

Triangles

2016

Very Short Answer Type Question [1 Mark]

Question 1.

If $\triangle ABC \sim \triangle RPQ$, $AB = 3$ cm, $BC = 5$ cm, $AC = 6$ cm, $RP = 6$ cm and $PQ = 10$ cm, then find QR

Solution:

$\triangle ABC \sim \triangle RPQ$,

(Given)

$\therefore \frac{AB}{RP} = \frac{BC}{PQ} = \frac{AC}{RQ}$ (Proportional sides of similar triangles)

$$\frac{3}{6} = \frac{5}{10} = \frac{6}{QR}$$

$$\frac{1}{2} = \frac{1}{2} = \frac{6}{QR}$$

$\Rightarrow QR = 12$ cm.

Short Answer Type Question I [2 Marks]

Question 2.

R and S are points on the sides DE and EF respectively of a $\triangle DEF$ such that $ER = 5$ cm, $RD = 2.5$ cm, $SE = 1.5$ cm and $FS = 3.5$ cm. Find whether $RS \parallel DF$ or not.

Solution:

Construction: Join RS

To find:

$RS \parallel DF$ or not

Proof: We have

$RE = 5$ cm

and

$RD = 2.5$ cm

Now

$$\frac{RE}{RD} = \frac{5}{2.5} = \frac{2}{1}$$

Similarly, we have,

$ES = 1.5$ cm

and

$FS = 3.5$ cm

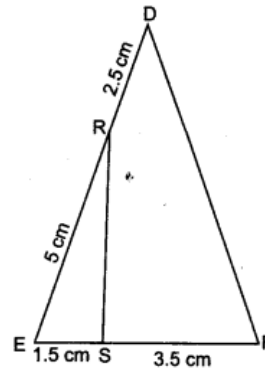
Now,

$$\frac{SF}{ES} = \frac{3.5}{1.5} = \frac{7}{3}$$

Here

$$\frac{RE}{RD} \neq \frac{SF}{ES}$$

$\Rightarrow RS$ is not parallel to DF .



Short Answer Type Questions II [3 Marks]

Question 3.

From airport two aeroplanes start at the same time. If the speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h, then find the distance between two aeroplanes after 2 hours.

Solution:

Speed of aeroplane along north = 500 km/h
 Speed of aeroplane along east = 650 km/h
 Distance travelled by aeroplane in 2 hours in North direction.

$$= OB = 500 \times 2$$

$$= 1000 \text{ km.}$$

Distance travelled by aeroplane in 2 hours in East direction.

$$= OA = 650 \times 2$$

$$= 1300 \text{ km}$$

Distance between both aeroplanes after 2 hours = AB.

$$\therefore AB^2 = OB^2 + OA^2$$

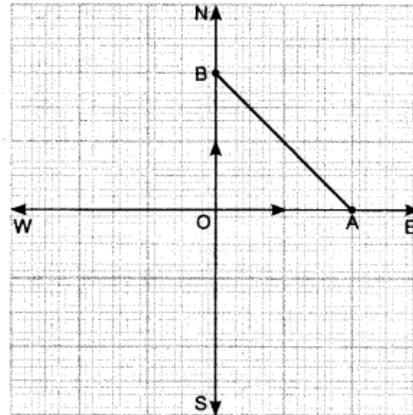
[By Pythagoras theorem in $\triangle AOB$]

$$= (1000)^2 + (1300)^2$$

$$= 1000000 + 1690000$$

$$= 2690000$$

$$AB = 100\sqrt{269} \text{ km}$$



Question 4.

$\triangle ABC$, is right angled at C. If p is the length of the perpendicular from C to AB and a, b, c are

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}.$$

the lengths of the sides opposite $\angle A, \angle B, \angle C$ respectively then prove that

Solution:

Given: In $\triangle ACB$, $\angle C = 90^\circ$, and $CD \perp AB$.

Also, $AB = c$, $BC = a$, $CA = b$ and $CD = p$

To prove: $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Proof: In $\triangle ABC$, $\angle C = 90^\circ$

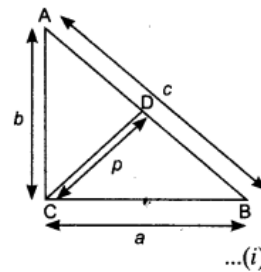
Apply Pythagoras theorem

$$AB^2 = BC^2 + AC^2$$

$$\Rightarrow c^2 = a^2 + b^2$$

Now, area of $\triangle ABC = \frac{1}{2} \times AB \times CD = \frac{1}{2} \times p \times c$... (ii)

Also area of $\triangle ABC = \frac{1}{2} \times AC \times BC = \frac{1}{2} \times b \times a$... (iii)



Equating (iii) and (ii), we have

$$\frac{1}{2}pc = \frac{1}{2}ab$$

$$pc = ab$$

$$c = \frac{ab}{p} \text{ and } c^2 = \frac{a^2b^2}{p^2}$$

Put value of c^2 in equation (i)

$$\frac{a^2b^2}{p^2} = a^2 + b^2$$

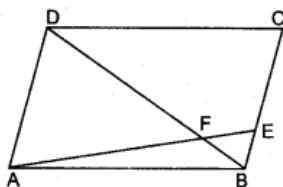
$$\frac{1}{p^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2} \quad \text{(dividing both sides by } a^2b^2)$$

$$\frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$$

Question 5.

In the figure, ABCD is a parallelogram and E divides BC in the ratio 1: 3. DB and AE intersect at F. Show that $DF = 4FB$ and $AF = 4FE$

Solution:



Given: ABCD is a parallelogram and $BE : EC :: 1 : 3$

To show: $DF = 4FB$ and $AF = 4FE$

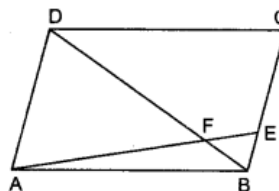
Proof: In $\triangle ADF$ and $\triangle EBF$

$$\angle ADF = \angle EBF \quad \text{(Alternate angles)}$$

$$\angle AFD = \angle EFB \quad \text{(V.O.A.)}$$

$$\triangle ADF \sim \triangle EBF \quad \text{(by AA)}$$

$$\frac{DF}{BF} = \frac{AF}{EF} = \frac{AD}{BE} \quad \dots(i)$$



as

$$\frac{BE}{EC} = \frac{1}{3} \Rightarrow EC = 3BE$$

\therefore

$$BC = BE + CE = BE + 3BE$$

\Rightarrow

$$BC = 4BE$$

as

$$AD = BC$$

\therefore

$$AD = 4BE$$

Put in (i), we get

$$\frac{DF}{BF} = \frac{AF}{EF} = \frac{4}{1}$$

and

$$DF = 4BF$$

$$AF = 4EF$$

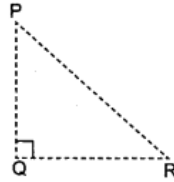
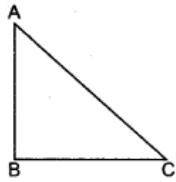
Long Answer Type Questions [4 Marks]

Question 6.

State and prove Converse of Pythagoras' Theorem.

Solution:

Statement: In a triangle, if the square of one side is equal to the sum of the squares of the other two sides then the angle opposite to the first side is right angle.



Given: In $\triangle ABC$, $AC^2 = AB^2 + BC^2$

To prove: $\angle ABC = 90^\circ$

Construction: Construct a $\triangle PQR$, such that $AB = PQ$ and $BC = QR$ and $\angle Q = 90^\circ$

Proof: In $\triangle PQR$, $\angle Q = 90^\circ$ (Given)

$\therefore PR^2 = PQ^2 + QR^2$ (By Pythagoras)

$$PR^2 = AB^2 + BC^2$$

(Using $PQ = AB$ and $QR = BC$ by construction) ... (i)

but $AC^2 = AB^2 + BC^2$ (Given) ... (ii)

Equating (i) and (ii) $PR^2 = AC^2$

$$\Rightarrow PR = AC$$

Now, in $\triangle ABC$ and $\triangle PQR$

$AB = PQ$ (By construction)

$BC = QR$ (by construction)

$AC = PR$ (Proved)

$\therefore \triangle ABC \cong \triangle PQR$ (By SSS)

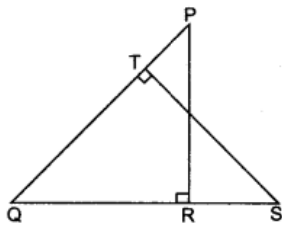
Hence $\angle B = \angle Q = 90^\circ$

This shows that, $\triangle ABC$ is a right-angled triangle.

Question 7.

In the figure, $\triangle PQR$ and $\triangle STQ$ respectively. Prove that $QR \times QS = QP \times QT$

Solution:



Given: In $\triangle PQR$, $\angle R = 90^\circ$ and in $\triangle STQ$, $\angle T = 90^\circ$

To prove: $QR \times QS = QP \times QT$

Proof: In $\triangle PQR$ and $\triangle STQ$

$$\angle PQR = \angle SQT$$

(Common)

$$\angle PRQ = \angle STQ = 90^\circ$$

(Given)

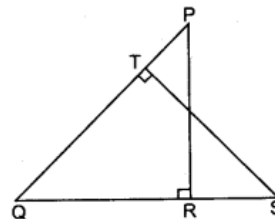
$$\triangle PQR \sim \triangle STQ$$

(By AA)

$$\Rightarrow \frac{QR}{QT} = \frac{QP}{QS}$$

(Corresponding sides of similar triangles are proportion)

$$\Rightarrow QR \times QS = QP \times QT$$



Very Short Answer Type Question [1 Mark]

Question 8.

In $\triangle DEW$, $AB \parallel EW$. If $AD = 4$ cm, $DE = 12$ cm and $DW = 24$ cm, then find the value of DB .

Solution:

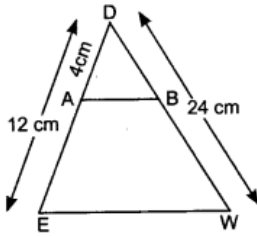
In $\triangle DEW$, $AB \parallel EW$

So, by B.P.T.,

$$\frac{AD}{DE} = \frac{DB}{DW}$$

$$\Rightarrow \frac{4}{12} = \frac{DB}{24}$$

$$\Rightarrow DB = 8 \text{ cm}$$



Short Answer Type Question I [2 Marks]

Question 9.

A ladder is placed against a wall such that its foot is at distance of 5 m from the wall and its top reaches a window $5\sqrt{3}$ m above the ground. Find the length of the ladder

Solution:

$\therefore ABC$ is a right triangle, right angled at B .

So, by Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

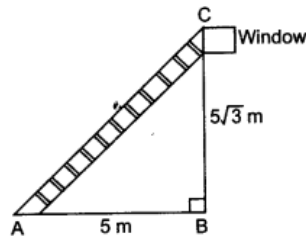
$$= 5^2 + (5\sqrt{3})^2 = 25 + 75$$

$$AC^2 = 100$$

$$\Rightarrow AC = \sqrt{100}$$

$$\Rightarrow AC = 10 \text{ m}$$

Hence, length of the ladder = 10 m.



Short Answer Type Questions II [3 Marks]

Question 10.

In figure, if $\angle CAB = \angle CED$, then prove that $AB \times DC = ED \times BC$.

Solution:

Given: $\angle CAB = \angle CED$

i.e. $\angle 1 = \angle 2$

To prove: $AB \times DC = ED \times BC$

Proof: In $\triangle CAB$ and $\triangle CED$

$$\angle 1 = \angle 2$$

$$\angle C = \angle C$$

So, $\triangle CAB \sim \triangle CED$

$$\Rightarrow \frac{BC}{DC} = \frac{AB}{ED}$$

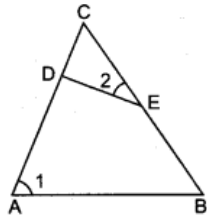
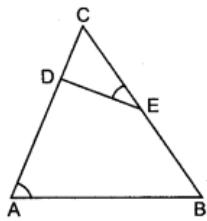
$$\Rightarrow AB \times DC = BC \times ED$$

(Given)

(Common)

(By AA similarity)

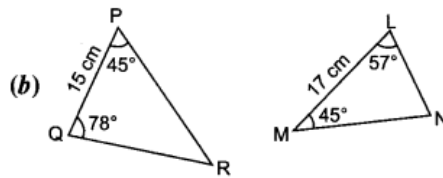
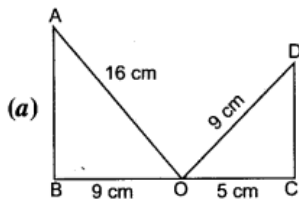
Hence Proved



Question 11.

State whether the given pairs of triangles are similar or not. In mention the criterion.

Solution:



(a) In $\triangle ABO$, $\angle B = 90^\circ$

By Pythagoras theorem,

$$AB = \sqrt{16^2 - 9^2} = \sqrt{256 - 81} = \sqrt{175} = 5\sqrt{7} \text{ cm.}$$

In $\triangle DCO$, $\angle C = 90^\circ$

So, by Pythagoras theorem,

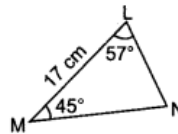
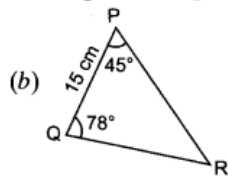
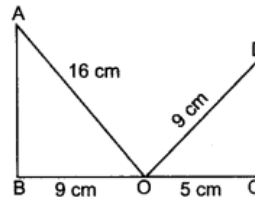
$$DC = \sqrt{9^2 - 5^2} = \sqrt{81 - 25} = \sqrt{56} = 2\sqrt{14}$$

\therefore In $\triangle ABO$, $AB = 5\sqrt{7}$ cm, $OB = 9$ cm; $AO = 16$ cm

and In $\triangle DCO$, $DC = 2\sqrt{14}$ cm; $OC = 5$ cm; $DO = 9$ cm

Clearly the sides of both the triangles are not proportional

So, given triangles are not similar.



In $\triangle PQR$,

$$\angle P + \angle Q + \angle R = 180^\circ$$

$$\Rightarrow 45^\circ + 78^\circ + \angle R = 180^\circ$$

$$\Rightarrow \angle R = 180^\circ - 123^\circ$$

$$\Rightarrow \angle R = 57^\circ$$

In $\triangle LMN$

$$\angle L + \angle M + \angle N = 180^\circ$$

$$\Rightarrow 57^\circ + 45^\circ + \angle N = 180^\circ$$

$$\Rightarrow \angle N = 180^\circ - 102^\circ$$

$$\Rightarrow \angle N = 78^\circ$$

Now, in $\triangle PQR$ and $\triangle LMN$,

$$\therefore \angle P = \angle M = 45^\circ$$

$$\angle Q = \angle N = 78^\circ$$

$$\angle R = \angle L = 57^\circ$$

So, $\triangle PQR \sim \triangle LMN$

(By AAA criterion)

Long Answer Type Questions [4 Marks]

Question 12.

In $\triangle ABC$, from A and B altitudes AD and BE are drawn. Prove that $\triangle ADC \sim \triangle BEC$. Is $\triangle ADB$

~ $\triangle AEB$ and $\triangle ADB \sim \triangle ADC$?

Solution:

In $\triangle ADC$ and $\triangle BEC$,

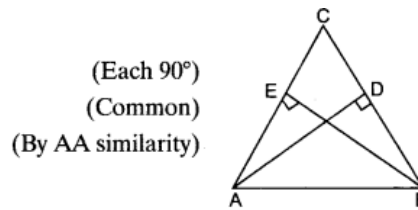
$$\angle D = \angle E$$

$$\angle ACD = \angle BCE$$

So, $\triangle ADC \sim \triangle BCE$

$\triangle ADB$ is not similar to $\triangle AEB$.

and $\triangle ADB$ is not similar to $\triangle ADC$.



Question 13.

In $\triangle ABC$, if $\angle ADE = \angle B$, then prove that $\triangle ADE \sim \triangle ABC$. Also, if $AD = 7.6$ cm, $AE = 7.2$ cm, $BE = 4.2$ cm and $BC = 8.4$ cm, then find DE .

Solution:

Given: $\angle ADE = \angle B$, i.e. $\angle 1 = \angle 2$

To prove: $\triangle ADE \sim \triangle ABC$

Proof: In $\triangle ADE$ and $\triangle ABC$

$$\angle 1 = \angle 2$$

$$\angle A = \angle A$$

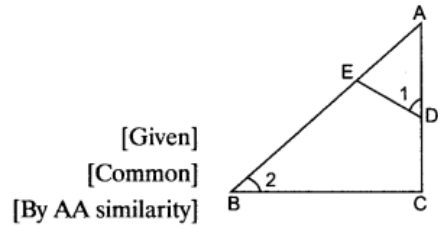
So, $\triangle ADE \sim \triangle ABC$

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{7.6}{7.2 + 4.2} = \frac{DE}{8.4}$$

$$\Rightarrow \frac{7.6}{11.4} = \frac{DE}{8.4} \Rightarrow DE = \frac{7.6 \times 8.4}{11.4} = 5.6$$

Hence, $DE = 5.6$ cm.



$$\{\because AB = AE + BE = 7.2 + 4.2\}$$

2014

Very Short Answer Type Questions [1 Mark]

Question 14.

If $\triangle ABC \sim \triangle RPQ$, $AB = 3$ cm, $BC = 5$ cm, $AC = 6$ cm, $RP = 6$ cm and $PQ = 10$ cm, then find QR .

Solution:

It is given that $\triangle ABC \sim \triangle RPQ$

$$\therefore \frac{AB}{RP} = \frac{AC}{QR}$$

Now, $AB = 3$ cm, $RP = 6$ cm, $QR = ?$, $AC = 6$ cm

$$\text{So, } \frac{3}{6} = \frac{6}{QR}$$

$$QR = \frac{36}{3} = 12 \text{ cm}$$

Question 15.

Let $\triangle ABC \sim \triangle DEF$, ar $(\triangle ABC) = 169$ cm² and ar $(\triangle DEF) = 121$ cm². If $AB = 26$ cm then find DE .

Solution:

It is given that $\Delta ABC \sim \Delta DEF$

$$\therefore \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AB^2}{DE^2} \quad [\text{When two triangles are similar then the ratio of their areas is equal to ratio of squares of any of corresponding sides}]$$

$$\begin{aligned} \text{So, } \frac{169}{121} &= \left(\frac{26}{DE}\right)^2 \\ \frac{26}{DE} &= \sqrt{\frac{169}{121}} \\ \frac{26}{DE} &= \frac{13}{11} \\ DE &= \frac{26 \times 11}{13} \\ DE &= 22 \text{ cm} \end{aligned}$$

Question 16.

If in an equilateral triangle, the length of the median is $\sqrt{3}$ cm, then find the length of the side of equilateral triangle.

Solution:

Let a be the side of equilateral triangle. Median is also the altitude in an equilateral triangle.

$$\begin{aligned} \therefore (\text{Altitude})^2 + \left(\frac{a}{2}\right)^2 &= (a)^2 \\ (\sqrt{3})^2 + \frac{a^2}{4} &= a^2 & [\because \text{Altitude} = \text{Median} = \sqrt{3} \text{ cm}] \\ \frac{12 + a^2}{4} &= a^2 \\ 12 + a^2 &= 4a^2 \\ 3a^2 &= 12 \\ a^2 &= 4 \\ a &= 2 \end{aligned}$$

\therefore Side of triangle = 2 cm

Short Answer Type Questions I [2 Marks]

Question 17.

In the figure, D and E are points on AB and AC respectively such that $DE \parallel BC$. If $AD = \frac{1}{3} BD$ and $AE = 4.5$ cm, find AC.

Solution:

$$DE \parallel BC \quad \text{5} \quad [\text{Given}]$$

$$AD = \frac{1}{3} BD$$

$$AE = 4.5 \text{ cm}$$

$$\text{Now, } \therefore AD = \frac{1}{3} BD$$

$$\therefore \frac{AD}{BD} = \frac{1}{3} \quad \dots(i)$$

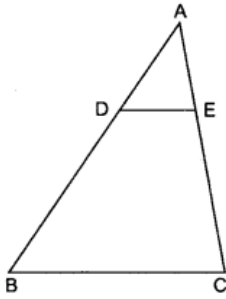
$$DE \parallel BC,$$

$$\therefore \frac{AD}{BD} = \frac{AE}{EC}$$

$$\text{So, } \frac{1}{3} = \frac{4.5}{EC}$$

$$EC = 13.5 \text{ cm}$$

$$AC = AE + EC = 4.5 + 13.5 = 18 \text{ cm}$$



[By B.P.T.]

Question 18.

Determine whether the triangle having sides $(a - 1)$ cm, $2\sqrt{a}$ cm and $(a + 1)$ cm is a right angled triangle.

Solution:

Given three sides of triangle are $(a - 1)$ cm, $2\sqrt{a}$ cm and $(a + 1)$ cm

In the given triangle, $(a + 1)$ cm is the longest side.

$$\text{Now, } (a + 1)^2 = a^2 + 1 + 2a \quad \dots(i)$$

$$\begin{aligned} \text{and sum of squares of other two sides} &= (2\sqrt{a})^2 + (a - 1)^2 \\ &= 4a + a^2 + 1 - 2a = a^2 + 1 + 2a \quad \dots(ii) \end{aligned}$$

\therefore From (i) and (ii)

$$(a + 1)^2 = (2\sqrt{a})^2 + (a - 1)^2$$

\therefore By converse of Pythagoras Theorem, given triangle is a right angled triangle.

Question 19.

In an equilateral triangle of side $3\sqrt{3}$ cm, find the length of the altitude.

Solution:

In ΔABC , AD is altitude

AD is median also

[$\because \Delta ABC$ is equilateral]

So,

$$BD = \frac{3\sqrt{3}}{2} \text{ cm}$$

In ΔABD ,

$$(AD)^2 + (BD)^2 = (AB)^2$$

$$\therefore (AD)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2 = (3\sqrt{3})^2$$

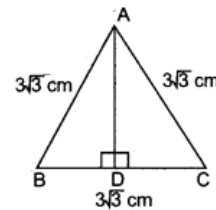
$$AD^2 + \frac{27}{4} = 27$$

$$AD^2 = 27 - \frac{27}{4}$$

$$AD^2 = \frac{108 - 27}{4} \Rightarrow AD^2 = \frac{81}{4} \Rightarrow AD = \frac{9}{2} = 4.5 \text{ cm}$$

\therefore

$$\text{Altitude} = 4.5 \text{ cm}$$



Short Answer Type Questions II [3 Marks]

Question 20.

In the figure, ABC is a triangle and $BD \perp AC$. Prove that $AB^2 + CD^2 = AD^2 + BC^2$

Solution:

Given: A triangle ABC in which $BD \perp AC$.

To prove: $AB^2 + CD^2 = AD^2 + BC^2$

Proof: In right $\triangle ADB$,

$$AB^2 = AD^2 + BD^2$$

[By Pythagoras Theorem] ... (i)

In right $\triangle CDB$,

$$BC^2 = CD^2 + BD^2$$

[By Pythagoras Theorem] ... (ii)

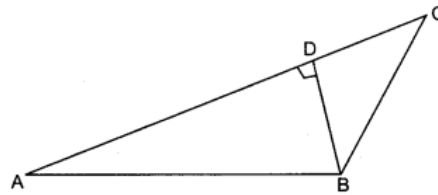
Subtracting equation (ii) from (i), we get,

$$AB^2 - BC^2 = AD^2 + BD^2 - CD^2 - BD^2$$

$$AB^2 - BC^2 = AD^2 - CD^2$$

$$\therefore AB^2 + CD^2 = AD^2 + BC^2$$

Hence proved.



Question 21.

Right angled triangles BAC and BDC are right angled at A and D and they are on same side of BC. If AC and BD intersect at P, then prove that $AP \times PC = PB \times DP$.

Solution:

In $\triangle APB$ and $\triangle DPC$,

$$\angle BAP = \angle CDP = 90^\circ$$

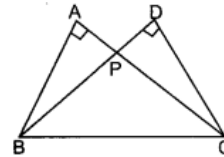
$$\angle APB = \angle DPC$$

$$\therefore \triangle APB \sim \triangle DPC$$

$$\therefore \frac{AP}{DP} = \frac{PB}{PC}$$

$$AP \times PC = PB \times DP$$

[Given]
[Vertically opposite angles]
[by AA similarity of triangles.]



Hence proved.

Question 22.

In the given figure, if $LM \parallel CB$ and $LN \parallel CD$, prove that $AM \times AD = AB \times AN$.

Solution:

Given: $LM \parallel CB$

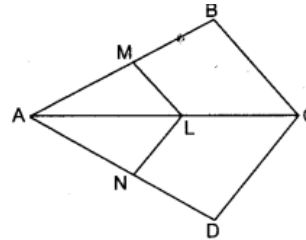
$LN \parallel CD$

To prove: $AM \times AD = AB \times AN$

Proof: In $\triangle BAC$,

$$\therefore \frac{AM}{MB} = \frac{AL}{LC}$$

... (i) [By Thales Theorem]



In $\triangle DAC$,

$$\therefore \frac{AN}{ND} = \frac{AL}{LC}$$

... (ii) [By Thales Theorem]

\therefore From (i) and (ii)

$$\frac{AM}{MB} = \frac{AN}{ND}$$

or
$$\frac{MB}{AM} = \frac{ND}{AN}$$

$$\frac{MB}{AM} + 1 = \frac{ND}{AN} + 1$$

[Adding 1 on both sides]

$$\frac{MB + AM}{AM} = \frac{ND + AN}{AN}$$

$$\frac{AB}{AM} = \frac{AD}{AN}$$

$$\therefore AM \times AD = AB \times AN$$

Hence proved.

Question 23.

In the given figure, ABC is a triangle, right angled at B and $BD \perp AC$. If $AD = 4$ cm and $CD = 5$ cm, find BD and AB.

Solution:

Given: A right $\triangle ABC$ in which $BD \perp AC$.

$$AD = 4 \text{ cm}, CD = 5 \text{ cm}$$

In $\triangle ABC$,

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + BC^2 = (4 + 5)^2 = (9)^2 = 81 \dots(i) \text{ [By Pythagoras Theorem]}$$

In $\triangle ADB$,

$$AD^2 + BD^2 = AB^2 \dots(ii) \text{ [By Pythagoras Theorem]}$$

In $\triangle CDB$,

$$CD^2 + BD^2 = BC^2 \dots(iii) \text{ [By Pythagoras Theorem]}$$

Adding equations (ii) and (iii), we get

$$AD^2 + CD^2 + 2BD^2 = AB^2 + BC^2$$

$$(4)^2 + (5)^2 + 2BD^2 = 81$$

$$2BD^2 + 16 + 25 = 81$$

$$2BD^2 = 81 - 41$$

$$2BD^2 = 40$$

$$BD^2 = 20$$

$$BD = \sqrt{20} = 2\sqrt{5} \text{ cm}$$

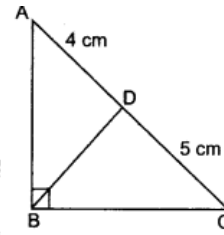
Putting, $BD = 2\sqrt{5}$ and $AD = 4$ cm in equation (ii), we get

$$(4)^2 + (2\sqrt{5})^2 = AB^2$$

$$AB^2 = 16 + 20$$

$$AB^2 = 36$$

$$AB = \sqrt{36} = 6 \text{ cm}$$



Question 24.

Equiangular triangles are drawn on sides of right angled triangle in which perpendicular is double of its base. Show that area of triangle on the hypotenuse is the sum of areas of the other two triangles?

Solution:

Given: A right angled triangle ABC with right angled at B.

Equiangular triangles PAB, QBC and RAC are described on sides AB, BC and CA respectively.

Let $BC = x$ and $AB = 2x$

To prove: $\text{ar}(\triangle PAB) + \text{ar}(\triangle QBC) = \text{ar}(\triangle RAC)$

Proof: \because Equiangular triangles are equilateral also,

$$\therefore \text{Area of } \triangle PAB = \frac{\sqrt{3}}{4} \times (2x)^2$$

$$= \sqrt{3}x^2$$

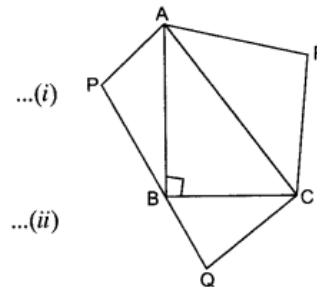
$$\text{Area of } \triangle QBC = \frac{\sqrt{3}}{4} \times (x)^2$$

$$= \frac{\sqrt{3}}{4}x^2$$

$$\text{Area of } \triangle RAC = \frac{\sqrt{3}}{4} \times (AC)^2$$

$$= \frac{\sqrt{3}}{4} \times 5x^2$$

$$= \frac{5\sqrt{3}x^2}{4}$$



...(i)

...(ii)

$$[\because \text{In } \triangle ABC, AB^2 + BC^2 = AC^2 \\ AC^2 = (2x)^2 + x^2 = 5x^2]$$

...(iii)

Adding (i) and (ii)

$$\text{ar}(\triangle PAB) + \text{ar}(\triangle QBC) = \sqrt{3}x^2 + \frac{\sqrt{3}x^2}{4} = \frac{4\sqrt{3}x^2 + \sqrt{3}x^2}{4} = \frac{5\sqrt{3}x^2}{4}$$

$$= \text{ar}(\triangle RAC)$$

[From (iii)]

$$\therefore \text{ar}(\triangle PAB) + \text{ar}(\triangle QBC) = \text{ar}(\triangle RAC)$$

Hence proved.

Question 25.

If in a right angle AABC, right angled at A, $AD \perp BC$, then prove that $AB^2 + CD^2 = BD^2 +$

AC^2

Solution:

In right $\triangle ADB$, $\angle D = 90^\circ$

$$AB^2 = AD^2 + BD^2 \quad \dots(i) \text{ [By Pythagoras Theorem]}$$

In right $\triangle ADC$, $\angle D = 90^\circ$

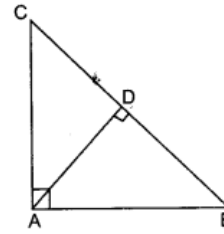
$$AC^2 = AD^2 + CD^2 \quad \dots(ii) \text{ [By Pythagoras Theorem]}$$

Subtracting equation (ii) from (i), we get

$$AB^2 - AC^2 = AD^2 + BD^2 - AD^2 - CD^2$$

$$AB^2 - AC^2 = BD^2 - CD^2$$

Hence proved.



Long Answer Type Questions [4 Marks]

Question 26.

Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Solution:

Given: Two triangles ABC and DEF such that $\triangle ABC \sim \triangle DEF$.

$$\text{To prove: } \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

Construction: Draw $AL \perp BC$ and $DM \perp EF$

Proof: In $\triangle ALB$ and $\triangle DME$,

$$\angle ALB = \angle DME = 90^\circ \text{ [By Construction]}$$

$$\angle B = \angle E \quad [\because \triangle ABC \sim \triangle DEF]$$

\therefore By AA criterion of similarity,

$$\triangle ALB \sim \triangle DME$$

$$\text{So, } \frac{AL}{DM} = \frac{AB}{DE} \quad \dots(i)$$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \quad \dots(ii) \text{ [Ratio of corresponding sides of similar triangles are equal]}$$

From (i) and (ii), we get

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AL}{DM} \quad \dots(iii)$$

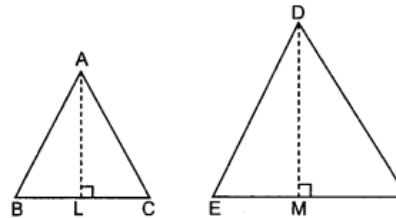
$$\text{Now, } \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{\frac{1}{2}(BC \times AL)}{\frac{1}{2}(EF \times DM)}$$

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{BC \times AL}{EF \times DM}$$

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{BC \times BC}{EF \times EF} = \frac{BC^2}{EF^2} \quad \text{[From (iii)]}$$

$$\text{And } \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} \quad \text{[From (iii)]}$$

$$\text{Hence, } \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} \quad \text{Hence proved.}$$



Question 27.

In $\triangle ABC$, $AX \perp BC$ and Y is middle point of BC. Then prove that,

$$(i) \quad AB^2 = AY^2 + \frac{BC^2}{4} - BC \cdot XY$$

$$(ii) \quad AC^2 = AY^2 + \frac{BC^2}{4} + BC \cdot XY$$

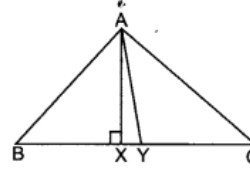
Solution:

In $\triangle ABC$, $AX \perp BC$ and Y is middle point of BC .

Then prove that,

$$(i) AB^2 = AY^2 + \frac{BC^2}{4} - BC \cdot XY$$

$$(ii) AC^2 = AY^2 + \frac{BC^2}{4} + BC \cdot XY$$



Given: A $\triangle ABC$ in which $AX \perp BC$ and Y is mid-point of BC .

To prove: (i) $AB^2 = AY^2 + \frac{BC^2}{4} - BC \cdot XY$

$$(ii) AC^2 = AY^2 + \frac{BC^2}{4} + BC \cdot XY$$

Proof: (i) In $\triangle ABX$,

$$AB^2 = AX^2 + BX^2 \quad \text{[By Pythagoras Theorem]}$$

$$AB^2 = AX^2 + (BY - XY)^2$$

$$AB^2 = AX^2 + \left(\frac{BC}{2} - XY\right)^2 \quad \text{[}\because Y \text{ is mid point of } BC\text{]}$$

$$AB^2 = AX^2 + \frac{BC^2}{4} + XY^2 - 2\left(\frac{BC}{2}\right)(XY)$$

$$AB^2 = (AX^2 + XY^2) + \frac{BC^2}{4} - \frac{2BC}{2} \cdot XY$$

$$AB^2 = AY^2 + \frac{BC^2}{4} - BC \cdot XY \quad \text{[}\because \text{ In } \triangle AXY, AX^2 + XY^2 = AY^2\text{]} \\ \text{Hence proved.}$$

(ii) In $\triangle AXC$,

$$AC^2 = AX^2 + XC^2 \quad \text{[By Pythagoras Theorem]}$$

$$AC^2 = AX^2 + (XY + YC)^2$$

$$AC^2 = AX^2 + \left(XY + \frac{BC}{2}\right)^2 \quad \text{[}\because Y \text{ is mid-point of } BC\text{]}$$

$$AC^2 = (AX^2 + XY^2) + \frac{BC^2}{4} + 2(XY) \cdot \left(\frac{BC}{2}\right)$$

$$AC^2 = AY^2 + \frac{BC^2}{4} + BC \cdot XY \quad \text{[}\because \text{ In } \triangle AXY, AX^2 + XY^2 = AY^2\text{]} \\ \text{Hence proved.}$$

Question 28.

Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then other two sides are divided in the same ratio.

Solution:

Given: A triangle ABC in which $DE \parallel BC$ and DE intersects AB in D and AC in E .

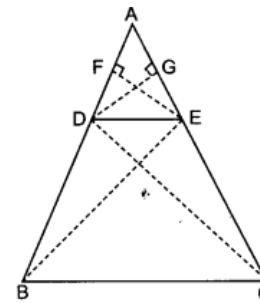
To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE , CD and draw $EF \perp BA$ and $DG \perp CA$

Proof: \because EF is perpendicular to AB .
 \therefore EF is height of triangles ADE and DBE .

$$\text{Now, } \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DBE)} = \frac{\frac{1}{2}(AD \times EF)}{\frac{1}{2}(DB \times EF)} = \frac{AD}{DB} \quad \dots(i)$$

Similarly, $DG \perp CA$, so DG is height of $\triangle ADE$ and $\triangle DEC$.



$$\therefore \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DEC)} = \frac{\frac{1}{2}(\text{AE} \times \text{DG})}{\frac{1}{2}(\text{EC} \times \text{DG})} = \frac{\text{AE}}{\text{EC}} \quad \dots(ii)$$

But, $\triangle DBE$ and $\triangle DEC$ are on same base DE and between same parallels DE and BC .

$$\therefore \frac{\text{ar}(\triangle DBE)}{\text{ar}(\triangle DEC)} = \frac{1}{1}$$

Multiplying both sides by $\text{ar}(\triangle ADE)$

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DBE)} = \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DEC)}$$

Thus,
$$\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}} \quad \text{[From (i) and (ii)]}$$

Question 29.

Prove that the sum of square of the sides of a rhombus is equal to the sum of squares of its diagonals.

Solution:

Given: ABCD is a rhombus in which diagonals AC and BD intersect at O.

To prove: $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Proof: As diagonals of a rhombus intersect/bisect each other at 90° .

$\therefore \triangle AOD, \triangle BOA, \triangle COB$ and $\triangle DOC$ are right triangles

and $OD = OB = \frac{BD}{2} \quad \dots(i)$

$OA = OC = \frac{AC}{2} \quad \dots(ii)$

Now by Pythagoras Theorem,

In $\triangle AOD,$
 $AD^2 = OD^2 + OA^2 \quad \dots(iii)$

In $\triangle BOA,$
 $AB^2 = OA^2 + OB^2 \quad \dots(iv)$

In $\triangle COB,$
 $BC^2 = OC^2 + OB^2 \quad \dots(v)$

In $\triangle DOC,$
 $CD^2 = OC^2 + OD^2 \quad \dots(vi)$

Adding equations (iii), (iv), (v) and (vi), we get

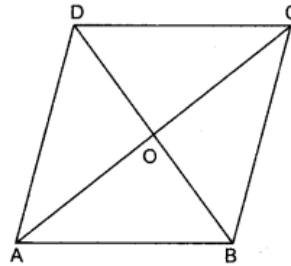
$$AD^2 + AB^2 + BC^2 + CD^2 = 2OD^2 + 2OA^2 + 2OB^2 + 2OC^2$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2(OD^2 + OA^2 + OB^2 + OC^2)$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2 \left[\left(\frac{BD}{2} \right)^2 + \left(\frac{AC}{2} \right)^2 + \left(\frac{BD}{2} \right)^2 + \left(\frac{AC}{2} \right)^2 \right]$$

$$AB^2 + BC^2 + CD^2 + AD^2 = 2 \left[\frac{2(BD^2 + AC^2)}{4} \right]$$

$\therefore AB^2 + BC^2 + CD^2 + AD^2 = BD^2 + AC^2$ Hence proved.



Question 30.

In $\triangle ABC$, X is any point on AC. If Y, Z, U and V are the middle points of AX, XC, AB and BC respectively, then prove that $UY \parallel VZ$ and $UV \parallel YZ$.

Solution:

Given: In $\triangle ABC$, Y, Z, U and V are mid-points of AX, XC, AB and BC.

To prove: $UY \parallel VZ$ and $UV \parallel YZ$

Construction: Join BX

Proof: In $\triangle ABX$,

$$AU = UB, \text{ i.e. } \frac{AU}{UB} = \frac{1}{1}$$

and $AY = YX, \text{ i.e. } \frac{AY}{YX} = \frac{1}{1}$

$$\therefore \frac{AU}{UB} = \frac{AY}{YX}$$

So, $UY \parallel BX$

Similarly, in $\triangle BCX$,

$$BX \parallel VZ$$

\therefore From (i) and (ii)

$$UY \parallel VZ$$

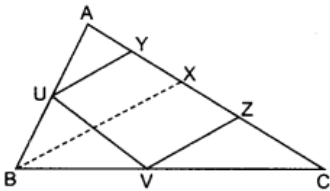
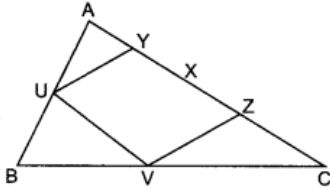
In $\triangle ABC$,

$$\frac{BU}{UA} = \frac{BV}{VC} = \frac{1}{1}$$

\therefore By converse of B.P.T.,

$$UV \parallel YZ$$

From equation (iii) and (iv), $UY \parallel VZ$ and $UV \parallel YZ$.



Question 31.

In $\triangle ABC$, $\angle B = 90^\circ$, $BD \perp AC$, ar $(\triangle ABC) = A$ and $BC = a$, then prove that

$$BD = \frac{2Aa}{\sqrt{4A^2 + a^4}}$$

Solution:

In $\triangle ABC$, $\angle B = 90^\circ$, $BD \perp AC$, ar $(\triangle ABC) = A$ and $BC = a$, then prove that

$$BD = \frac{2Aa}{\sqrt{4A^2 + a^4}}$$

Given: Area of $\triangle ABC = A$

$$BC = a \text{ and } BD \perp AC$$

To prove: $BD = \frac{2Aa}{\sqrt{4A^2 + a^4}}$

Proof: $A = \text{Area of } \triangle ABC$

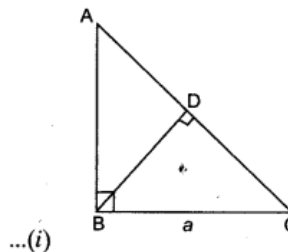
$$A = \frac{1}{2} BC \times AB = \frac{1}{2} \times a \times AB$$

$$AB = \frac{2A}{a}$$

In $\triangle ADB$ and $\triangle ABC$

$$\angle ADB = \angle ABC = 90^\circ$$

$$\angle BAD = \angle BAC$$



[Common]

∴ By AA similarity criterion, $\triangle ADB \sim \triangle ABC$

$$\therefore \frac{AB}{AC} = \frac{BD}{BC} \quad \dots(ii)$$

In $\triangle ABC$, $AB^2 + BC^2 = AC^2$ [By Pythagoras Theorem]

$$\frac{4A^2}{a^2} + a^2 = AC^2$$

$$AC = \sqrt{\frac{4A^2 + a^4}{a^2}} = \frac{\sqrt{4A^2 + a^4}}{a}$$

From (i) and (ii), we get

$$\frac{2A}{a \times AC} = \frac{BD}{a}$$

$$BD = \frac{2A}{AC} = \frac{2Aa}{\sqrt{4A^2 + a^4}}$$

Hence proved.

2013

Short Answer Type Questions I [2 Marks]

Question 32.

In the figure, PQR and SQR are two right triangles with common hypotenuse QR. If PR and SQ intersect at M such that PM = 3 cm, MR = 6 cm and SM = 4 cm, find the length of MQ.

Solution:

Consider the triangles MPQ and MSR,

$$\angle P = \angle S \quad (\text{Each } 90^\circ)$$

$$\angle 1 = \angle 2 \quad (\text{Vertically opposite angles})$$

So, $\triangle MPQ \sim \triangle MSR$ (By AA similarity)

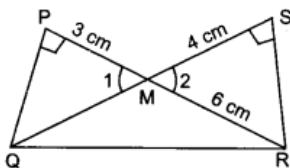
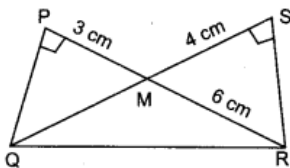
$$\frac{PM}{SM} = \frac{MQ}{MR}$$

$$\frac{3}{4} = \frac{MQ}{6}$$

$$\frac{3}{4} = \frac{MQ}{6}$$

$$MQ = \frac{3}{4} \times 6 = \frac{9}{2} = 4.5 \text{ cm}$$

Hence, MQ = 4.5 cm.



Question 33.

Find the length of each altitude of an equilateral triangle

Solution:

$\triangle ABC$ is an equilateral triangle.

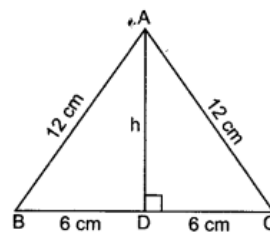
∴ AD is the altitude of height 'h' (say) then AD is median of BC also.

$$\Rightarrow CD = BD = 6 \text{ cm.}$$

In right $\triangle ADC$, $\angle D = 90^\circ$

$$AC^2 = AD^2 + CD^2 \quad (\text{By Pythagoras Theorem})$$

$$12^2 = h^2 + 6^2$$



$$\begin{aligned}
 h^2 &= 12^2 - 6^2 \\
 &= 144 - 36 = 108 \\
 h &= \sqrt{108} \Rightarrow h = 6\sqrt{3} \text{ cm.}
 \end{aligned}$$

Hence, length of altitude is $6\sqrt{3}$ cm.

Short Answer Type Questions II [3 Marks]

Question 34.

In the figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$ prove that

$$\frac{BE}{DE} = \frac{AC}{BC}$$

Solution:

Given $DB \perp BC$, i.e. $\angle 1 + \angle 2 = 90^\circ$

$DE \perp AB$, i.e. $\angle E = 90^\circ$

$AC \perp BC$, i.e. $\angle C = 90^\circ$

To prove: $\frac{BE}{DE} = \frac{AC}{BC}$

Proof: In $\triangle ABC$, $\angle C = 90^\circ$

$$\angle A + \angle 2 = 90^\circ \quad \dots(i)$$

Also, $\angle 1 + \angle 2 = 90^\circ$ (given) $\dots(ii)$

From (i) and (ii), we get

$$\angle 1 + \angle 2 = \angle 2 + \angle A$$

$$\Rightarrow \angle 1 = \angle A$$

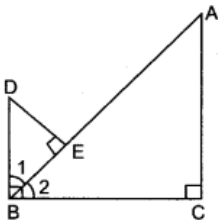
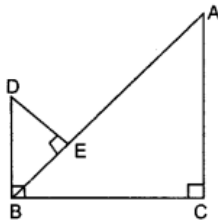
Now, in $\triangle DEB$ and $\triangle BCA$

$$\angle E = \angle C \quad \text{(Each } 90^\circ)$$

$$\angle 1 = \angle A \quad \text{(Proved above)}$$

$$\therefore \triangle DEB \sim \triangle BCA$$

$$\Rightarrow \frac{BE}{AC} = \frac{DE}{BC} \Rightarrow \frac{BE}{DE} = \frac{AC}{BC}$$



Question 35.

In the given figure, ABCD is a rectangle. P is the mid-point of DC. If $QB = 7$ m, $AD = 9$ cm and $DC = 24$ cm, then prove that $\angle APQ = 90^\circ$

Solution:

Given: ABCD is a rectangle.

P is the mid-point of DC, i.e. DP = PC

BQ = 7 cm, AD = 9 cm; CD = 24 cm

To prove: $\angle APQ = 90^\circ$

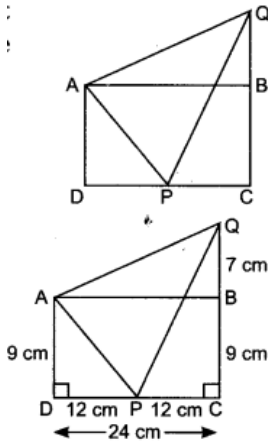
Proof. In $\triangle QCP$, $\angle C = 90^\circ$

$$\begin{aligned}PQ^2 &= PC^2 + QC^2 && \text{(By Pythagoras theorem)} \\ &= 12^2 + 16^2 = 144 + 256 = 400\end{aligned}$$

$$\Rightarrow PQ = 20 \text{ cm}$$

Now, in $\triangle ADP$, $\angle D = 90^\circ$

$$\text{So, } AP^2 = AD^2 + DP^2 \quad \text{(By Pythagoras theorem)}$$



$$= 9^2 + 12^2 = 81 + 144 = 225$$

$$\Rightarrow AP = 15 \text{ cm}$$

Now, in $\triangle QBA$, $\angle B = 90^\circ$

$$\begin{aligned}\Rightarrow AQ^2 &= AB^2 + QB^2 && \text{(By Pythagoras theorem)} \\ &= 24^2 + 7^2 = 576 + 49 = 625\end{aligned}$$

$$\Rightarrow AQ = 25 \text{ cm}$$

$$\therefore 625 = 400 + 225$$

$$\Rightarrow AQ^2 = PQ^2 + AP^2$$

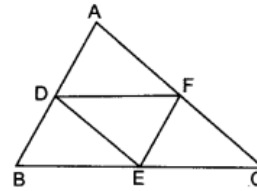
$$\Rightarrow \angle APQ = 90^\circ \quad \text{(By converse of Pythagoras theorem)}$$

Question 36.

In $\triangle ABC$, D, E and F are the mid-points of AB, BC and AC respectively. Find the ratio of area of $\triangle ABC$ and area of $\triangle DEF$.

Solution:

Given: D, E and F are the mid-points of AB, BC and AC respectively of $\triangle ABC$.
 Since, D and F are the mid-points of AB and AC respectively
 So, by mid-point theorem, $DF = \frac{1}{2}BC$ and $DF \parallel BC$



$\Rightarrow DF = BE$ and $DF \parallel BE$

Similarly, $EF = BD$ and $EF \parallel BD$

$\Rightarrow BEFD$ is a parallelogram

So, $\triangle BDE \cong \triangle FED$... (i) [Diagonal divides a parallelogram into two congruent triangles]

Similarly,

$EFCF$ and $ADEF$ are also a parallelogram

$\Rightarrow \triangle ECF \cong \triangle FDE$... (ii) and $\triangle FED \cong \triangle DAF$... (iii)

From (i), (ii) and (iii)

$$\triangle BDE \cong \triangle FED \cong \triangle EFC \cong \triangle DAF$$

We know that congruent triangles have equal areas.

So, $\text{ar}(\triangle BDE) = \text{ar}(\triangle FED) = \text{ar}(\triangle EFC) = \text{ar}(\triangle DAF)$

Let $\text{ar}(\triangle FED) = x$ sq units then $\text{ar}(\triangle ABC) = 4x$ sq units.

Now, $\angle A = \angle E$ [Opposite angles of parallelogram ADEF are equal]

$\angle B = \angle F$ [Opposite angle of parallelogram BEFD are equal]

$\Rightarrow \triangle ABC \sim \triangle EFD$ [By AA similarity]

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{4x}{x} = \frac{4}{1}$$

$$\text{ar}(\triangle ABC) : \text{ar}(\triangle DEF) = 4 : 1$$

Long Answer Type Questions [4 Marks]

Question 37.

In the figure, ABC is a right triangle, right angled at B. AD and CF are two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm. Find the length of CE.

Solution:

In right-triangle ABD, $\angle B = 90^\circ$

$$\therefore AD^2 = AB^2 + BD^2 \quad \text{[By Pythagoras Theorem]}$$

$$\Rightarrow AD^2 = AB^2 + \left(\frac{BC}{2}\right)^2 \quad [\because BD = DC]$$

$$\Rightarrow AD^2 = AB^2 + \frac{BC^2}{4} \quad \dots(i)$$

Now, in right-triangle EBC, $\angle B = 90^\circ$,

$$\therefore CE^2 = BC^2 + BE^2 \quad \text{[By Pythagoras Theorem]}$$

$$\Rightarrow CE^2 = BC^2 + \left(\frac{AB}{2}\right)^2$$

$$\Rightarrow CE^2 = BC^2 + \frac{AB^2}{4} \quad \dots(ii)$$

Adding (i) and (ii), we get

$$AD^2 + CE^2 = AB^2 + \frac{BC^2}{4} + BC^2 + \frac{AB^2}{4}$$

$$AD^2 + CE^2 = \frac{5}{4}(AB^2 + BC^2)$$

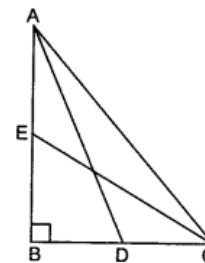
$$AD^2 + CE^2 = \frac{5}{4}AC^2 \quad [\because \text{By Pythagoras Theorem in } \triangle ABC \quad AC^2 = AB^2 + BC^2]$$

$$\left(\frac{3\sqrt{5}}{2}\right)^2 + CE^2 = \frac{5}{4} \times 25$$

$$CE^2 = \frac{125}{4} - \frac{45}{4}$$

$$CE^2 = 20$$

$$\therefore CE = \sqrt{20} = 2\sqrt{5} \text{ cm}$$



$[\because BE = AE]$

Question 38.

ABC is a right triangle, right-angled at B. D and E trisect BC, prove that $8AE^2 = 3AC^2 + 5AD^2$

Solution:

Given: A right-triangle ABC right angled at B.
Points D and E trisect BC. i.e. $BD = DE = EC$

To prove: $8AE^2 = 3AC^2 + 5AD^2$

Proof: Let $BD = x$

and $BD = DE = EC$

$\therefore BD = DE = EC = x.$

Now, in $\triangle ABD$.

$$AD^2 = AB^2 + BD^2$$

$$AD^2 = AB^2 + x^2$$

[By Pythagoras Theorem]

...(i)

In $\triangle ABE$

$$AE^2 = AB^2 + BE^2$$

$$AE^2 = AB^2 + 4x^2$$

[By Pythagoras Theorem]

...(ii) [$\because BE = BD + DE = x + x = 2x$]

In $\triangle ABC$

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = AB^2 + 9x^2$$

[By Pythagoras Theorem]

...(iii) [$\because BC = BD + DE + EC = 3x$]

Now, multiplying (i) by 5, (ii) by 8 and (iii) by 3, we get

$$5AD^2 = 5AB^2 + 5x^2$$

...(iv)

$$8AE^2 = 8AB^2 + 32x^2$$

...(v)

$$3AC^2 = 3AB^2 + 27x^2$$

...(vi)

Adding (iv) and (vi), we get

$$5AD^2 + 3AC^2 = 5AB^2 + 5x^2 + 3AB^2 + 27x^2$$

$$5AD^2 + 3AC^2 = 8AB^2 + 32x^2$$

...(vii)

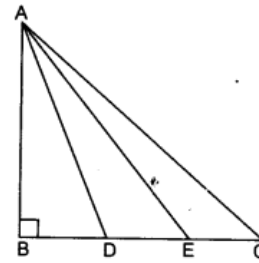
Subtracting (v) from (vii), we get

$$5AD^2 + 3AC^2 - 8AE^2 = 8AB^2 + 32x^2 - 8AB^2 - 32x^2$$

$$5AD^2 + 3AC^2 - 8AE^2 = 0$$

$$\therefore 5AD^2 + 3AC^2 = 8AE^2$$

Hence proved.



Question 39.

Prove that in a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Solution:

Given: $\triangle ABC$ in which $AC^2 = AB^2 + BC^2$

To prove: $\angle B = 90^\circ$

Construction: Construct a $\triangle PQR$ right angled at Q such that $PQ = AB$ and $QR = BC$.

Proof:

Now, in $\triangle PQR$, we have

$$PR^2 = PQ^2 + QR^2$$

or $PR^2 = AB^2 + BC^2$

But $AC^2 = AB^2 + BC^2$

So, $AC = PR$

Now, in $\triangle ABC$ and $\triangle PQR$

$$AB = PQ$$

$$BC = QR$$

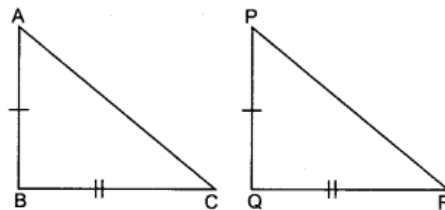
$$AC = PR$$

So, $\triangle ABC \cong \triangle PQR$

Therefore, $\angle B = \angle Q$

So, $\angle B = 90^\circ$

Hence, angle B is right angle.



[By Pythagoras theorem as $\angle Q = 90^\circ$]

[By construction] ... (i)

[Given] ... (ii)

[From (i) and (ii)] ... (iii)

[By construction]

[By construction]

[Proved in (iii) above]

[By SSS]

[$\because \angle Q = 90^\circ$]

Question 40.

In $\triangle ABC$, AD is the median to BC and in $\triangle PQR$, PM is the median to QR.

If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$, prove that $\triangle ABC \sim \triangle PQR$.

Solution:

Given: In $\triangle ABC$, AD is the median, i.e. $BD = DC$

In $\triangle PQR$, PM is the median, i.e. $QM = MR$

and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

To prove: $\triangle ABC \sim \triangle PQR$

Proof: $\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$\Rightarrow \triangle ABD \sim \triangle PQM$

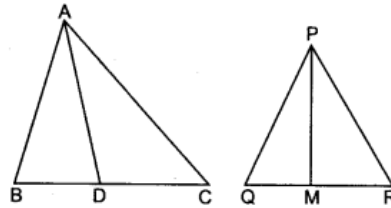
$\Rightarrow \angle B = \angle Q$

Now, in $\triangle ABC$ and $\triangle PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$$\angle B = \angle Q$$

$\therefore \triangle ABC \sim \triangle PQR$



[By SAS]

[Corresponding angles of similar angles]

[Given]

[Proved above]

[By SAS]

2012

Short Answer Type Question I [2 Marks]

Question 41.

In the given figure, if $AB \parallel DC$, find the value of x.

Solution:

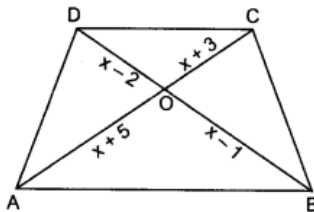
Given: $AB \parallel DC \therefore \triangle DOC \sim \triangle BOA$

$$\therefore \frac{OD}{OB} = \frac{OC}{OA} \Rightarrow \frac{x-2}{x-1} = \frac{x+3}{x+5}$$

$$(x-2)(x+5) = (x+3)(x-1)$$

$$\Rightarrow x^2 + 3x - 10 = x^2 + 2x - 3$$

$$\Rightarrow x = 7$$



Short Answer Type Question II [3 Marks]

Question 42.

In the given figure $PQ \parallel BA$; $PR \parallel CA$. If $PD = 12$ cm. Find $BD \times CD$.

Solution:

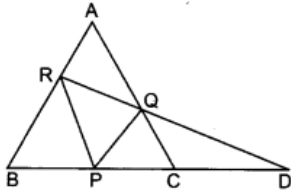
In $\triangle BRD$,

$$\frac{BR}{PD} = \frac{PQ}{QD}$$

\therefore

In $\triangle RDP$, $PR \parallel QC$ [Given]
 $\therefore \frac{RD}{QD} = \frac{PD}{CD}$... (ii)

From (i) and (ii), we get
 $\frac{PD}{CD} = \frac{BD}{PD}$
 $\Rightarrow BD \times CD = PD \times PD = 12 \times 12 = 144 \text{ cm}^2$



2011

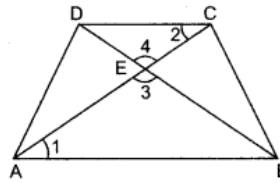
Short Answer Type Question I [2 Marks]

Question 43.

If one diagonal of a trapezium divides the other diagonal in the ratio 1 : 3. Prove that one of the parallel sides is three times the other.

Solution:

$DE : EB = 1 : 3$
 In $\triangle AEB$ and $\triangle CED$, $\angle 1 = \angle 2$ (Alternate angles)
 $\angle 3 = \angle 4$ (V.O.A.)
 $\therefore \triangle AEB \sim \triangle CED$
 $\Rightarrow \frac{AB}{CD} = \frac{BE}{DE}$
 $\Rightarrow \frac{AB}{CD} = \frac{3}{1}$ [$\because DE : BE = 1 : 3$]
 $\Rightarrow AB = 3CD$

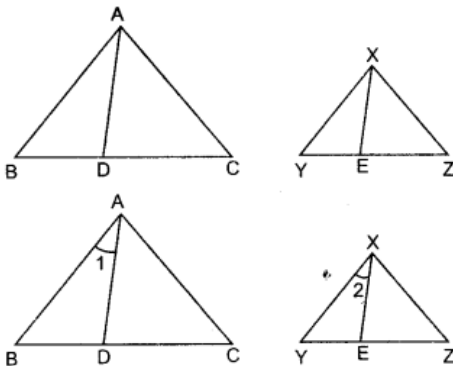


Short Answer Type Questions II [3 Marks]

Question 44.

In given figure $\triangle ABC$ is similar to $\triangle XYZ$ and AD and XE are angle bisectors of $\angle A$ and $\angle X$ respectively such that AD and XE in centimetres are 4 and 3 respectively, find the ratio of area of $\triangle ABD$ and area of $\triangle XYE$.

Solution:



AD bisects $\angle A \therefore \angle 1 = \frac{1}{2}\angle A$

Similarly $\angle 2 = \frac{1}{2}\angle X$

$\therefore \Delta ABC \sim \Delta XYZ$

$\therefore \angle A = \angle X$

$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle X \Rightarrow \angle 1 = \angle 2$

Also $\angle B = \angle Y$

$\therefore \Delta ABD \sim \Delta XYE$

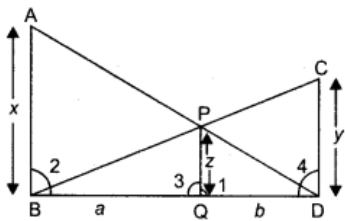
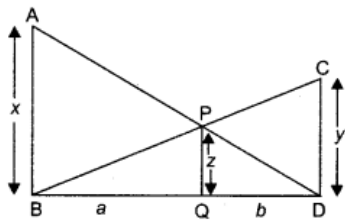
$$\frac{\text{Area } \Delta ABD}{\text{Area } \Delta XYE} = \frac{AD^2}{XE^2} = \frac{4^2}{3^2} = \frac{16}{9}$$

Question 45

In figure, $AB \parallel PQ \parallel CD$, $AB = x$ units, $CD = y$ units and $PQ = z$ units, prove that

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

Solution:



Let $BQ = a$ units, $DQ = b$ units

In ΔADB and ΔPDQ ,

$\therefore PQ \parallel AB \therefore \angle 1 = \angle 2$,

and $\angle ADB = \angle PDQ$ (Com

$\therefore \Delta ADB \sim \Delta PDQ$

Similarly $\Delta BCD \sim \Delta BPQ$

$\therefore \Delta ADB \sim \Delta PDQ$

$$\therefore \frac{AB}{PQ} = \frac{BD}{DQ}$$

$$\frac{x}{z} = \frac{a+b}{b} \Rightarrow \frac{x}{z} = \frac{a}{b} + 1$$

$$\Rightarrow \frac{x}{z} - 1 = \frac{a}{b}$$

Also, $\triangle ABC \sim \triangle PQR$

$$\frac{BD}{BQ} = \frac{CD}{PQ} \Rightarrow \frac{a+b}{a} = \frac{y}{z}$$

$$1 + \frac{b}{a} = \frac{y}{z} \Rightarrow \frac{b}{a} = \frac{y}{z} - 1 \Rightarrow \frac{b}{a} = \frac{y-z}{z}$$

$$\Rightarrow \frac{a}{b} = \frac{z}{y-z} \quad \dots(ii)$$

From (i) and (ii), we get

$$\frac{x}{z} - 1 = \frac{z}{y-z} \Rightarrow \frac{x}{z} = \frac{z}{y-z} + 1$$

$$\frac{x}{z} = \frac{z+y-z}{y-z}$$

$$\frac{x}{z} = \frac{y}{y-z} \Rightarrow \frac{z}{x} = \frac{y-z}{y}$$

$$\frac{z}{x} = 1 - \frac{z}{y}$$

$$z\left(\frac{1}{x}\right) = z\left(1 - \frac{1}{y}\right)$$

$$\Rightarrow \frac{1}{x} = \frac{1}{z} - \frac{1}{y} \Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z} \quad \text{Hence proved.}$$

Long Answer Type Questions [4 Marks]

Question 46.

The area of two similar triangles are 49 cm^2 and 64 cm^2 respectively. If the difference of the corresponding altitudes is 10 cm , then find the lengths of altitudes (in centimetres).

Solution:

$\triangle ABC \sim \triangle DEF$ (Given)

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{49}{64} = \frac{BC^2}{EF^2} \Rightarrow \frac{BC}{EF} = \frac{7}{8}$$

Also $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \frac{\frac{1}{2}BC \times AM}{\frac{1}{2}EF \times DN} \Rightarrow \frac{49}{64} = \frac{BC}{EF} \times \frac{AM}{DN}$

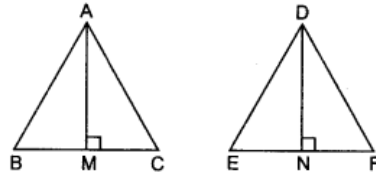
$$\Rightarrow \frac{49}{64} = \frac{7}{8} \times \frac{AM}{DN} \Rightarrow \frac{7}{8} = \frac{AM}{DN} \Rightarrow DN = \frac{8}{7}AM$$

Also $DN - AM = 10$ (Given)

$$\Rightarrow \frac{8}{7}AM - AM = 10 \Rightarrow \frac{1}{7}AM = 10$$

$$\therefore AM = 70 \text{ cm}$$

$$\therefore DN = 80 \text{ cm}$$



Question 47.

In an equilateral triangle ABC, D is a point on side BC such that $4BD = BC$. Prove that $AD^2 = BC^2$.

Solution:

In equilateral $\triangle ABC$,

$$4BD = BC$$

Construction: Draw $AE \perp BC$.

\therefore

$$BE = \frac{1}{2}BC.$$

In right $\triangle AED$,

$$AD^2 = DE^2 + AE^2$$

\Rightarrow

$$AE^2 = AD^2 - DE^2$$

In right $\triangle AEB$,

$$AB^2 = AE^2 + BE^2$$

\Rightarrow

$$AB^2 = AD^2 - DE^2 + BE^2$$

\Rightarrow

$$AB^2 + DE^2 - BE^2 = AD^2$$

\Rightarrow

$$AB^2 + (BE - BD)^2 - BE^2 = AD^2$$

\Rightarrow

$$AB^2 + BE^2 + BD^2 - 2BE \cdot BD - BE^2 = AD^2$$

\Rightarrow

$$AB^2 + BD^2 - 2BE \cdot BD = AD^2$$

\Rightarrow

$$AB^2 + \left(\frac{1}{4}BC\right)^2 - 2 \times \frac{1}{2}BC \times \frac{1}{4}BC = AD^2$$

\Rightarrow

$$AB^2 + \frac{1}{16}BC^2 - \frac{1}{4}BC^2 = AD^2$$

\Rightarrow

$$BC^2 - \frac{3}{16}BC^2 = AD^2$$

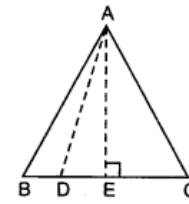
($\because AB = BC$)

\Rightarrow

$$\frac{13BC^2}{16} = AD^2$$

\Rightarrow

$$13 BC^2 = 16 AD^2$$



... (i)

[Using (i)]

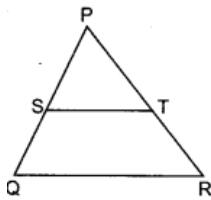
2010

Very Short Answer Type Questions [1 Mark]

Question 48.

In figure, S and T are points on the sides PQ and PR, respectively of $\triangle PQR$, such that $PT = 2$ cm, $TR = 4$ cm and ST is parallel to QR . Find the ratio of the areas of $\triangle PST$ and $\triangle PQR$.

Solution:



S and T are points on the sides PQ and PR of $\triangle PQR$ and $PT = 2$ cm, $TR = 4$ cm, $ST \parallel QR$

In $\triangle PST$ and $\triangle PQR$

$$\angle S = \angle Q$$

[Corresponding angles]

$$\angle P = \angle P$$

\therefore

$$\triangle PST \sim \triangle PQR$$

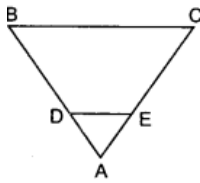
\therefore

$$\frac{\text{ar}(\triangle PST)}{\text{ar}(\triangle PQR)} = \frac{PT^2}{PR^2} = \frac{2^2}{6^2} = \frac{4}{36} = \frac{1}{9}$$

Question 49.

In figure, $DE \parallel BC$ in $\triangle ABC$ such that $BC = 8$ cm, $AB = 6$ cm and $DA = 1.5$ cm. Find DE .

Solution:



DE || BC

In $\triangle ADE$ and $\triangle ABC$,

$$\angle ADE = \angle ABC \quad [\text{Corresponding angles}]$$

$$\angle A = \angle A \quad [\text{Common}]$$

$$\therefore \triangle ADE \sim \triangle ABC \quad [\text{AA similarity}]$$

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$$

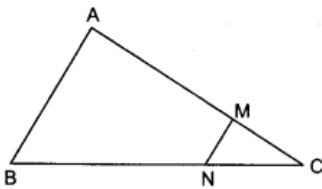
$$\text{Now, } \frac{1.5}{6} = \frac{DE}{8}$$

$$DE = \frac{1.5 \times 8}{6} = 2 \text{ cm}$$

Question 50.

In figure, MN || AB, BC = 7.5 cm, AM = 4 cm and MC = 2 cm. Find the length BN.

Solution:



In $\triangle ABC$, MN || AB $\Rightarrow \triangle ABC \sim \triangle MNC$

$$\Rightarrow \frac{MC}{AM} = \frac{NC}{BN}$$

$$\Rightarrow \frac{2}{4} = \frac{7.5 - x}{x} \quad [\text{Let } x = \text{BN}]$$

$$\Rightarrow x = 15 - 2x$$

$$\Rightarrow 3x = 15 \Rightarrow x = 5$$

Hence, BN = 5 cm.

Short Answer Type Questions I [2 Marks]

Question 51.

Triangle ABC is right angled at B, and D is mid-point of BC. Prove that $AC^2 = 4AD^2 - 3AB^2$.

Solution:

Given: $\triangle ABC$ with $\angle B = 90^\circ$

D is the mid-point of BC.

To prove: $AC^2 = 4AD^2 - 3AB^2$

Proof: In $\triangle ABC$, $\angle B = 90^\circ$

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= AB^2 + (2BD)^2 \\ &= AB^2 + 4BD^2 \end{aligned}$$

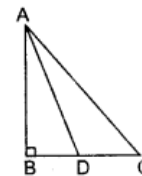
$$\text{In } \triangle ABD, \quad AD^2 = AB^2 + BD^2$$

$$\Rightarrow BD^2 = AD^2 - AB^2$$

From (i) and (ii), we get

$$AC^2 = AB^2 + 4(AD^2 - AB^2) = AB^2 + 4AD^2 - 4AB^2$$

$$AC^2 = 4AD^2 - 3AB^2$$



[Given]
[By Pythagoras theorem]

...(i)

[Using Pythagoras theorem]

...(ii)

Hence proved.

Question 52.

If BL and CM are medians of a triangle ABC right angled at A, then prove that $4(BL^2 + CM^2) =$

$5BC^2$.

Solution:

Given: A right angled triangle ABC, right angled at A.

BL and CM are the medians.

To prove: $4(BL^2 + CM^2) = 5BC^2$

Proof: In right angled triangle CAB,

$$BC^2 = AC^2 + AB^2 \quad [\text{By Pythagoras theorem}] \dots(i)$$

In right-angled triangle CAM,

$$CM^2 = AC^2 + AM^2$$

Also, $AM = \frac{1}{2}AB$

[As CM is median]

$$\therefore CM^2 = AC^2 + \left(\frac{1}{2}AB\right)^2$$

$$\Rightarrow 4CM^2 = 4AC^2 + AB^2 \quad \dots(ii)$$

In right-angled triangle LAB,

$$BL^2 = AL^2 + AB^2$$

Also, $AL = \frac{1}{2}AC$ [As BL is median]

$$\therefore BL^2 = \left(\frac{1}{2}AC\right)^2 + AB^2$$

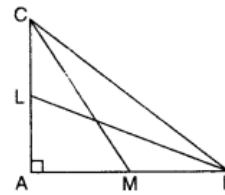
$$\Rightarrow 4BL^2 = AC^2 + 4AB^2 \quad \dots(iii)$$

Adding (ii) and (iii), we get

$$4BL^2 + 4CM^2 = 5AC^2 + 5AB^2$$

$$\Rightarrow 4(BL^2 + CM^2) = 5(AC^2 + AB^2)$$

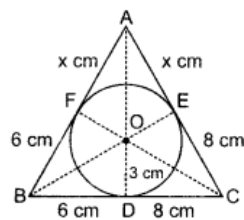
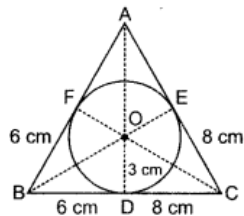
$$\Rightarrow 4(BL^2 + CM^2) = 5BC^2 \quad [\text{From (i)}] \text{ Hence proved.}$$



Question 53.

In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 6 cm and 8 cm respectively. Find the side AB if the area of $\Delta ABC = 63 \text{ cm}^2$.

Solution:



Area of $\Delta ABC = \text{area of } \Delta OBC + \text{area of } \Delta OAB + \text{area of } \Delta OAC$

$$\Rightarrow 63 = \frac{1}{2} \times 14 \times 3 + \frac{1}{2} \times (6 + x) \times 3 + \frac{1}{2} \times (8 + x) \times 3$$

$$\Rightarrow 63 = \frac{3}{2} (14 + 6 + x + 8 + x)$$

$$\Rightarrow 42 = 28 + 2x$$

$$\Rightarrow 2x = 14 \Rightarrow x = 7$$

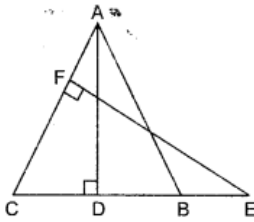
$$\therefore AB = (6 + 7) \text{ cm} = 13 \text{ cm.}$$

Short Answer Type Question II [3 Marks]

Question 54.

In figure, ABC is an isosceles triangle in which $AB = AC$. E is a point on the side CB produced, such that $FE \perp AC$. If $AD \perp CB$, prove that $AB \times EF = AD \times EC$.

Solution:



In $\triangle ADB$ and $\triangle EFC$,

$$\angle D = \angle F \quad \text{[Each } 90^\circ\text{]}$$

and $\angle B = \angle C$

[Angles opp. to equal sides of a triangle are equal]

$$\Rightarrow \triangle ABD \sim \triangle ECF \quad \text{[AA similarity]}$$

$$\therefore \frac{AB}{EC} = \frac{AD}{EF} \quad \text{[Corresponding]}$$

$$\therefore AB \times EF = AD \times EC$$

Long Answer Type Questions [4 Marks]

Question 55.

Prove that in a right angle triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. Point D is the mid-point of the side BC of a right triangle ABC, right angled at C. Prove that, $4AD^2 = 4AC^2 + BC^2$.

Solution:

Given: In $\triangle ABC$, $\angle B = 90^\circ$

To prove: $AC^2 = AB^2 + BC^2$

Construction: Draw $BD \perp AC$

Proof: Since, in $\triangle ABC$, $\angle B = 90^\circ$ and $BD \perp AC$

so, $\triangle ADB \sim \triangle ABC$ [If a \perp is drawn from the vertex of the rt. angle of rt. \triangle to the hypotenuse then \triangle 's on both sides of the \perp are similar to the whole \triangle and to each other]

and $\triangle BDC \sim \triangle ABC$

Now, $\triangle ADB \sim \triangle ABC$

$$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC} \Rightarrow AB^2 = AC \cdot AD \quad \dots(i)$$

Again, $\triangle BDC \sim \triangle ABC$

$$\Rightarrow \frac{CD}{BC} = \frac{BC}{AC} \Rightarrow BC^2 = AC \cdot CD \quad \dots(ii)$$

Adding equation (i) and (ii), we get

$$\begin{aligned} AB^2 + BC^2 &= AC \cdot AD + AC \cdot CD \\ &= AC(AD + CD) \\ &= AC \cdot AC \\ &= AC^2 \end{aligned}$$

$$\therefore AB^2 + BC^2 = AC^2$$

Hence proved

Other part:

In $\triangle ADC$, $\angle C = 90^\circ$

$$\therefore AD^2 = AC^2 + CD^2$$

D is mid point of BC

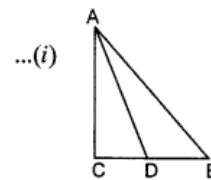
$$\therefore CD = \frac{1}{2}BC$$

Putting this value in equation (i), we get

$$AD^2 = AC^2 + \left(\frac{BC}{2}\right)^2$$

$$AD^2 = AC^2 + \frac{BC^2}{4}$$

$$4AD^2 = 4AC^2 + BC^2$$



Question 56.

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Using the above, prove the following:

If the areas of two similar triangles are equal, then prove that the triangles are congruent.

Solution:

To prove: $\triangle ABC \cong \triangle PQR$

Proof: Using the above result, we have

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{BC^2}{QR^2}$$

Also $\text{ar}(\triangle ABC) = \text{ar}(\triangle PQR)$ [Given]

$$\therefore 1 = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2} = \frac{BC^2}{QR^2}$$

$$\Rightarrow AB = PQ, AC = PR, BC = QR$$

$$\Rightarrow \triangle ABC \cong \triangle PQR \quad \text{[SSS]}$$

Question 57.

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite the first side is a right angle.

Using the above, do the following:

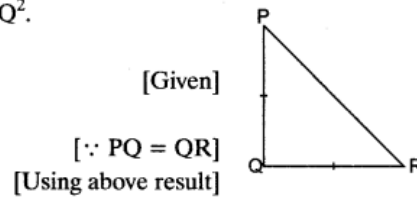
In an isosceles triangle PQR, $PQ = QR$ and $PR^2 = 2PQ^2$. Prove that $\angle Q$ is a right angle.

Solution:

Given: In isosceles ΔPQR , $PQ = QR$ and $PR^2 = 2PQ^2$.

To prove: $\angle Q$ is a right angle.

Proof: $PR^2 = 2PQ^2$
 $= PQ^2 + PQ^2$
 $\Rightarrow PR^2 = PQ^2 + QR^2$
 $\Rightarrow \Delta PQR$ is right angled at Q
 $\therefore \angle Q$ is a right angle.



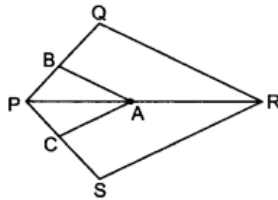
Question 58.

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Using the above, do the following:

In figure, $BA \parallel QR$, and $CA \parallel SR$, prove

Solution:



In ΔPQR and ΔPSR

$QR \parallel BA$

\therefore By BPT $\frac{PB}{BQ} = \frac{PA}{AR}$...*(i)*

In ΔPSR

$CA \parallel SR$
 $\frac{PC}{CS} = \frac{PA}{AR}$

\therefore [by BPT] ...*(ii)*

Equating *(i)* and *(ii)*

$$\frac{PB}{BQ} = \frac{PC}{CS}$$

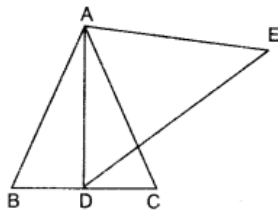
$$\Rightarrow \frac{QB}{BP} = \frac{SC}{CP}$$

Question 59.

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides. Using the above, do the following:

AD is an altitude of an equilateral triangle ABC . On AD as base, another equilateral triangle ADE is constructed. Prove that, $\text{area}(\Delta ADE) : \text{area}(\Delta ABC) = 3:4$

Solution:



$\triangle ABC$ and $\triangle ADE$ are equilateral triangles.

$$\frac{\text{ar} \triangle ABC}{\text{ar} \triangle ADE} = \frac{(AB)^2}{(AD)^2} \quad \dots(i)$$

where

$$AD = \frac{\sqrt{3}}{2} AB$$

Putting in equation (i)

$$\frac{\text{ar} \triangle ABC}{\text{ar} \triangle ADE} = \left[\frac{(AB)^2}{\left(\frac{\sqrt{3}}{2} AB\right)^2} \right] = \left(\frac{2}{\sqrt{3}} \cdot \frac{AB}{AB} \right)^2$$

$$\frac{\text{ar} \triangle ABC}{\text{ar} \triangle ADE} = \frac{4}{3}$$

Question 60.

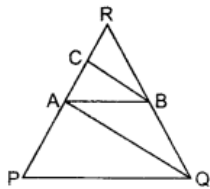
If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Using the above, do the following:

In figure, $PQ \parallel AB$ and $AQ \parallel CB$.

Prove that $AR^2 = PR \cdot CR$.

Solution:



$$\Rightarrow \frac{AR}{PR} = \frac{RB}{RQ} \quad \text{(Using above result) } \dots(i)$$

In $\triangle AQR$, $BC \parallel AQ$

$$\Rightarrow \frac{CR}{AR} = \frac{RB}{RQ} \quad \text{(Using above result) } \dots(ii)$$

From (i) and (ii), we have

$$\frac{AR}{PR} = \frac{CR}{AR} \Rightarrow AR^2 = PR \cdot CR$$

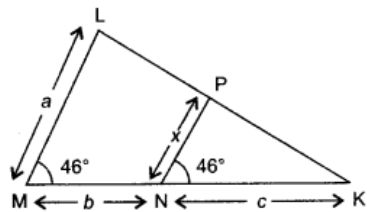
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Very Short Answer Type Questions [1 Mark]

Question 61.

In fig. $\angle M = \angle N = 46^\circ$, express JC in terms of a , b and c , where a , b and c are lengths of LM , MN and NK respectively.

Solution:



In $\triangle LMK$ and $\triangle PNK$

$$\angle M = \angle N = 46^\circ \quad \text{[Given]}$$

$$\angle K = \angle K \quad \text{[Common]}$$

$$\triangle LMK \sim \triangle PNK \quad \text{[By AA similarity]}$$

$$\frac{ML}{NP} = \frac{MK}{NK}$$

$$\frac{a}{x} = \frac{b+c}{c} \Rightarrow x = \frac{ac}{b+c}$$

Question 62.

If the areas of two similar triangles are in ratio 25 : 64, write the ratio of their corresponding sides.

Solution:

$$\frac{\text{ar of triangle I}}{\text{ar of triangle II}} = \left(\frac{\text{Corresponding side of triangle I}}{\text{Corresponding side of triangle II}} \right)^2$$

$$\Rightarrow \frac{25}{64} = \left(\frac{\text{Side of triangle I}}{\text{Side of triangle II}} \right)^2 \Rightarrow \frac{\text{Side of triangle I}}{\text{Side of triangle II}} = \frac{5}{8}$$

Question 63.

In a $\triangle ABC$, $DE \parallel BC$. If $DE = \frac{3}{2} BC$ and area of $\triangle ABC = 81 \text{ cm}^2$, find the area of $\triangle ADE$.

Solution:

In $\triangle ABC$ and $\triangle ADE$

$$\angle A = \angle A$$

$$\angle B = \angle D$$

$$\triangle ABC \sim \triangle ADE$$

$$\frac{\text{ar} \triangle ABC}{\text{ar} \triangle ADE} = \left(\frac{BC}{DE} \right)^2 = \left(\frac{3}{2} \cdot \frac{DE}{DE} \right)^2 = \frac{9}{4}$$

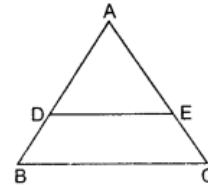
$$\frac{\text{ar} \triangle ABC}{\text{ar} \triangle ADE} = \frac{9}{4}$$

$$\Rightarrow \frac{81}{\text{ar} \triangle ADE} = \frac{9}{4}$$

$$36 \text{ cm}^2 = \text{ar} \triangle ADE$$

$$\Rightarrow \text{ar} \triangle ADE = 36 \text{ cm}^2$$

[Common]
[DE || BC]
[By AA similarity]

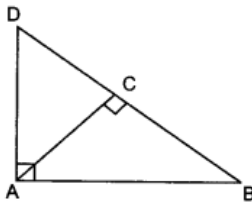


Short Answer Type Questions I [2 Marks]

Question 64.

In figure, $\triangle ABD$ is a right triangle, right angled at A and $AC \perp BD$. Prove that $AB^2 = BC \cdot BD$.

Solution:



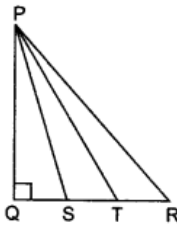
In $\triangle BAD$ and $\triangle BCA$

$$\begin{aligned} \angle B &= \angle B && \text{[Common]} \\ \angle BAD &= \angle BCA && \text{[90° each]} \\ \triangle BAD &\sim \triangle BCA && \text{[By AA similarity]} \\ \therefore \frac{BA}{BC} &= \frac{BD}{BA} \\ (BA)^2 &= BD \cdot BC \\ (AB)^2 &= BC \cdot BD \end{aligned}$$

Question 65.

In figure, S and T trisect the side QR of a right triangle PQR. Prove that $8PT^2 = 3PR^2 + 5PS^2$.

Solution:



Given: Right triangle PQR. S and T trisect QR.

To prove: $8PT^2 = 3PR^2 + 5PS^2$

Proof: $QS = ST = TR = \frac{1}{3}QR$

In right triangle PQS,

$$PS^2 = PQ^2 + QS^2$$

In right triangle PQT,

$$PT^2 = PQ^2 + QT^2$$

In right triangle PQR,

$$PR^2 = PQ^2 + QR^2$$

Subtracting (iii) from (ii), we get

$$PS^2 - PT^2 = QS^2 - QT^2$$

$$\begin{aligned} \Rightarrow PS^2 - PT^2 &= \left(\frac{1}{3}QR\right)^2 - \left(\frac{2}{3}QR\right)^2 \\ &= \frac{1}{9}QR^2 - \frac{4}{9}QR^2 = \frac{-1}{3}QR^2 \end{aligned}$$

$$\Rightarrow 3PS^2 - 3PT^2 = -QR^2$$

Subtracting (iv) from (iii), we get

$$PT^2 - PR^2 = QT^2 - QR^2 = \left(\frac{2}{3}QR\right)^2 - QR^2$$

$$\Rightarrow PT^2 - PR^2 = \frac{4}{9}QR^2 - QR^2 = \frac{-5}{9}QR^2$$

$$\Rightarrow 9PT^2 - 9PR^2 = -5QR^2$$

Substituting for $(-QR^2)$ from (v) in (vi), we get

$$\Rightarrow 9PT^2 - 9PR^2 = 5(3PS^2 - 3PT^2)$$

$$\Rightarrow 9PT^2 - 9PR^2 = 15PS^2 - 15PT^2$$

$$\Rightarrow 24PT^2 = 15PS^2 + 9PR^2$$

$$\Rightarrow 8PT^2 = 5PS^2 + 3PR^2$$

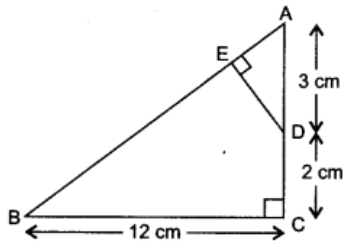
Hence proved.

Short Answer Type Questions II [3 Marks]

Question 66.

In figure, $\triangle ABC$ is right angled at C and $DE \perp AB$. Prove that $\triangle ABC \sim \triangle ADE$ and hence find the lengths of AE and DE.

Solution:



Given: $\triangle ABC$ and $\triangle ADE$ right angled at C and E.

Proof: In $\triangle ABC$ and $\triangle ADE$

$$\angle C = \angle E \quad [90^\circ \text{ Each}]$$

$$\angle A = \angle A \quad [\text{Common angle}]$$

$$\triangle ABC \sim \triangle ADE \quad [\text{By AA similarity}]$$

Since,

$$\triangle ABC \sim \triangle ADE$$

In $\triangle ABC$,

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = 25 + 144 = 169$$

\Rightarrow

$$AB = 13$$

Now,

$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$$\frac{13}{3} = \frac{12}{DE} = \frac{5}{AE}$$

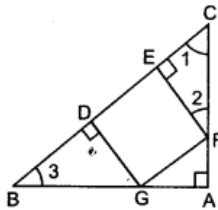
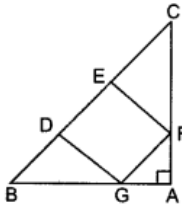
Then,

$$AE = \frac{15}{13}, DE = \frac{36}{13}$$

Question 67.

In figure, DEFG is a square and $\angle BAC = 90^\circ$. Show that $DE^2 = BD \times EC$.

Solution:



Given: DEFG is a square and $\angle BAC = 90^\circ$.

To prove: $DE^2 = BD \times EC$

Proof: In $\triangle BDG$ and $\triangle CEF$

$$\angle 1 + \angle 3 = 90^\circ$$

[Because $\angle A = 90^\circ$]

$$\angle 1 + \angle 2 = 90^\circ$$

[Because $\angle CEF = 90^\circ$]

$$\angle 1 + \angle 2 = \angle 1 + \angle 3$$

So,

$$\angle 2 = \angle 3$$

and

$$\angle CEF = \angle BDG$$

[90° each]

Hence,

$$\triangle BDG \sim \triangle FEC$$

[AA Similarity]

\therefore

$$\frac{BD}{FE} = \frac{DG}{EC} = \frac{BG}{FC}$$

Now, DEFG is a square

\therefore

$$DE = EF = FG = DG$$

Then,

$$\frac{BD}{DE} = \frac{DE}{EC}$$

$$DE^2 = BD \cdot EC.$$

Question 68.

In figure, $AD \perp BC$ and $BD = \frac{1}{3} CD$.

Prove that $2CA^2 = 2AB^2 + BC^2$.

Solution:

In figure, $AD \perp BC$ and $BD = \frac{1}{3} CD$.

Prove that $2CA^2 = 2AB^2 + BC^2$.

Given: In $\triangle ABC$, $AD \perp BC$ and $BD = \frac{1}{3} CD$

To prove: $2CA^2 = 2AB^2 + BC^2$

Proof: $\because BC = BD + CD$ and $BD = \frac{1}{3} CD$ [Given]

$$\therefore BC = \frac{1}{3} CD + CD = \frac{4}{3} CD$$

$$\Rightarrow CD = \frac{3}{4} BC \quad \dots(i)$$

In right angled $\triangle ADC$,

$$AC^2 = CD^2 + AD^2$$

[By Pythagoras theorem] $\dots(ii)$

In right angled $\triangle ADB$,

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow AD^2 = AB^2 - BD^2$$

Substituting in (ii), we get

$$\Rightarrow AC^2 = CD^2 + AB^2 - BD^2$$

$$\Rightarrow AC^2 = CD^2 + AB^2 - \left(\frac{1}{3} CD\right)^2 \quad \text{[Put } BD = \frac{1}{3} CD]$$

$$\Rightarrow AC^2 = CD^2 - \frac{1}{9} CD^2 + AB^2$$

$$\Rightarrow AC^2 = \frac{8}{9} CD^2 + AB^2$$

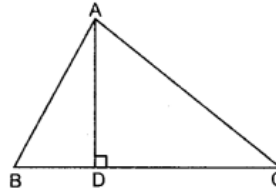
$$\Rightarrow AC^2 = \frac{8}{9} \left(\frac{3}{4} BC\right)^2 + AB^2 \quad \text{[Using (i)]}$$

$$\Rightarrow AC^2 = \frac{8}{9} \times \frac{9}{16} BC^2 + AB^2$$

$$\Rightarrow AC^2 = \frac{1}{2} BC^2 + AB^2$$

$$\Rightarrow 2AC^2 = BC^2 + 2AB^2 \quad \text{Hence proved.}$$

[All India]



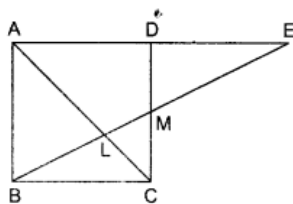
[Given]

Question 69.

In figure, M is mid-point of side CD of a parallelogram ABCD. The line BM is drawn

intersecting AC at L and AD produced at E. Prove that $EL = 2BL$.

Solution:



Proof: In $\triangle MDE$ and $\triangle MCB$

$$DM = CM$$

[Given]

$$\angle 1 = \angle 2$$

[Vertically opposite]

$$\angle 3 = \angle 4 \quad [\because BC \parallel AE \text{ and } DC \text{ is a transversal}]$$

[Alt. int. \angle s]

$$\therefore \triangle MDE \cong \triangle MCB$$

[ASA Congruency]

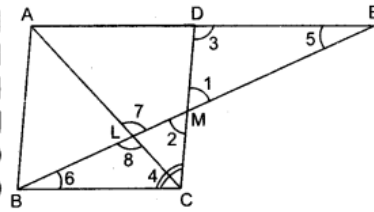
$$DE = BC$$

[CPCT] ... (i)

$$\text{Also } BC = AD$$

... (ii)

(Opposite sides of the parallelogram)



$$\therefore AD = DE$$

[On equating (i) and (ii)]

$$\text{Now, } AE = AD + DE$$

$$\Rightarrow AE = 2AD$$

[Put $DE = AD$]

In $\triangle BLC$ and $\triangle ELA$,

$$\angle 5 = \angle 6$$

[Alt. int. angles]

$$\text{and } \angle 7 = \angle 8$$

[Vertically opposite angles]

$$\therefore \triangle BLC \sim \triangle ELA$$

[AA similarity]

$$\Rightarrow \frac{BL}{EL} = \frac{LC}{LA} = \frac{BC}{AE} \Rightarrow \frac{BL}{EL} = \frac{BC}{AE} \Rightarrow \frac{BL}{EL} = \frac{BC}{2AD}$$

$$\Rightarrow \frac{BL}{EL} = \frac{AD}{2AD}$$

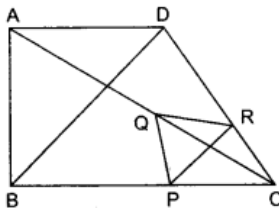
[$\because BC = AD$]

$$\Rightarrow \frac{BL}{EL} = \frac{1}{2} \Rightarrow EL = 2BL$$

Question 70.

In figure, two triangles ABC and DBC lie on the same side of base BC . P is a point on BC such that $PQ \parallel BA$ and $PR \parallel BD$. Prove that $QR \parallel AD$.

Solution:



Given: In $\triangle ABC$, $PQ \parallel AB$ and $PR \parallel BD$

To prove: $QR \parallel AD$

Proof: In $\triangle ABC$, $PQ \parallel AB$

$$\Rightarrow \text{By BPT, } \frac{CP}{BP} = \frac{CQ}{AQ}$$

Now in $\triangle BCD$, $PR \parallel BD$

\Rightarrow By using BPT

$$\frac{CP}{BP} = \frac{CR}{RD}$$

From (i) and (ii), we get

$$\frac{CQ}{AQ} = \frac{CR}{RD}$$

\Rightarrow By converse of BPT, $QR \parallel AD$

Long Answer Type Questions [4 Marks]

Question 71.

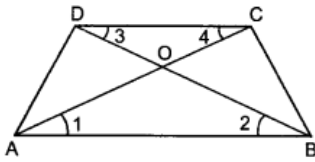
Prove that the ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Using the above, do the following:

In a trapezium $ABCD$, AC and BD intersecting at O , $AB \parallel DC$ and $AB = 2CD$, if area of $\triangle AOB$

= 84 cm², find the area of ΔCOD

Solution:



Given: ABCD is a trapezium

$$AB \parallel CD$$

Also

$$AB = 2CD$$

To find: Area of ΔCOD

Now, in ΔAOB and ΔCOD

$$\angle 1 = \angle 4$$

$$\angle 2 = \angle 3$$

\therefore

$$\Delta AOB \sim \Delta COD$$

$$\frac{\text{ar}\Delta AOB}{\text{ar}\Delta COD} = \frac{(AB)^2}{(CD)^2}$$

$$\frac{84}{\text{ar}\Delta COD} = \frac{(2CD)^2}{(CD)^2}$$

$$\frac{84}{\text{ar}\Delta COD} = \frac{4(CD)^2}{(CD)^2}$$

$$\frac{84}{\text{ar}\Delta COD} = \frac{4}{1}$$

$$\text{ar}\Delta COD = 21 \text{ cm}^2$$

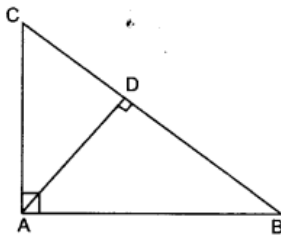
Question 72.

Prove that in a right angle triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Using the above, do the following:

Prove that in a ΔABC , if AD is perpendicular to BC , then $AB^2 + CD^2 = AC^2 + BD^2$.

Solution:



In ΔABC , $AD \perp BC$

To prove: $AB^2 + CD^2 = AC^2 + BD^2$

Proof: In ΔABD ,

$$\angle D = 90^\circ$$

$$AB^2 = BD^2 + AD^2$$

[By Pythagoras theorem]

$$\therefore AD^2 = AB^2 - BD^2 \quad \dots(i)$$

In ΔADC , $\angle D = 90^\circ$

$$\therefore AD^2 = AC^2 - CD^2$$

...(ii) [By Pythagoras theorem]

On equating equation (i) and (ii), we get

$$AB^2 - BD^2 = AC^2 - CD^2$$

$$AB^2 + CD^2 = AC^2 + BD^2$$

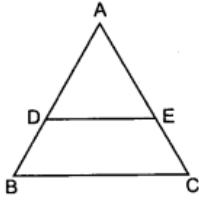
Question 73.

Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Using the above result, do the following:

In figure, $DE \parallel BC$ and $BD = CE$. Prove that $\triangle ABC$ is an isosceles triangle.

Solution:



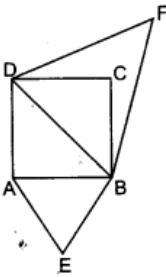
$$\begin{aligned} \therefore \quad & BD = CE && \text{[Given]} \\ \text{So,} \quad & AD = AE && \left\{ \because \frac{AD}{DB} = \frac{AE}{EC} \text{ Proved above} \right\} \\ \therefore \quad & AD + BD = AE + CE \\ \Rightarrow \quad & AB = AC \\ \therefore \quad & \triangle ABC \text{ is an isosceles triangle.} \end{aligned}$$

Question 74.

Prove that the ratio of areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Using the above, prove the following: The areas of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described, on its diagonal.

Solution:



Equilateral triangle ABE is described on side AB of square ABCD.

Equilateral triangle BDF is described on diagonal BD of square ABCD.

$$\begin{aligned} \therefore \quad & \triangle ABE \sim \triangle BDF \\ \frac{\text{ar} \triangle ABE}{\text{ar} \triangle BDF} &= \frac{(AB)^2}{(BD)^2} = \frac{(AB)^2}{(\sqrt{2} AB)^2} && \text{[as } BD = \sqrt{2}AB \text{]} \\ \frac{\text{ar} \triangle ABE}{\text{ar} \triangle BDF} &= \frac{AB^2}{2AB^2} = \frac{1}{2} \\ (\text{ar} \triangle ABE) &= \frac{1}{2} (\text{ar} \triangle BDF) \end{aligned}$$

Question 75.

$\triangle ABC$ is an isosceles triangle in which $AC = BC$. If $AB^2 = 2AC^2$ then, prove that $\triangle ABC$ is right triangle.

Solution:

$$\begin{aligned} & AB^2 = 2AC^2 && \text{[Given]} \\ \therefore \quad & AB^2 = AC^2 + AC^2 \\ \text{Also} \quad & AC = BC && \text{[Given]} \\ \therefore \quad & AB^2 = BC^2 + AC^2 \end{aligned}$$

By converse of Pythagoras theorem, $\triangle ABC$ is a right-angled triangle where $\angle C = 90^\circ$.