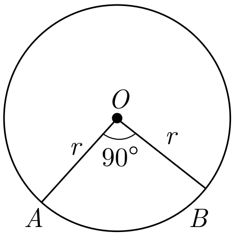
**Ans :** [Board Term-2 2012]

A line intersecting the circle at two distinct points is called a secant.

**51.** What is the perimeter of a sector of a circle whose central angle is $90^\circ$ and radius is 7 cm?

**Ans :** [Board Term-2 2012]

As per question the digram is shown below.



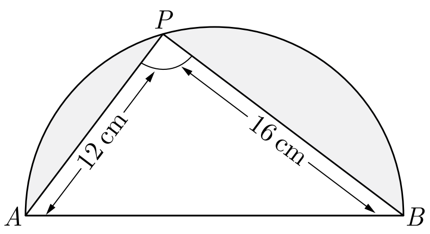

Perimeter of the sector,

$$p = 2r + \frac{2\pi r\theta}{360^\circ}$$

$$= 2 \times 7 + 2 \times \frac{22}{7} \times 7 \times \frac{90}{360}$$

$$14 + 11 = 25 \text{ cm}$$

**52.** In the given figure, $AB$ is the diameter where $AP = 12$ cm and $PB = 16$ cm. Taking the value of $\pi$ as 3, find the perimeter of the shaded region.




**Ans :** [Board Term-2 2012]

From Pythagoras theorem we have

$$AB = \sqrt{(16)^2 + (12)^2}$$

$$= \sqrt{256 + 144}$$

$$= \sqrt{400} = 20 \text{ cm}$$

$$\text{Radius of circle} = 10 \text{ cm.}$$

Perimeter of shaded region

$$\pi r + AP + PB = 3 \times 10 + 12 + 16$$
$$= 30 + 12 + 16 = 58 \text{ cm}$$

**53.** Find the area of circle that can be inscribed in a square of side 10 cm.

**Ans :** [Board Term-2 2012]

$$\text{Radius of the circle} = \frac{10}{2} = 5 \text{ cm}$$

Area of the circle,

$$\pi r^2 = \pi \times (5)^2 = 25\pi\, cm^2$$

**54.** A thin wire is in the shape of a circle of radius 77 cm. It is bent into a square. Find the side of the square (Taking, $\pi = \frac{22}{7}$).

**Ans :** [Board Term-2 2012]

Let side of square be $x$.

Perimeter of the circle = Perimeter of square

$$2\pi r = 4x$$
$$2 \times \frac{22}{7} \times 77 = 4x$$
$$x = \frac{2 \times 22 \times 11}{4} = 121$$

Thus side of the square is 121 cm.

**55.** What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40 cm and 9 cm?

**Ans :** [Board Term-2 2012]

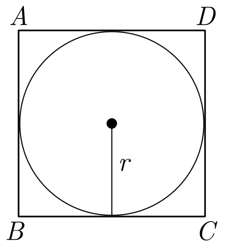
Area of the circle = sum of areas of two circles

$$\pi R^2 = \pi \times (40)^2 + \quad (9)$$
$$R^2 = 1600 + 81$$
$$R = \sqrt{1681} = 41 \text{ cm}$$

Thus diameter of given circle = $41 \times 2 = 82$ cm

**56.** Find the area (in $cm^2$) of the circle that can be inscribed in a square of side 8 cm.

**Ans :** [board Term-2, 2012 Set (28, 32, 33)]




Side of square = diameter of circle = 8 cm

Radius of circle, $r = \frac{8}{2} = 4$ cm

Area of circle, $\pi r^2 = \pi \times 4 \times 4 = 16\pi \text{ cm}^2$

**57.** If the radius of a circle is doubled, what about its area?

**Ans :** [Board Term-2 2012]

Let the radius of the circle be $r$, then area will be $\pi r^2$

Now if radius is doubled,

Area $= \pi(2r)^2 = 4\pi r^2 = 4 \times \pi r^2$

The area will be 4 times the area of the first circle.

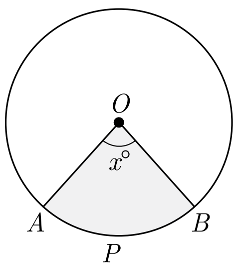
**58.** If the perimeter and the area of the circle are numerically equal, then find the radius of the circle.

**Ans :** [Board Term-2, 2012 Set(13)]

Perimeter of the circle = area of the circle.

$$2\pi r = \pi r^2$$
$$r = 2 \text{ units}$$

**59.** In given fig., $O$ is the centre of a circle. If the area of the sector $OAPB$ is $\frac{5}{36}$ times the area of the circle, then find the value of $x$.




**Ans :** [Board Term-2 2012]

Area of the sector,

$$A_{\frown} = \frac{\pi r^2 \theta}{360^\circ}$$

Area of sector $OAPB$ is $\frac{5}{36}$ times the area of circle.

Thus $\pi r^2 \times \frac{x}{360} = \frac{5}{36}\pi r^2$

$$\frac{x}{360} = \frac{5}{36}$$

$$x = 50^\circ$$

**60.** If circumference of a circle is 44 cm, then what will be the area of the circle?

**Ans :** [Board Term-2 2012]

Circumference of a circle = 44 cm

Radius of the circle $= \frac{22}{2 \times \frac{22}{7}} = 7$ cm

Area of the circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\, cm^2$

**61.** A steel wire when bent in the form of a square encloses an area or 121 cm$^2$. If the same wire is bent in the form of a circle, then find the circumference of the circle.

**Ans :** [Board Term-2 2012]

Area of square = (side)$^2$ = 121 cm$^2$

Side of square $= \sqrt{121} = 11$ cm

Parameter of square$= 4 \times 11 = 44$ cm

Circumference of the circle = Perimeter of the square

= 44 cm

**62.** Find the radius of a circle whose circumference is equal to the sum of the circumference of two circles of diameter 36 cm and 20 cm

**Ans :** [Board Term-2 2012]

Circumference of the circle,

$$2\pi r = 2\pi \times 18 + 2 \quad 10$$

$$r = 18 + 10 = 28 \text{ cm}$$

Hence radius of given circle is 28 cm.

**63.** Find the diameter of a circle whose area is equal to the sum of areas of two circles of diameter 16 cm and 12 cm.

**Ans :** [Board Term-2 2012]

Let $r$ be the radius of the circle. Since area of the circle is equal to the sum of areas of two circles,

$$\pi r^2 = \pi \times (8)^2 + \pi(6)^2$$

$$\pi r^2 = \pi(64 + 36)$$

$$r^2 = 100 \text{ or, } r = 10 \text{ cm}$$

Diameter of the circle = 2 × 10 = 20 cm.

**64.** If the circumference of a circle increases from $4\pi$ to $8\pi$, then what about its area ?

**Ans :** [Board Term-2 Delhi 2013]

Circumference of the circle

$$2\pi r = 4\pi \text{ cm or } r = 2 \text{ cm.}$$

Increased circumference

$$2\pi R = 8\pi \text{ cm or } R = 4 \text{ cm.}$$

Area of the 1$^{st}$ circle

$$\pi r^2 = \pi \times (2)^2 = 4\pi \text{ cm}$$

Area of the new circle

$$\pi R^2 = \pi(4)^2 = 16\pi = 4 \times 4\pi$$

Area of the new circle= 4 times the area of first circle.

**65.** If the radius of the circle is 6 cm and the length of an arc 12 cm. Find the area of the sector.

**Ans :** [Board Term-2 2014]

Area of the sector$= \frac{1}{2} \times$(length of the corresponding arc)$\times$ radius

$$= \frac{1}{2} \times l \times r = \frac{1}{2} \times 12 \times 6$$

$$= 36 \text{ cm}^2$$

**66.** A chord of a circle of radius 10 cm subtends a right angle at the centre. Find area of minor segment. ($\pi = 3.14$)

**Ans :** [Board Term-2 2012]

Radius of circle $r = 10$ cm, central angle $= 90^\circ$

Area of minor segment,

$$= \frac{1}{2} \times 10^2 \times \left[\frac{3.14 \times 90}{180} - \sin 90^\circ\right.$$

$$= \frac{1}{2} \times 100 \times [1.57 - 1] = 28.5 \text{ cm}^2$$

**67.** If the perimeter of a semi-circular protractor is 36 cm, find its diameter. (Use $\pi = \frac{22}{7}$).

**Ans :** [Board Term-2 2012]

Perimeter $\pi r + 2r = (\pi + 2)r = 36$

or, $\left(\frac{22}{7} + 2\right)r = 36$ or, $r = 7$

Diameter $= 14$ cm.

## TWO MARKS QUESTIONS

**68.** The areas of two circles are in the ratio 9 : 4, then what is the ratio of their circumferences?

**Ans :** [Board 2020 Delhi Basic]

Given,

$$\frac{\text{Area of } 1^{\text{st}} \text{circle}}{\text{Area of } 2^{\text{nd}} \text{circle}} = \frac{9}{4}$$

i.e.,
$$\frac{\pi r_1^2}{\pi r_2^2} = \frac{9}{4}$$

$$\frac{r_1^2}{r_2^2} = \frac{9}{4}$$

$$\frac{r_1}{r_2} = \frac{3}{2}$$

Ratio of their circumference

$$\frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{3}{2}$$

Hence, the ratio of their circumference is 3 : 2.

**69.** The length of the minute hand of clock is 14 cm. Find the area swept by the minute hand in 15 minutes.

**Ans :** [Board 2020 OD Basic]

Minute hand completes full circle degree in 1 hour. So, degree swept by minute hand in 1 hour (60 minutes) is $360^\circ$.

Degree swept by minute hand in 1 minute is $\frac{360^\circ}{60} = 6^\circ$ and degree swept by minute hand in 15 minutes,

$$\theta = 6^\circ \times 15 = 90^\circ$$

Hence, $\theta = 90^\circ$

and $r = 14$ cm

Area swept by minute hand

$$\frac{\theta}{360^\circ} \times \pi r^2 = \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (14)^2$$

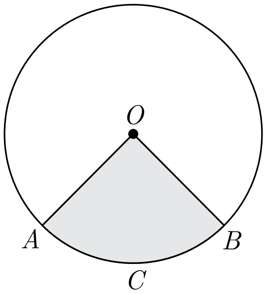
$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = 154 \text{ cm}^2$$

**70.** The perimeter of a sector of a circle with radius 6.5 cm is 31 cm, then find the area of the sector.

**Ans :** [Board 2020 Delhi Basic]

Given, Radius $= 6.5$ cm

Let $O$ be the centre of a circle with radius 6.5 cm and $OACBO$ be its sector with perimeter 31 cm.




Thus, we have

$$OA + OB + \widehat{ACB} = 31 \text{ cm}$$

$$6.5 + 6.5 + \widehat{ACB} = 31 \text{ cm}$$

$$\widehat{ACB} = 18 \text{ cm}$$

Now, area of sector $OACBO$

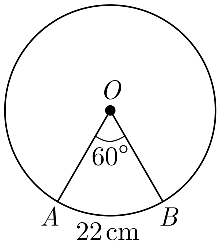
$$= \frac{1}{2} \times \text{radius} \times \widehat{ACB}$$

$$= \frac{1}{2} \times 6.5 \times 18 = 58.5 \text{ cm}^2$$

**71.** A piece of wire 22 cm long is bent into the form an arc of a circle subtending an angle of $60^\circ$ at its centre. Find the radius of the circle. [Use $\pi = \frac{22}{7}$]

**Ans :** [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.




Here $AB$ is an arc of a circle of radius $r$.

$$\text{Length of arc} = \frac{2\pi r\theta}{360^\circ}$$

$$22 = \frac{2 \times 22 \times r \times 60^\circ}{7 \times 360^\circ}$$

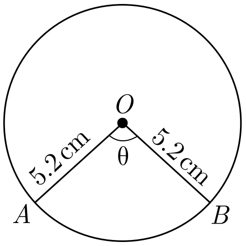
$$22 = \frac{22 \times r}{21} \Rightarrow r = 21$$

Hence, the radius of the circle is 21 cm.

**72.** The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

**Ans :** [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.




Perimeter of the sector

$$p = 2r + \frac{2\pi r\theta}{360^\circ}$$

$$16.4 = 2 \times 5.2 + \frac{2\pi \times 5.2 \times \theta}{360^\circ}$$

$$16.4 = 10.4 + \frac{2\pi \times 5.2 \times \theta}{360^\circ}$$

$$6 = \frac{2\pi \times 5.2 \times}{360^\circ}$$

$$\frac{3}{5.2} = \frac{\theta \times \pi}{360^\circ}$$

Now, area of sector $= \frac{\theta}{360^\circ} \times \pi r^2 = \left(\frac{\theta \times \pi}{360^\circ}\right) r^2$

$$= \frac{3}{5.2} \times (5.2)^2 = 15.6 \text{ sq. units.}$$

**73.** The area of a circular play ground is 22176 cm$^2$. Find the cost of fencing this ground at the rate of 50 per metre.

**Ans :** [Board 2020 OD Standard]

Area of a circular play ground,

$$A = 22176 \text{ cm}^2$$

i.e., $$\pi r^2 = 22176 \text{ cm}^2$$

$$r^2 = 22176 \times \frac{7}{22}$$

$$= 7056$$

$$r = 84 \text{ cm} = 0.84 \text{ m}$$

Perimeter of ground,

$$p = 2\pi r$$

Cost of fencing this ground,

$$= ₹\, 50 \times 2\pi r$$

$$= ₹\, 50 \times 2 \times \frac{22}{7} \times 0.84 = ₹\, 264$$

**74.** The wheel of a motorcycle is of radius 35 cm. How many revolutions are required to travel a distance of 11 m?

**Ans :** [Board 2020 OD Basic]

Given, radius of wheel, $r = 35$ cm

Circumference of the wheel,

$$2\pi r = 2 \times \frac{22}{7} \times 35 = 220 \text{ cm}$$

Number of revolutions required to cover 11 m or 1100 cm,

$$= \frac{1100}{220} = 5 \text{ revolutions}$$

**75.** The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand from 9 a.m. to 9.35 a.m.

**Ans :** [Board Term-2 2012]

Angle subtended by minute hand in 60 minute = 360 ∘

Angle subtended in 1 minute $= \frac{360^\circ}{60} = 6^\circ$

Angle subtended in 35 minutes,

$$\theta = 35 \times 6^\circ = 210^\circ$$

Area swept by the minute hand

= Area of a sector

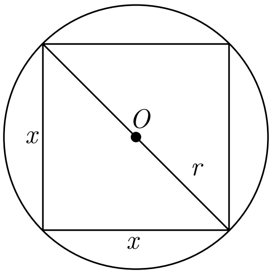
$$= \pi r^2 \frac{\theta}{360^\circ} = \frac{22}{7} \times 14 \times 14 \times \frac{210^\circ}{360^\circ}$$

$$= \frac{1078}{3} = 259.33 \text{ cm}^2$$

**76.** Find the area of the square that can be inscribed in a circle of radius 8 cm.

**Ans :** [Board Term-2 2015]

As per question the digram is shown below.




Let the side of square be $x$ and radius of circle be $r$.

Radius of the circle, $r = 8$ cm

Diameter of circle, $2r = 16$ cm

Diagonal of square $2r = 16$ cm

From Pythagoras theorem we have

$$x^2 + x^2 = (2r)^2$$

$$x^2 + x^2 = (16)^2$$

$$2x^2 = 16 \times 16$$

$$x^2 = \frac{16 \times 16}{2} = 128$$

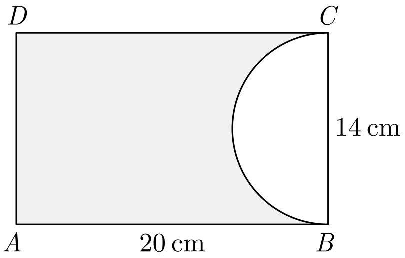
Thus area of square,

$$x^2 = 128\,\text{cm}^2$$

**77.** A paper is in the form of a rectangle $ABCD$ in which $AB = 20$ cm, $BC = 14$ cm. A semi-circular portion with $BC$ as diameter is cut off. Find the area of the part. Use $\pi = \frac{22}{7}$.

**Ans :** [Board Term-2 2012, Foreign 2014 ]

As per question the digram is shown below.




Area of remaining part,

$$= \text{Area of rectangle} - \text{Area of semi-circle}$$

$$= 20 \times 14 - \frac{1}{2}\pi 7^2$$

$$= 280 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

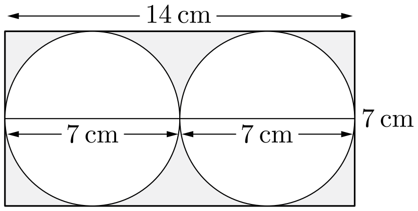
$$= 280 - 77 = 203 \text{ cm}$$

Hence, area of remaining part is 203 cm.

**78.** Two circular pieces of equal radii and maximum areas, touching each other are cut out from a rectangular cardboard of dimensions 14 cm $\times$ 7 cm. find the area of the remaining cardboard. (Use $\pi = \frac{22}{7}$)

**Ans :** [Board Term-2 Delhi 2013]

As per question the digram is shown below.




Area of the remaining cardboard

= Area of rectangular cardboard $- 2 \times$ Area of circle

$$= 14 \times 7 - 2\pi\left(\frac{7}{2}\right)^2$$

$$= 14 \times 7 - 2 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$$

$$= 98 - \frac{44}{7} \times \frac{49}{4} = 98 - 77 = 21$$

Hence, area of remaining card board is 21 cm$^2$

**79.** If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = \frac{22}{7}$, find the circumference (in cm) of the circle.

**Ans :** [Board Term-2 Delhi 2012]

Let $r$ be the radius of the circle.

Now, circumference $-$ radius $= 37$

$$2\pi r - r = 37$$

$$2 \times \frac{22}{7} r - r = 37$$

$$r\left(\frac{22-7}{7}\right) = 37$$

$$r \times \frac{37}{7} = 37$$

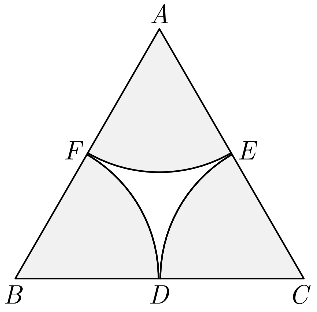
$$r = \frac{37 \times 7}{37} = 7 \text{ cm}$$

Circumference of the circle,

$$2\pi r = 2 \times \frac{22}{7} \times 7$$

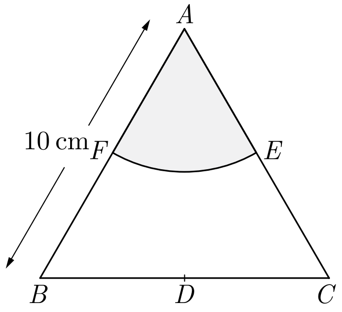
$$= 44 \text{ cm.}$$

**80.** In fig. arcs are drawn by taking vertices $A, B$ and $C$ of an equilateral triangle of side 10 cm, to intersect the side $BC$, $CA$ and $AB$ at their respective mid-points $D, E$ and $F$. Find the area of the shaded region. (Use $\pi = 3.14$).




**Ans :** [Board Term-2 2011]

Figure given below shows the single sector.




Since $\Delta ABC$ is an equilateral triangle

$$\angle A = \angle B = \angle C = 60^\circ$$

Here we have 3 sector and area of all three sector is equal.

Area of sector $AFEA$,

$$\text{Area}_{AFEA} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{60^\circ}{360^\circ} \times \pi (5)^2 = \frac{25}{6}\pi \text{ cm}^2$$

Thus total area of shaded region

$$\text{Area} = 3\left(\frac{25}{6}\pi\right) = \frac{25 \times 3.14}{2}$$

$$= 39.25 \text{ cm}^2$$

**81.** If the perimeter of a protractor is 72 cm, calculate its area. Use $\pi = \frac{22}{7}$.

**Ans :** [Board Term-2, 2012 Set (22)]

Perimeter of semi-circle

$$\pi r + 2r = 72 \text{ cm}$$

$$(\pi + 2)r = 72 \text{ cm}$$

$$\left(\frac{22}{7} + 2\right)r = 72 \text{ cm}$$

$$r\left(\frac{22+14}{7}\right) = 72 \text{ cm}$$

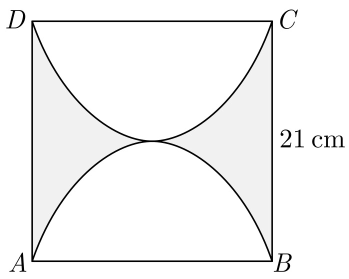
$$\frac{36}{7}r = 72 \Rightarrow r = 14 \text{ cm}$$

Area of protractor,

$$\frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$$

$$= 308 \text{ cm}^2$$

**82.** Find the perimeter of the shaded region if $ABCD$ is a square of side 21 cm and $APB$ and $CPD$ are semicircle. Use $\pi = \frac{22}{7}$.




**Ans :** [Board Term-2 SQP 2016]

It may be seen easily that perimeter of the shaded region include $AD$, $BC$ and two semi circle arc.

Thus perimeter of the shaded region, $= AD + BC +$

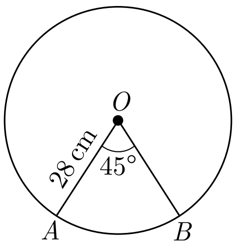
$+$ lengths of the arcs of semi circles $APB$ and $CPD$

$$= 21 + 21 + 2\left(\frac{22}{7} \times \frac{21}{2}\right) = 42 + 66 = 108 \text{ cm.}$$

**83.** Find the area of the corresponding major sector of a circle of radius 28 cm and the central angle $45^\circ$.

**Ans :** [Board Term-2 2015]

As per question statement figure is shown below;




Area of major sector,

$$= \text{area of circle} - \text{area of minor sector}$$

$$= \pi r^2\left(1 - \frac{\theta}{360^\circ}\right)$$

$$= \frac{22}{7} \times 28 \times 28\left(1 - \frac{45^\circ}{360^\circ}\right)$$

$$= \frac{22}{7} \times 28 \times 28 \times \frac{7}{8}$$

$$= 2156 \text{ cm}^2$$

**84.** The diameters of the front and rear wheels of a tractor are 80 cm and 200 cm respectively. Find the number of revolutions of rear wheel to cover the distance which the front wheel covers in 800 revolutions.

**Ans :** [Board Term-2 Delhi 2013]

Circumference of front wheel

$$\pi d = \frac{22}{7} \times 80 = \frac{1760}{7} \text{ cm}$$

Distance covered by front wheel in 800 revolutions

$$= \frac{1760}{7} \times 800$$

Circumference of rear wheel

$$= \frac{22}{7} \times 200 \quad \frac{4400}{7} \text{ cm}$$

Revolutions made by rear wheel

$$= \frac{\frac{1760}{7} \times 800}{\frac{4400}{7}} = \frac{1760 \times 800}{4400} = 320 \text{ revolutions}$$

## THREE MARKS QUESTIONS

**85.** A road which is 7 m wide surrounds a circular park whose circumference is 88 m. Find the area of the road.

**Ans :** [Board 2020 Delhi Basic]

Let $w = 7$ m be the width of road.

Circumference of a circular park,

$$2\pi r = 88 \text{ m}$$

Inner radius of park,

$$r = \frac{88}{2\pi} = \frac{88 \times 7}{2 \times 22}$$

$$= 2 \times 7 = 14 \text{ m}$$

Outer radius of park including road width,

$$R = r + w$$

$$= 14 + 7 = 21 \text{ m}$$

Area of the road,

$$\pi(R^2 - r^2) = \pi(R + r)(R - r)$$

$$= \frac{22}{7}(21 + 14)(21 - 14)$$

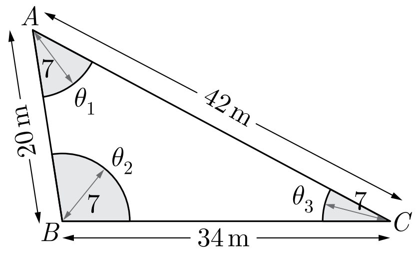
$$= \frac{22}{7} \times 35 \times 7 = 770 \text{ m}^2$$

Hence, the area of the road is $770 \text{ m}^2$.

**86.** Three horses are tied each with 7 m long rope at three corners of a triangular field having sides 20 m, 34 m and 42 m. Find the area of the plot which can be grazed by the horses.

**Ans :** [Board 2020 Delhi Basic]

As per information given in question we have drawn the figure below.




Let $\angle A = \theta_1$, $\angle B = \theta_2$ and $\angle C = \theta_3$.

Now, area which can be grazed by the horses is the sum of the areas of three sectors with central angles $\theta_1$, $\theta_2$ and $\theta_3$ each with radius $r = 7$ m.

$$\frac{\pi r^2 \theta_1}{360^\circ} + \frac{\pi r^2 \theta_2}{360^\circ} + \frac{\pi r^2 \theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ}(\theta_1 + \theta_2 + \theta_3) \quad ...(1)$$

From angle sum property of a triangle we have

$$\theta_1 + \theta_2 + \theta_3 = 180^\circ$$

Substituting above in equation (1) we have

$$\frac{\pi r^2\theta_1}{360^\circ}+\frac{\pi r^2\theta_2}{360^\circ}+\frac{\pi r^2\theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ}\times 180^\circ = \frac{\pi r^2}{2}$$

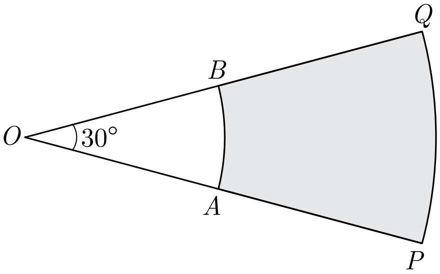
$$= \frac{22}{7}\times\frac{1}{2}\times(7)^2$$

$$= \frac{22}{7}\times\frac{1}{2}\times 7\times 7$$

$$= 77\text{ m}^2$$

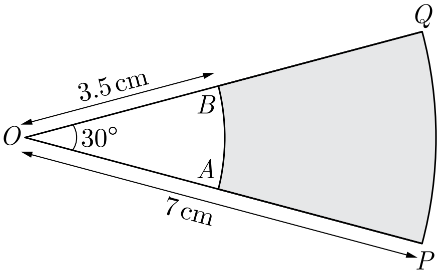
Hence, the area grazed by the horses is $77\text{ m}^2$

**87.** In Figure, $PQ$ and $AB$ are two arcs of concentric circles of radii 7 cm and 3.5 cm respectively, with centre $O$. If $\angle POQ = 30^\circ$, then find the area of shaded region.




**Ans :** [Board 2020 OD Basic]

We redraw the given figure as below.




Area of shaded region

$$\pi[R^2-r^2]\frac{\theta}{360^\circ} = \frac{22}{7}[7^2-(3.5)^2]\frac{30^\circ}{360^\circ}$$

$$= \frac{22}{7}(7+3.5)(7-3.5)\times\frac{1}{12}$$

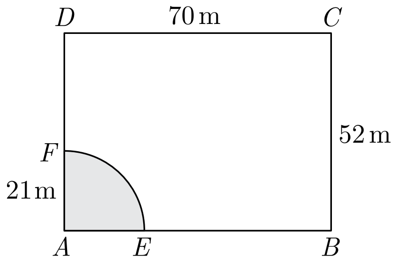
$$= \frac{22}{7}\times 10.5\times 3.5\times\frac{1}{12}$$

$$= 9.625\text{ cm}^2$$

**88.** A horse is tethered to one corner of a rectangular field of dimensions $70\text{ m}\times 52\text{ m}$, by a rope of length 21 m. How much area of the field can it graze?

**Ans :** [Board 2020 OD Basic]

As per information given in question we have drawn the figure below.




Length of the rope is 21 cm.

Shaded portion $AEFA$ indicates the area in which the horse can graze. Clearly it is the area of a quadrant of a circle of radius, $r = 21$ m.

Area of quadrant,

$$\frac{1}{4}\pi r^2 = \frac{1}{4}\times\frac{22}{7}\times(21)^2$$

$$= \frac{1}{4}\times\frac{22}{7}\times 21\times 21$$

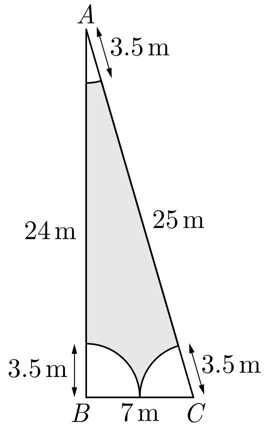
$$= 346.5\text{ m}^2$$

Hence, the graze area is $346.5\text{ m}^2$

**89.** Sides of a right triangular field are 25 m, 24 m and 7 m. At the three corners of the field, a cow, a buffalo and a horse are tied separately with ropes of 3.5 m each to graze in the field. Find the area of the field that cannot be grazed by these animals.

**Ans :** [Board 2020 SQP Standard]

As per information given in question we have drawn the figure below.




Let $\angle A = \theta_1$, $\angle B = \theta_2$ and $\angle C = \theta_3$.

Now, area which can be grazed by the animals is the sum of the areas of three sectors with central angles $\theta_1$, $\theta_2$ and $\theta_3$ each with radius $r = 3.5$ m.

$$\frac{\pi r^2\theta_1}{360^\circ}+\frac{\pi r^2\theta_2}{360^\circ}+\frac{\pi r^2\theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ}(\theta_1+\theta_2+\theta_3) \qquad ...(1)$$

From angle sum property of a triangle we have

$$\theta_1+\theta_2+\theta_3 = 180^\circ$$

Substituting above in equation (1) we have

$$\frac{\pi r^2\theta_1}{360^\circ}+\frac{\pi r^2\theta_2}{360^\circ}+\frac{\pi r^2\theta_3}{360^\circ} = \frac{\pi r^2}{360^\circ}\times 180^\circ = \frac{\pi r^2}{2}$$

$$= \frac{22}{7}\times\frac{1}{2}\times(3.5)^2$$

$$= 19.25$$

Hence, the area grazed by the horses is $19.25\,\text{m}^2$.

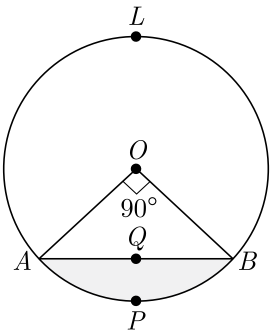
$$\text{Area of } \Delta\,ABC = \frac{1}{2}\times AB\times BC$$

$$= \frac{1}{2}\times 24\times 7 = 84\,\text{m}^2$$

Area of the field that cannot be grazed by these animals = Area of triangle − Area of three sectors

$$= 84 - 1925 = 64.75\,\text{m}^2$$

**90.** In the given figure, a chord $AB$ of the circle with centre $O$ and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment $AQBP$. Hence find the area of major segment $ALBQA$. (Use $\pi = 3.14$)




**Ans :** [Board Term-2 Foreign 2016]

Area of sector $OAPB$,

$$= \frac{90}{360}\pi(10)^2 = 25\pi\text{ cm}^2$$

Area of $\Delta AOB$,

$$= \frac{1}{2}\times 10\times 10 = 50\text{ cm}^2$$

Area of minor segment $AQBP$,

$$= (25\pi - 50)\text{ cm}^2$$

$$= 25\times 3.14 - 50$$

$$= 78.5 - 50$$

$$= 28.5\text{ cm}^2$$

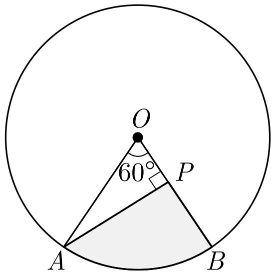
$$\text{Also area of circle} = \pi(10)^2$$

$$= 3.14\times 100 = 314\text{ cm}^2$$

Area of major segment $ALBQA = 314 - 28.5$

$$= 285.5\text{ cm}^2$$

**91.** In the given figure, $AOB$ is a sector of angle $60^\circ$ of a circle with centre $O$ and radius 17 cm. If $AP\perp OB$ and $AP = 15$ cm, find the area of the shaded region.




**Ans :** [Board Term-2 2016]

Here $OA = 17$ cm $AP = 15$ cm and $\Delta OPA$ is right triangle

Using Pythagoras theorem, we have

$$OP = \sqrt{17^2 - 15^2} = 8\text{ cm}$$

Area of the shaded region

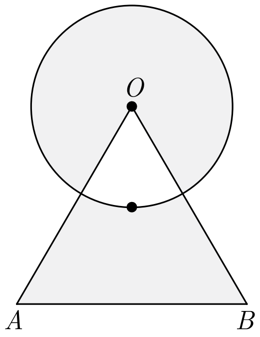
= Area of the sector $\Delta\,OAB$ − Area of $\Delta OPA$

$$= \frac{60}{360}\times\pi r^2 - \frac{1}{2}\times b\times h$$

$$= \frac{60^\circ}{360^\circ}\times\frac{22}{7}\times 17\times 17 - \frac{1}{2}\times 8\times 15$$

$$= 151.38 - 60 = 91.38\text{ cm}^2$$

**92.** Find the area of shaded region shown in the given figure where a circular arc of radius 6 cm has been drawn with vertex $O$ of an equilateral triangle $OAB$ of side 12 cm as centre.



**Ans :** [Board Term-2 Foreign SQP 2016]

Since $OAB$ is an an equilateral triangle,we have

$$\angle AOB = 60^\circ$$

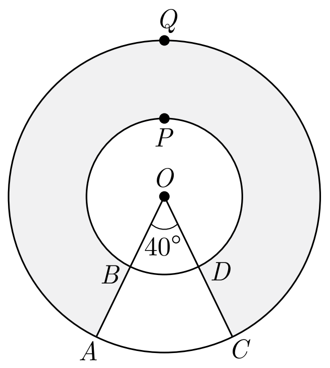
Area of shaded region = Area of major sector + (Area of $\Delta AOB-$ Area of minor sector)

$$= \frac{300^\circ}{360^\circ} \times \frac{22}{7} \times (6)^2 + \left(\frac{\sqrt{3}}{4}(12)^2 - \frac{60}{360} \times \frac{22}{7} \times 6^2\right)$$

$$= \frac{660}{7} + 36\sqrt{3} - \frac{132}{7}$$

$$= 36\sqrt{3} + \frac{528}{7} \text{ cm}^2$$

**93.** In the given figure, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40^\circ$. Use $\pi = \frac{22}{7}$.




**Ans :** [Board Term-2 OD 2016]

Radii of two concentric circle is 7 cm and 14 cm.
Angle $\angle AOC = 40^\circ$,
Angle $\angle AOC = 360^\circ - 40^\circ = 320^\circ$
Area of shaded region,

$$\frac{\theta}{360^\circ}\pi[R^2 - r^2] = \frac{320^\circ}{360^\circ} \times \frac{22}{7}[14^2 - 7^2]$$

$$= \frac{8}{9} \times 22 \times (14 \times 2 - 7)$$

$$= \frac{8}{9} \times 22 \times 21 = \frac{8}{3} \times 22 \times 7$$

$$= \frac{8 \times 154}{3} \text{ cm}^2$$

Required area $= \frac{1232}{3}$ cm$^2$

$= 410.67$ cm$^2$

**94.** Find the area of minor segment of a circle of radius 14 cm, when its centre angle is $60^\circ$. Also find the area of corresponding major segment. Use $\pi = \frac{22}{7}$.

**Ans :** [Board Term-2 OD 2015]

Here, $r = 14$ cm, $\theta = 60^\circ$

Area of minor segment,

$$\pi r^2 \frac{\theta}{360^\circ} - \frac{1}{2}r^2 \sin\theta = \pi(14)^2 \frac{60^\circ}{360^\circ} - \frac{1}{2} \times (14)^2 \times \frac{\sqrt{3}}{2}$$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{60^\circ}{360^\circ} - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2}$$

$$= \left(\frac{308}{3} - 49\sqrt{3}\right) = 17.9 \text{ cm}^2 \text{ approx.}$$

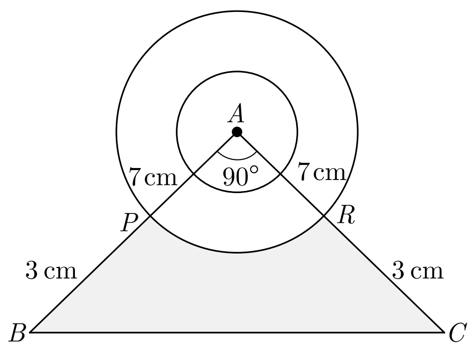
Area of major segment $= \pi r^2 - \left(\frac{308}{3} - 49\sqrt{3}\right)$

$$= \frac{22}{7} \times 14 \times 14 - \frac{308}{3} + 49\sqrt{3}$$

$$= \frac{1540}{3} + 49\sqrt{3} = 598.10$$

$$= 598 \text{ cm}^2 \text{ approx.}$$

**95.** A momento is made as shown in the figure. Its base $PBCR$ is silver plate from the front side. Find the area which is silver plated. Use $\pi = \frac{22}{7}$.




**Ans :** [Board Term-2 2015]

From the given figure area of right-angled $\Delta ABC$,

$$\frac{1}{2}AC \times AB = \frac{1}{2} \times 10 \times 10 = 50$$

Area of quadrant $APR$ is the $\frac{1}{4}$ of the circle of radii 7 cm.

Thus area of quadrant $APR$ of the circle of radii 7 cm

$$\frac{1}{4}\pi(7)^2 = \frac{1}{4} \times \frac{22}{7} \times 49 = 38.5 \text{ cm}^2$$

Area of base $PBCR$

$$= \text{Area of } \Delta ABC - \text{ Area of quadrant } APR$$

$$= 50 - 38.5 = 11.5 \text{ cm}^2$$

**96.** The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle. Use $\pi = \frac{22}{7}$.

**Ans :** [Board Term-2 2015]

Let radius of the circle be $r$.

Now as per question statement we have

$$\text{Circumference} = \text{Diameter} + 16.8 \text{ cm}$$

$$2\pi r = 2r + 16.8 \text{ cm}$$

$$2\left(\frac{22}{7}\right)r = 2r + 16.8$$

$$\frac{44}{7}r = 2r + 16.8$$

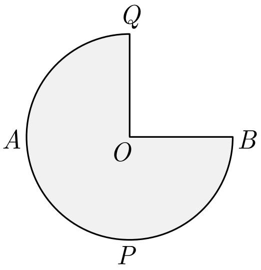
$$44r = 14r + 16.8 \times 7$$

$$30r = 177.6$$

$$r = \frac{117.6}{30} = 3.92$$

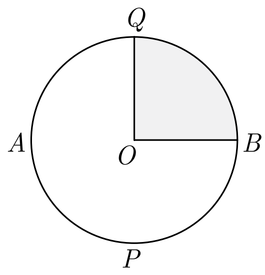
Thus $r = 3.92$ cm

**97.** In fig. $APB$ and $AQP$ are semi-circle, and $AO = OB$. If the perimeter of the figure is 47 cm, find the area of the shaded region. Use $\pi = \frac{22}{7}$.




**Ans :** [Board Term-2 Delhi 2015]

We have redrawn the given figure as shown below;




Let $r$ be the radius of given circle. It is given that perimeter of given figure is 47 cm.

$$2\pi r - \frac{1}{4}(2\pi r) + 2r = 47$$

$$\frac{3\pi r}{2} + 2r = 47$$

$$r\left(\frac{3}{2} \times \frac{22}{7} + 2\right) = 47$$

$$r\left(\frac{33}{7} + 2\right) = 47$$

$$r = \frac{47 \times 7}{47} = 7 \text{ cm}$$

Now, area of shaded region

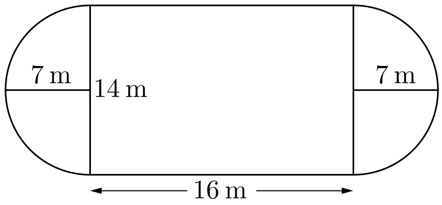
$$A = \text{area of circle } - \frac{1}{4} \text{ area of circle}$$

$$= \frac{3}{4} \text{ area of circle}$$

$$= \frac{3}{4}\pi r^2 = \frac{3}{4} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{3}{2} \times 77 = 115.5 \text{ cm}^2$$

**98.** Find the area of the adjoining diagram.




**Ans :** [Board Term-2, 2014]

The given figure is combination of one rectangle and two semicircle of same radii .

Required area,

$=$ area of two semi-circles $+$ area of rectangle

$=$ area of one circle $+$ area of rectangle

$= \pi r^2 + (l \times b)$

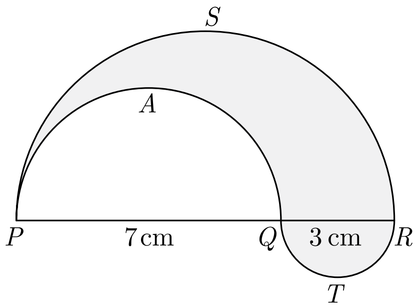
(where $r$ is radius of circle and $l$ and $b$ are length and breadth of rectangle)

$= \frac{22}{7} \times 7^2 + (16 \times 14)$

$= \frac{22}{7} \times 7 \times 7 + (16 \times 14)$

$= 154 + 224 = 378 \text{ m}^2$

**99.** In the fig., $PSR, RTQ$ and $PAQ$ are three semi-circles of diameters 10 cm, 3 cm and 7 cm region respectively. Find the perimeter of shaded region. Use $\pi = \frac{22}{7}$.




**Ans :** [Board Term-2 Delhi 2014]

Perimeter of shaded region

$= $ Perimeter of semi-circles $PSR + RTQ + PAQ$

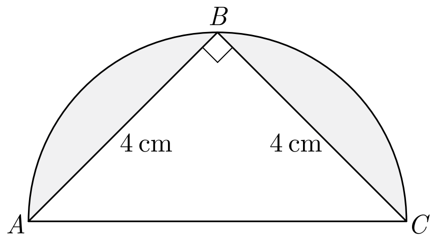
$= \pi(5) + \pi(1.5) + \pi(3.5)$

$= \pi(10)$

$= \frac{22}{7} \times 10 = \frac{220}{7} = 31.4 \text{ cm}$

Perimeter of shaded region is 31.4 cm approx.

**100.** In the figure, $\Delta ABC$ is in the semi-circle, find the area of the shaded region given that $AB = BC = 4$ cm.(Use $\pi = 3.14$)




**Ans :** [Board Term-2 Delhi 2014]

As $\Delta ABC$ is a triangle in semi-circle, $\angle B$ is right angle,

$$AC = \sqrt{4^2 + 4^2} = 4\sqrt{2} \text{ cm}$$

Radius of circle $\frac{4\sqrt{2}}{2} = 2\sqrt{2}$ cm

Area of shaded portion,

$= $ Area of the semi-circle$-$(Area of $\Delta ABC$)

$= \left\{\frac{1}{2}\pi \times (2\sqrt{2})^2\right\} - \left\{\frac{1}{2} \times 4 \times 4\right\}$

$= \left\{\frac{1}{2} \times 3.14 \times 8\right\} - 8$

$= 12.56 - 8 = 4.56 \text{ cm}^2$

**101.** In a circle of radius 21 cm, an arc subtends an angle of $60^\circ$ at the centre. Find the area of sector formed by the arc.

**Ans :** [Board Term-2 Delhi Compt. 2017]

We have $r = 21$ cm and $\theta = 60^\circ$

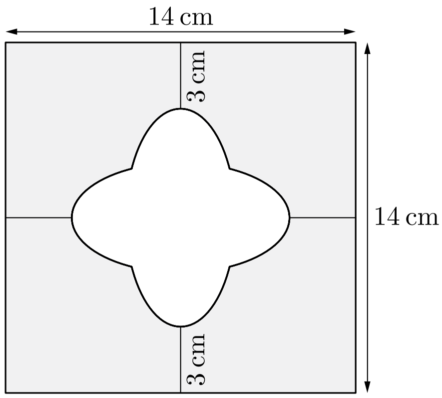
Area formed the sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$

$= \frac{1}{6} \times 22 \times 3 \times 21$

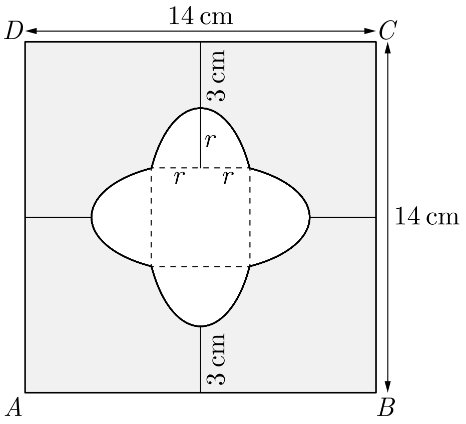
$= 11 \times 21 = 231 \text{ cm}^2$

**102.** In fig., find the area of the shaded region $(\pi = 3.14)$




**Ans :** [Board Term-2 2011, Delhi 2015]

We have redrawn the given figure as shown below.




$$3 + r + 2r + r + 3 = 14$$

$$4r + 6 = 14 \Rightarrow r = 2$$

Thus radius of the semi-circle formed inside is 2 cm and length of the side of square formed inside the semi-circle is 4 cm.

Area of square $ABCD$

$$= 14 \times 14 = 196 \text{ cm}^2$$

Thus area of 4 semi circle $= 4 \times \frac{1}{2}\pi r^2$

$$= 2 \times 3.14 \times 2 \times 2 = 25.12 \text{ cm}^2$$

Area of the square formed inside the semi-circle

$$(2r)^2 = 4 \times 4 = 16 \text{ cm}^2$$

Area of the shaded region,

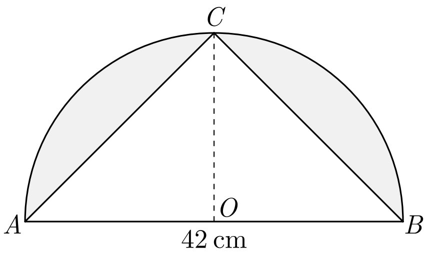
$$= \text{area of square } ABCD$$

$$- (\text{Area of 4 semi-circle} + \text{Area of square})$$

$$= 196 - (25.12 + 16)$$

$$= 196 - 41.12 = 154.88 \text{ cm}^2$$

**103.** In the figure, $\Delta ACB$ is in the semi-circle. Find the area of shaded region given that $AB = 42$ cm.




**Ans :** [Board Term-2 2014]

Here base of triangle is equal to the diameter of semicircle which is 42 cm.

$$\text{Base of triangle} = \text{diameter of semicircle}$$

$$= 42 \text{ cm}$$

$$\text{and its heigh} = \text{radius of semicircle}$$

$$= \frac{42}{2} = 21 \text{ cm}$$

Area of shaded portion,

$$= \text{Area of semicircle} - \text{area of } \Delta ABC$$

$$= \frac{1}{2}\pi r^2 - \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times \frac{22}{7} \times (21)^2 - \frac{1}{2} \times 42 \times 21$$

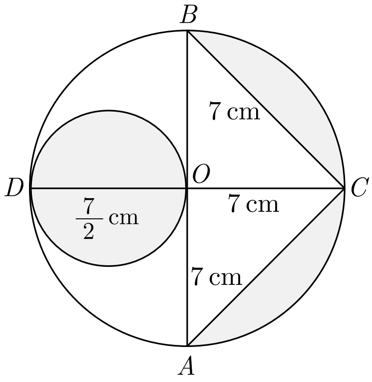
$$= \frac{1}{2} \times \frac{22}{7} \times 21 \times 21 - \frac{1}{2} \times 42 \times 21$$

$$= 11 \times 3 \times 21 - 21 \times 21$$

$$= 693 - 441 = 252$$

Hence, the area of shaded portion $= 252 \text{ cm}^2$

**104.** $AB$ and $CD$ are two diameters of a circle perpendicular to each other and $OD$ is the diameter of the smaller circle. If $OA = 7$ cm, find the area of the shaded region.




**Ans :** [Board Term-2, 2012]

Area of a circle with $DO$ as diameter

$$\pi r^2 = \pi\left(\frac{7}{2}\right)^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{2} \text{ sq.cm}$$

Area of semi-circle with $AB$ as diameter

$$\frac{\pi r_1^2}{2} = \frac{1}{2}\pi(7)^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ sq.cm}$$

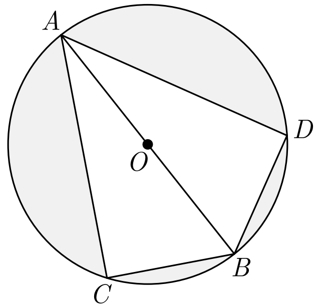
Area of $\Delta ABC$ $= \frac{1}{2} \times 14 \times 7 = 49$ sq.cm

Area of shaded region

$= \text{Area of circle} + \text{Area of semi-circle} - \text{Area of } \Delta ABC$

$= \frac{77}{2} + 77 - 49 = 66.5 \text{ cm}^2$

**105.** Find the area of the shaded region in figure, if $BC = BD = 8$ cm, $AC = AD = 15$ cm and $O$ is the centre of the circle. (Take $\pi = 3.14$)




**Ans :** [Board Term-2 2012]

Since $\angle ADB$ and $\angle ACB$ are angle in a semicircle,

$$\angle ADB = \angle ACB = 90^\circ$$

Since $\Delta ADB \cong \Delta ACB$

Thus $\text{ar}\Delta ADB = \text{ar}\Delta ACB$

$$= \frac{1}{2} \times 15 \times 8 = 60 \text{ cm}^2$$

and $\text{ar}\Delta ADB + \text{ar}\Delta ACB = 2 \times 60 = 120 \text{ cm}^2$

Now in $\Delta ABC$, we have

$$AB = \sqrt{AC^2 + BC^2}$$

$$= \sqrt{15^2 + 8^2} = \sqrt{225 + 64}$$

$$= 17 \text{ cm}$$

Area of circle $\pi r^2 = \frac{22}{7} \times \frac{17}{2} \times \frac{17}{2}$

$$= 226.87 \text{ cm}^2$$

Area of shaded portion,

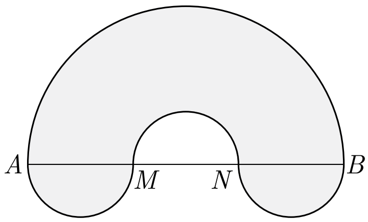
$= \text{area of circle} - \text{area of sum of } \Delta ACB \text{ and } \Delta ADB.$

$= 226.87 - 120 = 106.87 \text{ cm}^2$

Hence, area of shaded region

$$= 106.87 \text{ cm}^2$$

**106.** In the given figure, $AB$ is the diameter of the largest semi-circle. $AB = 21$ cm, $AM = MN = NB$. Semi-circle are drawn with $AM, MN$ and $NB$ as shown. Using $\pi = \frac{22}{7}$, calculate the area of the shaded region.




**Ans :** [Board Term-2 2012]

We have $AB = 21$ cm

Radius of semi-circle with diameter $AB$,

$$R = \frac{21}{2}$$

Here $AM = MN = NB = \frac{21}{3} = 7$ cm

Thus radii of smaller semi circle $r = \frac{7}{2}$ cm

Area of semi-circle with radius $R$

$$\frac{1}{2}\pi R^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{4} \text{ cm}^2$$

Area of semi-circle with diameter $AM$, $MN$ and $NB$ are equal

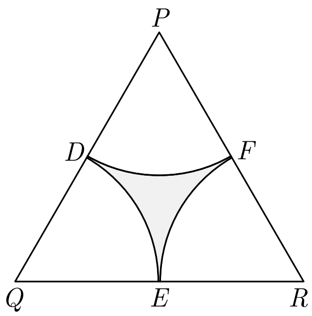
$$\frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{4} \text{ cm}^2$$

Area of shaded region

= Area largest semicircle + smallest semicircle

$$= \frac{693}{4} + \frac{77}{4} = \frac{770}{4} = 192.5 \text{ cm}^2$$

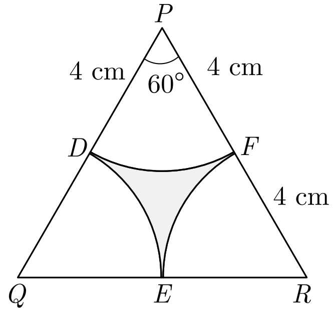
**107.** In the given figure, $\Delta PQR$ is an equilateral triangle of side 8 cm and $D, E, F$ are centres of circular arcs, each of radius 4 cm. Find the area of shaded region. (Use $\pi = 3.14$) and $\sqrt{3} = 1.732$




**Ans :** [Board Term-2, 2012]

Here angle $\angle P = \angle Q = \angle R = 60^\circ$ because triangle is equilateral. side of triangle is 8 cm.

Consider circular section $PDE$. Radius of circular arc is 4 cm.




Area of sector $PDF$,

$$\frac{\theta}{360^\circ} \times \pi r^2 = \frac{60^\circ}{360^\circ} \times 3.14 \times 4 \times 4$$

$$= \frac{1}{6} \times 3.14 \times 16 = 8.373$$

Area of shaded region

$$= \text{Area of } \Delta PQR - 3(\text{area of sector})$$

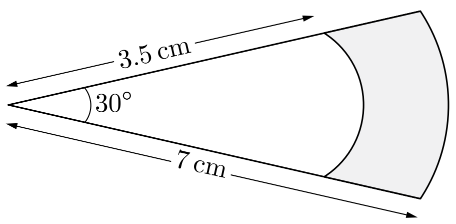
$$= \frac{\sqrt{3}}{4}(8)^2 - 3 \times 8.373$$

$$= 16\sqrt{3} - 3 \times 8.373$$

$$= 16 \times 1.732 - 25.12$$

$$= 27.712 - 25.12 = 2.59 \text{ cm}^2$$

**108.** In fig., sectors of two concentric circles of radii 7 cm and 3.5 cm are given. Find the area of shaded region. Use $\pi = \frac{22}{7}$.




**Ans :** [Board Term-2 2012]

Area of shaded region,

$$\frac{\theta}{360^\circ}\pi(R^2 - r^2) = \frac{30^\circ}{360}$$

$$= \frac{1}{12} \times \frac{22}{7}$$

$$= \frac{1}{12} \times \frac{22}{7} \times 10.5 \times 3.5$$

$$= \frac{1}{12} \times \frac{22}{7} \times \frac{21}{2} \times \frac{7}{2} = \frac{77}{8} = 9.62 \text{ cm}^2$$

**109.** A wire when bent in the form of an equilateral triangle encloses an area of $121\sqrt{3}$ cm². If the wire is bent in the form of a circle, find the area enclosed by the circle. Use $\pi = \frac{22}{7}$.

**Ans :** [Board Term-2 OD 2017]

Let $l$ be length of wire. If it is bent in the form of an equilateral triangle, side of triangle will be $\frac{l}{3}$.

Area enclosed by the triangle,

$$\frac{\sqrt{3}}{4} \times \left(\frac{l}{3}\right)^2 = 121\sqrt{3}$$

$$\frac{1}{4} \times \left(\frac{l}{3}\right)^2 = 121$$

$$\frac{1}{2} \times \frac{l}{3} = 11$$

$$l = 66 \text{ cm}$$

Same wire is bent in the form of circle. Thus circumference of circle will be 66.

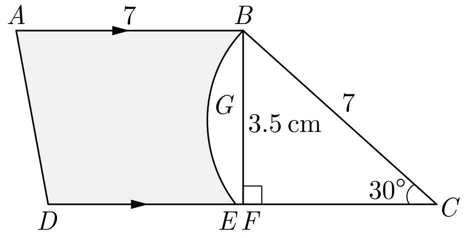
$$2\pi r = 66$$

$$r = \frac{66}{2\pi} = \frac{66}{2 \times \frac{22}{7}} = \frac{21}{2}$$

Area enclosed by the circle

$$\pi r^2 = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{2} = 346.5 \text{ cm}^2$$

**110.** Adjoining fig, $ABCD$ is a trapezium with $AB || DC$ and $\angle BCD = 30^\circ$. Fig. $BGEC$ is a sector of a circle with centre $C$ and $AB = BC = 7$ cm, $DE = 4$ cm and $BF = 3.5$ cm, then find the area of the shaded region. Use $\pi = \frac{22}{7}$.




**Ans :** [Board Term-2 OD Compt. 2017]

We have

$$AB = 7 \text{ cm}$$

$$DE = 4 \text{ cm, and}$$

$$BF = 3.5 \text{ cm}$$

Now $DC = DE + EC = 4 + 7 = 11$ cm

Area of Trapezium $ABCD$

$$\text{Area}_{\square} = \frac{1}{2}(DC + AB)(BF)$$

$$= \frac{1}{2}(11+7) \times 3.5 = \frac{1}{2} \times 18 \times 3.5$$

$$= 31.5 \text{ cm}^2$$

Area of circular sector,

$$\text{Area}_{\frown} = \frac{30^o}{360^o} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{1}{12} \times 22 \times 7$$

$$= 12.83 \text{ cm}^2$$

Area of shaded region,

$$= \text{Area}_{\square} - \text{Area}_{\frown}$$

$$= 31.5 - 12.83 = 18.67 \text{ cm}^2$$

Let, $\angle A = \theta$, $\angle B = \theta_2, \angle C = \theta_3$ and $\angle D = \theta_4$

Area of sector $A$,

$$\frac{\theta_1}{360^\circ}\pi r^2 = \frac{\theta_1}{360^\circ} \times \frac{22}{7} \times 7 \times 7$$

$$= \frac{\theta_1}{360^\circ} \times 154 \text{ cm}^2$$

$$\text{area of sector } B = \frac{\theta_2}{360^\circ} \times 154 \text{ cm}^2$$

$$\text{area of sector } C = \frac{\theta_3}{360^\circ} \times 154 \text{ cm}^2$$

$$\text{area of sector } D = \frac{\theta_4}{360^\circ} \times 154 \text{ cm}^2$$

$$\text{area of 4 sectors} = \frac{\theta_1 + \theta_2 + \theta_3 + \theta_4}{360^\circ} \times 154$$

$$\text{Area}_{4\frown} = \frac{360^\circ}{360^\circ} \times 154 = 154 \text{ cm}^2$$

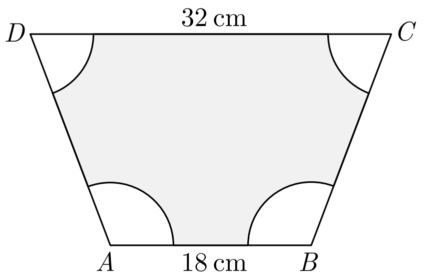
Thus area of shaded region,

$$= \text{Area}_{\square} - \text{Area}_{4\frown}$$

$$= 350 - 154 = 196 \text{ cm}^2$$

## FOUR MARKS QUESTIONS

**111.** In the given figure $ABCD$ is a trapezium with $AB || DC, AB = 18$ cm and $DC = 32$ cm and the distance between $AB$ and $AC$ is 14 cm. If arcs of equal radii 7 cm taking $A, B, C$ and $D$ have been drawn, then find the area of the shaded region.




**Ans :** [ Board Term-2 Foreign 2017]

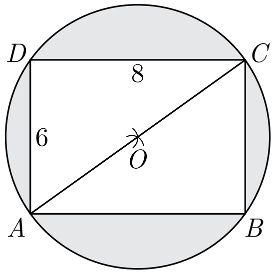
In trapezium $ABCD$, we have $AB = 18$ cm, $CD = 32$ cm $AB || CD$ and distance between || lines = 14 cm and the radius of each sector = 7 cm.

Area of trapezium $ABCD$,

$$\text{Area}_{\square} = \frac{1}{2}(18+32) \times 14 = \frac{1}{2} \times 50 \times 14$$

$$= 350 \text{ cm}^2$$

**112.** Find the area of the shaded region in Figure, if $ABCD$ is a rectangle with sides 8 cm and 6 cm and $O$ is the centre of circle. (Take $\pi = 3.14$)




**Ans :** [Board 2019 Delhi]

In $\Delta ABC$, $\angle B = 90^\circ$

Using Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$= 8^2 + 6^2$$

$$= 64 + 36 = 100$$

$$AC = 10 \text{ cm}$$

Since, $AC$ is the diameter of circle,

Radius of circle, $r = 5$ cm

Area of the shaded region

$= \text{(area of the circle)} - \text{(area of the rectangle)}$

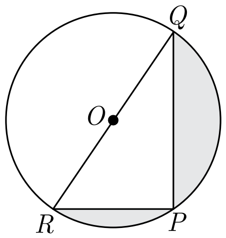
$= \pi r^2 - (AB \times BC)$

$= 3.14 \times 5^2 - (8 \times 6)$

$= 78.5 - 48$

$= 30.5 \text{ cm}^2$

**113.** Find the area of the shaded region in Figure, if $PQ = 24$ cm, $PR = 7$ cm and $O$ is the centre of the circle.




**Ans :** [Board 2020 OD Standard]

We have $PQ = 24$ cm

$PR = 7$ cm

The angle in the semicircle is right angle, therefore

$\angle RPQ = 90^\circ$

In $\Delta RPQ$, $RQ^2 = PR^2 + PQ^2$

$RQ^2 = (7)^2 + (24)^2$

$= 49 + 576 = 625$

$RQ = 25 \text{ cm}$

Area of $\Delta RPQ = \frac{1}{2} \times RP \times PQ$

$= \frac{1}{2} \times 7 \times 24$

$= 84 \text{ cm}^2$

area of semi-circle $= \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times \left(\frac{25}{2}\right)^2$

$= \frac{11 \times 625}{7 \times 4} = \frac{6875}{28}$ cm

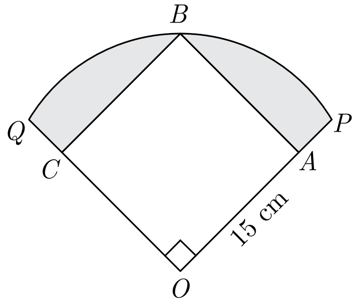
Now, area of shaded region

$=$ area of semi-circle $-$ area of $\Delta RPQ$

$= \frac{6875}{28} - 84 = \frac{6875 - 2352}{28}$

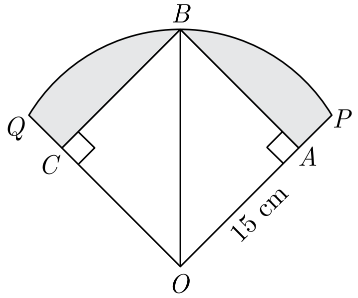
$= \frac{4523}{28} = 161.54 \text{ cm}^2$

**114.** In Figure, a square $OABC$ is inscribed in a quadrant $OPBQ$. If $OA = 15$ cm, find the area of the shaded region. (Use $\pi = 3.14$).




**Ans :** [Board 2019 OD]

We have redrawn the figure given below.




Using Pythagoras theorem in $\Delta BAO$,

$OB^2 = OA^2 + AB^2 = 15^2 + 15^2$

$= 225 + 225 = 450$

$OB = \sqrt{450} = 15\sqrt{2}$

Thus radius $OB = 15\sqrt{2}$ cm.

Area of square $= (OA)^2 = (15)^2 = 225 \text{ cm}^2$

Now, area of quadrant,

$\frac{\pi r^2}{4} = \frac{1}{4} \times 3.14 \times (15\sqrt{2})^2$

$= \frac{1}{4} \times 3.14 \times 225 \times 2$

$= \frac{3.14 \times 225}{2}$

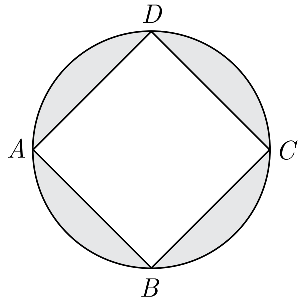
$= 353.25 \text{ cm}^2$

Therefore, area of shaded region

= Area of quadrant $OPBQ$ − area of square $OABC$

$= 353.25 - 225 = 128.25\,\text{cm}^2$

**115.** In Figure, $ABCD$ is a square with side $2\sqrt{2}$ cm and inscribed in a circle. Find the area of the shaded region.(Use $\pi = 3.14$).




**Ans :** [Board 2019 OD]

Side of square, $a = 2\sqrt{2}$ cm.

Area of square $a^2 = (2\sqrt{2})^2 = 8\,\text{cm}^2$

Length of the diagonal of a square is given by,

$$d = a\sqrt{2}$$

$$= 2\sqrt{2} \times \sqrt{2} = 4\,\text{cm}$$

Since, the square is inscribed in a circle, hence the diagonal of square will be the diameter of the circle,

Radius, $r = \dfrac{d}{2} = \dfrac{4}{2} = 2\,\text{cm}$

Area of the circle,

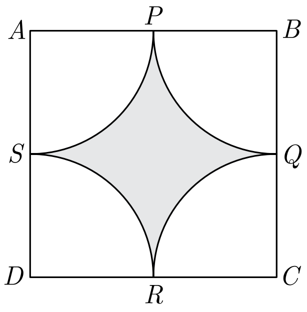
$$\pi r^2 = 3.14 \times (2)^2 = 12.56\,\text{cm}^2$$

Therefore, area of shaded region

= Area of circle − Area of the square

$$= (12.56 - 8) = 4.56\,\text{cm}^2$$

**116.** Find the area of the shaded region in Figure, where arcs drawn with centres $A$, $B$, $C$ and $D$ intersect in pairs at midpoint $P$, $Q$, $R$ and $S$ of the sides $AB$, $BC$, $CD$ and $DA$ respectively of a square $ABCD$ of side 12 cm. [Use $\pi = 3.14$]




**Ans :** [Board 2018]

Radius of each arc drawn is $r = \frac{12}{2} = 6$ cm.

Area of one quadrant is $\frac{1}{4}\pi r^2$, thus area of four quadrants,

$$4 \times \tfrac{1}{4}\pi r^2 = \pi \times 6^2 = 3.14 \times 36$$

$$= 113.04\,\text{cm}^2$$

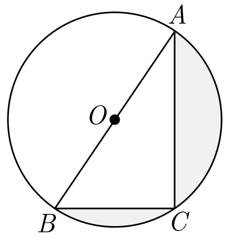
Area of square $ABCD$,

$$= 12 \times 12 = 144\,\text{cm}^2$$

Hence Area of shaded region

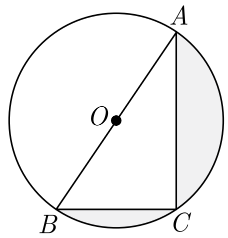
$$= 144 - 113.04 = 30.96\,\text{cm}^2$$

**117.** In the figure, $O$ is the centre of circle such that diameter $AB = 13$ cm and $AC = 12$cm. $BC$ is joined. Find the area of the shaded region. ($\pi = 3.14$)




**Ans :** [Board Term-2 OD 2016]

We redraw the given figure as below.




Radius of semi circle $ACB$,

$$r = \frac{13}{2}\text{ cm}$$

Area of semicircle,

$$\frac{\pi}{2}r^2 = \frac{3.14}{2} \times \frac{13}{2} \times \frac{13}{2}$$

$$= \frac{3.14 \times 169}{8} = \frac{530.66}{8}\text{ cm}^2$$

The angle subtended on a semicircle is a right angle, thus $\angle ACB = 90^\circ$

In $\Delta ABC$,

$$AC^2 + BC^2 = AB^2$$

$$12^2 + BC^2 = 169$$

$$BC^2 = (160 - 144) = 25$$

$$BC = 5 \text{ cm}$$

Also area of triangle $\Delta ABC$,

$$\Delta = \frac{1}{2} \times \text{Base} \times \text{Hight}$$

$$= \frac{1}{2} \times AC \times BC$$

$$= \frac{1}{2} \times 12 \times 5$$

$$= 30 \text{ cm}^2$$

Area of shaded region,

$$\frac{\pi}{2} r^2 - \Delta = \frac{530.66}{8} - 30$$

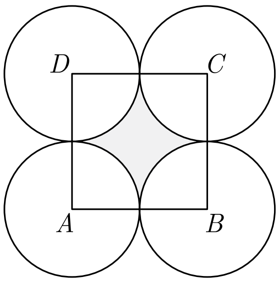
$$= (66.3325 - 30) \text{ cm}^2$$

$$= 36.3325 \text{ cm}^2$$

**118.** Four equal circles are described at the four corners of a square so that each touches two of the others. The shaded area enclosed between the circle is $\frac{24}{7}$ cm$^2$. Find the radius of each circle.

**Ans :** [Board Term-2 SQP 2017]

As per question statement the figure is shown below.




Let $r$ be the radius of each circle.

Area of square – Area of 4 sectors $= \frac{24}{7}$ cm$^2$

$$(2r)^2 - 4\left(\pi r^2 \times \frac{90^\circ}{360^\circ}\right) = \frac{24}{7}$$

$$4r^2 - \frac{22}{7} r^2 = \frac{24}{7}$$

$$\frac{28r^2 - 22r^2}{7} = \frac{24}{7}$$

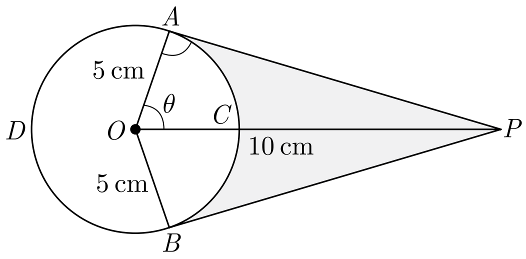
$$6r^2 = 24$$

$$r^2 = 4 \Rightarrow r = \pm 2$$

Thus radius of each circle is 2 cm.

**119.** An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point $C$ on the belt elastic belt is pulled directly away from the centre $O$ of the pulley until it is at $P$, 10 cm from the point $O$. Find the length of the belt that is still in contact with the pulley. Also find the shaded area.

(Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)




**Ans :** [Board Term-2 Delhi 2016]

Here $AP$ is tangent at point $A$ on circle.

Thus $\angle OAP = 90^\circ$

Now $\cos\theta = \frac{OA}{OP} = \frac{5}{10} = \frac{1}{2} = \cos 60^\circ$

Thus $\theta = 60^\circ$

Reflex $\angle AOB = 360^\circ - 60^\circ - 60^\circ = 240^\circ$

Now arc $ADB = \frac{2 \times 3.14 \times 5 \times 120^\circ}{360^\circ}$

$$= 20.93 \text{ cm}$$

Hence length of elastic in contact is 20.93 cm.

Now, $AP = 5\sqrt{3}$ dm

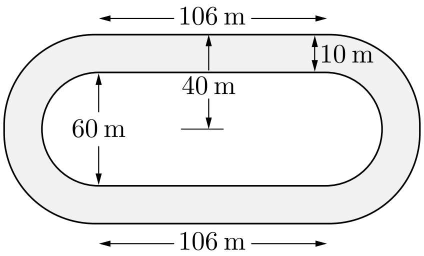
Area $(\Delta\, OAP + \Delta\, OBP) = 25\sqrt{3} = 43.25$ cm$^2$

Area of sector $OACB$,

$$= 25 \times 3.14 \times \frac{120^\circ}{360^\circ} = 26.16 \text{ cm}^2.$$

Shaded Area $= 43.25 - 26.16 = 17.09$ cm$^2$

**120.** Fig. depicts a racing track whose left and right ends are semi-circular. The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide everywhere, find the area of the track.




**Ans :** [Board Term-2 2011]

Width of the inner parallel lines = 60 m

Width of the outer lines = $40 \times 2 = 80$ m

Radius of the inner semicircles $= \frac{60}{2} = 30$ m

Radius of the outer semicircles $= \frac{80}{2} = 40$ m

Area of inner rectangle $= 106 \times 60 = 3180 \text{ m}^2$

Area of outer rectangle $= 106 \times 80 = 4240 \text{ m}^2$.

Area of the inner semicircles

$$= 2 \times \frac{1}{2} \times \frac{22}{7} \times 30 \times 30 = \frac{19800}{7} \text{ m}^2$$

Area of outer semicircles

$$= 2 \times \frac{1}{2} \times \frac{22}{7} \times 40 \times 40 = \frac{35200}{7} \text{ m}^2$$

Area of racing track

= (area of outer rectangle + area of outer semicircles)

– (area of inner rectangle + area of inner semicircles)

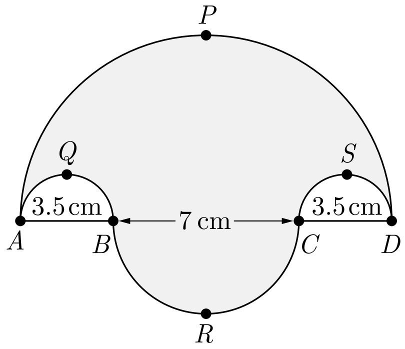
$$= 4240 + \frac{35200}{7} - \left(\frac{3180 + 19800}{7}\right)$$

$$= 1060 + \frac{15400}{7} = \frac{7420 + 15400}{7}$$

$$= \frac{22820}{7} = 3260 \text{ m}^2$$

Hence, area of track is $3260 \text{ m}^2$

**121.** Find the area of the shaded region in Figure, $\overset{\frown}{APD}, \overset{\frown}{AQB}, \overset{\frown}{BRC}$ and $\overset{\frown}{CSD}$, are semi-circles of diameter 14 cm, 3.5 cm, 7 cm and 3.5 cm respectively. Use $\pi = \frac{22}{7}$.




**Ans :** [ Board Term-2 Foreign 2016]

Diameter of the largest semi circle = 14 cm

$$\text{Radius} = \frac{14}{2} = 7 \text{ cm}$$

Diameter of two equal unshaded semicircle = 3.5 cm

$$\text{Radius of each circle} = \frac{3.5}{2} \text{ cm}$$

Diameter of smaller shaded semi-circle = 7 cm

$$\text{Radius} = 3.5 \text{ cm}$$

Area of shaded portion

= area of largest semi-circle+

+ area of smaller shaded semicircle+

– area of two unshaded semicircles

$$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$- 2 \times \frac{1}{2} \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}$$

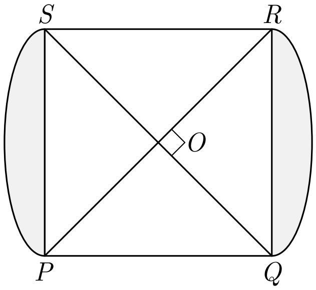
$$= \frac{1}{2} \times \frac{22}{7}\left[7^2 + \left(\frac{7}{2}\right)^2 - 2\left(\frac{7}{4}\right)^2\right] \text{cm}^2$$

$$= \frac{1}{2} \times \frac{22}{7} \times (7)^2\left[1 + \frac{1}{4} - \frac{1}{8}\right]$$

$$= 11 \times 7\left[\frac{9}{8}\right]$$

$$= \frac{693}{8} \text{ sq. cm or } 86.625 \text{ cm}^2$$

**122.** In figure, $PQRS$ is square lawn with side $PQ = 42$ metre. Two circular flower beds are there on the sides $PS$ and $QR$ with centre at $O$, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts).




**Ans :** [Board Term-2 OD 2015]

Radius of circle with centre $O$ is $OR$.

Let $OR$ be $x$ then using Pythagoras theorem we have

$$x^2 + x^2 = (42)^2 \Rightarrow x = 21\sqrt{2} \text{ m}$$

Area of segment of circle with centre angle $90^\circ$

$$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times (21\sqrt{2})^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 21 \times 21 \times 2$$

$$= 11 \times 3 \times 21 = 693$$

Area of triangle $\triangle ROQ$,

$$= \frac{1}{2} \times (21\sqrt{2})^2 = 21 \times 21 = 441$$

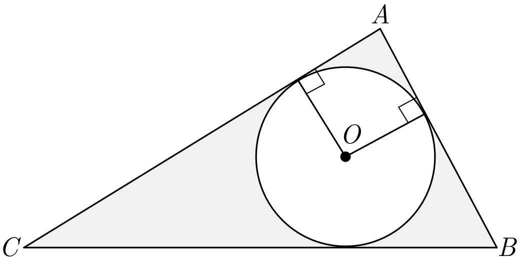
Area of the one side flower bed

$$= 693 - 441 = 252 \text{ m}^2$$

Area of flower bed of both

$$= 2 \times 252 = 504 \text{ m}^2$$

**123.** In the figure, $ABC$ is a right angled triangle right angled at $\angle A$. Find the area of the shaded region, if $AB = 6$ cm, $BC = 10$ cm and $O$ is the centre of the circle of the triangle $ABC$.




**Ans :** [Board Term-2 2015]

Let $r$ be the radius of incircle.

Using the tangent properties we have

$$BC = 8 - r + 6 - r$$

$$10 = 14 - 2r$$

, $$2r = 4 \Rightarrow r = 2 \text{ cm}$$

Area of circle $$\pi r^2 = \frac{22}{7} \times 2 \times 2 = \frac{88}{7} = 12.57 \text{ cm}^2$$

Now,area of $\Delta ABC$,

$$\Delta_{ABC} = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$$

Area of shaded region

$$= \text{Area of } \Delta ABC - \text{ Area of the circle}$$

$$= 24 - 12.57 \text{ cm}^2 = 11.43 \text{ cm}^2$$

**124.** Two circular beads of different sizes are joined together such that the distance between their centres is 14 cm. The sum of their areas is $130\pi$ cm². Find the radius of each bead.

**Ans :** [Board Term-2, 2015]

Let the radii of the circles are $r_1$ and $r_2$.

$$r_1 + r_2 = 14 \quad ...(1)$$

Sum, of their areas,

$$\pi(r_1^2 + r_2^2) = 130\pi$$

$$r_1^2 + r_2^2 = 130 \quad ...(2)$$

Now $$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1 r_2$$

$$(14)^2 = 130 + 2r_1 r_2$$

$$2r_1 r_2 = 196 - 130 = 66$$

$$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1 r_2$$

$$= 130 - 66 = 64$$

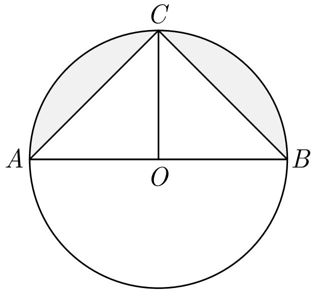
Thus $$r_1 - r_2 = 8 \quad ...(3)$$

From (1) and (3), we get

$$2r_1 = 22 \Rightarrow r_1 = 11 \text{ cm}$$

$$r_2 = 14 - 11 = 3 \text{ cm.}$$

**125.** A round thali has 2 inbuilt triangular for serving vegetables and a separate semi-circular area for keeping rice or chapati. If radius of thali is 21 cm, find the area of the thali that is shaded in the figure.




**Ans :** [Board Term-2 2014]

Since $AOB$ is the diameter of the circle, area of shaded region,

$$= (\text{Area of semi-circle} - \text{Area of } \Delta ABC)$$

Area of semi-circle

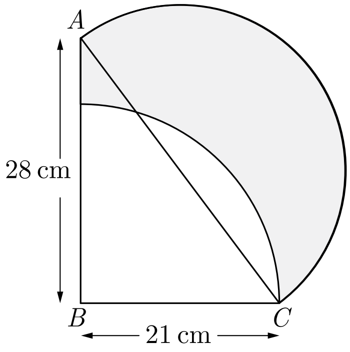
$$\frac{\pi r^2}{2} = \frac{1}{2} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$$

$$= \frac{1386}{2} = 693 \text{ cm}^2$$

$$\text{Area of triangle } = \frac{1}{2} \times 21 \times 42 = 441 \text{ cm}^2$$

$$\text{Area of shaded region } = 693 - 441 = 252 \text{ cm}^2$$

**126.** In the fig., $ABC$ is a right-angle triangle, $\angle B = 90^\circ$, $AB = 28$ cm and $BC = 21$ cm. With $AC$ as diameter, a semi-circle is drawn and with $BC$ as radius a quarter circle is drawn. Find the area of the shaded region.




**Ans :** [Board Term -2 2011, Foreign 2014]

In right angled triangle $\Delta ABC$ using Pythagoras theorem we have

$$AC^2 = AB^2 + BC^2$$

$$= 28^2 + 21^2$$

$$= 784 + 441$$

or $$AC^2 = 1225 \Rightarrow AC = 35 \text{ cm}$$

Area of shaded region,

$= \text{area of } \Delta ABC +$

$+ \text{ area of semi-circle with diameter } AC +$

$- \text{ area of quadrant with radius } BC$

$$= \frac{1}{2}(21 \times 28) + \frac{1}{2} \times \frac{22}{7} \times \left(\frac{35}{2}\right)^2 - \frac{1}{4} \times \frac{22}{7} \times (21)^2$$

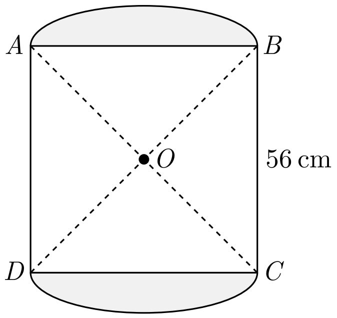
$$= 21 \times 14 + \frac{11}{7} \times \frac{35}{2} \times \frac{35}{2} - \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$$

$$= 21 \times 14 + \frac{55}{2} \times \frac{35}{2} - \frac{11}{2} \times 3 \times 21$$

$$= 294 + 481.25 - 346.5$$

$$= 775.25 - 346.5 = 428.75 \text{ cm}^2.$$

**127.** In fig., two circular flower beds have been shown on two sides of a square lawn $ABCD$ of side 56 m. If the centre of each circular flower bed is the point of intersection $O$ of the diagonals of the square lawn, find the sum of the areas of the lawn and flower beds.




**Ans :** [Board Term-2 2011]

Side of square $= 56$

Diagonal of square $= 56\sqrt{2}$

Radius of circle $= \frac{1}{2} \times 56\sqrt{2} = 28\sqrt{2}$

Total area $=$ Area of sector $OAB +$

$+$ Area of sector $ODC +$

$+$ Area of $\Delta\, OAD +$

$+$ Area of $\Delta\, OBC$

$$= \frac{22}{7} \times (28\sqrt{2})^2 \times \frac{90^\circ}{360^\circ} + \frac{22}{7} \times (28\sqrt{2})^2 \times \frac{90^\circ}{360^\circ} +$$

$$+\frac{1}{4}\times 56\times 56+\frac{1}{4}\times 56\times 56$$

$$=\frac{1}{4}\times\frac{22}{7}\times(28\sqrt{2})^2+\frac{1}{4}\times\frac{22}{7}\times(28\sqrt{2})^2+$$

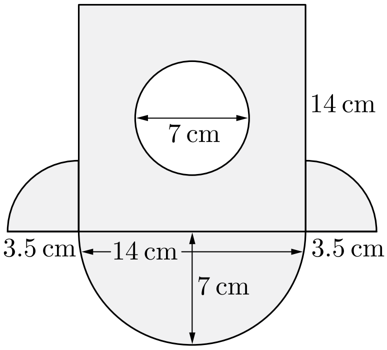
$$+\frac{1}{4}\times 56\times 56+\frac{1}{4}\times 56\times 56$$

$$=\frac{1}{4}\times 28\times 56\left(\frac{22}{7}+\frac{22}{7}+2+2\right)\text{ m}^2$$

$$=7\times 56\left(\frac{22+22+14+14}{7}\right)\text{ m}^2$$

$$=56\times 72=4032\text{ m}^2.$$

**128.** In fig., find the area of the shaded region. Use $\pi=\frac{22}{7}$.




**Ans :** [Board Term-2 2011]

Area of square $=(14)^2=196\text{ cm}^2$

Area of internal circle $=\frac{22}{7}\times\left(\frac{7}{2}\right)^2\text{ cm}^2$

$$=\frac{77}{2}=38.5\text{ cm}^2$$

Area of semi-circle with 14 cm diameter

$$=\frac{1}{2}\times\frac{22}{7}\times 7^2\text{ cm}^2$$

$$=77\text{ cm}^2$$

Area of two quarter circles of radius $\frac{7}{2}$ cm

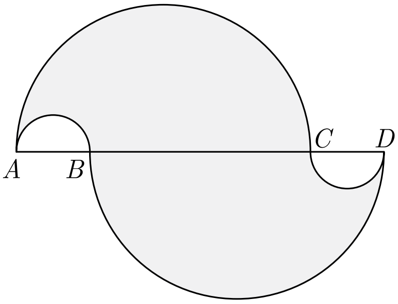
$$=2\times\frac{1}{4}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2=\frac{77}{4}=19.25\text{ cm}^2$$

$$\text{Shaded area }=196-38.5+77+19.25$$

$$=292.25-38.5$$

$$=253.75\text{ cm}^2.$$

**129.** In fig., $AC=BD=7$ cm and $AB=CD=1.75$ cm. Semi-circles are drawn as shown in the figure. Find the area of the shaded region. Use $\pi=\frac{22}{7}$.




**Ans :** [Board Term-2 2011]

Area of shaded region

= 2(Area of semi-circle of radius $\frac{7}{2}$ cm)

−2(Area of semi-circle of radius $\frac{7}{8}$ cm)

$$=2\left[\frac{1}{2}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\right]-2\ \frac{1}{2}\times\frac{22}{7}\times\ \frac{7}{8}\ ^2$$

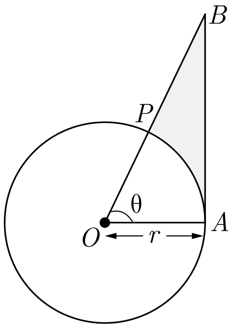
$$=2\times\frac{1}{2}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\left[1-\left(\frac{1}{4}\right)^2\right]$$

$$=\frac{77}{2}\left[1-\frac{1}{16}\right]=\frac{77}{2}\times\frac{15}{16}=\frac{1155}{32}\text{ cm}^2$$

$$=36.09\text{ cm}^2$$

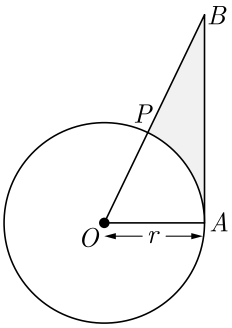
**130.** The given fig. is shown a sector $OAP$ of a circle with centre $O$, containing $\angle\theta$. $AB$ is perpendicular to the radius $OA$ and meets $OP$ produced at $B$. Prove that the perimeter of shaded region is

$r = \left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$




**Ans :** [Board Term-2 OD 2015, 2016]

As per question statement we have redrawn this figure as given below.




Here $OAP$ is sectors of circle with centre $O$, $\angle POA = \theta$ and $OA \perp AB$.

Perimeter of shaded region $= BP + AB + \widehat{AP}$ (1)

Now $\tan\theta = \frac{AB}{r} \Rightarrow r\tan\theta = AB$ ...(2)

$$\sec\theta = \frac{OB}{r} \Rightarrow r\sec\theta = OB$$

$$OB - OP = BP \Rightarrow r\sec\theta - r = OP \quad ...(3)$$

Length of arc $AP$,

$$\widehat{AP} = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{\theta}{360} \times 2\pi r = \frac{\theta\pi r}{180} \quad ...(4)$$

Putting value from equation (2), (3), (4) in equation (1) we get perimeter of shaded region as

$$= r\tan\theta + r\sec\theta - r + \frac{\theta\pi r}{180}$$

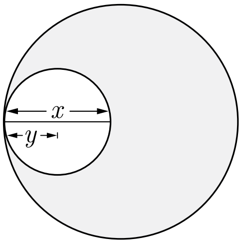
$$= r\left[\tan\theta + \sec\theta + \frac{\theta\pi}{180} - 1\right]$$

Hence, Proved.

**131.** Two circles touch internally. The sum of their areas is $116\pi$ and the difference between their centres is 6 cm. Find the radii of the circles.

**Ans :** [ Board Term-2 Foreign 2017]

Let the radius of larger circle be $x$ and the radius of smaller circle be $y$. As per question statement we have shown diagram below.




Now $x - y = 6$ ...(1)

and $\pi x^2 + \pi y^2 = 116\pi$

$$\pi(x^2 + y^2) = 116\pi$$

$$x^2 + y^2 = 116 \quad ...(2)$$

From (1) and (2) we have

$$x^2 + (x-6)^2 = 116$$

$$x^2 + x^2 - 12x + 36 = 116$$

$$x^2 - 6x - 40 = 0$$

$$x^2 - 10x + 4x - 40 = 0$$

$$x(x-10) + 4(x+10) = 0$$

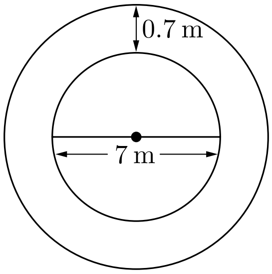
$$x = 10, \text{ and } y = 10 - 6 = 4$$

Hence, radii of the circles are 10 cm and 4 cm.

**132.** A park is of the shape of a circle of diameter 7 m. It is surrounded by a path of width of 0.7 m. Find the expenditure of cementing the path. If its cost is Rs.110 per sq. m.

**Ans :** [ Board Term-2 Foreign 2017]

As per question statement we have shown diagram below.




The inner diameter of park $= 7$ m

$$\text{radius} = \frac{7}{2} = 3.5 \text{ m}$$

Width of path $= 0.7$ m

Radius of park with path

$$= 3.5 + 0.7 = 4.2 \text{ m}$$

$$\text{Area of the path} = \pi(4.2)^2 - \pi(3.5)^2$$

$$= \frac{22}{7}(17.64 - 12.25)$$

$$= \frac{22}{7} \times 5.39 = 22 \times 0.77$$

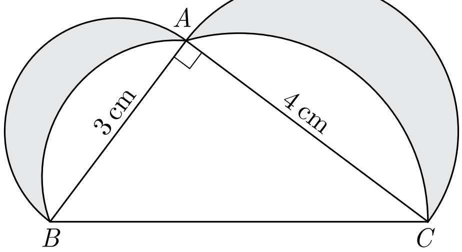
$$= 16.94 \text{ m}^2$$

Cost of the cementing the path

$$= 16.94 \times 110$$

$$= Rs.1863.40$$

**133.** In the given figure, $\Delta ABC$ is a right angled triangle in which $\angle A = 90^\circ$. Semicircles are drawn on $AB, AC$ and $BC$ as diameters. Find the area of the shaded region.




**Ans :** [Board Term-2 OD 2017]

In $\Delta ABC$ we have

$\angle A = 90^\circ$, $AB = 3 = 3$ cm, and $AC = 4$ cm

Now $\quad BC = \sqrt{AB^2 + AC^2} = \sqrt{3^2 + 4^2} = 5$ cm.

Area of shaded Area

= Area of semicircle with radius $\frac{3}{2}$ cm

+ area of semi circle with radius $\frac{4}{2}$ cm

+ Area of triangle $\Delta ABC$)

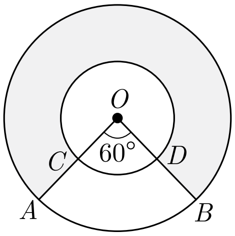
− Area of semicircle with radius $\frac{5}{2}$ cm

$$= \frac{\pi}{2}\left(\frac{3}{2}\right)^2 + \frac{\pi}{2}(2)^2 + \frac{1}{2} \times 3 \times 4 - \frac{\pi}{2}\left(\frac{5}{2}\right)^2$$

$$= \frac{9\pi}{8} + 2\pi + 6 - \frac{25\pi}{8} = \frac{9\pi + 16\pi - 25\pi}{8} + 6$$

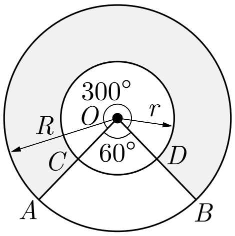
$$= 6 \text{ cm}^2$$

**134.** In the given figure, two concentric circle with centre $O$ have radii 21 cm and 42 cm. If $\angle AOB = 60^\circ$, find the area of the shaded region. Use $\pi = \frac{22}{7}$.




**Ans :**

We have redrawn the given figure as shown below.




Here $\angle AOB = 60^\circ$ and $\angle COD = 60^\circ$

$$R = 42 \text{ cm}, \; r = 21 \text{ cm}$$

Reflex of $\angle AOB$,

$$\theta = (360^\circ - 60^\circ) = 300^\circ$$

Now, area of shaded region

$$\pi R^2 \frac{\theta}{360^\circ} - \pi r^2 \frac{\theta}{360^\circ} = \frac{\theta\pi}{360^\circ}(R^2 - r^2)$$

$$= \frac{300^\circ}{360^\circ} \times \frac{22}{7} \times (42^2 - 21^2)$$

$$= \frac{5}{6} \times \frac{22}{7} \times (42-21)(42+21)$$

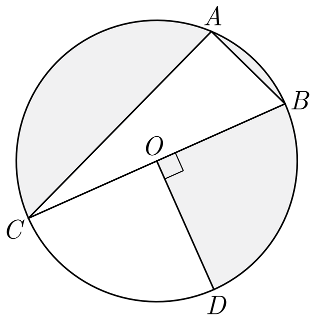
$$= \frac{5}{6} \times \frac{22}{7} \times 21 \times 63$$

$$= 5 \times 11 \times 63$$

$$= 3465 \text{ cm}^2$$

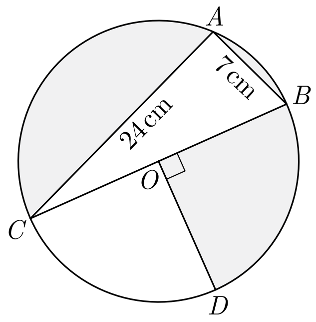
Thus area of shaded region is 3465 cm$^2$.

**135.** In the given figure, $O$ is the centre of the circle with $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90^\circ$. Find the area of the shaded region.




**Ans :**

We have redrawn the given figure as shown below.




Here $\Delta\, CAB$ is right angle triangle with $\angle CAB = 90^\circ$

In right $\Delta\, CAB$, by Pythagoras theorem, we have

$$BC^2 = AC^2 + AB^2$$

$$= 24^2 + 7^2$$

$$= 576 + 49 = 625$$

Thus $BC = 25$ cm which is diameter. Now radius is $\frac{25}{2}$ or 12.5 cm.

Area of shaded region,

$=$ area of semicircle $+$area of quadrant $-$ area of $\Delta\, ACB$

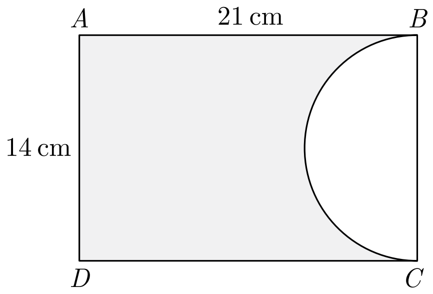
$$= \frac{1}{2}\pi r^2 + \frac{1}{4}\pi r^2 - \frac{1}{2} \times AB \times AC$$

$$= \frac{3}{4}\pi r^2 - \frac{1}{2} \times 7 \times 24 = \frac{3}{4} \times \frac{22}{7} \times \frac{625}{4} - 7 \times 12$$

$$= 368.3035 - 84 = 284.3 \text{ cm}^2$$

Thus area of shaded region $= 284.3035$ cm$^2$

**136.** In the given figure, $ABCD$ is a rectangle of dimensions 21 cm $\times$ 14 cm. A semicircle is drawn with $BC$ as diameter. Find the area and the perimeter of the shaded region in the figure.




**Ans :** [Board Term-2 OD 2017]

Area of shaded region,

$=$ Area of rectangle $ABCD$ $-$ area of semicircle

$$= 21 \times 14 - \frac{\pi}{2} \times 7^2$$

$$= 294 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

$= 294 - 77 = 217 \text{ cm}^2$

Perimeter of shaded area

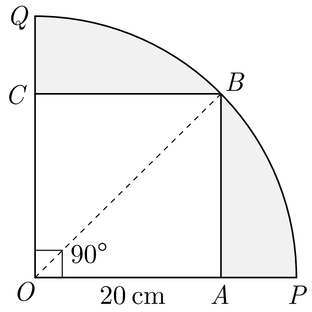
$$= AB + AD + CD + \widehat{CB}$$

$$= 21 + 14 + 21 + \frac{22}{7} \times 7$$

$$= 21 + 14 + 21 + 22 = 78 \text{ cm}$$

Hence, area of shaded region is $217 \text{ cm}^2$ and perimeter is 78 cm.

**137.** A square $OABC$ is inscribed in a quadrant $OPBQ$ of a circle. If $OA = 20$ cm, find the area of the shaded region. [Use $\pi = 3.14$]




**Ans :** [Board Term-2 Delhi 2014]

We have $OB = \sqrt{OA^2 + AB^2}$

$$= \sqrt{20^2 + 20^2} = \sqrt{800}$$

Thus $OB = 20\sqrt{2}$ cm

radius $r = 20\sqrt{2}$

Area of shaded region

= Area of sector $OQBPO$ − Area of square $OABC$

$$= \pi r^2 \frac{90^\circ}{360^\circ} - (20)^2$$

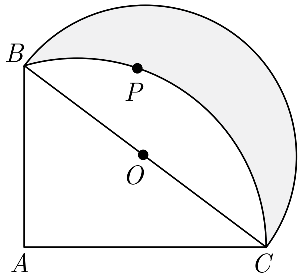
$$= 3.14 \times (20\sqrt{2})^2 \times \frac{90^\circ}{360^\circ} - (20)^2$$

$$= 3.14 \times 200 - 400$$

$$= 628 - 400 = 228$$

Required area is $228 \text{ cm}^2$.

**138.** In given figure $ABPC$ is a quadrant of a circle of radius 14 cm and a semicircle is drawn with $BC$ as diameter. Find the are of the shaded region.




**Ans :** [Board Term-2 SQP 2017]

Radius of the quadrant $AB = AC = 14$ cm

$$BC = \sqrt{14^2 + 14^2} = 14\sqrt{2} \text{ cm}$$

Radius of semicircle $= \frac{14\sqrt{2}}{2} = 7\sqrt{2}$ cm

Area of semicircle $= \frac{1}{2}\pi(7\sqrt{2})^2$

$$= \frac{1}{2} \times \frac{22}{7} \times 98$$

$$= 154 \text{ cm}^2$$

Area of segment $BPCO$

$$\frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2}r^2 = r^2\left(\frac{\pi\theta}{360^\circ} - \frac{1}{2}\right)$$

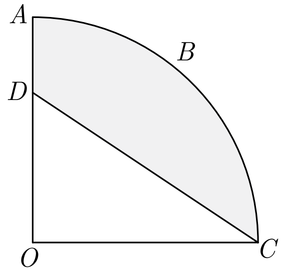
$$= 14 \times 14\left(\frac{22}{7} \times \frac{90}{360} - \frac{1}{2}\right)$$

$$= 14 \times 14\left(\frac{11}{14} - \frac{1}{2}\right)$$

$$= 14 \times 14 \times \frac{2}{7} = 56 \text{ cm}^2$$

Hence, area of shaded region is $56 \text{ cm}^2$.

**139.** In the figure $OABC$ is a quadrant of a circle of radius 7 cm. If $OD = 4$ cm, find the area of shaded region.



**Ans :** [Board Term-2 Foreign 2014]

Area of shaded region,

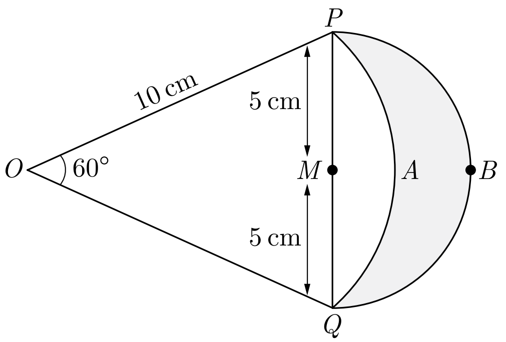
$$= \text{Area of sector } OCBAD - \text{ Area of } \Delta\, ODC$$

$$= \pi \times 7^2 \times \frac{90^\circ}{360^\circ} - \frac{1}{2} \times 7 \times 4$$

$$= \pi \times 49 \times \frac{1}{4} - 14$$

$$= \frac{49\pi}{4} - 14 = 24.5 \text{ cm}^2$$

**140.** Figure shows two arcs $PAQ$ and $PQB$. Arc $PAQ$ is a part of circle with centre $O$ and radius $OP$ while arc $PBQ$ is a semi-circle drawn on $PQ$ as diameter with centre $M$. If $OP = PQ = 10$ cm show that area of shaded region is $25\left(\sqrt{3} - \frac{\pi}{6}\right)$ cm$^2$.




**Ans :** [Board Term-2 Delhi 2016]

We have $\angle POQ = 60^\circ$

and $OP = OQ = PQ = 10$

Area of segment $PAQM$,

$$= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right.$$

cm$^2$

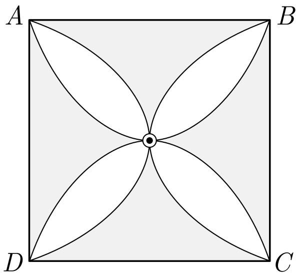
Area of semicircle $= \frac{\pi 5^2}{2} = \frac{25\pi}{2}$ cm$^2$

Area of shaded region,

$$= \frac{25\pi}{2} - \left(\frac{50\pi}{3} - 25\sqrt{3}\right)$$

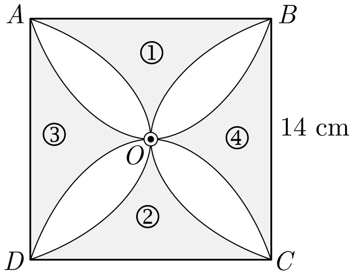
$$= 25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{ cm}^2.$$

**141.** In fig. $ABCD$ is a square of side 14 cm. Semi-circle are drawn with each side of square as diameter. Find the area of the shaded region. Use $\pi = \frac{22}{7}$.




**Ans :** [Board Term-2 Delhi 2016]

We have redrawn the given figure as shown below.




If we subtract area of two semicircle $AOD$ and $COB$ , from square $ABCD$ we will get area of part 1 and part 2.

Area of square $= 14 \times 14 = 196$ cm$^2$

Radius of semicircle $= \frac{14}{2} = 7$ cm

Area of semicircle $AOB + DOC$

$$= \frac{22}{7} \times 7^2 = 154 \text{ cm}^2$$

So, area of each of two shaded part

$$196 - 154 = 42 \text{ cm}^2$$

Hence, area of four shaded parts is 84 cm$^2$.

**142.** The long and short hands of a clock are 6 cm and 4 cm long respectively. Find the sum of distances travelled by their tips in 24 hours. (Use $\pi = 3.14$)

**Ans :** [Board Term-2 Foreign 2015]

Long hand makes 24 rounds in 24 hours and short hand makes 2 round in 24 hours. Distance travelled by tips of hands in one round is equal to the circumference of circle.

Radius of the circle formed by long hand = 6 cm. and radius of the circle formed by short hand = 4 cm.

Distance travelled by long hand in one round

$$= \text{circumference of the circle } 2 \times 6 \times \pi$$

Distance travelled by long hand in 24 rounds

$$= 24 \times 12\pi = 288\pi$$

Distance travelled by short hand in a round $= 2 \times 4\pi$

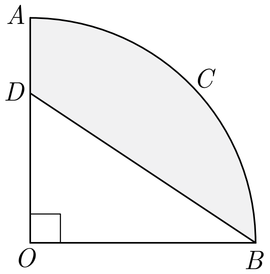
Distance travelled by short hand in 2 round

$$= 2 \times 8\pi = 16\pi$$

Sum of the distance $= 288\pi + 16\pi = 304\pi$

$$= 304 \times 3.14 = 954.56 \text{ cm}$$

**143.** In the given figure $DACB$ is a quadrant of a circle with centre $O$ and radius 3.5 cm. If $OD = 2$ find the area of the region.




**Ans :** [Board Term-2 Delhi 2017]

Area of shaded region,

$= $ area of quadrant $OACB -$ area $\Delta\, DOB$

$$= \frac{1}{4}\pi r^2 - \frac{1}{2} \times \text{base} \times \text{height}$$

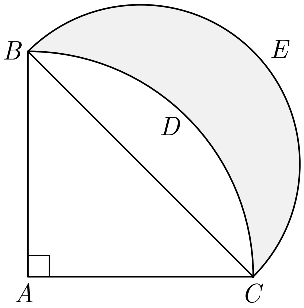
$$= \frac{1}{4} \times \frac{22}{7} \times 3.5^2 - \frac{1}{2} \times 2 \times 3\ 5$$

$$= 3.5\Big(\frac{1}{4} \times \frac{22}{7} \times 3.5 - 1$$

$$= 3.5\Big(\frac{11}{4} - 1\Big) = 3.5 \times \frac{7}{4} = 6.125$$

Hence the area of shaded region is 6.125 cm.

**144.** As $ABDC$ is a quadrant of a circle of radius 28 cm and a semi-circle $BEC$ is drawn with $BC$ as diameter. Find the area of the shaded region. Use $\pi = \frac{22}{7}$.




**Ans :** [Board Term-2 SQP 2017]

As $ABC$ is a quadrant of the circle, $\angle BAC$ will be $90^\circ$.

In $\Delta\, ABC$, $\quad BC^2 = AC^2 + AB^2$

$$= (28)^2 + (28)^2 = 2 \times (28)^2$$

$$BC = 28\sqrt{2} \text{ cm}$$

Radius of semi-circle drawn on $BC$,

$$= \frac{28\sqrt{2}}{2} = 14\sqrt{2}$$

Area of semi-circle $= \frac{1}{2}\pi(14\sqrt{2})^2$

$$= \frac{1}{2} \times \frac{22}{7} \times 14 \times 14 \times 2$$

$$= 616 \text{ cm}^2$$

Area of $\Delta\, ABC$ $= \frac{1}{2} \times 28 \times 28 = 392 \text{ cm}^2$

Area of quadrant $= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28$

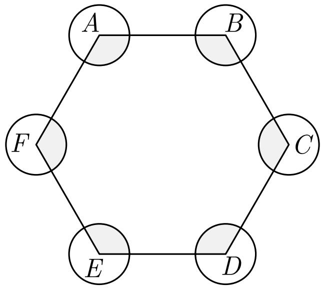
$$= 616 \text{ cm}^2$$

Area of the shaded region

$=$ Area of semi-circle $+$area of $\Delta$ $-$ Area of quadrant

$= 616 + 392 - 616 = 392 \text{ cm}^2$.

**145.** In fig., $ABCDEF$ is any regular hexagon with different vertices $A, B, C, D, E$ and $F$ as the centres of circle with same radius $r$ are drawn. Find the area of the shaded portion.




**Ans :** [Board Term-2 2011]

Let $n$ be number of sides.

Now $\qquad n \times \text{each angle} = (n-2) \times 180^\circ$

$$6 \times \text{each angle} = 4 \times 180^\circ$$

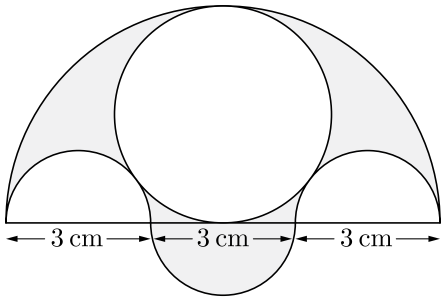
$$\text{each angle} = 120^\circ$$

$$\text{Area of a sector} = \pi r^2 \times \frac{120^\circ}{360^\circ}$$

$$\text{Area of 6 shaded regions} = 6\pi r^2 \times \frac{120^\circ}{360^\circ}$$

$$= 2\pi r^2$$

**146.** Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.




**Ans :**

Area of shaded region

$= \text{Area of semicircle with } d = 9 \text{ cm}$

$\quad + \text{Area of semicircle with } d = 3 \text{ cm}$

$\quad\quad - 2 \times \text{area of semicircle with } d = 3 \text{ cm}$

$\quad\quad\quad - \text{area of circle with } d = 4.5 \text{ cm}$

$$= \frac{1}{2} \times \pi \times \left(\frac{9}{2}\right)^2 + \frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2 - 2 \times \frac{1}{2} \times \pi \times \left(\frac{3}{2}\right)^2 - \pi \times \left(\frac{4.5}{2}\right)^2$$

$$= \frac{\pi}{8}[(9)^2 + (3)^2 - 2(3)^2 - 2(4.5)^2]$$

$$= \frac{\pi}{8}[4(4.5)^2 + (3)^2 - 2(3)^2 - 2(4.5)^2]$$

$$= \frac{\pi}{8}[2(4.5)^2 - (3)^2] = \frac{\pi}{8}[2(3 \times 1.5)^2 - (3)^2]$$

$$= \frac{\pi(3)^2}{8}[2(1.5)^2 - 1] = \frac{9\pi}{8}[4.5 - 1]$$

$$= \frac{9 \times 22}{8 \times 7} \times 3.5 = \frac{99}{8} = 12.375 \text{ cm}^2$$

Thus area of shaded region is 12.375 $\text{cm}^2$