Coordinate Geometry

2016

Short Answer Type Questions I [2 Marks]

Question 1.

Find the ratio in which y-axis divides the line segment joining the points A(5, -6) and B(-l, -4). Also find the coordinates of the point of division.

Solution:

Let the point on y-axis be $P(0, y)$ and $AP : PB = k : 1$.

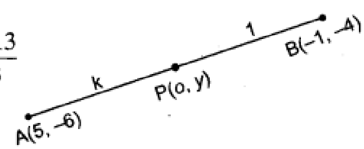
$\therefore$ Co-ordinates of P given by: $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

Then, taking x-axis of A, B; $\frac{5 \times 1 + k(-1)}{k+1} = 0 \Rightarrow \frac{5-k}{k+1} = 0 \Rightarrow k = 5$

Hence the required ratio is 5 : 1

Now, taking y-axis, $y = \frac{(-4)(5) + (1)(-6)}{5+1} = \frac{-13}{3}$

Hence point on y-axis is $\left(0, \frac{-13}{3}\right)$



Question 2.

The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q(2, -5) and R(-3, 6), find the coordinates of P.

Solution:

Let the required point be $(2y, y)$. Let $Q(2, -5)$ and $R(-3, 6)$ are given points.

Now, $PQ = PR \Rightarrow \sqrt{(2y-2)^2 + (y+5)^2} = \sqrt{(2y+3)^2 + (y-6)^2}$

[$\because$ using Distance formula, $\sqrt{(x-2)^2 + (y+5)^2} = \sqrt{(x+3)^2 + (y-6)^2}$]

Squaring both sides we get

$4y^2 + 4 - 8y + y^2 + 10y + 25 = 4y^2 + 9 + 12y + y^2 - 12y + 36$

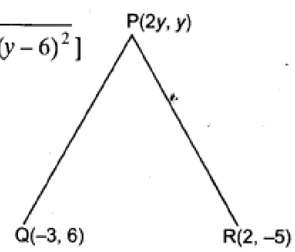
$\Rightarrow \quad 2y + 29 = 45$

$\Rightarrow \quad 2y = 45 - 29 = 16$

$\Rightarrow \quad y = 8$

$\Rightarrow \quad 2y = 16$

Hence coordinates of P are (16, 8)

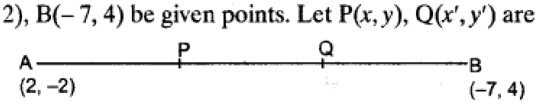


Question 3.

Let P and Q be the points of trisection of the line segment joining the points A(2, -2) and B(-7, 4) such that P is nearer to A. Find the coordinates of P and Q.

Solution:

Let $A(2, -2)$, $B(-7, 4)$ be given points. Let $P(x, y)$, $Q(x', y')$ are point of trisection.



P divides AB in the ratio 1 : 2

Coordinates of P are $\left(\frac{2 \times 2 + 1(-7)}{1+2}, \frac{(-2)(2) + 1(4)}{1+2}\right)$ or $(-1, 0)$

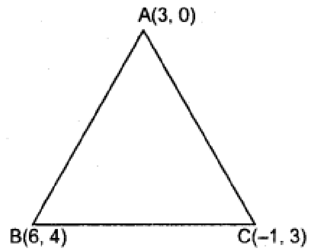
Q is mid point of PB. So using mid point formula coordinates of Q are $\left(\frac{-1-7}{2}, \frac{0+4}{2}\right)$ or $(-4, 2)$

Question 4.

Prove that the points (3,0), (6,4) and (-1,3) are the vertices of a right angled isosceles triangle.

Solution:

Let the triangle be ΔABC as shown in figure. Distances are:



Using distance formula,

$AB = \sqrt{(3-6)^2+(0-4)^2} = 5$

$BC = \sqrt{(6+1)^2+(4-3)^2} = 5\sqrt{2}$

$CA = \sqrt{(-1-3)^2+(3-0)^2} = 5$

Here, $AB = AC \Rightarrow \Delta ABC$ is isosceles triangle

Consider, $AB^2 + AC^2 = (5)^2 + (5)^2 = 25 + 25 = 50$

$\Rightarrow$ and, $BC^2 = (5\sqrt{2})^2 = 50$

$\therefore$ Here, $AB^2 + AC^2 = BC^2$

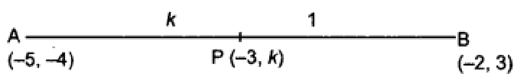
$\Rightarrow \Delta ABC$ is a right angled triangle.

[$\because$ In right Δ, using Pythagoras theorem $(H)^2 = (P)^2 + (B)^2$]
where H = hypotenuse, B = base, P = perpendiculars

Question 5.

Find the ratio in which the point (-3, k) divides the line-segment joining the points (-5, -4) and (-2,3). Also find the value of k.

Solution:



Let P divides AB in $k : 1$.

Then $-3 = \dfrac{k\times(-2)+1(-5)}{k+1}$ [Using section formula, $(x, y) = \left(\dfrac{mx_2+nx_1}{m+n}, \dfrac{my_2+ny_1}{m+n}\right)$]

$\Rightarrow -3k - 3 = -2k - 5$

$\Rightarrow -k = -2$

$\Rightarrow k = 2$

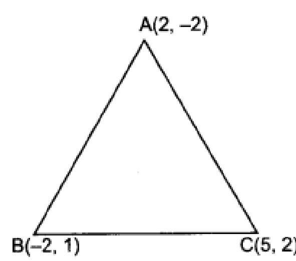
Hence the required ratio is 2 : 1

Question 6.

Prove that the points (2, -2), (-2, 1) and (5, 2) are the vertices of a right angled triangle. Also find the area of this triangle.

Solution:

Let A(2, –2), B(–2, 1) and C(5, 2) be the given points. So,



Using Distance formula

$AB^2 = (2 + 2)^2 + (-2 -1)^2 = 16 + 9 = 25$

$\therefore AB = 5$

$BC^2 = (-2 -5)^2 + (1 - 2)^2 = 49 + 1 = 50$

$BC = 5\sqrt{2}$

$AC^2 = (5 - 2)^2 + (2 + 2)^2 = 9 + 16 = 25$

$AC = 5$

$\therefore BC^2 = AC^2 + AB^2$, so ΔABC is a right angled triangle in which BC is hypotenuse.

$\therefore$ $ar(\Delta ABC) = \dfrac{1}{2} \times AB \times AC = \dfrac{1}{2} \times 5 \times 5 = \dfrac{25}{2}$ sq. units

Short Answer Type Questions II [3 Marks]

Question 7.

In figure ABC is a triangle coordinates of whose vertex A are (0, -1). D and E respectively are the mid-points of the sides AB and AC and their coordinates are (1, 0) and (0,1) respectively. If F is the mid-point of BC, find the areas of ΔABC and ΔDEF.

Solution:

Let coordinates of B are (x, y). Then using mid point formula we

$$\frac{x+0}{2} = 1 \quad \Rightarrow \quad x = 2$$

$$\frac{y-1}{2} = 0 \quad \Rightarrow \quad y = 1$$

Coordinates of B are (2,1)

Let coordinates of C are (p, q)

Similarly coordinates of C we have

$$\frac{p+0}{2} = 0 \quad \Rightarrow \quad p = 0$$

$$\frac{q-1}{2} = 1 \quad \Rightarrow \quad q = 3$$

Coordinates of C are (0, 3)

Area of ΔABC

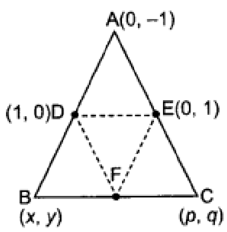
$$\Rightarrow \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = \frac{1}{2}[0(1-3) + 2(3+1) + 0(-1-1)]$$

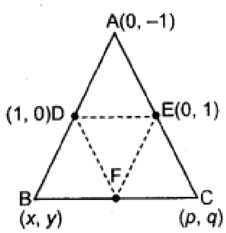
$$= \frac{1}{2} \times 8 = 4 \text{ sq. units}$$

Coordinates of F are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$ i.e. (1, 2) [∵ Using mid-point formula]

$$\text{Area of } \Delta DEF = \frac{1}{2}[1(1-2) + 0(2-0) + 1(0-1)] = \frac{1}{2}[-1+0-1]$$

$$= \frac{1}{2} \times (-2) = [-1] = 1 \text{ sq. units} \quad [\because \text{Area cannot be negative}]$$





Question 8.

If the point P(x, y) is equidistant from the points A (a + b, b – a) and B(a -b,a+ b). Prove that bx = ay.

Solution:

Given, $PA = PB \Rightarrow PA^2 = PB^2$

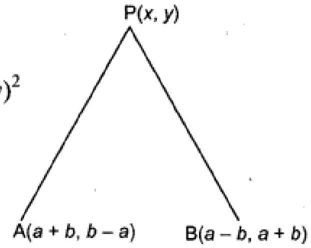
Applying distance formula,

$$\Rightarrow (a+b-x)^2 + (b-a-y)^2 = (a-b-x)^2 + (a+b-y)^2$$

$$\Rightarrow (a+b)^2 + x^2 - 2ax - 2bx + (b-a)^2 + y^2 - 2by + 2ay$$

$$= (a-b)^2 + x^2 - 2ax + 2bx + (a+b)^2 + y^2 - 2ay - 2by$$

$\Rightarrow 4ay = 4bx \Rightarrow ay = bx$ or $bx = ay$ Hence proved.



Question 9.

If the point C(-l, 2) divides internally the line-segment joining the points A(2, 5) and B(x,y) in the ratio 3 : 4, find the value of $x^2 + y^2$.

Solution:

Using section formula,

$$-1 = \frac{3 \times x + 4 \times 2}{3+4}$$

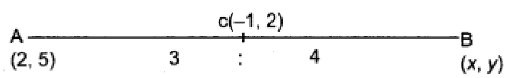
$$-1 = \frac{3x+8}{7}$$

$$3x + 8 = -7 \Rightarrow 3x = -15 \Rightarrow x = -5$$

Similarly, $2 = \dfrac{3 \times y + 4 \times 5}{3+4}$

$$14 = 3y + 20 \Rightarrow 3y = -6 \Rightarrow y = -2$$

Hence; $x^2 + y^2 = (-5)^2 + (-2)^2 = 25 + 4 = 29$



Long Answer Type Questions [4 Marks]

Question 10.

Prove that the area of a triangle with vertices (t, t – 2), (t + 2, t + 2) and (t + 3, t) is independent of t.

Solution:

Given vertices of triangle are $\{t, t-2\}, \{t+2, t+2\}, \{t+3, t\}$

Let $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are vertices of the triangle.

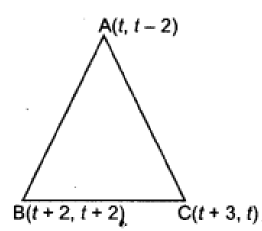
$$\text{Area of the triangle} = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$

$$= \frac{1}{2}[t(t+2-t) + (t+2)\{t-t+2\} + (t+3)\{t-2-t-2\}]$$

$$= \frac{1}{2}[2t + 2t + 4 - 4t - 12]$$

$$= \frac{1}{2} \times (-8) = 4 \text{ sq units, since area can't be negative.}$$

Hence, area is independent of t.

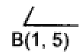


Question 11.

In fig., the vertices of AABC are A(4, 6), B(l, 5) and C(7, 2). A line-segment DE is drawn to intersect the sides AB and AC

at D and E respectively such that AD/AC=AE/AC=1/3. Calculate the area of ΔADE and compare it with area of ΔABC.

Solution:



Given: $\frac{AD}{AB} = \frac{1}{3}$

$3AD = AB$

$\therefore$ $3AD = AD + DB$

$2AD = DB$

$\frac{AD}{DB} = \frac{1}{2}$

Similarly, $\frac{AE}{EC} = \frac{1}{2}$

Calculated using section formula

Coordinates of D are $\left(\frac{1(1)+2(4)}{1+2}, \frac{1(5)+2(6)}{1+2}\right)$ i.e. $\left(3, \frac{17}{3}\right)$

Coordinates of E are $\left(\frac{1(7)+2(4)}{1+2}, \frac{1(2)+z(6)}{1+2}\right)$ i.e. $\left(5, \frac{14}{3}\right)$

$$\text{Area of } \Delta ADE = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$

$$= \frac{1}{2}\left[4\left(\frac{17}{3}-\frac{14}{3}\right)+3\left(\frac{14}{3}-6\right)+6\left(6-\frac{17}{3}\right)\right]$$

$$= \frac{1}{2}\left[4+3\left(\frac{-4}{3}\right)+5\left(\frac{1}{3}\right)\right]$$

$$= \frac{1}{2}\left[4-4+\frac{5}{3}\right] = \frac{5}{6} \text{ sq. units}$$

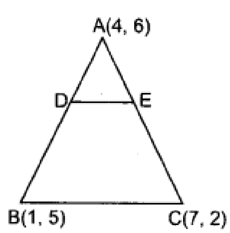
$$\text{Area of } \Delta ABC = \frac{1}{2}[4(5-2) + 1(2-6) + 7(6-5)]$$

$$= \frac{1}{2}[4 \times 3 + (-4) + 7 \times 1]$$

$$= \frac{1}{2}[12 - 4 + 7]$$

$$= \frac{1}{2} \times 15 = \frac{15}{2} \text{ sq. units}$$

Hence, $\frac{\text{Area } (\Delta ADE)}{\text{Area } (\Delta ABC)} = \frac{5/6}{15/12} = \frac{5}{6} \div \frac{15}{12} = \frac{5}{6} \times \frac{12}{15} = \frac{2}{3}$



Question 12.

The coordinates of the points A, B and C are (6,3), (-3,5) and (4, -2) respectively.P(JC, y) is any point in the plane. Show that $\frac{ar(\Delta PBC)}{ar(\Delta ABC)} = \left|\frac{x+y-2}{7}\right|$.

Solution:

Taking points P, B, C. Firstly,

$$\text{area}(\Delta PBC) = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$
$$= \frac{1}{2}[x(7) - 3(-2-y) + 4(y-5)]$$
$$= \frac{1}{2}[7x + 7y - 14] \text{ sq. units}$$

Now,
$$\text{area}(\Delta ABC) = \frac{1}{2}[6 \times 7 - 3(-5) + 4(3-5)]$$
$$= \frac{1}{2}[42 + 15 - 8] = \frac{1}{2} \times 49 \text{ sq. units}$$

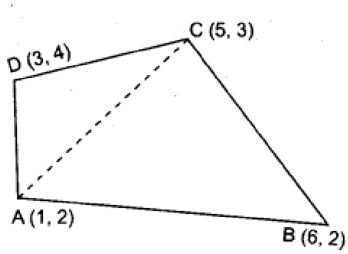
Hence,
$$\left|\frac{\text{area}(\Delta PBC)}{\text{area}(\Delta ABC)}\right| = \left|\frac{7x+7y-14}{49}\right| = \left|\frac{x+y-2}{7}\right|$$

Question 13.

Find the area of the quadrilateral ABCD, the coordinate of whose vertices are A(1, 2), B(6,2), C(5,3) and D(3,4).

Solution:

$$\text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$
$$= \frac{1}{2}[1(2-3) + 6(3-2) + 5(2-2)]$$
$$= \frac{1}{2}[-1 + 6 + 0] = \frac{5}{2} \text{ sq. units}$$



$$\text{Now, Area of } (\Delta ACD) = \frac{1}{2}[1(3-4) + 5(4-2) + 3(2-3)]$$
$$= \frac{1}{2}[-1 + 10 - 3]$$
$$= \frac{1}{2} \times 6 = 3 \text{ sq. units}$$

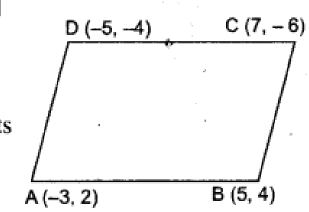
$$\text{Hence, Area (quadrilateral ABCD)} = \frac{5}{2} + 3 = \frac{11}{2} \text{ sq. units}$$

Question 14.

Find the area of a quadrilateral ABCD, the coordinates of whose vertices are A(—3,2), B(5,4), C(7, -6) and D(-5, -4).

Solution:

$$\text{Area of } \Delta ABD = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$
$$= \frac{1}{2}[-3(8) + 5(-6) + (-5)(2-4)]$$
$$= \frac{1}{2}[-24 - 30 + 10]$$
$$= \frac{1}{2} \times (-44) = (-22) = 22 \text{ sq. units}$$



Since area can't be negative

$$\text{area of } \Delta BCD = \frac{1}{2}[5(-2) + 7(-8) - 5(10)]$$

$$= \frac{1}{2}[-10 - 56 - 50]$$
$$= \frac{1}{2}(-116) = (-58) = 58 \text{ sq. units}$$

Since area cannot be negative.

$$\text{Area of quadrilateral ABCD} = \text{Area}(\Delta ABD) + \text{area}(\Delta BCD)$$
$$= 22 + 58 = 80 \text{ sq. units.}$$

2015

Short Answer Type Questions I [2 Marks]

Question 15.

If A(5, 2), B(2, -2) and C(-2, t) are the vertices of a right angled triangle with ∠B = 90°, then find the value of t.

Solution:

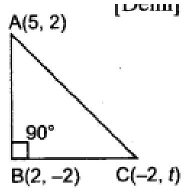
Using distance formula in right triangle ABC,

$$AB^2 = (5-2)^2 + (2-(-2))^2 = 9 + 16 = 25$$
$$AC^2 = (5-(-2))^2 + (2-t)^2 = 49 + 4 - 4t + t^2 = t^2 - 4t + 53$$
$$BC^2 = (2+2)^2 + (-2-t)^2 = 16 + t^2 + 4t + 4 = t^2 + 4t + 20$$

Now ΔABC is a right triangle, right angled at B.

So, $$AC^2 = AB^2 + BC^2$$
$$t^2 - 4t + 53 = 25 + t^2 + 4t + 20$$
$$8t = 8 \Rightarrow t = \frac{8}{8} = 1$$

Hence, $t = 1$



(By Pythagoras theorem)

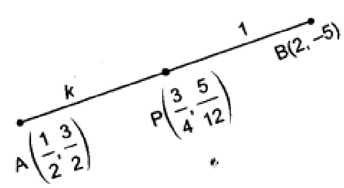
Question 16.

Find the ratio in which the point P P(3/4,5/12) divides the line segment joining the points A(1/2,3/2) and 3(2, -5).

Solution:

Let point P divides the line segment AB in the ratio $k : 1$. Using section formula,

then the coordinates of P are $\left(\frac{2k + \frac{1}{2}}{k+1}, \frac{-5k + \frac{3}{2}}{k+1}\right)$



A.T.Q. $$\frac{2k + \frac{1}{2}}{k+1} = \frac{3}{4}$$

$\Rightarrow$ $$8k + 2 = 3k + 3$$

$\Rightarrow$ $$5k = 1$$

$\Rightarrow$ $$k = \frac{1}{5}$$

Hence, P divides the line segment AB in the ratio 1 : 5.

Question 17.

The points A(4,7), B(p, 3) and C(7,3) are the vertices of a right triangle, right-angled at B. Find the value of p.

Solution:

In right ΔABC,

Using distance formula,

$$AC = \sqrt{3^2 + (-4)^2} = 5$$
$$AB = \sqrt{(p-4)^2 + 16}$$
$$BC = \sqrt{(p-7)^2 + 0}$$

Now, $AC^2 = AB^2 + BC^2$ [$\because$ Pythagoras theorem]

$\Rightarrow$ $25 = (p-4)^2 + 16 + (p-7)^2$

$25 = p^2 - 8p + 16 + 16 + p^2 - 14p + 49$

$\Rightarrow$ $2p^2 - 22p + 56 = 0$

$\Rightarrow$ $p^2 - 11p + 28 = 0$

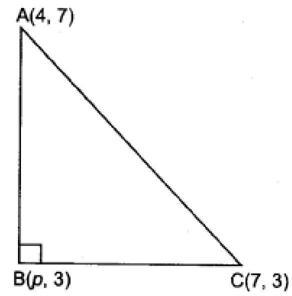
$(p-4)(p-7) = 0 \quad \Rightarrow \quad p = 4$ or $p = 7$

If $p = 7$, then $B = (7, 3)$

It coincides with C

$\therefore$ $p \neq 7$

Hence, $p = 4$



Question 18.

Find the relation between x and y if the points A(x, y), B(-5, 7) and C(-4, 5) are collinear

Solution:

$\because$ A, B and C are collinear. So, area (ΔABC) = 0

$\therefore$ $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow$ $\frac{1}{2}[x(7-5) + (-5)(5-y) + (-4)(y-7)] = 0$

$2x - 25 + 5y - 4y + 28 = 0$

$\Rightarrow$ Required relation between x & y is $2x + y + 3 = 0$

Question 19.

If A(4,3), B(-l,y) and C(3,4) are the vertices of right triangle ABC, right-angled at A, then find the value ofy.

Solution:

In right ΔCAB, using distance formula,

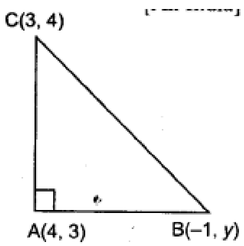
$$BC^2 = (3+1)^2 + (4-y)^2 = 16 + (4-y)^2$$
$$AB^2 = (-1-4)^2 + (y-3)^2 = 25 + (y-3)^2$$
$$AC^2 = (4-3)^2 + (3-4)^2 = 2$$

Also $BC^2 = AB^2 + AC^2$ [$\because$ Pythagoras theorem]

$\Rightarrow$ $16 + (4-y)^2 = 25 + (y-3)^2 + 2$

$16 + 16 + y^2 - 8y = 25 + y^2 - 6y + 9 + 2$

$-2y = 4 \quad \Rightarrow \quad y = -2$



Question 20.

Show that the points (a,a), (-a,-a) and (-√3a, √3 a) are the vertices of an equilateral triangle.

Solution:

Let $P(a, a)$, $Q(-a, -a)$, $R(-\sqrt{3}a, \sqrt{3}a)$. Using distance formula,

$$PQ = \sqrt{(a+a)^2 + (a+a)^2} = \sqrt{4a^2 + 4a^2} = 2\sqrt{2}a$$

$$QR = \sqrt{(-a+\sqrt{3}a)^2 + (-a-\sqrt{3}a)^2}$$
$$= \sqrt{a^2 + 3a^2 - 2\sqrt{3}a^2 + a^2 + 3a^2 + 2\sqrt{3}a^2} = \sqrt{8a^2} = 2\sqrt{2}a$$

$$RP = \sqrt{(a+\sqrt{3}a)^2 + (a-\sqrt{3}a)^2}$$
$$= \sqrt{a^2 + 3a^2 + 2\sqrt{3}a^2 + a^2 + 3a^2 - 2\sqrt{3}a^2} = \sqrt{8a^2} = 2\sqrt{2}a$$

$\Rightarrow$ Here, $PQ = QR = RP$

$\therefore$ P, Q, R are vertices of an equilateral triangle.

Question 21.

For what values of k are the points (8,1), (3, -2k) and (k, -5) collinear?

Solution:

For collinear points area of Δ made by these points will be zero.

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]=0$$

$$\Rightarrow \quad 8(-2k+5)+3(-5-1)+k(1+2k)=0$$

$$\Rightarrow \quad -16k+40-18+k+2k^2=0$$

$$2k^2-15k+22=0$$

$$2k^2-11k-4k+22=0$$

$$\Rightarrow \quad k(2k-11)-2(2k-11)=0$$

$$(2k-11)(k-2)=0$$

$$\Rightarrow \quad k=2, k=\frac{11}{2}$$

Short Answer Type Questions II [3 Marks]

Question 22.

Find the area of the triangle ABC with A(l, -4) and mid-points of sides through A being (2, -1) and (0, -1).

Solution:

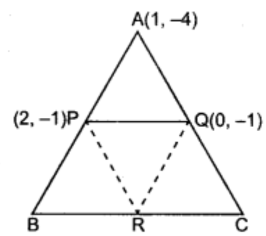
1. Firstly, $ar(\Delta APQ)=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$=\frac{1}{2}|1(-1+1)+2(-1+4)+0(-4+1)|$$

$$=\frac{1}{2}|0+6+0|=3 \text{ sq. units}$$

∵ P, Q and R are the mid point of sides AB, AC and BC respectively

So, $ar(\Delta ABC)=4\, ar(\Delta APQ)=4\times 3=12$ sq. units.



Question 23.

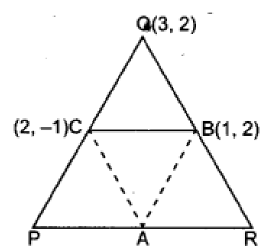
Find the area of the triangle PQR with Q (3, 2) and the mid-points of the sides through Q being (2, -1) and (1,2).

Solution:

Firstly, $ar(\Delta QCB)=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$=\frac{1}{2}|3(-1-2)+2(2-2)+1(2+1)|$$

$$=\frac{1}{2}|-9+0+3|=3 \text{ sq.units}$$



∵ A, B and C are the mid-points of sides PR, RQ and QP respectively.

So, $ar(\Delta QPR)=4\times ar(\Delta QCB)$

$$=4\times 3=12 \text{ sq. units}$$

Question 24.

If the coordinates of points A and B are (-2, -2) and (2, – 4) respectively, find the coordinates of P such that AP = 3/7 AB, where P lies on the line segment AB.

Solution:

Given, $AP = \frac{3}{7} AB \Rightarrow AP : PB = 3 : 4$

$\Rightarrow$ P divides AB in the ratio 3 : 4

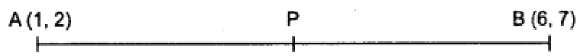
Using section formula,

$\therefore$ Coordinates of point P are $= \left(\frac{3\times2+4\times(-2)}{3+4}, \frac{3\times(-4)+4\times(-2)}{3+4}\right) = \left(\frac{-2}{7}, \frac{-20}{7}\right)$

Question 25.

Find the coordinates of a point P on the line segment joining A(l, 2) and B(6,7) such that AP=2/5AB

Solution:



Given, $AP = \frac{2}{5}AB$

$\therefore$ $AP : PB = 2 : 3 \Rightarrow$ P divides AB in ratio 2 : 3.

Using section formula,

Coordinates of point P are $\left(\frac{2\times6+3\times1}{2+3}, \frac{2\times7+3\times2}{2+3}\right)$ i.e. (3, 4)

$\therefore$ Coordinates of P are (3, 4)

Question 26.

Point A lies on the line segment PQ joining P(6, -6) and Q(-4, -1) in such a way that PA/PQ=2/5. If point P also lies on the line 3x + k(y + 1) = 0, find the value of k

Solution:

Coordinates of P are (6, –6). Given that:

$\because$ P(6, –6) lies on the line. So,

$$3x + k(y + 1) = 0$$
$$\Rightarrow 3 \times 6 + k(-6 + 1) = 0$$
$$\Rightarrow 18 - 5k = 0$$
$$\Rightarrow k = \frac{18}{5}.$$

Long Answer Type Questions [4 Marks]

Question 27.

If A(-4,8), B(-3, -4), C(0, – 5) and D(5,6) are the vertices of a quadrilateral ABCD, find its area.

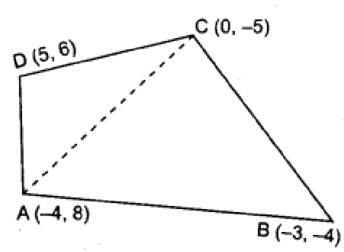
Solution:

Firstly, $ar(\Delta ABC) = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

$= \frac{1}{2}|-4(-4+5) - 3(-5-8) + 0(8+4)|$

$= \frac{1}{2}|-4 + 39 + 0| = \frac{1}{2} \times 35$

$= \frac{35}{2}$ sq. units



Now, $ar(\Delta ACD) = \frac{1}{2}|-4(-5-6) + 0(6-8) + 5(8+5)|$

$= \frac{1}{2}|44 + 0 + 65| = \frac{109}{2}$ sq. units

So, ar(quadrilateral ABCD) = ar(ΔABC) + ar(ΔACD)

$= \frac{35}{2} + \frac{109}{2} = \frac{144}{2} = 72$ sq. units

Question 28.

If P(-5, -3), Q (-4, -6), R(2, -3) and S(l, 2) are the vertices of a quadrilateral PQRS, find its area.

Solution:

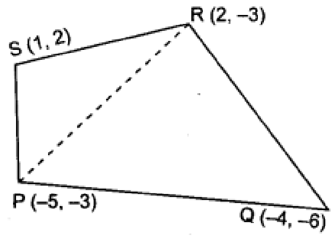
Firstly, $ar(\Delta PQR) = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

$= \frac{1}{2}|-5(-6+3) -4(-3-3) + 2(-3+6)|$

$= \frac{1}{2}|15+0+6| = \frac{21}{2}$ sq. units

Now, $ar(\Delta PRS) = \frac{1}{2}|-5(-3-2) + 2(2+3) + 1(-3+3)|$

$= \frac{1}{2}|25+10+0| = \frac{35}{2}$ sq. units



So, ar(quad PQRS) = $ar(\Delta PQR) + ar(\Delta PRS)$

$= \frac{21}{2} + \frac{35}{2} = \frac{56}{2} = 28$ sq. units.

Question 29.

Find the values of k so that the area of the triangle with vertices (1, -1), (-4,2k) and (-k, – 5) is 24 sq. units.

Solution:

Given that, Area of Δ = 24 sq. units

$\Rightarrow \frac{1}{2}|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)| = 24$

$|1(2k+5) - 4(-5+1) - k(-1-2k)| = 48$

$\Rightarrow |2k+5+16+k+2k^2| = 48$

$\Rightarrow |2k^2+3k+21| = 48$

$\Rightarrow 2k^2+3k+21 = \pm 48$

$\Rightarrow 2k^2+3k+21 = 48$ or $2k^2+3k+21 = -48$

$2k^2+3k-27 = 0$

$2k^2+9k-6k-27 = 0$

$k(2k+9)-3(2k+9) = 0$

$(2k+9)(k-3) = 0$

$\Rightarrow k = \frac{-9}{2}$ or $k = 3$

$2k^2+3k+69 = 0$

Discriminant, D $= (3)^2 - 4\times 2\times 69\ [\because b^2-4ac]$

$= -ve$

$\therefore$ No solution

Question 30.

Find the values of k for which the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 1, 5k) are collinear.

Solution:

∵ The points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 15k) are collinear. So, ar (ΔABC) = 0

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$\frac{1}{2}[(k+1)(2k+3-5k)+3k(5k-2k)+(5k-1)(2k-2k-3)] = 0$$

$$\Rightarrow \quad (k+1)(-3k+3)+3k\times 3k+(5k-1)(-3) = 0$$

$$\Rightarrow \quad -3k^2+3k-3k+3+9k^2-15k+3 = 0$$

$$\Rightarrow \quad 6k^2-15k+6 = 0$$

$$\Rightarrow \quad 2k^2-5k+2 = 0$$

$$2k^2-4k-k+2 = 0$$

$$2k(k-2)-1(k-2) = 0$$

$$\Rightarrow \quad (k-2)(2k-1) = 0$$

$$\Rightarrow \quad k = 2 \text{ or } k = \frac{1}{2}.$$

Question 31.

The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point C are (0, -3). The origin is the mid-point of the base. Find the coordinates of the points A and B. Also find the coordinates of another point D such that BACD is a rhombus.

Solution:

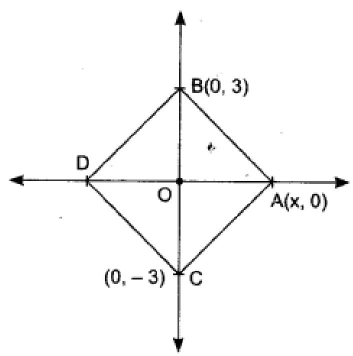
Given that, ∵ O is mid point of BC and coordinates of C are (0, – 3)

∴ coordinate of B are (0, 3)

Now AO will be the perpendicular bisector of BC. Therefore A will lie on x-axis. let coordinates of A are $(x, 0)$

Now, in equilateral ΔABC, AB= BC

Using distance formula,



$$\Rightarrow \quad \sqrt{(x-0)^2+(0-3)^2} = 6$$

$$\sqrt{x^2+9} = 6$$

$$x^2+9 = 36 \Rightarrow x^2 = 27$$

$$x = \pm 3\sqrt{3}$$

∴ coordinates of A are $(3\sqrt{3}, 0)$ or $(-3\sqrt{3}, 0)$

When A is $(3\sqrt{3}, 0)$ then D will be $(-3\sqrt{3}, 0)$ so that BACD is a rhombus, since opposite sides are equal.

2014

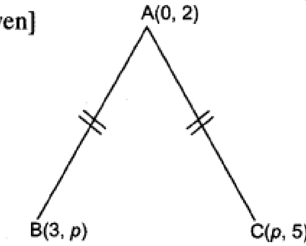
Short Answer Type Questions II [3 Marks]

Question 32.

If the point A(0, 2) is equidistant from the points B(3, p) and C(p, 5), find p. Also find the length of AB.

Solution:

Here, $AB = AC$ [Given]



$\Rightarrow AB^2 = AC^2$

Using distance formula,

$\Rightarrow (3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$\Rightarrow 9 + p^2 - 4p + 4 = p^2 + 9$

$\Rightarrow -4p + 4 = 0$

$\Rightarrow 4p = 4$

$\Rightarrow p = 1$

So, point B is (3, 1); point C is (1, 5)

Now, length of $AB = \sqrt{(3-0)^2+(1-2)^2}$ [$\because$ use distance formula]

$= \sqrt{9+1} = \sqrt{10}$ units

Question 33.

If the points A(-2,1), B (a, b) and C(4, -1) are collinear and a – b = 1, find the value of a and b.

Solution:

Since, the points A(–2, 1), B(a, b) and C(4, –1) are collinear,

So, area of triangle, $ar(\Delta ABC) = 0$

$\frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$

$\Rightarrow \frac{1}{2}\ |-2(b+1) + a(-1-1) + 4(1-b)| = 0$

$\Rightarrow -2b - 2 - 2a + 4 - 4b = 0 \quad \Rightarrow 2a + 6b = 2$

$\Rightarrow a + 3b = 1$ ...(*i*)

Also, given that $a - b = 1$ ...(*ii*)

On solving the equations (*i*) and (*ii*), we get

$a = 1, b = 0$

Question 34.

If the points P(-3,9), Q(a, b) and R(4, -5) are collinear and a + b = 1, find the value of a and b.

Solution:

Since, the points P(–3, 9), Q(a, b) and R(4, –5) are collinear,

So, area of triangle $ar(\Delta PQR) = 0$

$\frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$

$\Rightarrow \frac{1}{2}\ |-3(b+5) + a(-5-9) + 4(9-b)| = 0$

$\Rightarrow -3b - 15 - 14a + 36 - 4b = 0 \quad \Rightarrow 14a + 7b - 21 = 0$

$\Rightarrow 2a + b = 3$ ...(i)

Also, given that $a + b = 1$ ...(ii)

On solving the equations (i) and (ii), we get

$a = 2, b = -1$

Question 35.

Points A(-l, y) and B(5,7) lie on a circle with centre 0(2, -3y). Find the values ofy. Hence, find the radius of the circle.

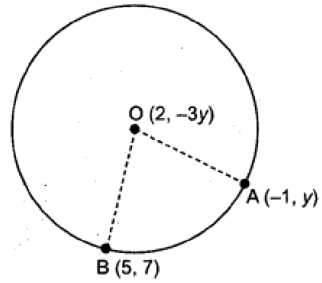
Solution:

Given, O is the centre of the circle and the points A and B lie on the circle.

So, $OA = OB (= r)$ [$\because$ radius of same circle]

$\Rightarrow OA^2 = OB^2$

Using distance formula,



$\Rightarrow (2+1)^2 + (-3y-y)^2 = (2-5)^2 + (-3y-7)^2$

$\Rightarrow 9 + 16y^2 = 9 + 9y^2 + 42y + 49$

$\Rightarrow 7y^2 - 42y - 49 = 0$

$\Rightarrow y^2 - 6y - 7 = 0$

$\Rightarrow (y-7)(y+1) = 0$

$\Rightarrow y = -1 \text{ or } 7$

When $y = -1$, then co-ordinates are: O(2, 3) and A(–1, –1)

$$\text{Radius of circle, } r = OA = \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{9+16} = 5 \text{ units}$$

When $y = 7$, then coordinates are: O(2, – 21) and A(–1, 7)

$$\text{Radius of circle, } r = OA = \sqrt{(2+1)^2 + (-21-7)^2} = \sqrt{9+784} = \sqrt{793} \text{ units.}$$

Question 36.

If the point A(-l, -4); B(b, c) and C(5, -1) are collinear and 2b+ c = 4, find the value of b and c.

Solution:

Since, the points A(–1, –4), B(b, c) and C(5, –1) are collinear,

So, area of triangle, ar (ΔABC) = 0

$$\frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$$

$$\Rightarrow \frac{1}{2}|-1(c+1) + b(-1+4) + 5(-4-c)| = 0$$

$\Rightarrow -c - 1 + 3b - 20 - 5c = 0 \quad \Rightarrow 3b - 6c = 21$

$\Rightarrow b - 2c = 7$ ...(*i*)

Also, given that $2b + c = 4$ ...(*ii*)

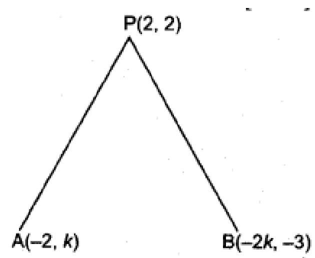
Question 37.

If the point P(2, 2) is equidistant from the points A(-2, k) and B(-2k, -3), find k.

Solution:

Since, given that $PA = PB \Rightarrow PA^2 = PB^2$

Using distance formula,



$\Rightarrow (+2+2)^2 + (2-k)^2 = (2+2k)^2 + (2+3)^2$

$\Rightarrow 16 + 4 - 4k + k^2 = 4 + 8k + 4k^2 + 25$

$\Rightarrow 16 - 4k + k^2 = 8k + 4k^2 + 25$

$\Rightarrow 3k^2 + 12k + 9 = 0$

$\Rightarrow k^2 + 4k + 3 = 0$

$\Rightarrow (k+3)(k+1) = 0$

$\Rightarrow k = -1 \text{ or } -3$

When $k = -1$, then point A is (–2, –1)

$$AP = \sqrt{(2+2)^2 + (2+1)^2} = \sqrt{16+9} = 5 \text{ units}$$

When $k = -3$, then point A is (–2, –3)

$$AP = \sqrt{(2+2)^2 + (2+3)^2} = \sqrt{16+25} = \sqrt{41} \text{ units}$$

Question 38.

If the point P(k – 1, 2) is equidistant from the points A(3, k) and B(k, 5), find the values of k.

Solution:

Given that point P(k – 1, 2) is equidistant from the points A(3, k) and B(k, 5), so

$$\therefore \quad AP = BP \quad \Rightarrow AP^2 = BP^2$$

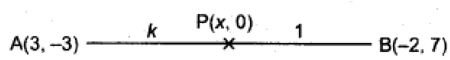
Using distance formula,

$$\Rightarrow \quad (k-1-3)^2 + (2-k)^2 = (k-1-k)^2 + (2-5)^2$$
$$\Rightarrow \quad (k-4)^2 + (2-k)^2 = (-1)^2 + (-3)^2$$
$$\Rightarrow \quad k^2 - 8k + 16 + 4 - 4k + k^2 = 1 + 9$$
$$\Rightarrow \quad 2k^2 - 12k + 10 = 0$$
$$\Rightarrow \quad k^2 - 6k + 5 = 0$$
$$\Rightarrow \quad k^2 - 5k - k + 5 = 0$$
$$\Rightarrow \quad k(k-5) - 1(k-5) = 0$$
$$\Rightarrow \quad (k-1)(k-5) = 0$$
$$\Rightarrow \text{ Either} \quad k - 1 = 0 \text{ or } k - 5 = 0$$
$$\Rightarrow \quad k = 1 \text{ or } 5$$

Question 39.

Find the ratio in which the line segment joining the points A(3, -3) and B(-2, 7) is divided by x-axis. Also find the coordinates of the point of division.

Solution:



Let point P(x, 0) on x-axis divides the join of A(3, –3) and B(–2, 7) in the ratio k : 1

Then coordinates of P are $\left(\frac{-2k+3}{k+1}, \frac{7k-3}{k+1}\right)$

If point P lies on x-axis, then y coordinate of P is 0.

$$\Rightarrow \quad \frac{7k-3}{k+1} = 0 \quad \Rightarrow \quad 7k - 3 = 0$$
$$\Rightarrow \quad k = \frac{3}{7}$$

$\therefore$ Ratio is $\frac{3}{7} : 1$, i.e. 3 : 7.

Putting $\quad k = \frac{3}{7}$ in (i), we get

the coordinates of point P = $\left(\frac{-\frac{6}{7}+3}{\frac{3}{7}+1}, 0\right)$, i.e. $\left(\frac{3}{2}, 0\right)$.

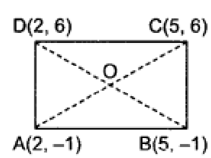
Question 40.

Prove that the diagonals of a rectangle ABCD, with vertices A(2, – 1), B(5, – 1), C(5,6) and D(2,6), are equal and bisect each other.

Solution:

Given; A(2, –1), B(5, –1), C(5, 6) and D(2, 6) are the vertices of a rectangle ABCD.

Using distance formula,



$$\therefore \quad AC = \sqrt{(5-2)^2 + (6+1)^2} = \sqrt{9+49} = \sqrt{58} \text{ units}$$
$$BD = \sqrt{(5-2)^2 + (-1-6)^2} = \sqrt{9+49} = \sqrt{58} \text{ units}$$
$$\therefore \quad AC = BD, \text{ i.e. diagonals are equal}$$

Now, the coordinates of mid-point of AC are $\left(\frac{2+5}{2}, \frac{6-1}{2}\right)$, i.e. $\left(\frac{7}{2}, \frac{5}{2}\right)$

The coordinates of mid-point of BD are $\left(\frac{5+2}{2}, \frac{-1+6}{2}\right)$, i.e. $\left(\frac{7}{2}, \frac{5}{2}\right)$

As the coordinates of the mid-points of AC and BD are same, hence diagonals bisect each other.

Question 41.

Find a point P on they-axis which is equidistant from the points A(4,8) and B(- 6, 6). Also find the distance AP.

Solution:

Let point $P(0, y)$ on y-axis is equidistant from the points A(4, 8) and B(–6, 6).

$\therefore \quad AP = BP \quad \Rightarrow \quad AP^2 = BP^2$

Using distance formula,

$\Rightarrow \quad (4-0)^2 + (8-y)^2 = (-6-0)^2 + (6-y)^2$

$\Rightarrow \quad 16 + 64 - 16y + y^2 = 36 + 36 - 12y + y^2$

$\Rightarrow \quad -4y = -8 \quad \Rightarrow y = 2$

$\therefore$ Point is P(0, 2)

Distance $AP = \sqrt{(4-0)^2 + (8-2)^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$ units

Question 42.

Find the value(s) of k for which the points (3k – 1, k – 2), (k, k-1) and (k – 1, – k – 2) are collinear

Solution:

Since the points $(3k-1, k-2)$, $(k, k-7)$ and $(k-1, -k-2)$ are collinear, so area of triangle formed by these points is zero.

$\Rightarrow \quad \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$

$\Rightarrow \quad \frac{1}{2}[(3k-1)(k-7+k+2) + k(-k-2-k+2) + (k-1)(k-2-k+7)] = 0$

$\Rightarrow \quad \frac{1}{2}[(3k-1)(2k-5) + k(-2k) + (k-1)(5)] = 0$

$\Rightarrow \quad \frac{1}{2}[6k^2 - 15k - 2k + 5 - 2k^2 + 5k - 5] = 0$

$\Rightarrow \quad \frac{1}{2}[4k^2 - 12k] = 0$

$\Rightarrow \quad 2k^2 - 6k = 0$

$\Rightarrow \quad 2k(k-3) = 0$

$\Rightarrow$ Either $\quad k = 0$ or $k - 3 = 0$

$\Rightarrow \quad k = 0$ or $k = 3$.

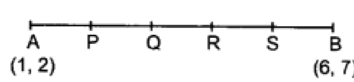
Question 43.

points P, Q, R and S divide the line segment joining the points A(l, 2) and B(6, 7) in 5 equal parts. Find the coordinates of the points P, Q and R.

Solution:

Line segment that joins points A(1, 2) and B(6, 7) is divided by points P, Q, R, S into 5 equal parts

$\therefore \quad AP = PQ = QR = RS = SB$



Use section formula $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

P divides the join of A and B in ratio 1 : 4.

$\therefore$ The coordinates of P are $\left(\frac{6+4}{1+4}, \frac{7+8}{1+4}\right)$, i.e. P(2, 3).

R divides the join of A and B in the ratio 3 : 2

$\therefore$ The coordinates of R are $\left(\frac{18+2}{3+2}, \frac{21+4}{3+2}\right)$, i.e. R(4, 5).

Now, Q is the midpoint PR

$\therefore$ The coordinates of Q are $\left(\frac{12+3}{5}, \frac{14+6}{5}\right)$, i.e. (3, 4)

Question 44.

Find the value(s) of p for which the points (p + 1, 2p – 2), (p – 1 ,p) and (p -6, 2p – 6) are collinear.

Solution:

Since, the points $(p + 1, 2p - 2)$, $(p - 1, p)$ and $(p - 3, 2p - 6)$ are collinear, so, area of triangle formed by these points is zero.

$$\Rightarrow \quad \frac{1}{2}[(p + 1)(p - 2p + 6) + (p - 1)(2p - 6 - 2p + 2) + (p - 3)(2p - 2 - p)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(p + 1)(6 - p) + (p - 1)(-4) + (p - 3)(p - 2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(6p - p^2 + 6 - \text{p} - 4p + 4 + p^2 - 2p - 3p + 6] = 0$$

$$\Rightarrow \quad \frac{1}{2}[-4p + 16] = 0 \quad \Rightarrow -2p + 8 = 0$$

$$\Rightarrow \quad -2p = -8 \Rightarrow p = 4$$

Question 45.

Find the value(s) of p for which the points (3p + 1, p), (p + 2, p – 5) and (p + 1, -p) are collinear.

Solution:

Since the points $(3p + 1, p)$, $(p + 2, p - 5)$ and $(p + 1, -p)$ are collinear, so area of the triangle formed by these points is zero.

$$\Rightarrow \quad \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(3p + 1)(p - 5 + p) + (p + 2)(-p - p) + (p + 1)(p - p + 5)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(3p + 1)(2p - 5) + (p + 2)(-2p) + 5(p + 1)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(6p^2 - 15p + 2p - 5 - 2p^2 - 4p + 5p + 5] = 0$$

$$\Rightarrow \quad \frac{1}{2}[(4p^2 - 12p] = 0$$

$$\Rightarrow \quad 2p^2 - 6p = 0$$

$$\Rightarrow \quad 2p(p - 3) = 0$$

$\Rightarrow$ Either $\quad p = 0$ or $p - 3 = 0 \Rightarrow p = 0, 3$

Long Answer Type Questions [4 Marks]

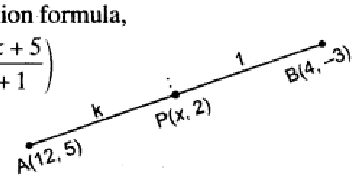
Question 46.

Find the ratio in which the point P(x, 2) divides the line segment joining the points A(12,5) and B(4, -3). Also, find the value of x.

Solution:

Let point P$(x, 2)$ divides AB in the ratio $k : 1$. Using section formula,

then the coordinates of P in terms of k is $\text{P}\left(\frac{4k + 12}{k + 1}, \frac{-3k + 5}{k + 1}\right)$

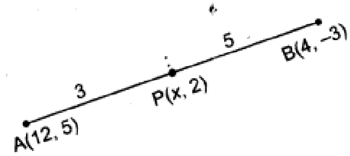


A.T.Q. $\quad \frac{-3k + 5}{k + 1} = 2$

$$\Rightarrow \quad -3k + 5 = 2k + 2$$

$$\Rightarrow \quad -5k = -3$$

$$\Rightarrow \quad k = \frac{3}{5}$$



Thus, P divides AB in the ratio 3 : 5

Now, $x = \frac{3 \times 4 + 5 \times 12}{3 + 5} = \frac{12 + 60}{8} = \frac{72}{8} = 9$

Hence $\quad x = 9$

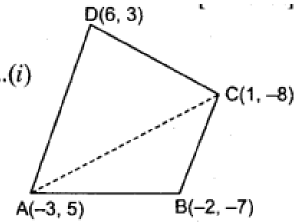
Question 47.

If A(-3,5), B(-2, -7), C(l, -8) and D(6,3) are the vertices of a quadrilateral ABCD, find its area.

Solution:

Area of quadrilateral ABCD

= Area of triangle ABC + Area of triangle ACD ...(*i*)



Now, $\text{ar(ABC)} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-3(-7+8) - 2(-8-5) + 1(5+7)]$

$= \frac{1}{2}[-3 + 26 + 12] = \frac{35}{2}$ sq. units ...(*ii*)

Also, $\text{ar(ACD)} = \frac{1}{2}[-3(-8-3) + 1(3-5) + 6(5+8)]$

$= \frac{1}{2}[33 - 2 + 78] = \frac{109}{2}$ sq. units ...(*iii*)

From (*i*), (*ii*), (*iii*), we get

$\text{ar(ABCD)} = \frac{35}{2} + \frac{109}{2} = \frac{144}{2} = 72$ sq. units

Question 48.

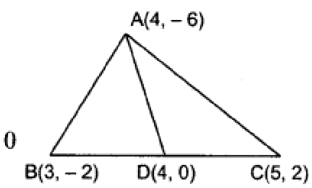
A(4, -6), B(3, -2) and C(5, 2) are the vertices of a AABC and AD is its median. Prove that the median AD divides AABC into two triangles of equal areas.

Solution:

A(4, –6), B(3, –2) and C(5, 2) are the vertices of ΔABC.

∵ AD is the median, so, D is the mid-point of BC.

∴ Coordinates of D $= \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4, 0)$



Now, $\text{ar}(ABD) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$= \left|\frac{1}{2}[3(0+6) + 4(-6+2) + 4(-2-0)]\right|$

$= \left|\frac{1}{2}[18 - 16 - 8]\right| = \frac{1}{2}|-6| = 3$ sq. units

$\text{ar(ADC)} = \left|\frac{1}{2}[4(2+6) + 5(-6-0) + 4(0-2)]\right|$

$= \left|\frac{1}{2}[32 - 30 - 8]\right| = \frac{1}{2}|-6| = 3$ sq. units

∴ Clearly, ar(ABD) = ar(ADC).

Thus, median AD divides ΔABC into 2 triangles of equal areas.

Question 49.

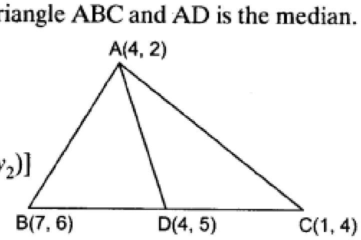
If A(4,2), B(7,6) and C(l, 4) are the vertices of a ΔABC and AD is its median, prove that the median AD divides ΔABC into two triangles of equal areas

Solution:

Given; A(4, 2), B(7, 6) and C(1, 4) are the vertices of a triangle ABC and AD is the median.

∴ D is the mid-point of BC as AD is median

∴ The coordinates of D are $\left(\frac{7+1}{2}, \frac{6+4}{2}\right)$, i.e. (4, 5).



Now, $\text{ar}(ABD) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \left|\frac{1}{2}[4(6-5) + 7(5-2) + 4(2-6)]\right|$

$= \left|\frac{1}{2}[4 + 21 - 16]\right| = \frac{9}{2}$ sq. units

$\text{ar}(ADC) = \left|\frac{1}{2}[4(5-4) + 4(4-2) + 1(2-5)]\right| = \left|\frac{1}{2}[4 + 8 - 3]\right| = \frac{9}{2}$ sq. units

∴ Clearly, ar(ABD) = ar(ADC).

Thus, median AD divides ΔABC into 2 triangles of equal areas.

Question 50.

The mid-point P of the line segment joining the points A(- 10, 4) and B(- 2, 0) lies on the line segment joining the points C(- 9, – 4) and D(- 4, y). Find the ratio in which P divides CD. Also find the value of y.

Solution:

$\because$ P is the mid-point of the line segment joining A(–10, 4) and B(–2, 0).

$\therefore$ The coordinates of P are $\left(\frac{-10-2}{2}, \frac{4+0}{2}\right)$, i.e. P(–6, 2). ...(*i*)

Let P(–6, 2) divides the join of C(–9, –4) and D(–4, *y*) in the ratio *k* : 1. Using section formula,

$\therefore$ The coordinates of P are $\left(\frac{-4k-9}{k+1}, \frac{ky-4}{k+1}\right)$ ...(*ii*)

$\therefore$ From (*i*), (*ii*)

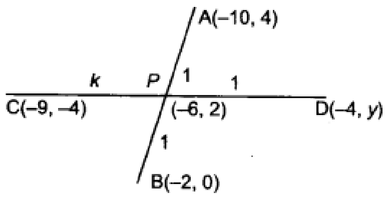
A.T.Q. $\frac{-4k-9}{k+1} = -6$ and $\frac{ky-4}{k+1} = 2$ ...(*iii*)

Consider, $\frac{-4k-9}{k+1} = -6$

$\Rightarrow$ $-4k-9 = -6k-6$

$\Rightarrow$ $2k = 3$

$\Rightarrow$ $k = \frac{3}{2}$



So, P divides CD in the ratio 3 : 2

From (*iii*), $\frac{\frac{3}{2}y-4}{\frac{3}{2}+1} = 2$

$\Rightarrow$ $\frac{3y-8}{3+2} = 2 \Rightarrow 3y-8 = 10$

$\Rightarrow$ $3y = 18 \Rightarrow y = 6$

2013

Short Answer Type Questions II [3 Marks]

Question 51.

Prove that the points (7,10), (-2,5) and (3, -4) are the vertices of an isosceles right triangle.

Solution:

Let A (7, 10); B(–2, 5); C(3, –4) be vertices of isosceles right triangle.

Now, using distance formula,

$$AB = \sqrt{(7+2)^2+(10-5)^2} = \sqrt{81+25} = \sqrt{106} \text{ units}$$

$$BC = \sqrt{(-2-3)^2(5+4)^2} = \sqrt{25+81} = \sqrt{106} \text{ units}$$

$$CA = \sqrt{(3-7)^2+(-4-10)^2} = \sqrt{16+196} = \sqrt{212} \text{ units}$$

Clearly,

$\therefore$ $(\sqrt{212})^2 = (\sqrt{106})^2 + (\sqrt{106})^2$

$\Rightarrow$ $AC^2 = AB^2 + BC^2$

$\Rightarrow$ $\angle ABC = 90°$ [$\because$ Follows converse of Pythagoras theorem]

Here, $AB = BC$ and $\angle ABC = 90°$

So, ΔABC is an isosceles right triangle.

Question 52.

Find, the ratio in which the y-axis divides the line segment joining the points (-4, -6) and (10,12). Also find the coordinates of the point of division.

Solution:

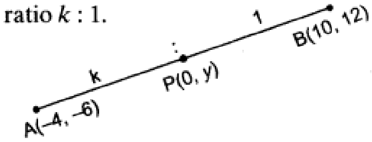
Let point P (0, y) which lies on y-axis divides AB in the ratio $k : 1$.

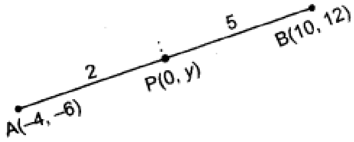
Then by section formula, in x-co-ordinates,

$$\frac{10k - 4}{k+1} = 0 \Rightarrow 10k - 4 = 0$$

$$\Rightarrow \quad k = \frac{4}{10} = \frac{2}{5}$$

Hence, point P divides AB in the ratio 2 : 5





$$\text{Now, } y = \frac{2 \times 12 - 5 \times (-6)}{2+5} = \frac{24-30}{7} = -\frac{6}{7}.$$

[Applying section formula for y-coordinates]

Hence, Coordinates of P are $\left(0, \frac{-6}{7}\right)$.

Question 53.

Prove that the points A(0, -1), B(-2, 3), C(6, 7) and D(8, 3) are the vertices of a rectangle ABCD?

Solution:

Using distance formula,

$$AB = \sqrt{(-2-0)^2 + (3+1)^2} = \sqrt{4+16} = \sqrt{20} \text{ units}$$

$$BC = \sqrt{(6+2)^2 + (7-3)^2} = \sqrt{64+16} = \sqrt{80} \text{ units}$$

$$CD = \sqrt{(8-6)^2 + (3-7)^2} = \sqrt{4+16} = \sqrt{20} \text{ units}$$

$$DA = \sqrt{(8-0)^2 + (3+1)^2} = \sqrt{64+16} = \sqrt{80} \text{ units}$$

Also $\quad AC = \sqrt{(6-0)^2 + (7+1)^2} = \sqrt{36+64} = \sqrt{100} = 10$ units

and $\quad BD = \sqrt{(8+2)^2 + (3-3)^2} = \sqrt{100+0} = 10$ units

Since, $\quad AB = CD$ and $BC = DA$

and diagonal $AC = BD$

In quadrilateral ABCD, opposite sides are equal and both the diagonals are equal. Therefore, ABCD is a rectangle.

Question 54.

Show that the points (-2,3) (8,3) and (6, 7) are the vertices of a right triangle.

Solution:

Using distance formula,

Consider A(–2, 3), B(8, 3) and C(6, 7)

Now, $\quad AB^2 = (8 + 2)^2 + (3 - 3)^2 = 100$ units

$BC^2 = (6 - 8)^2 + (7 - 3)^2 = 4 + 16 = 20$ units

$AC^2 = (6 + 2)^2 + (7 - 3)^2 = 64 + 16 = 80$ units

Clearly, $\quad 100 = 20 + 80$

$\Rightarrow \quad AB^2 = BC^2 + AB^2$

So, by converse of Pythagoras theorem, ΔABC is a right triangle.

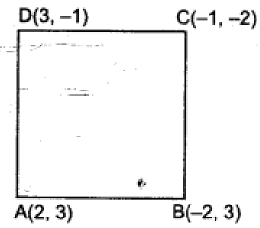
Question 55.

Prove that the points A(2, 3), B(-2, 2), C(-l, -2) and D(3, -1) are the vertices of a square ABCD.

Solution:

Using distance formula,

$$AB = \sqrt{(-2-2)^2+(2-3)^2} = \sqrt{(-4)^2+(-1)^2}$$
$$= \sqrt{16+1} = \sqrt{17} \text{ units}$$
$$BC = \sqrt{(-2+1)^2+(2+2)^2} = \sqrt{(-1)^2+(4)^2}$$
$$= \sqrt{1+16} = \sqrt{17} \text{ units}$$
$$CD = \sqrt{(-1-3)^2+(-2+1)^2} = \sqrt{(-4)^2+(-1)^2}$$
$$= \sqrt{16+1} = \sqrt{17} \text{ units}$$
$$DA = \sqrt{(3-2)^2+(-1-3)^2} = \sqrt{(1)^2+(-4)^2}$$
$$= \sqrt{1+16} = \sqrt{17} \text{ units}$$



Also, $$AC = \sqrt{(-1-2)^2+(-2-3)^2} = \sqrt{(-3)^2+(-5)^2}$$
$$= \sqrt{9+25} = \sqrt{34} \text{ units}$$
$$BD = \sqrt{(-2-3)^2+(2+1)^2} = \sqrt{(-5)^2+(3)^2}$$
$$= \sqrt{25+9} = \sqrt{34} \text{ units}$$

Since, AB = BC = CD = DA and AC = BD

⇒ ABCD is a square, because all sides are equal. Diagonals are also equal.

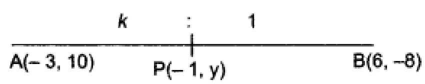
Question 56.

Find the ratio in which point P(-l, y) lying on the line segment joining points A(-3,10) and B(6, -8) divides it. Also find the value of y.

Solution:

Let point P(–1, y) divides AB in ratio $k : 1$.

Using sections formula for x-coordinates;



$$\therefore \quad -1 = \frac{6k-3}{k+1}$$
$$-k-1 = 6k-3$$
$$-7k = -2$$
$$k = \frac{2}{7}$$

Hence point P divides AB in the ratio 2 : 7.

Now, again using section formula for y-coordinates,

$$y = \frac{-8k+10}{k+1}$$
$$\Rightarrow \quad y = \frac{-8\left(\frac{2}{7}\right)+10}{\frac{2}{7}+1} = \frac{\frac{-16+70}{7}}{\frac{2+7}{7}} = \frac{70-16}{2+7} = \frac{54}{9} = 6$$

∴ Coordinates of P are (–1, 6).

Question 57.

Prove that the points A(2, -1), B(3, 4), C(-2, 3) and D(-3, -2) are the vertices of a rhombus ABCD. Is ABCD a square?

Solution:

Using distance formula,

$$AB = \sqrt{(3-2)^2+(4+1)^2} = \sqrt{(1)^2+(5)^2} = \sqrt{1+25} = \sqrt{26} \text{ units}$$
$$BC = \sqrt{(3+2)^2+(4-3)^2} = \sqrt{(5)^2+(1)^2} = \sqrt{25+1} = \sqrt{26} \text{ units}$$
$$CD = \sqrt{(-2+3)^2+(3+2)^2} = \sqrt{(1)^2+(5)^2} = \sqrt{25+1} = \sqrt{26} \text{ units}$$
$$DA = \sqrt{(-3-2)^2+(-2+1)^2} = \sqrt{(-5)^2+(-1)^2} = \sqrt{25+1} = \sqrt{26} \text{ units}$$

Also, $$AC = \sqrt{(-2-2)^2+(3+1)^2} = \sqrt{(-4)^2+(4)^2} = \sqrt{16+16} = \sqrt{32} \text{ units}$$
$$BD = \sqrt{(3+3)^2+(4+2)^2} = \sqrt{(6)^2+(6)^2} = \sqrt{36+36} = \sqrt{72} \text{ units}$$

$\because$ AB = BC = CD = DA. But AC $\neq$ BD

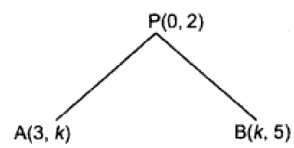
$\Rightarrow$ ABCD is a rhombus, not a square.

Question 58.

Find that value of k for which the point (0, 2) is equidistant from two points (3, k) and (k, 5).

Solution:

Consider point P(0, 2) is equidistant from A(3, k) and B(k, 5)



Given that, PA = PB

Using distance formula,

$$\sqrt{(0-3)^2+(2-k)^2} = \sqrt{(k-0)^2+(5-2)^2}$$
$$\sqrt{9+4+k^2-4k} = \sqrt{k^2+9}$$

Squaring both sides

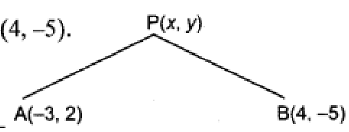
$$k^2 - 4k + 13 = k^2 + 9$$
$$k^2 - 4k + 13 - 9 - k^2 = 0$$
$$-4k = -4 \quad \Rightarrow k = 1$$

Question 59.

If the point P(x,y) is equidistant from two points A (-3,2) and B (4, -5), prove that y = x- 2.

Solution:

Point P(x, y) is equidistant from the points A(–3, 2) and B(4, –5).



$\therefore$ PA = PB [Given]

Using distance formula,

$$\Rightarrow \quad \sqrt{(-3-x)^2+(2-y)^2} = \sqrt{(4-x)^2+(-5-y)^2}$$

Squaring both sides, we get

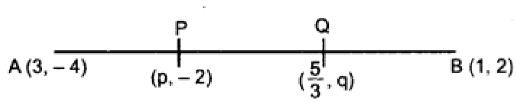
$$9 + x^2 + 6x + 4 + y^2 - 4y = 16 + x^2 - 8x + 25 + y^2 + 10y$$
$$\Rightarrow \quad 6x - 4y + 13 = -8x + 10y + 41$$
$$\Rightarrow \quad -4y - 10y = 41 - 13 - 8x - 6x$$
$$\Rightarrow \quad -14y = -14x + 28$$
$$\Rightarrow \quad -14y = -14(x-2)$$
$$\Rightarrow \quad y = x - 2$$

Question 60.

The line segment AB joining the points A(3, – 4), and B(l, 2) is trisected at the points P(p, – 2) and Q(5/3, q). Find the values of p and q.

Solution:

Now, again AP : PB = 1 : 2. Using distance formula,



$$\therefore \quad p = \frac{1\times1+2\times3}{1+2}$$
$$\Rightarrow \quad p = \frac{7}{3}$$

Also, AQ : QB = 2 : 1. Again using section formula,

$$\Rightarrow \quad q = \frac{2\times2+1\times-4}{1+2} = 0 \quad \Rightarrow \quad q = 0$$

Question 61.

If the point A (x, y) is equidistant from two points P (6, -1) and Q (2,3), prove that y = x – 3.

Solution:

Point A(x, y) is equidistant from P(6, –1) and Q(2, 3). Using distance formula,

$$PA = AQ$$

$$\Rightarrow \quad \sqrt{(6-x)^2+(-1-y)^2} = \sqrt{(2-x)^2+(3-y)^2}$$

Squaring both sides, we get

$$(6-x)^2 + (-1-y)^2 = (2-x)^2 + (3-y)^2$$

$$\Rightarrow \quad 36 + x^2 - 12x + 1 + y^2 + 2y = 4 + x^2 - 4x + 9 + y^2 - 6y$$

$$\Rightarrow \quad -12x + 2y + 37 = -4x - 6y + 13$$

$$\Rightarrow \quad 2y + 6y = 13 - 4x + 12x - 37$$

$$\Rightarrow \quad 8y = 8x - 24$$

$$\Rightarrow \quad y = x - 3$$

Hence, proved.

Question 62.

If the point R (x, y) is equidistant from two points P (- 3, 4) and Q (2, – 1), prove that y = x + 2.

Solution:

Point R(x, y) is equidistant from the points P(–3, 4) and Q(2, –1). Using distance formula,

$$\therefore \quad PR = RQ$$

$$\sqrt{(x+3)^2+(y-4)^2} = \sqrt{(x-2)^2+(y+1)^2}$$

Squaring both sides, we get

$$(x+3)^2 + (y-4)^2 = (x-2)^2 + (y+1)^2$$

$$\Rightarrow \quad x^2 + y^2 + 6x - 8y + 9 + 16 = x^2 + y^2 - 4x + 2y + 4 + 1$$

$$\Rightarrow \quad 6x - 8y + 25 = -4x + 2y + 5$$

$$\Rightarrow \quad -8y - 2y = -4x - 6x + 5 - 25$$

$$\Rightarrow \quad -10y = -10x - 20$$

$$\Rightarrow \quad -10y = -10(x + 2)$$

$$\Rightarrow \quad y = x + 2$$

Hence, proved.

Long Answer Type Questions [4 Marks]

Question 63.

If the area of AABC formed by A(x, y), B(l, 2) and C(2, 1) is 6 square units, then prove that x +y= 15.

Solution:

Point R(x, y) is equidistant from the points P(–3, 4) and Q(2, –1). Using distance formula,

$$\therefore \quad PR = RQ$$

$$\sqrt{(x+3)^2+(y-4)^2} = \sqrt{(x-2)^2+(y+1)^2}$$

Squaring both sides, we get

$$(x+3)^2 + (y-4)^2 = (x-2)^2 + (y+1)^2$$

$$\Rightarrow \quad x^2 + y^2 + 6x - 8y + 9 + 16 = x^2 + y^2 - 4x + 2y + 4 + 1$$

$$\Rightarrow \quad 6x - 8y + 25 = -4x + 2y + 5$$

$$\Rightarrow \quad -8y - 2y = -4x - 6x + 5 - 25$$

$$\Rightarrow \quad -10y = -10x - 20$$

$$\Rightarrow \quad -10y = -10(x + 2)$$

$$\Rightarrow \quad y = x + 2$$

Hence, proved.

Question 64.

Find the value of x for which the points (x – 1), (2,1) and (4,5) are collinear

Solution:

Since the point A(x, –1), B(2, 1) and C(4, 5) are collinear

So, area of triangle, ar(ΔABC) = 0

$$\Rightarrow \quad \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}\,|x(1-5)+2(5+1)+4(-1,-1)| = 0$$

$$\Rightarrow \quad |-4x+12-8| = 0$$

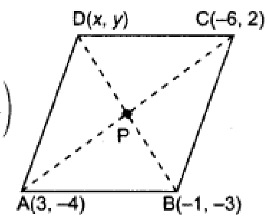
$$\Rightarrow \quad |-4x+4| = 0 \Rightarrow -4x+4=0 \Rightarrow 4x=4$$

$$\Rightarrow \quad x = 1$$

Question 65.

The three vertices of a parallelogram ABCD are A(3, -4), B(-l, -3) and C(-6, 2). Find the coordinates of vertex D and find the area of parallelogram ABCD.

Solution:



Given ABCD is a parallelogram

Let coordinates of point D be (x, y).

The coordinates of mid-point of AC $= \left(\frac{-6+3}{2}, \frac{2-4}{2}\right) = \left(\frac{-3}{2}, -1\right)$

The coordinates of mid-point of BD $= \left(\frac{x-1}{2}, \frac{y-3}{2}\right)$

Since, diagonals of a parallelogram bisect each other, so, P is the mid-point of AC as well as BD.

$$\Rightarrow \quad \left(\frac{x-1}{2}, \frac{y-3}{2}\right) = \left(\frac{-3}{2}, -1\right)$$

Comparing both sides, we get

$$\Rightarrow \quad \frac{x-1}{2} = \frac{-3}{2} \text{ and } \frac{y-3}{2} = -1$$

$$\Rightarrow \quad x-1 = -3 \text{ and } y-3 = -2$$

$$\Rightarrow \quad x = -2 \text{ and } y = 1$$

Therefore, coordinates of D are (– 2, 1).

Now, $\text{ar}(\Delta ABC) = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$= \frac{1}{2}\,|3(-3-2)-1(2+4)-6(-4+3)|$$

$$= \frac{1}{2}\,|-15-6+6| = \frac{15}{2} \text{ sq. units}$$

Area of parallelogram(ABCD) = 2 ar (ΔABC)

$$= 2 \times \frac{15}{2} = 15 \text{ sq. units}$$

Question 66.

If the points A(l, -2), B(2,3), C(-3,2) and D(-4, -3) are the vertices of parallelogram ABCD, then taking AB as the base, find the height of this parallelogram

Solution:

Using distance formula,

$$AB = \sqrt{(2-1)^2 + \{3-(-2)\}^2} = \sqrt{26} \text{ units}$$

$$\text{Area } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}|1(3-2) + 2\{2-(-2)\} + (-3)(-2-3)|$$

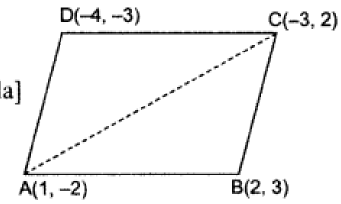
$$= \frac{1}{2} \times 24 = 12 \text{ sq. units}$$

$\therefore$ Area of ||gm ABCD = 2 × area of ΔABC

= 2 × 12 = 24 sq. units

Now, area ||gm ABCD = Base × h [∵ By formula]

= AB × h



$\Rightarrow$ AB × h = 24

$$\Rightarrow \quad h = \frac{24}{\sqrt{26}}$$

$$\Rightarrow \quad h = \frac{24}{26} \times \sqrt{26}$$

$\Rightarrow$ Height of parallelogram = $\frac{12}{13}\sqrt{26}$ units

Question 67.

For the AABC formed by the points A(4, -6), B(3, -2) and C(5,2), verify that median divides the triangle into two triangles of equal area.

Solution:

Consider AD is median of ΔABC.

Here D is mid point of BC as AD is median

$\therefore$ Coordinate of D are $x = \frac{3+5}{2} = 4$

$$y = \frac{-2+2}{2} = 0$$

Coordinates of D are (4, 0). Now

B(3,

$$\text{Area of } \Delta ABD = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}|4(0+2) + 4(-2+6) + 3(-6-0)|$$

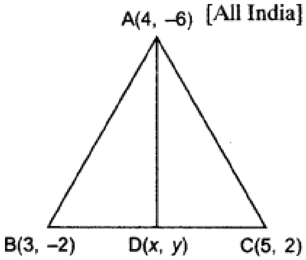
$$= \frac{1}{2}|8 + 16 - 18| = \left|\frac{6}{2}\right| = 3 \text{ sq. units}$$

Now, $\text{Area of } \Delta ACD = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}|4(0-2) + 4(2+6) + 5(-6-0)|$$

$$= \frac{1}{2}|-8 - 32 - 30| = \frac{1}{2}|32 - 38| = \frac{1}{2}|-6| = 3 \text{ sq. units}$$

[All India]



Clearly, ar ΔABD = ar ΔACD.

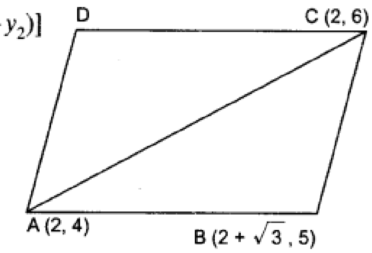
Thus, median AD divides triangle in q triangles of equal area.

Question 68.

Find the area of a parallelogram ABCD if three of its vertices are A(2, 4), B(2 + √3,5) and C(2, 6).

Solution:

ABCD is a parallelogram, A(2, 4), B(2 + $\sqrt{3}$, 5), C(2, 6) form the vertices of ΔABC.



$\therefore$ Area of ΔABC $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}|2(5-6) + (2+\sqrt{3})(6-4) + 2(4-5)|$

$= \frac{1}{2}|(-2 + 4 + 2\sqrt{3} - 2|$

$= \frac{1}{2}|2\sqrt{3}| = \sqrt{3}$ sq. units.

Diagonal AC divides the parallelogram in two triangles of equal area.

$\because$ Area of parallelogram ABCD = 2(Area of ΔABC) = $2(\sqrt{3}) = 2\sqrt{3}$ sq. units.

Question 69.

If the area of the triangle formed by points A (x,y), B (1,2) and C (2,1) is 6 square units, then show that x + y = 15.

Solution:

The points are A(x, y), B(1, 2) and C(2, 1)

Area of ΔABC $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 6$ sq. units [Given]

$\Rightarrow \quad \frac{1}{2}|x(2-1) + 1(1-y) + 2(y-2)| = 6$

$\Rightarrow \quad |x + 1 - y + 2y - 4| = 12$

$\Rightarrow \quad |x + y - 3| = 12$

$\Rightarrow \quad x + y - 3 = 12$

$\Rightarrow \quad x + y = 15$

Hence, proved.

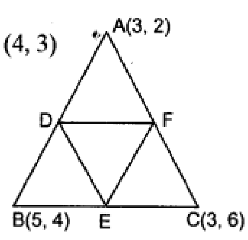
Question 70.

Find the area of the triangle formed by joining the mid-points of the sides of a triangle whose vertices are (3,2), (5,4) and (3, 6).

Solution:

Consider, the points are A(3, 2), B(5, 4) and C(3, 6) form the vertices of ΔABC.

Let D, E and F be the mid-points of the sides AB, BC and AC respectively of the triangle ABC.



$\therefore$ Coordinates of D are $\left(\frac{3+5}{2}, \frac{2+4}{2}\right)$, i.e. the coordinates of D are (4, 3)

Coordinates of E are $\left(\frac{5+3}{2}, \frac{4+6}{2}\right)$

$\Rightarrow$ Coordinates of E = (4, 5)

Coordinates of F are $\left(\frac{3+3}{2}, \frac{6+2}{2}\right)$

$\Rightarrow$ Coordinates of F are (3, 4).

Coordinates of D(4, 3), E(4, 5), F(3, 4)

Now, Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Area of ΔDEF $= \frac{1}{2}|4(5-4) + 4(4-3) + 3(3-5)| = \frac{2}{2} = 1$ sq. unit

Question 71.

If the area of the triangle formed by joining the points A (x, y), B (3, 2) and C (- 2, 4) is 10 square units, then show that 2x + 5y + 4 = 0.

Solution:

We know that,

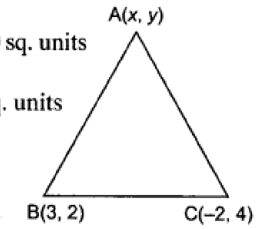
$$\text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 10 \text{ sq. units}$$

$$= \frac{1}{2}|x(2-4)+(3)(4-y)-2(y-2)| = 10 \text{ sq. units}$$

$$20 = (-2x-5y+16)$$

$$-2x-5y+16-20 = 0$$

$$\Rightarrow \quad 2x+5y+4 = 0 \quad \text{Hence proved}$$



2010

Very Short Answer Type Questions [1 Mark]

Question 72.

If a point A(0, 2) is equidistant from the points B(3, p) and C(p, 5), then find the value of p.

Solution:

Given points are A(0, 2), B(3,p), C(p, 5)

According to question,

$$AB = AC$$

Using distance formula

$$\sqrt{(3-0)^2+(p-2)^2} = \sqrt{(p-0)^2+(5-2)^2}$$

Squaring both sides

$$9+p^2+4-4p = p^2+9$$

$$4-4p = 0$$

$$p = 1$$

Question 73.

Find the value of k, if the point P(2,4) is equidistant from the points A(5, k) and B(k, 7).

Solution:

Now, given that, $AP = PB$. Using distance formula,

$$\Rightarrow \sqrt{(5-2)^2+(k-4)^2} = \sqrt{(k-2)^2+(7-4)^2}$$

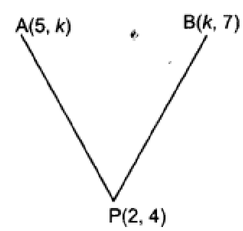
Squaring both sides we get

$$\Rightarrow (3)^2+(k-4)^2 = (k-2)^2+(3)^2$$

$$\Rightarrow 9+k^2-8k+16 = k^2-4k+4+9$$

$$\Rightarrow -4k = -12$$

$$\Rightarrow k = 3$$



Question 74.

Find the ratio in which the line segment joining the points (1,-3) and (4,5) is divided by x-axis.

Solution:

Let the ratio in which the line segment joining (1, –3) and (4, 5) is divided by x-axis be $k : 1$

Therefore, the coordinates of the point of division is $\left(\frac{4k+1}{k+1}, \frac{5k-3}{k+1}\right)$

[∵ Using section formula]

We know that y-coordinate of any point on x-axis is 0.

$$\therefore \quad \frac{5k-3}{k+1} = 0$$

$$5k-3 = 0$$

$$5k = 3$$

$$k = \frac{3}{5}$$

Ratio $= k : 1 = 3 : 5$

Short Answer Type Questions II [3 Marks]

Question 75.

If the vertices of a triangle are (1, – 3) (4,p) and (-9,7) and its area is 15 sq. units, find the value(s) of p.

Solution:

Let the triangle be ABC with vertices A(1, –3), B(4, p), C(–9, 7).

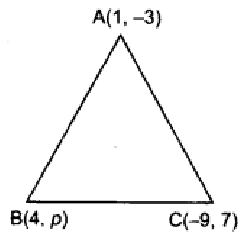
Area of ΔABC = 15 sq. units.

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 15$$

$$\frac{1}{2}[1(p-7) + 4(7+3) - 9(-3-p)] = 15$$

$$p - 7 + 40 + 27 + 9p = 30$$

$$p = -3.$$



Question 76.

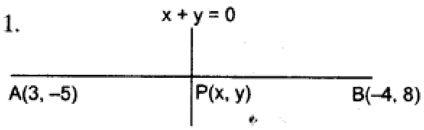
A point P divides the line segment joining the points A(3, -5) and B(-4, 8) such AP/PB=k/1. If P lies on the line x + y = 0, then find the value of K.

Solution:

AB is a line with A(3, –5) and B(–4, 8).

P(x, y) is any point on AB such that AP : PB = K : 1.

Using section formula.



$$(x, y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

Here, $x_1 = 3,\ x_2 = -4,\ y_1 = -5, y_2 = 8,\ m_1 = \text{K},\ m_2 = 1.$

$$(x, y) = \left(\frac{\text{K}(-4) + 1(3)}{\text{K}+1}, \frac{\text{K}(8) + 1(-5)}{\text{K}+1}\right)$$

$$(x, y) = \left(\frac{-4\text{K}+3}{\text{K}+1}, \frac{8\text{K}-5}{\text{K}+1}\right)$$

On equating the coordinates both sides, we get

$$x = \frac{-4\text{K}+3}{\text{K}+1},\quad y = \frac{8\text{K}-5}{\text{K}+1}$$

Given that, $x + y = 0$

$$\frac{-4\text{K}+3}{\text{K}+1} + \frac{8\text{K}-5}{\text{K}+1} = 0$$

$$\frac{-4\text{K}+3+8\text{K}-5}{\text{K}+1} = 0$$

$$4\text{K} - 2 = 0$$

$$4\text{K} = 2$$

$$\text{K} = \frac{2}{4} = \frac{1}{2}$$

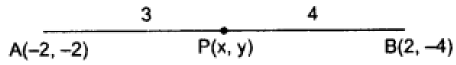
$$\text{K} = \frac{1}{2}.$$

Question 77.

Find the coordinates of a point P, which lies on the line segment joining the points A(-2, -2) and B(2, -4) such that AP =3/7 AB.

Solution:

Given that, $AP = \frac{3}{7}AB \Rightarrow \frac{AP}{AB} = \frac{3}{7} \Rightarrow AP : PB = 3 : 4$



Using section formula,

$$x = \frac{3(2)+4(-2)}{3+4} = \frac{6-8}{7} = -\frac{2}{7}$$

$$y = \frac{3(-4)+4(-2)}{3+4} = \frac{-12-8}{7} = \frac{-20}{7}$$

Coordinates of P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$

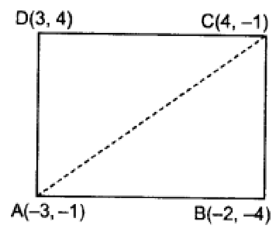
Question 78.

Find the area of the quadrilateral ABCD whose vertices are A(-3, -1), B(-2, -4), C(4, – 1) and D(3, 4).

Solution:

Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

where area of triangle is $\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$



$$= \frac{1}{2}[-3(-4+1)+(-2)(-1+1)+4(-1+4)] + \frac{1}{2}[-3(-1-4)+4(4+1)+3(-1+1)]$$

$$= \frac{1}{2}[-3(-3)+(-2)(0)+4(3)] + \frac{1}{2}[-3(-5)+4(5)+3(0)]$$

$$= \frac{1}{2}[9+0+12] + \frac{1}{2}[15+20] = \frac{1}{2}\times 21 + \frac{1}{2}\times 35$$

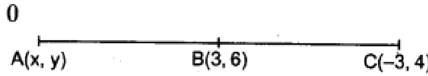
$$= \frac{1}{2}\times 56 = 28 \text{ Sq. units.}$$

Question 79.

If the points A(x, y), B(3, 6) and C(-3,4) are collinear, show that x – 3y + 15 = 0.

Solution:

If A, B and C are collinear then area of $\Delta ABC = 0$



$$\Rightarrow \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)| = 0$$

$$\Rightarrow \frac{1}{2}[x(6-4)+3(4-y)+(-3)(y-6)] = 0$$

$$\Rightarrow 2x+12-3y-3y+18 = 0$$

$$\Rightarrow 2x-6y+30 = 0$$

$$\Rightarrow x-3y+15 = 0$$

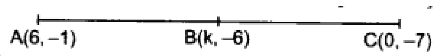
Hence, proved.

Question 80.

Find the value of£, for which the points A(6, -1), B(& – 6) and C(0, -7) are collinear.

Solution:

Given points are A(6, –1), B(k, –6), C(0, –7).



As A, B, C are collinear points, so, $Ar\ \Delta ABC = 0$

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$$

i.e.,
$$\frac{1}{2}|6(-6+7)+k(-7+1)+0(-1+6)|=0$$
$$\frac{1}{2}|6-6k+0|=0$$
$$\frac{1}{2}|6-6k|=0$$
$$|6-6k|=0$$
$$6-6k=0$$
$$6(1-k)=0$$
$$1-k=0$$
$\therefore$
$$k=1.$$

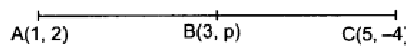
Question 81.

Find the value ofp, if the points A(l, 2), B(3,p) and C(5, -4) are collinear.

Solution:

If three points are collinear, then area bounded by them is zero.

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]=0$$



i.e.,
$$\frac{1}{2}|1(p+4)+3(-4-2)+5(2-p)|=0$$
$$\frac{1}{2}|p+4-18+10-5p|=0$$
$$|-4p-4|=0$$
$$4p+4=0$$
$$p=-1$$

Question 82.

Find the area of the triangle whose vertices are (-7, -3), (1, -7) and (3,0).

Solution:

The area of the $\Delta = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

Here,
$$x_1 = -7,\quad x_2 = 1,\quad x_3 = 3$$
$$y_1 = -3,\quad y_2 = -7,\ y_3 = 0$$

$$\text{Area of the triangle} = \frac{1}{2}[-7(-7-0)+1\{0-(-3)\}+3\{-3-(-7)\}]$$
$$= \frac{1}{2}[(-7)(-7)+(1)(3)+3(-3+7)]$$
$$= \frac{1}{2}[49+3+(3\times 4)]$$
$$= \frac{1}{2}[49+3+12] = \frac{1}{2}\times 64 = 32 \text{ sq. units.}$$

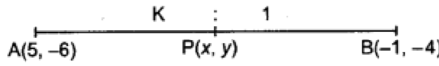
Question 83.

Find the ratio in which the y-axis divides the line segment joining the points (5, -6) and (-1, -4). Also find the coordinates of the point of intersection.

Solution:

Let the ratio be K : 1. Then by the section formula, the coordinates of the point which divides the line segment in the ratio K : 1 are $\left(\frac{-K+5}{K+1}, \frac{-4K-6}{K+1}\right)$ [$\because$ Using section formula]

This point lies on the y-axis and we know that on the y-axis, x is 0.



$$\therefore \quad \frac{-K+5}{K+1} = 0$$

$$-K+5 = 0$$

$$K = 5$$

The ratio is 5 : 1.

On putting the value of K = 5, we get the point of intersection $\left(\frac{-5+5}{5+1}, \frac{-4\times5-6}{5+1}\right)$

$$\Rightarrow \quad \left(0, -\frac{26}{6}\right)$$

$$\Rightarrow \quad \left(0, \frac{-13}{3}\right)$$

$\therefore$ coordinates of point of intersection are $\left(0, \frac{-13}{3}\right)$

>

Question 84.

Find the value of y for which the points (5, -4), (3, -1) and (1, y) are collinear.

Solution:

The points are collinear so area of triangle = 0

So, Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

Here, $x_1 = 5, \quad x_2 = 3, \quad x_3 = 1$

$y_1 = -4, \quad y_2 = -1, \quad y_3 = y$

$$\text{Area of triangle} = \frac{1}{2}[5(-1-y) + 3(y+4) + 1(-4+1)]$$

$$= \frac{1}{2}[-5 - 5y + 3y + 12 - 3]$$

$$= \frac{1}{2}[-5y + 3y - 5 + 9]$$

$$= \frac{1}{2}[-2y + 4] = \frac{1}{2}\times 2(-y+2) = (-y + 2)$$

As per condition, area of triangle must be zero.

$$-y + 2 = 0$$

$$-y = -2$$

$$y = 2$$

Question 85.

For what value of k, (k > 0), is the area of the triangle with vertices (-2, 5), (k, -4) and {2k + 1,10) equal to 53 sq. units?

Solution:

Vertices of the triangle are $(-2, 5)$, $(k, -4)$ and $(2k + 1, 10)$.

Then, Ar(Triangle) = 53 sq. units

$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 53$

$\Rightarrow \frac{1}{2}|-2(-4-10) + k(10-5) + (2k+1)(5-(-4))| = 53$

$\Rightarrow |28 + 5k + 18k + 9| = 106$

$\Rightarrow |23k + 37| = 106 \Rightarrow 23k + 37 = \pm 106$

$\Rightarrow 23k + 37 = 106$ or $23k + 37 = -106$

$\Rightarrow 23k = 69$ or $23k = -143$

$k = 3$ or $k = \frac{-143}{23}$

$\because$ Given, $k > 0$ $\therefore$ $k = 3$

2011

Short Answer Type Questions I [2 Marks]

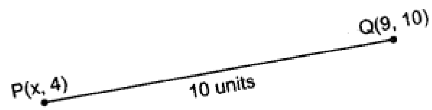
Question 86.

Find that value(s) of x for which the distance between the points P(JC, 4) and Q(9,10) is 10 units.

Solution:

Given that, $PQ = 10$ units

$\Rightarrow PQ^2 = 100$



Using distance formula,

$\Rightarrow (x-9)^2 + (4-10)^2 = 100$

$\Rightarrow (x-9)^2 + 36 = 100$

$\Rightarrow (x-9)^2 = 64$

$\Rightarrow x - 9 = \pm 8$

$\Rightarrow x - 9 = 8$ or $x - 9 = -8$

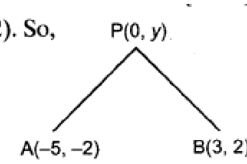
$\Rightarrow x = 17$ or $x = 1$.

Question 87.

Find the point ony-axis which is equidistant from the points (-5, -2) and (3, 2).

Solution:

Let point $P(0, y)$ on y-axis be equidistant from $A(-5, -2)$ and $B(3, 2)$. So,



$PA = PB$

$\Rightarrow PA^2 = PB^2$

Using distance formula,

$\Rightarrow 5^2 + (y+2)^2 = (-3)^2 + (y-2)^2$

$\Rightarrow 25 + y^2 + 4y + 4 = 9 + y^2 - 4y + 4$

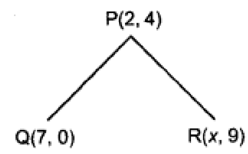
$\Rightarrow 8y = -16 \Rightarrow y = -2$

Hence, required points is $(0, -2)$

Question 88. .

If P(2,4) is equidistant from Q(7, 0) and R(x, 9), find the values of x. Also find the distance PQ.

Solution:



Given: $PQ = PR$

$\Rightarrow PQ^2 = PR^2$

Using distance formula,

$\Rightarrow (2-7)^2 + 4^2 = (2-x)^2 + (4-9)^2$

$\Rightarrow 25 + 16 = (2-x)^2 + 25$

$\Rightarrow (2-x)^2 = 16$

$\Rightarrow 2-x = \pm 4$

$\Rightarrow 2-x = 4 \quad \text{or} \quad 2-x = -4$

$\Rightarrow x = -2 \qquad x = 6$

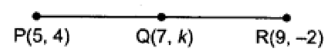
Distance, $PQ = \sqrt{(7-2)^2 + (0-4)^2}$

$= \sqrt{25+16} = \sqrt{41}$ units

Question 89.

Find the value of k, if the points P(5,4), Q(7, k) and R(9, -2) are collinear

Solution:



Since points P, Q, R are collinear.

So, area of $\Delta PQR = 0$

$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \frac{1}{2}[5(k+2) + 7(-2-4) + 9(4-k)] = 0$

$\Rightarrow \frac{1}{2}[5k + 10 - 42 + 36 - 9k] = 0$

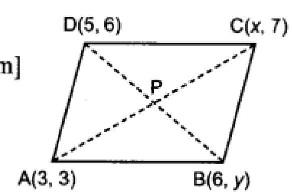
$\Rightarrow -4k + 4 = 0$

$\Rightarrow 4k = 4 \Rightarrow k = 1$

Question 90.

If (3, 3), (6, y), (x, 7) and (5, 6) are the vertices of a parallelogram taken in order, find the values of x and y.

Solution:



ABCD is a | |gm, [Given]

P is the mid-point of AC and BD. [∵ Property of parallelogram]

Taking AC, Coordinates of point P is $\left(\frac{x+3}{2}, 5\right)$ [∵ Using mid-point formula]

Taking BD, Also, coordinates of point P is $\left(\frac{11}{2}, \frac{y+6}{2}\right)$ [∵ Using mid-point formula]

A.T.Q $\frac{x+3}{2} = \frac{11}{2}$ and $\frac{y+6}{2} = 5$ [∵ As P is mid-point of AC & BD]

$x + 3 = 11 \qquad y + 6 = 10$

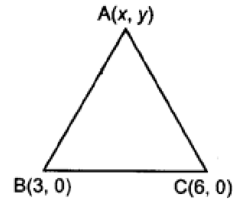
$x = 8 \qquad y = 4$

Question 91.

If two vertices of an equilateral triangle are (3,0) and (6,0), find the third vertex.

Solution:

Let ABC be an equilateral Δ.



Let coordinates of points A, B and C are (x, y), $(3, 0)$ and $(6, 0)$ respectively.

Now $\quad AB = AC = BC$

$[\because$ In equilateral Δ, all sides are equal$]$

$\Rightarrow \quad AB^2 = AC^2 = BC^2$

Using distance formula,

$\Rightarrow \quad (x-3)^2 + y^2 = (x-6)^2 + y^2 = 3^2$

$\Rightarrow \quad (x-3)^2 + y^2 = (x-6)^2 + y^2 \quad ...(i) \qquad$ and $(x-3)^2 + y^2 = 3^2 \quad ...(ii)$

$\Rightarrow \quad$ Solving (i); $(x-3)^2 = (x-6)^2$

$\Rightarrow \quad x^2 - 6x + 9 = x^2 - 12x + 36$

$\Rightarrow \quad 6x = 27$

$\Rightarrow \quad x = \frac{27}{6} = \frac{9}{2}$

Put $x = \frac{9}{2}$ in eqn (ii) we get

$$\left(\frac{9}{2} - 3\right)^2 + y^2 = 3^2$$

$\Rightarrow \quad \left(\frac{3}{2}\right)^2 + y^2 = 3^2$

$\Rightarrow \quad \frac{9}{4} + y^2 = 9$

$\Rightarrow \quad y^2 = 9 - \frac{9}{4} = \frac{27}{4} \quad \Rightarrow \quad y = \frac{3\sqrt{3}}{2}$

Hence, the third vertex is $\left(\frac{9}{2}, \frac{3\sqrt{3}}{2}\right)$

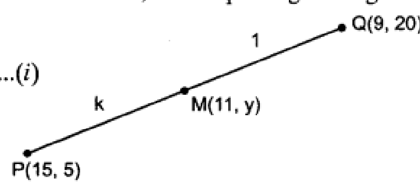
Question 92.

Point M(11,y) lies on the line segment joining the points P(15,5) and Q(9,20). Find the ratio in which point M divides the line segment PQ. Also find the value ofy

Solution:

Let point M divides the line segment PQ in the ratio $k : 1$

Then, Using section formula to calculate coordinates of M, and equating with given M-coordinates.



$$\frac{9k+15}{k+1} = 11 \text{ and } \frac{20k+5}{k+1} = y \quad ...(i)$$

$\Rightarrow \quad 9k + 15 = 11k + 11$

$\Rightarrow \quad 2k = 4 \quad \Rightarrow \quad k = 2$

Hence, point M divides the line segment PQ in the ratio 2 : 1. Then, using (i),

$$y = \frac{20 \times 2 + 5}{2+1} = \frac{45}{3} = 15$$

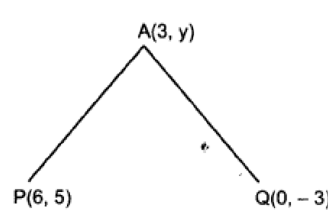
$\therefore \quad y = 15$

Question 93.

The point A(3,y) is equidistant from the points P(6,5) and Q(0, -3). Find the value of y.

Solution:

Given: $\quad AQ = AP$



$\Rightarrow \quad Q^2 = AP^2$

Using distance formula,

$\Rightarrow \quad (3-0)^2 + (y+3)^2 = (3-6)^2 + (y-5)^2$

$\Rightarrow \quad 9 + y^2 + 6y + 9 = 9 + y^2 - 10y + 25$

$\Rightarrow \quad 6y + 9 = -10y + 25$

$\Rightarrow \quad 16y = 16 \quad \Rightarrow \quad y = 1$

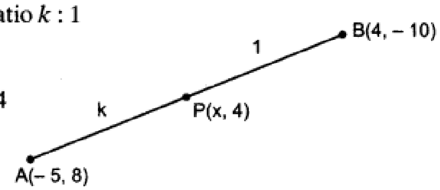
Question 94.

Point P(x, 4) lies on the line segment joining the points A(-5,8) and B(4, -10). Fmd the ratio in which point P divides the line segment AB. Also find the value of x

Solution:

Let point P divides the line segment AB in the ratio $k : 1$

Using distance formula



A.T.Q., $\frac{4k-5}{k+1} = x$ and $\frac{-10k+8}{k+1} = 4$

$\Rightarrow \quad -10k + 8 = 4k + 4$

$14k = 4$

$\Rightarrow \quad k = \frac{4}{14} = \frac{2}{7}$

$k = \frac{2}{7}$

So, point P divides the line segment AB in the ratio 2 : 7

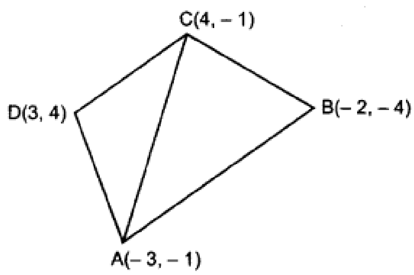
Now, $x = \frac{4.\left(\frac{2}{7}\right)-5}{\frac{2}{7}+1} = \frac{8-35}{2+7} = \frac{-27}{9} = -3$

$x = 3$

Question 95.

Find the area of the quadrilateral ABCD, whose vertices are A(-3, -1), B(-2, -4), C(4, -1) and D(3, 4).

Solution:



Firstly, $\text{ar}(\Delta ABC) = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

$= \frac{1}{2}[-3(-4+1) - 2(-1+1) + 4(-1+4)]$

$= \frac{1}{2}[9 - 0 + 12] = \frac{21}{2}$ sq. units

$\text{ar}(\Delta ACD) = \frac{1}{2}[-3(-1-4) + 4(4+1) + 3(-1+1)]$

$= \frac{1}{2}[15 + 20 + 0] = \frac{35}{2}$ sq. units

Now, $\text{ar(quadrilateral ABCD)} = \text{ar}(\Delta ABC) + \text{ar}(\Delta ACD)$

$= \frac{21}{2} + \frac{35}{2} = \frac{56}{2} = 28$ sq. units

Question 96.

Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are A(2,1), B(4,3) and C(2,5).

Solution:

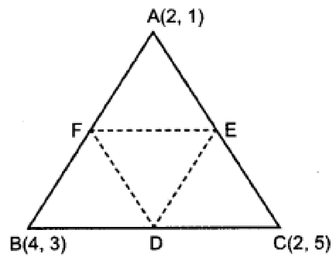
. D, E, F are the mid-points of the sides BC, CA and AB respectively.

So, coordinates of the points D, E, F are as

Using mid-point formula,

$D\left(\frac{4+2}{2}, \frac{3+5}{2}\right); E\left(\frac{2+2}{2}, \frac{5+1}{2}\right); F\left(\frac{4+2}{2}, \frac{3+1}{2}\right)$

i.e. D(3, 4); E(2, 3); F(3, 2)

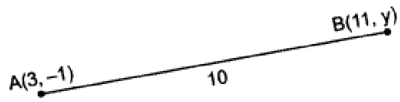


$$\text{Now, } ar(\Delta DEF) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$= \frac{1}{2}[3(3-2) + 2(2-4) + 3(4-3)]$$
$$= \frac{1}{2}[3 - 4 + 3] = \frac{2}{2} = 1 \text{ sq. unit}$$

Question 97.

Find the value of y for which the distance between the points A(3, -1) and B(11,y) is 10 units.

Solution:



. Given that: $AB = 10$ units

$\Rightarrow AB^2 = 100$

Using distance formula,

$\Rightarrow (11-3)^2 + (y+1)^2 = 100$

$\Rightarrow 64 + (y+1)^2 = 100$

$\Rightarrow (y+1)^2 = 36$

$\Rightarrow y + 1 = \pm 6$

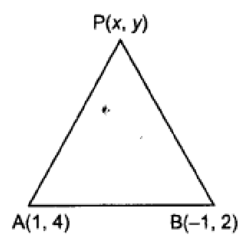
$\Rightarrow y + 1 = 6$ or $y + 1 = -6$

$\Rightarrow y = 5$ or $y = -7.$

Question 98.

Find a relation between jt and y such that the point P(x, y) is equidistant from the points A(1, 4) and B(-1, 2).

Solution:



Given that: $PA = PB$

$\Rightarrow$. $PA^2 = PB^2$

Using distance formula,

$\Rightarrow (x-1)^2 + (y-4)^2 = (x+1)^2 + (y-2)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 8y + 16 = x^2 + 2x + 1 + y^2 - 4y + 4$

$\Rightarrow -2x - 8y + 17 = 2x - 4y + 5$

$\Rightarrow 4x + 4y - 12 = 0$

$\Rightarrow$ Required relation: $x + y - 3 = 0.$

Question 99.

Find a point on x-axis which is equidistant from A(4, -3) and B(0,11).

Solution:

Let point P(x, 0) on x-axis be equidistant from the points A and B.

Then, $PA = PB$

$\Rightarrow PA^2 = PB^2$

Using distance formula,

$\Rightarrow (x-4)^2 + (0+3)^2 = (x-0)^2 + (0-11)^2$

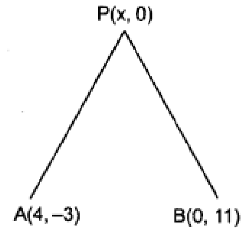
$\Rightarrow x^2 - 8x + 16 + 9 = x^2 + 121$

$\Rightarrow -8x = 121 - 25$

$\Rightarrow -8x = 96$

$\Rightarrow x = \frac{96}{-8} = -12 \Rightarrow x = -12$

Hence, coordinates of required point are (–12, 0)



Question 100.

If A(-2,3), B(6,5), C(x, – 5) and D(-4, – 3) are the vertices of a quadrilateral ABCD of area 80 sq. units, then find positive value of x.

Solution:

ar (quad ABCD) = ar (ΔABD) + ar (ΔBCD)

$80 = \left|\frac{1}{2}[-2(5+3)+6(-3-3)-4(3-5)]\right| + \left|\frac{1}{2}[6(-5+3)+x(-3-5)-4(5+5)]\right|$

$\left[\because \text{Area of triangle} = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\right]$

$\Rightarrow 80 = \left|\frac{1}{2}[-16-36+8]\right| + \left|\frac{1}{2}[-12-8x-40]\right|$

$\Rightarrow 80 = \left|\frac{1}{2}(-44)\right| + \left|\frac{1}{2}(-8x-52)\right|$

$\Rightarrow 80 = 22 + |26 + 4x|$

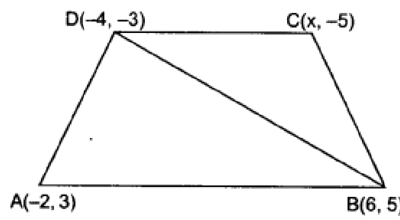
$\Rightarrow |26x + 4x| = 58 \Rightarrow 26 + 4x = \pm 58$

$[\because |x| = a \Rightarrow x = \pm a]$

$\Rightarrow 26 + 4x = 58$ or $26 + 4x = -58$

$\Rightarrow 4x = 32$ or $4x = -84$

$\Rightarrow x = 8$ or $x = -21$ $\therefore$ Positive value of $x = 8$



Question 101.

Find the area of the quadrilateral PQRS whose vertices are P(-l, – 3), Q(5, – 7), R(10, — 2) and S(5,17).

Solution:

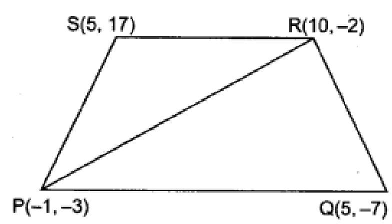
ar (quad PQRS) = ar (ΔPQR) + ar (ΔPRS)

$= \frac{1}{2}[-1(-7+2)+5(-2+3)+10(-3+7)] + \frac{1}{2}[-1(-2-17)+10(17+3)+5(-3+2)]$

$\left[\because \text{Area of triangle} = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\right]$

$= \frac{1}{2}[5+5+40] + \frac{1}{2}[19+200-5]$

$= 25 + 107 = 132$ sq. units.



Question 102.

Find the area of the quadrilateral ABCD whose vertices are A(3, – 1), B(9, -5), C(14,0) and D(9,19).

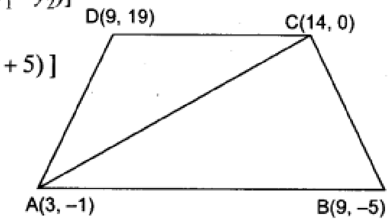
Solution:

Firstly, ar $(\Delta ABC) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$= \frac{1}{2}[3(-5-0)+9(0+1)+14(-1+5)]$$

$$= \frac{1}{2}[-15+9+56]$$

$$= \frac{1}{2} \times 50 = 25 \text{ sq. units.}$$



Now, ar $(\Delta ACD) = \frac{1}{2}[3(0-19)+14(19+1)+9(-1-0)]$

$$= \frac{1}{2}[-57+280-9]$$

$$= \frac{1}{2} \times 214 = 107 \text{ sq. units.}$$

Area of quadrilateral ABCD = ar(ΔABC) + ar(ΔACD) = 25 + 107 = 132 sq. units

Question 103.

Find the coordinates of the points which divide the line segment joining A (2, – 3) and B(-4, – 6) into three equal parts.

Solution:

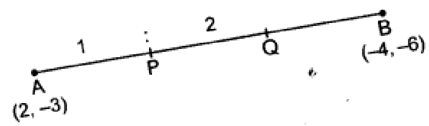
Let P and Q are the required point; which divides AB in three equal parts.

Point P divides the line segment AB in the ratio 1 : 2

So, coordinates of P are given by $\left(\frac{-4+2\times 2}{1+2}, \frac{1\times(-6)+2\times(-3)}{1+2}\right)$ [∵ Using section formula]

i.e. $\left(\frac{-4+4}{3}, \frac{-6-6}{3}\right)$

i.e. (0, –4)



Now point Q is the mid-point of PB.

So, coordinates of point Q are $\left(\frac{0-4}{2}, \frac{-4-6}{2}\right)$ i.e. (–2, –5).

2010

Very Short Answer Type Questions [1 Mark]

Question 104.

If P(2, p) is the mid-point of the line segment joining the points A(6, -5) and B(-2,11), find the value of p.

Solution:

P(2, p) is mid-point of A (6, –5) and B (–2,11). Using mid-point formula,

So, $\frac{-5+11}{2} = p$

$\Rightarrow p = \frac{6}{2} \Rightarrow p = 3.$

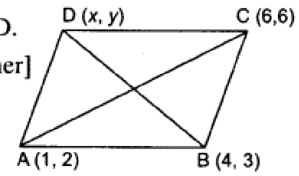
Question 105.

If A(l, 2), B(4, 3) and €(6,6) are three vertices of parallelogram ABCD, find co-ordinates of D.

Solution:

Let coordinates of D be (x, y) and P is mid-point of AC and BD.

[Diagonals of a parallelogram bisect each other]



$\therefore$ Using mid-point formula,

$$\left(\frac{x+4}{2}, \frac{y+3}{2}\right) = \left(\frac{1+6}{2}, \frac{2+6}{2}\right)$$

$$\Rightarrow \quad \frac{x+4}{2} = \frac{7}{2}, \frac{y+3}{2} = \frac{8}{2}$$

$$\Rightarrow \quad x + 4 = 7; y + 3 = 8$$

$$\Rightarrow \quad x = 3; y = 5$$

$\therefore$ Coordinates of D are (3, 5).

Question 106.

What is the distance between the points A(c, 0) and B(0, -c)?

Solution:

Distance $AB = \sqrt{(0-c)^2+(-c-0)^2}$ [$\because$ Using distance formula]

$$= \sqrt{c^2+c^2} = \sqrt{2c^2} = \sqrt{2}c \text{ units}$$

Question 107.

Find the distance between the points, A(2a, 6a) and B(2a + √3 a, 5a).

Solution:

Distance $AB = \sqrt{(2a+\sqrt{3}a-2a)^2+(5a-6a)^2}$ [$\because$ Using distance formula]

$$= \sqrt{3a^2+a^2} = \sqrt{4a^2} = 2a \text{ units}$$

Question 108.

Find the value of k if P(4, -2) is the mid point of the line segment joining the points A(5k, 3) and B(-k, —7).

Solution:

P(4, – 2) is mid point of A(5k, 3) and B(– k, – 7), Using mid-point formula,

$$\therefore \quad \frac{5k-k}{2} = 4 \quad \Rightarrow 4k = 8 \Rightarrow k = 2$$

Short Answer Type Questions II [3 Marks]

Question 109.

Point P divides the line segment joining the points A(2,1) and B(5, -8) such that AP /AB=1/3. If P lies on the line 2x – y + k = 0, find the value of k.

Solution:

P is the point of intersection of line segment AB and line $2x - y + k = 0$.

Here, given that, $\frac{AP}{AB} = \frac{1}{3} \Rightarrow 3AP = AB$

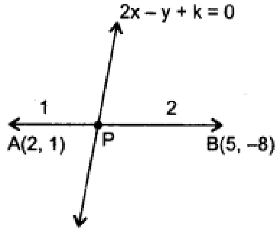
$\Rightarrow$ $3AP = AP + PB$

$\Rightarrow$ $2AP = PB$

$\Rightarrow$ $\frac{AP}{PB} = \frac{1}{2} \Rightarrow AP : PB = 1 : 2$

$\Rightarrow$ P divides the line segment joining A(2, 1) and B(5, – 8) in the ratio 1 : 2.

$\therefore$ Coordinates of point P are



$$x = \frac{1\times5+2\times2}{1+2} = 3 \quad [\because \text{Using section formula}]$$

$$y = \frac{1\times(-8)+2\times1}{1+2} = -2$$

i.e. P (3, – 2)

As point P lies on the line $2x - y + k = 0$, P must satisfy it.

$\Rightarrow$ $6 + 2 + k = 0 \Rightarrow k = -8$

Question 110.

If R(x, y) is a point on the line segment joining the points P(a, b) and Q(b, a), then prove that x + y = a + b.

Solution:

R(x, y) lies on the line segment joining the points P(a, b) and Q(b, a). Then P, Q, R are collinear, so ar(Δ PQR) = 0

$\Rightarrow$ $\frac{1}{2}|x(b-a) + a(a-y) + b(y-b)| = 0$ $\left[\begin{array}{l}\text{Using formula for area of a triangle} \\ \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\end{array}\right]$

$\Rightarrow$ $bx - ax + a^2 - ay + by - b^2 = 0$

$\Rightarrow$ $b(x+y) - a(x+y) + (a^2 - b^2) = 0$

$\Rightarrow$ $(b-a)(x+y) - (b-a)(b+a) = 0$

$\Rightarrow$ $(b-a)[(x+y) - (b+a)] = 0$

$\Rightarrow$ $x + y = b + a$

[Assuming $a \neq b$] Hence, proved.

Question 111.

Prove that the points P(a, b + c), Q(b, c + a) and R(c, a + b) are collinear.

Solution:

Area of Δ formed by the points P(a, $b + c$), Q(b, $c + a$) and R(c, $a + b$) is given by

$$= \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$$

or, $\text{ar}(\Delta PQR) = \frac{1}{2}|a(c+a-a-b) + b(a+b-b-c) + c(b+c-c-a)|$

$= \frac{1}{2}|a(c-b) + b(a-c) + c(b-a)|$

$\Rightarrow$ $= \frac{1}{2}|ac - ab + ab - bc + bc - ac| = 0$

$\because$ ar(ΔPQR) = 0

Hence, points are collinear.

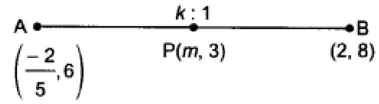
Question 112.

If the point P(m, 3) lies on the line segment joining the points A(-2/5,6) and B(2, 8), find the value of m.

Solution:

Let point P divides AB in ratio $k : 1$.

Using section formula,



$$3 = \frac{8 \times k + 6 \times 1}{k+1}$$

$$\Rightarrow \quad 3 = \frac{8k+6}{k+1} \quad \Rightarrow \quad 3k + 3 = 8k + 6 \quad \Rightarrow \quad -3 = 5k$$

$$\Rightarrow \quad k = \frac{-3}{5}$$

and, $$m = \frac{2k + \left(-\frac{2}{5}\right)}{k+1} = \frac{2\left(-\frac{3}{5}\right) - \frac{2}{5}}{\frac{-3}{5}+1} = \frac{-6-2}{2} \Rightarrow m = -\frac{8}{2} = -4 \quad \therefore m = -4$$

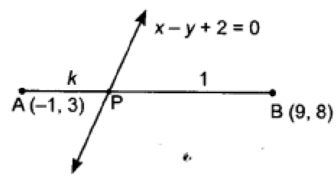
Question 113.

Point P divides the line segment joining the points A(-l, 3) and B(9, 8) such that AP/PB=k/1. If P lies on the line x -y + 2 = 0, find the value of k.

Solution:

P divides the joining of A(– 1, 3) and B(9, 8) such that $\frac{AP}{AB} = \frac{k}{1}$ i.e. AP : PB = k : 1.

Using section formula,



$\therefore$ Coordinates of P are: $\left(\frac{9k-1}{k+1}, \frac{8k+3}{k+1}\right)$

If P lies on $x - y + 2 = 0$, then P must satisfy it.

$$\frac{9k-1}{k+1} - \left(\frac{8k+3}{k+1}\right) + 2 = 0$$

$$\Rightarrow \quad 9k - 1 - 8k - 3 + 2k + 2 = 0$$

$$\Rightarrow \quad 3k - 2 = 0$$

$$\Rightarrow \quad k = \frac{2}{3}$$

Question 114.

Find the value of k, if the points A(7, -2), B(5,1) and C(3,2k) are collinear

Solution:

If points A(7, – 2), B(5, 1) and C(3, 2k) are collinear then, ar ΔABC = 0

$$\left[\because \text{Area of triangle } \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right]$$

$$\therefore \quad \text{ar } \Delta ABC = \frac{1}{2}|7(1-2k) + 5(2k+2) + 3(-2-1)| = 0$$

$$\Rightarrow \quad 7 - 14k + 10k + 10 - 9 = 0$$

$$\Rightarrow \quad -4k = -8 \quad \Rightarrow k = 2$$

Question 115.

If the points (p, q); (m, n) and (p-m,q-n) are collinear, show that pn = qm

Solution:

If P(p, q), Q(m, n), R($p - m, q - n$) are collinear then area of triangle formed by them is zero.

Hence, ar ΔPQR = 0

$$\left[\because \text{Area of triangle} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right]$$

$$\frac{1}{2}|pn - qm + mq - mn - pn + mn + pq - mq - qp + pn| = 0$$

$$\Rightarrow \quad |pn - qm| = 0$$

$$\Rightarrow \quad pn - qm = 0$$

$$\Rightarrow \quad pn = qm$$

Hence, proved.

Question 116.

Find the value of k, if the points A(8,1), B(3, -4) and C(2, k) are collinear

Solution:

Given points are A(8, 1), B(3, –4) and C(2, k).

As these points are collinear, so the area of triangle formed by these points is zero sq. units.

$$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$\therefore \quad \frac{1}{2}[8(-4-k)+3(k-1)+2(1+4)] = 0$$

$$\therefore \quad -32-8k+3k-3+10 = 0$$

$$-5k-25 = 0$$

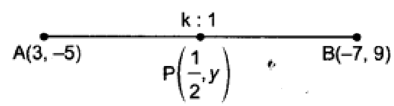
$$\therefore \quad k = -5$$

Question 117.

If point P (1/2, y)lies on the line segment joining the points A(3, -5) and B(-7,9) then find the ratio in which P divides AB. Also find the value of y.

Solution:

Let P divides AB in the ratio k : 1.



$$\therefore \quad \left(\frac{-7k+3}{k+1}, \frac{9k-5}{k+1}\right) = \left(\frac{1}{2}, y\right) \quad ...(i)$$

$$\Rightarrow \quad \frac{-7k+3}{k+1} = \frac{1}{2}$$

$$\Rightarrow \quad -14k+6 = k+1$$

$$\Rightarrow \quad -15k = -5$$

$$\Rightarrow \quad k = \frac{1}{3}$$

$\therefore$ Ratio is $k : 1$, i.e. $\frac{1}{3} : 1 \Rightarrow 1 : 3$

and, using (i), $$y = \frac{9k-5}{k+1} = \frac{9\times\frac{1}{3}-5}{\frac{1}{3}+1} = \frac{-6}{4} = \frac{-3}{2} \quad \therefore y = \frac{-3}{2}$$

Question 118.

Find the value of k for which the points A(9, k), B(4, -2) and C(3, -3) are collinear.

Solution:

If points A(9, k), B(4, – 2) and C(3, – 3) are collinear, so, ar (ΔABC) = 0

$$\left[\because \text{Area of triangle} = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\right]$$

$$\Rightarrow \quad \frac{1}{2}|9(-2+3)+4(-3-k)+3(k+2)| = 0$$

$$\Rightarrow \quad |9-12-4k+3k+6| = 0$$

$$\Rightarrow \quad -k = -3$$

$$\Rightarrow \quad k = 3$$

Question 119.

Find the value of k for which the points A(fc, 5), B(0,1) and C(2, -3) are collinear.

Solution:

If A(k, 5), B(0, 1), C(2, –3) are collinear then ar ΔABC = 0.

$$\left[\because \text{Area of triangle} = \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]\right]$$

$$\Rightarrow \quad \frac{1}{2}[k(1+3)+0(-3-5)+2(5-1)] = 0$$

$$\Rightarrow \quad |4k+8| = 0$$

$$\Rightarrow \quad 4k = -8$$

$$\Rightarrow \quad k = -2$$

Question 120.

Find the value of p for which the points A(-1, 3), B(2,p) and C(5, -1) are collinear.

Solution:

If points $A(-1, 3)$, $B(2, p)$ and $C(5, -1)$ are collinear, then ar $(\Delta ABC) = 0$

$$\left[\because \text{Area of triangle} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right]$$

$\Rightarrow \quad \frac{1}{2}|-1(p+1) + 2(-1-3) + 5(3-p)| = 0$

$\Rightarrow \quad |-p - 1 - 8 + 15 - 5p| = 0$

$\Rightarrow \quad 6p = 6$

$\Rightarrow \quad p = 1$