

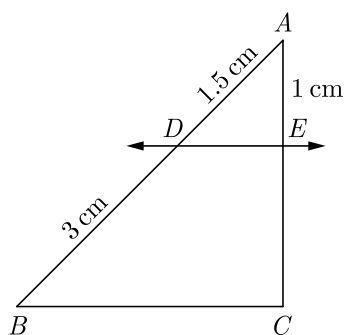
CHAPTER 6

TRIANGLES

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. In the given figure, $DE \parallel BC$. The value of EC is



- (a) 1.5 cm (b) 3 cm
(c) 2 cm (d) 1 cm

Ans :

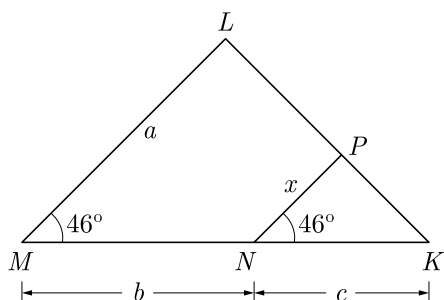
Since, $DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{EC} \Rightarrow EC = 2 \text{ cm}$$

Thus (c) is correct option.

2. In the given figure, x is



- (a) $\frac{ab}{a+b}$ (b) $\frac{ac}{b+c}$
(c) $\frac{bc}{b+c}$ (d) $\frac{ac}{a+c}$

Ans :

In $\triangle KPN$ and $\triangle KLM$, $\angle K$ is common and we have

$$\angle KNP = \angle KML = 46^\circ$$

Thus by $A - A$ criterion of similarity,

$$\triangle KNP \sim \triangle KML$$

Thus
$$\frac{KN}{KM} = \frac{NP}{ML}$$

$$\frac{c}{b+c} = \frac{x}{a} \Rightarrow x = \frac{ac}{b+c}$$

Thus (b) is correct option.

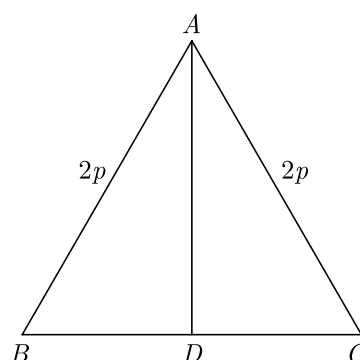
3. $\triangle ABC$ is an equilateral triangle with each side of length $2p$. If $AD \perp BC$ then the value of AD is

- (a) $\sqrt{3}$ (b) $\sqrt{3}p$
(c) $2p$ (d) $4p$

Ans :

We have $AB = BC = CA = 2p$

and $AD \perp BC$



In $\triangle ADB$, $AB^2 = AD^2 + BD^2$

$$(2p)^2 = AD^2 + p^2$$

$$AD^2 = \sqrt{3}p$$

Thus (b) is correct option.

4. Which of the following statement is false?

- (a) All isosceles triangles are similar.
- (b) All quadrilateral are similar.
- (c) All circles are similar.
- (d) None of the above

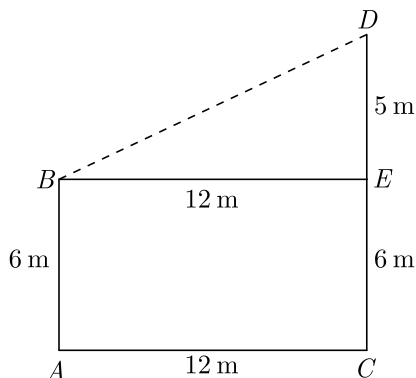
Ans :

Isosceles triangle is a triangle in which two side of equal length. Thus two isosceles triangles may not be similar. Hence statement given in option (a) is false. Thus (a) is correct option.

5. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, then distance between their tops is
- (a) 12 m
 - (b) 14 m
 - (c) 13 m
 - (d) 11 m

Ans :

Let AB and CD be the vertical poles as shown below.



We have $AB = 6$ m, $CD = 11$ m

and $AC = 12$ m

$$\begin{aligned} DE &= CD - CE \\ &= (11 - 6) \text{ m} = 5 \text{ m} \end{aligned}$$

In right angled, $\triangle BED$,

$$BD^2 = BE^2 + DE^2 = 12^2 + 5^2 = 169$$

$$BD = \sqrt{169} \text{ m} = 13 \text{ m}$$

Hence, distance between their tops is 13 m.

Thus (c) is correct option.

6. In a right angled $\triangle ABC$ right angled at B , if P and Q are points on the sides AB and BC respectively, then

- (a) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$
- (b) $2(AQ^2 + CP^2) = AC^2 + PQ^2$

$$(c) \quad AQ^2 + CP^2 = AC^2 + PQ^2$$

$$(d) \quad AQ + CP = \frac{1}{2}(AC + PQ)$$

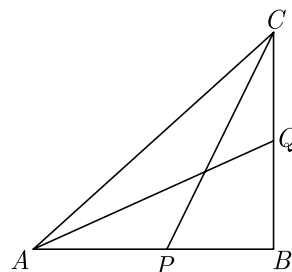
Ans :

In right angled $\triangle ABQ$ and $\triangle CPB$,

$$CP^2 = CB^2 + BP^2$$

and

$$AQ^2 = AB^2 + BQ^2$$



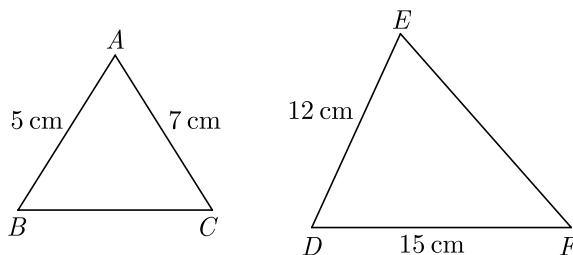
$$\begin{aligned} CP^2 + AQ^2 &= CB^2 + BP^2 + AB^2 + BQ^2 \\ &= CB^2 + AB^2 + BP^2 + BQ^2 \\ &= AC^2 + PQ^2 \end{aligned}$$

Thus (c) is correct option.

7. It is given that, $\triangle ABC \sim \triangle EDF$ such that $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm then the sum of the remaining sides of the triangles is
- (a) 23.05 cm
 - (b) 16.8 cm
 - (c) 6.25 cm
 - (d) 24 cm

Ans :

We have $\triangle ABC \sim \triangle EDF$



$$\text{Now} \quad \frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

Taking first and second ratios, we get

$$\frac{5}{12} = \frac{7}{EF} \Rightarrow EF = \frac{7 \times 12}{5}$$

$$= 16.8 \text{ cm}$$

Taking first and third ratios, we get

$$\frac{5}{12} = \frac{BC}{15} \Rightarrow BC = \frac{5 \times 15}{12}$$

$$= 6.25 \text{ cm}$$

Now, sum of the remaining sides of triangle,

$$EF + BC = 16.8 + 6.25 = 23.05 \text{ cm}$$

Thus (a) is correct option.

8. The area of a right angled triangle is 40 sq cm and its perimeter is 40 cm. The length of its hypotenuse is
- (a) 16 cm (b) 18 cm
- (c) 17 cm (d) data insufficient

Ans : (b) 18 cm

Let c be the hypotenuse of the triangle, a and b be other sides.

Now $c = \sqrt{a^2 + b^2}$

We have, $a + b + c = 40$ and $\frac{1}{2}ab = 40 \Rightarrow ab = 80$

$$c = 40 - (a + b) \text{ and } ab = 80$$

Squaring $c = 40 - (a + b)$ we have

$$c^2 = [40 - (a + b)]^2$$

$$a^2 + b^2 = 1600 - 2 \times 40(a + b) + (a + b)^2$$

$$a^2 + b^2 = 1600 - 2 \times 40(a + b) + a^2 + 2ab + b^2$$

$$0 = 1600 - 2 \times 40(a + b) + 2 \times 80$$

$$0 = 20 - (a + b) + 2$$

$$a + b = 22$$

$$c = 40 - (a + b) = 40 - 22 = 18 \text{ cm}$$

Thus (b) is correct option.

9. **Assertion :** In the $\triangle ABC$, $AB = 24$ cm, $BC = 10$ cm and $AC = 26$ cm, then $\triangle ABC$ is a right angle triangle.
Reason : If in two triangles, their corresponding angles are equal, then the triangles are similar.
- (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
- (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
- (c) Assertion (A) is true but reason (R) is false.

- (d) Assertion (A) is false but reason (R) is true.

Ans :

We have,

$$\begin{aligned} AB^2 + BC^2 &= (24)^2 + (10)^2 \\ &= 576 + 100 = 676 = AC^2 \end{aligned}$$

Thus $AB^2 + BC^2 = AC^2$ and ABC is a right angled triangle.

Also, two triangle are similar if their corresponding angles are equal.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

FILL IN THE BLANK QUESTIONS

10. A line drawn through the mid-point of one side of a triangle parallel to another side bisects the side.

Ans :

third

11. theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Ans :

Pythagoras

12. Line joining the mid-points of any two sides of a triangle is to the third side.

Ans :

parallel

13. All squares are

Ans :

similar

14. Two triangles are said to be if corresponding angles of two triangles are equal.

Ans :

equiangular

15. All similar figures need not be

Ans :

congruent

16. All circles are

Ans :

similar

17. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the side.

Ans :

third

18. If a line divides any two sides of a triangle in the same ratio, then the line is to the third side.

Ans :

parallel

19. All congruent figures are similar but the similar figures need be congruent.

Ans :

not

20. Two figures are said to be if they have same shape but not necessarily the same size.

Ans :

similar

21. theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Ans :

Basic proportionality

22. All triangles are similar.

Ans :

equilateral

23. Two figures having the same shape and size are said to be

Ans :

congruent

24. Two triangles are similar if their corresponding sides are

Ans :

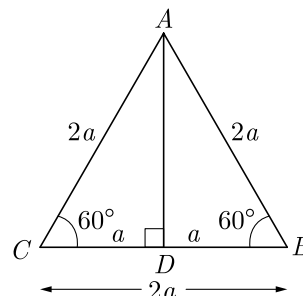
in the same ratio.

25. ΔABC is an equilateral triangle of side $2a$, then length of one of its altitude is

Ans :

[Board 2020 Delhi Standard]

ΔABC is an equilateral triangle as shown below, in which $AD \perp BC$.



Using Pythagoras theorem we have

$$AB^2 = (AD)^2 + (BD)^2$$

$$(2a)^2 = (AD)^2 + (a)^2$$

$$4a^2 - a^2 = (AD)^2$$

$$(AD)^2 = 3a^2$$

$$AD = a\sqrt{3}$$

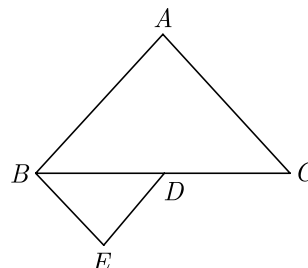
Hence, the length of altitude is $a\sqrt{3}$.

26. ΔABC and ΔBDE are two equilateral triangle such that D is the mid-point of BC . Ratio of the areas of triangles ABC and BDE is

Ans :

[Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



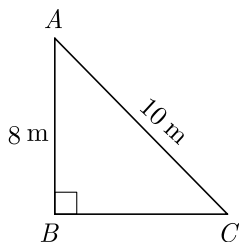
$$\begin{aligned} \frac{ar(\Delta ABC)}{ar(\Delta BDE)} &= \frac{\frac{\sqrt{3}}{4}(BC)^2}{\frac{\sqrt{3}}{4}(BD)^2} = \frac{(BC)^2}{(\frac{1}{2}BC)^2} \\ &= \frac{4BC^2}{BC^2} = \frac{4}{1} = 4:1 \end{aligned}$$

27. A ladder 10 m long reaches a window 8 m above the ground. The distance of the foot of the ladder from the base of the wall is m.

Ans :

[Board 2020 Delhi Standard]

Let AB be the height of the window above the ground and BC be a ladder.



Here, $AB = 8 \text{ m}$

and $AC = 10 \text{ m}$

In right angled triangle ABC ,

$$AC^2 = AB^2 + BC^2$$

$$10^2 = 8^2 + BC^2$$

$$BC^2 = 100 - 64 = 36$$

$$BC = 6 \text{ m}$$

28. In $\triangle ABC$, $AB = 6\sqrt{3} \text{ cm}$, $AC = 12 \text{ cm}$ and $BC = 6 \text{ cm}$, then $\angle B = \dots\dots\dots$

Ans : [Board 2020 OD Standard]

We have $AB = 6\sqrt{3} \text{ cm}$,

$AC = 12 \text{ cm}$ and

$BC = 6 \text{ cm}$

Now $AB^2 = 36 \times 3 = 108$

$$AC^2 = 144$$

and $BC^2 = 36$

It can be easily observed that above values satisfy Pythagoras theorem,

$$AB^2 + BC^2 = AC^2$$

$$108 + 36 = 144 \text{ cm}$$

Thus $\angle B = 90^\circ$

29. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of second triangle is $\dots\dots\dots$

Ans : [Board 2020 Delhi Basic]

Ratio of the perimeter of two similar triangles is equal to the ratio of corresponding sides.

$$\text{Thus } \frac{25}{15} = \frac{9}{\text{side}}$$

$$\text{side} = \frac{9 \times 15}{25} = 5.4 \text{ cm}$$

VERY SHORT ANSWER QUESTIONS

30. $\triangle ABC$ is isosceles with $AC = BC$. If $AB^2 = 2AC^2$, then find the measure of $\angle C$.

Ans :

[Board 2020 Delhi Basic]

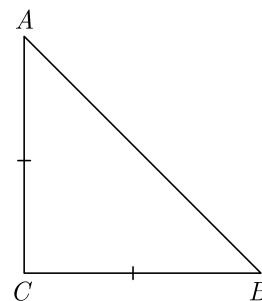
We have $AB^2 = 2AC^2$

$$AB^2 = AC^2 + AC^2$$

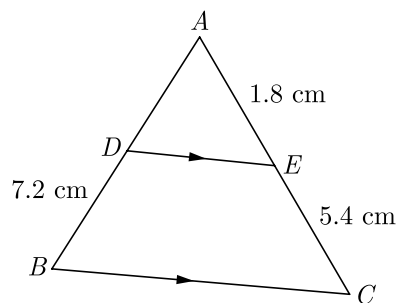
$$AB^2 = BC^2 + AC^2$$

($BC = AC$)

It satisfies the Pythagoras theorem. Thus according to converse of Pythagoras theorem, $\triangle ABC$ is a right angle triangle and $\angle C = 90^\circ$.



31. In Figure, $DE \parallel BC$. Find the length of side AD , given that $AE = 1.8 \text{ cm}$, $BD = 7.2 \text{ cm}$ and $CE = 5.4 \text{ cm}$.



Ans :

[Board 2019 OD]

Since $DE \parallel BC$ we have

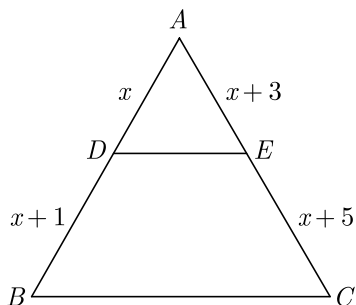
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substituting the values, we get

$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$AD = \frac{1.8 \times 7.2}{5.4} = \frac{12.96}{5.4} = 2.4 \text{ cm}$$

32. In $\triangle ABC$, $DE \parallel BC$, find the value of x .



Ans :

[Board Term-1 2016]

In the given figure $DE \parallel BC$, thus

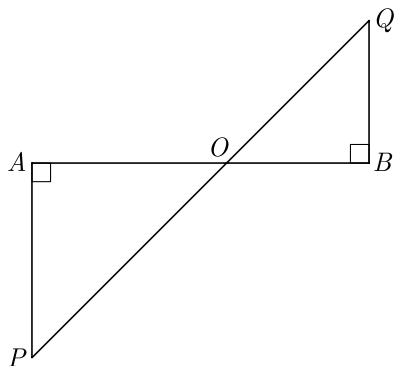
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{x+1} = \frac{x+3}{x+5}$$

$$x^2 + 5x = x^2 + 4x + 3$$

$$x = 3$$

33. In the given figure, if $\angle A = 90^\circ$, $\angle B = 90^\circ$, $OB = 4.5$ cm $OA = 6$ cm and $AP = 4$ cm then find QB .



Ans :

[Board Term-1, 2015]

In $\triangle PAO$ and $\triangle QBO$ we have

$$\angle A = \angle B = 90^\circ$$

Vertically opposite angle,

$$\angle POA = \angle QOB$$

Thus

$$\triangle PAO \sim \triangle QBO$$

$$\frac{OA}{OB} = \frac{PA}{QB}$$

$$\frac{6}{4.5} = \frac{4}{QB}$$

$$QB = \frac{4 \times 4.5}{6} = 3 \text{ cm}$$

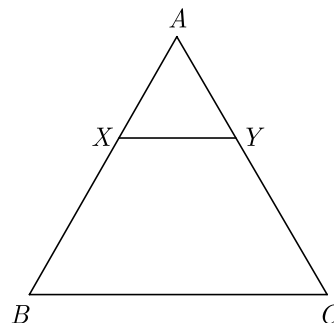
Thus $QB = 3$ cm

34. In $\triangle ABC$, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, $AY = 5$ and $YC = 9$, then state whether XY and BC parallel or not.

Ans :

[Board Term-1 2016, 2015]

As per question we have drawn figure given below.



In this figure we have

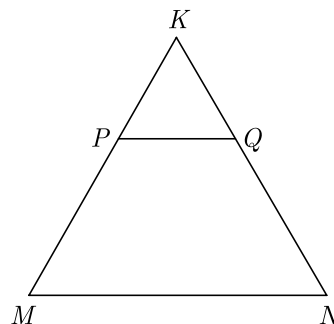
$$\frac{AX}{XB} = \frac{3}{4}, AY = 5 \text{ and } YC = 9$$

Now $\frac{AX}{XB} = \frac{3}{4}$ and $\frac{AY}{YC} = \frac{5}{9}$

Since $\frac{AX}{XB} \neq \frac{AY}{YC}$

Hence XY is not parallel to BC .

35. In the figure, PQ is parallel to MN . If $\frac{KP}{PM} = \frac{4}{13}$ and $KN = 20.4$ cm then find KQ .



Ans :

In the given figure $PQ \parallel MN$, thus

$$\frac{KP}{PM} = \frac{KQ}{QN} \quad (\text{By BPT})$$

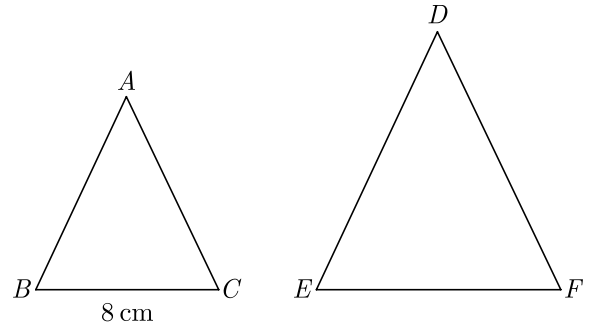
$$\frac{KP}{PM} = \frac{KQ}{KN - KQ}$$

$$\frac{4}{13} = \frac{KQ}{20.4 - KQ}$$

$$4 \times 20.4 - 4KQ = 13KQ$$

$$17KQ = 4 \times 20.4$$

$$KQ = \frac{20.4 \times 4}{17} = 4.8 \text{ cm}$$



Here we have $2AB = DE$ and $BC = 8 \text{ cm}$

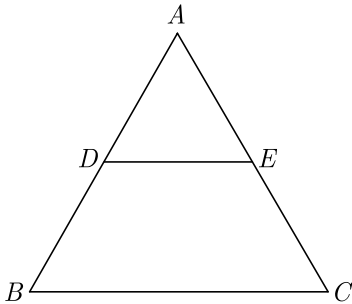
Since $\triangle ABC \sim \triangle DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{AB}{8} = \frac{2AB}{EF}$$

$$EF = 2 \times 8 = 16 \text{ cm}$$

36. In given figure $DE \parallel BC$. If $AD = 3c$, $DB = 4c \text{ cm}$ and $AE = 6 \text{ cm}$ then find EC .



Ans :

[Board Term-1 2016]

In the given figure $DE \parallel BC$, thus

$$\frac{AD}{BD} = \frac{AE}{EC}$$

$$\frac{3}{4} = \frac{6}{EC}$$

$$EC = 8 \text{ cm}$$

37. If triangle ABC is similar to triangle DEF such that $2AB = DE$ and $BC = 8 \text{ cm}$ then find EF .

Ans :

As per given condition we have drawn the figure below.

38. Are two triangles with equal corresponding sides always similar?

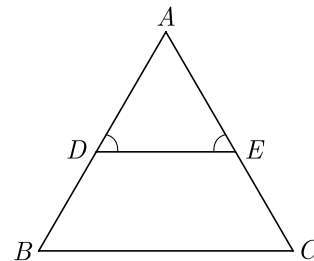
Ans :

[Board Term-1 2015]

Yes, Two triangles having equal corresponding sides are congruent and all congruent Δs have equal angles, hence they are similar too.

TWO MARKS QUESTIONS

39. In Figure $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, prove that $\triangle BAC$ is an isosceles triangle.



Ans :

[Board 2020 Delhi Standard]

We have, $\angle D = \angle E$

$$\text{and } \frac{AD}{DB} = \frac{AE}{EC}$$

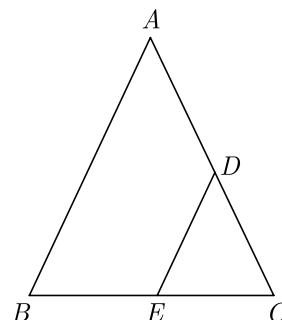
By converse of BPT, $DE \parallel BC$

Due to corresponding angles we have

$$\angle ADE = \angle ABC \text{ and}$$

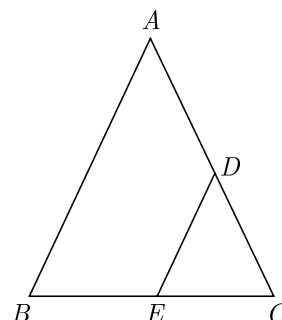
$\angle AED = \angle ACB$
 Given $\angle ADE = \angle AED$
 Thus $\angle ABC = \angle ACB$
 Therefore BAC is an isosceles triangle.

the sides CA, CB respectively such that $DE \parallel AB$,
 $AD = 2x, DC = x + 3, BE = 2x - 1$ and $CE = x$.
 Then, find x .

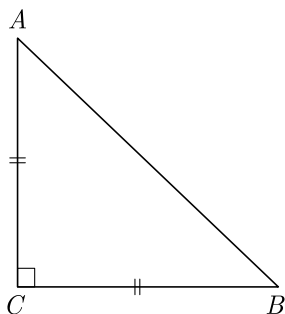


OR

In the figure of $\triangle ABC$, $DE \parallel AB$. If $AD = 2x$,
 $DC = x + 3, BE = 2x - 1$ and $CE = x$, then find the
 value of x .



40. In Figure, ABC is an isosceles triangle right angled at C with $AC = 4$ cm, Find the length of AB .



Ans :

[Board 2019 OD]

Since ABC is an isosceles triangle right angled at C ,

$$AC = BC = 4 \text{ cm}$$

$$\angle C = 90^\circ$$

Using Pythagoras theorem in $\triangle ABC$ we have,

$$AB^2 = BC^2 + AC^2$$

$$= 4^2 + 4^2 = 16 + 16 = 32$$

$$AB = 4\sqrt{2} \text{ cm.}$$

41. In the figure of $\triangle ABC$, the points D and E are on

Ans :

[Board Term-1 2015, 2016]

We have

$$\frac{CD}{AD} = \frac{CE}{BE}$$

$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

$$5x = 3 \text{ or, } x = \frac{3}{5}$$

Alternative Method :

In ABC , $DE \parallel AB$, thus

$$\frac{CD}{CA} = \frac{CE}{CB}$$

$$\frac{CD}{CA - CD} = \frac{CE}{CB - CE}$$

$$\frac{CD}{AD} = \frac{CE}{BE}$$

$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

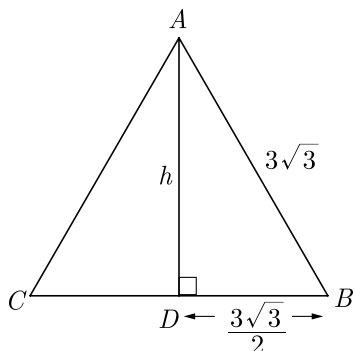
$$5x = 3 \text{ or, } x = \frac{3}{5}$$

42. In an equilateral triangle of side $3\sqrt{3}$ cm find the length of the altitude.

Ans :

[Board Term-1 2016]

Let $\triangle ABC$ be an equilateral triangle of side $3\sqrt{3}$ cm and AD is altitude which is also a perpendicular bisector of side BC . This is shown in figure given below.



Now

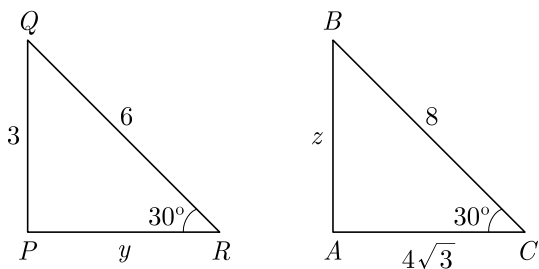
$$(3\sqrt{3})^2 = h^2 + \left(\frac{3\sqrt{3}}{2}\right)^2$$

$$27 = h^2 + \frac{27}{4}$$

$$h^2 = 27 - \frac{27}{4} = \frac{81}{4}$$

$$h = \frac{9}{2} = 4.5 \text{ cm}$$

43. In the given figure, $\triangle ABC \sim \triangle PQR$. Find the value of $y + z$.



Ans :

[Board Term-1 2010]

In the given figure $\triangle ABC \sim \triangle PQR$,

$$\text{Thus } \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

$$\frac{z}{3} = \frac{8}{6} \text{ and } \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

$$z = \frac{8 \times 3}{6} \text{ and } y = \frac{4\sqrt{3} \times 6}{8}$$

$$z = 4 \text{ and } y = 3\sqrt{3}$$

Thus

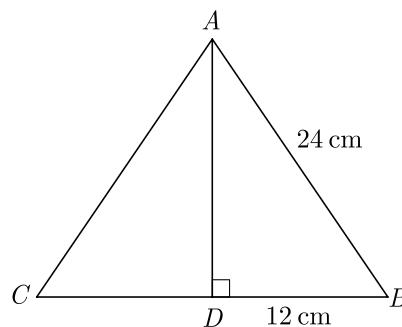
$$y + z = 3\sqrt{3} + 4$$

44. In an equilateral triangle of side 24 cm, find the length of the altitude.

Ans :

[Board Term-1 2015]

Let $\triangle ABC$ be an equilateral triangle of side 24 cm and AD is altitude which is also a perpendicular bisector of side BC . This is shown in figure given below.



Now

$$BD = \frac{BC}{2} = \frac{24}{2} = 12 \text{ cm}$$

$$AB = 24 \text{ cm}$$

$$AD = \sqrt{AB^2 - BD^2}$$

$$= \sqrt{(24)^2 - (12)^2}$$

$$= \sqrt{576 - 144}$$

$$= \sqrt{432} = 12\sqrt{3}$$

Thus $AD = 12\sqrt{3}$ cm.

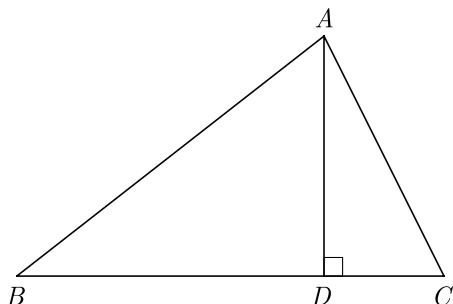
45. In $\triangle ABC$, $AD \perp BC$, such that $AD^2 = BD \times CD$. Prove that $\triangle ABC$ is right angled at A.

Ans :

[Board Term-1 2015]

As per given condition we have drawn the figure

below.



We have $AD^2 = BD \times CD$

$$\frac{AD}{CD} = \frac{BD}{AD}$$

Since $\angle D = 90^\circ$, by SAS we have

$$\triangle ADC \sim \triangle BDA$$

and $\angle BAD = \angle ACD$;

Since corresponding angles of similar triangles are equal

$$\angle DAC = \angle DBA$$

$$\angle BAD + \angle ACD + \angle DAC + \angle DBA = 180^\circ$$

$$2\angle BAD + 2\angle DAC = 180^\circ$$

$$\angle BAD + \angle DAC = 90^\circ$$

$$\angle A = 90^\circ$$

Thus $\triangle ABC$ is right angled at A .

46. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes. [Board 2020 SQP Standard]

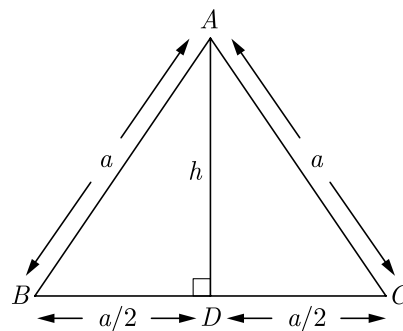
or

Find the altitude of an equilateral triangle when each of its side is a cm.

Ans :

[Board Term-1 2016]

Let $\triangle ABC$ be an equilateral triangle of side a and AD is altitude which is also a perpendicular bisector of side BC . This is shown in figure given below.



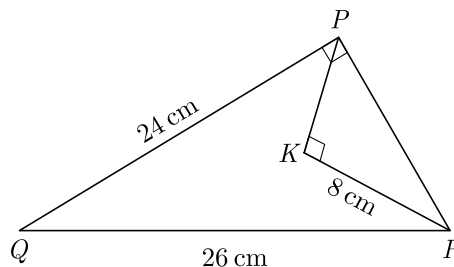
In $\triangle ABD$, $a^2 = \left(\frac{a}{2}\right)^2 + h^2$

$$h^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

Thus $h = \frac{\sqrt{3a}}{2}$

Thus $4h^2 = 3a^2$ Hence Proved

47. In the given triangle PQR , $\angle QPR = 90^\circ$, $PQ = 24$ cm and $QR = 26$ cm and in $\triangle PKR$, $\angle PKR = 90^\circ$ and $KR = 8$ cm, find PK .



Ans :

[Board Term-1 2012]

In the given triangle we have

$$\angle QPR = 90^\circ$$

Thus $QR^2 = QP^2 + PR^2$

$$PR = \sqrt{26^2 - 24^2}$$

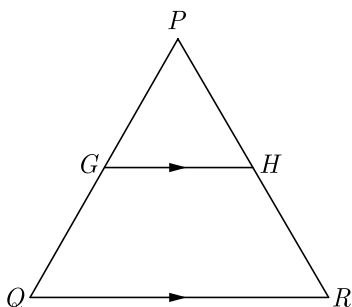
$$= \sqrt{100} = 10 \text{ cm}$$

Now $\angle PKR = 90^\circ$

Thus $PK = \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$

$$= \sqrt{36} = 6 \text{ cm}$$

48. In the given figure, G is the mid-point of the side PQ of $\triangle PQR$ and $GH \parallel QR$. Prove that H is the mid-point of the side PR or the triangle PQR .



Ans :

[Board Term-1 2012]

Since G is the mid-point of PQ we have

$$PG = GQ$$

$$\frac{PG}{GQ} = 1$$

We also have $GH \parallel QR$, thus by BPT we get

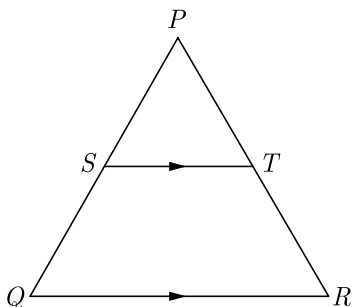
$$\frac{PG}{GQ} = \frac{PH}{HR}$$

$$1 = \frac{PH}{HR}$$

$$PH = HR. \quad \text{Hence proved.}$$

Hence, H is the mid-point of PR .

49. In the given figure, in a triangle PQR , $ST \parallel QR$ and $\frac{PS}{SQ} = \frac{3}{5}$ and $PR = 28$ cm, find PT .



Ans :

[Board Term-1 2011]

We have $\frac{PS}{SQ} = \frac{3}{5}$

$$\frac{PS}{PS + SQ} = \frac{3}{3 + 5}$$

$$\frac{PS}{PQ} = \frac{3}{8}$$

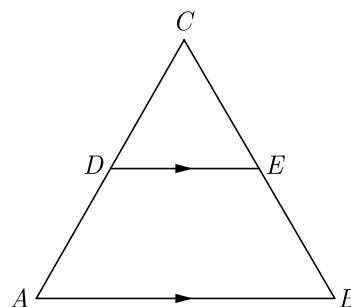
We also have, $ST \parallel QR$, thus by BPT we get

$$\frac{PS}{PQ} = \frac{PT}{PR}$$

$$PT = \frac{PS}{PQ} \times PR$$

$$= \frac{3 \times 28}{8} = 10.5 \text{ cm}$$

50. In the given figure, $\angle A = \angle B$ and $AD = BE$. Show that $DE \parallel AB$.



Ans :

[Board Term-1, 2012, set-63]

In $\triangle CAB$, we have

$$\angle A = \angle B \quad (1)$$

By isosceles triangle property we have

$$AC = CB$$

But, we have been given

$$AD = BE \quad (2)$$

Dividing equation (2) by (1) we get,

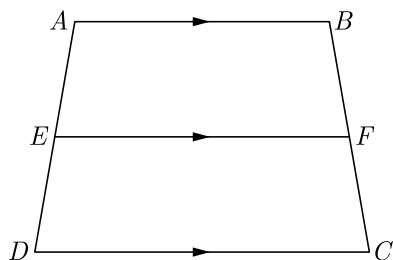
$$\frac{CD}{AD} = \frac{CE}{BE}$$

By converse of BPT,

$$DE \parallel AB. \quad \text{Hence Proved}$$

51. In the given figure, if $ABCD$ is a trapezium in which

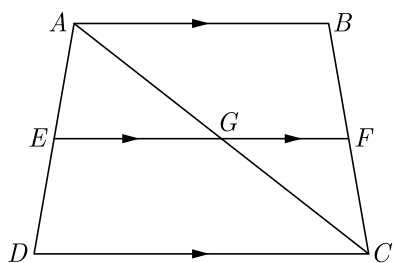
$AB \parallel CD \parallel EF$, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$



Ans :

[Board Term-1 2012]

We draw, AC intersecting EF at G as shown below.



In $\triangle CAB$, $GF \parallel AB$, thus by BPT we have

$$\frac{AG}{CG} = \frac{BF}{FC} \quad \dots(1)$$

In $\triangle ADC$, $EG \parallel DC$, thus by BPT we have

$$\frac{AE}{ED} = \frac{AG}{CG} \quad \dots(2)$$

From equations (1) and (2),

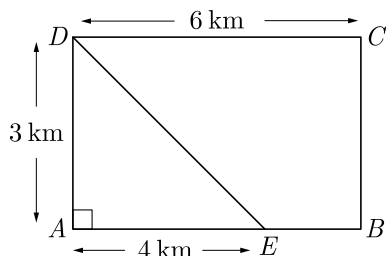
$$\frac{AE}{ED} = \frac{BF}{FC}. \quad \text{Hence Proved.}$$

- 52.** In a rectangle $ABCD$, E is a point on AB such that $AE = \frac{2}{3}AB$. If $AB = 6$ km and $AD = 3$ km, then find DE .

Ans :

[Board Term-1 2016]

As per given condition we have drawn the figure below.



We have $AE = \frac{2}{3}AB = \frac{2}{3} \times 6 = 4$ km

In right triangle ADE ,

$$DE^2 = (3)^2 + (4)^2 = 25$$

Thus

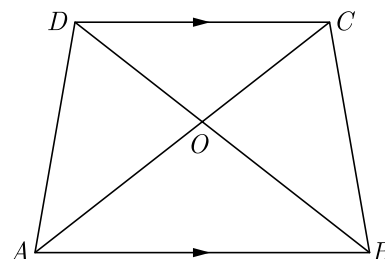
$$DE = 5 \text{ km}$$

- 53.** $ABCD$ is a trapezium in which $AB \parallel CD$ and its diagonals intersect each other at the point O . Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Ans :

[Board Term-1 2012]

As per given condition we have drawn the figure below.



In $\triangle AOB$ and $\triangle COD$, $AB \parallel CD$,

Thus due to alternate angles

$$\angle OAB = \angle DCO$$

and

$$\angle OBA = \angle ODC$$

By AA similarity we have

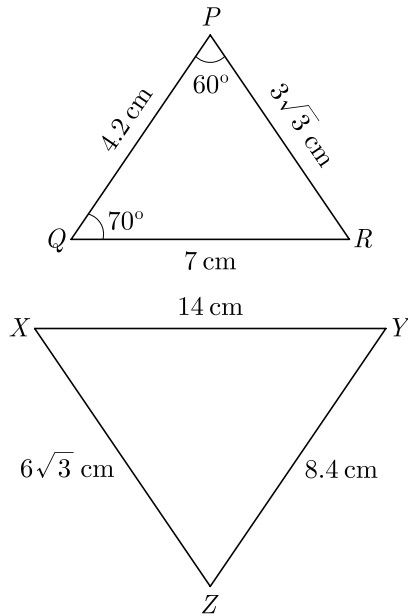
$$\Delta AOB \sim \Delta COD$$

For corresponding sides of similar triangles we have

$$\frac{AO}{CO} = \frac{BO}{DO}$$

$$\frac{AO}{BO} = \frac{CO}{DO}. \quad \text{Hence Proved}$$

54. In the given figures, find the measure of $\angle X$.



Ans :

[Board Term-1 2012]

From given figures,

$$\frac{PQ}{ZY} = \frac{4.2}{8.4} = \frac{1}{2},$$

$$\frac{PR}{ZX} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

and

$$\frac{QR}{YX} = \frac{7}{14} = \frac{1}{2}$$

Thus

$$\frac{QP}{ZY} = \frac{PR}{ZX} = \frac{QR}{YX}$$

By SSS criterion we have

$$\Delta PQR \sim \Delta ZYX$$

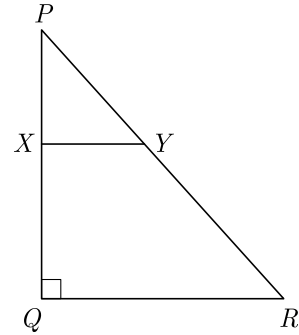
Thus

$$\begin{aligned} \angle X &= \angle R \\ &= 180^\circ - (60^\circ + 70^\circ) = 50^\circ \end{aligned}$$

Thus $\angle X = 50^\circ$

55. In the given figure, PQR is a triangle right angled at Q and $XY \parallel QR$. If $PQ = 6$ cm, $PY = 4$ cm and

$PX:XQ = 1:2$. Calculate the length of PR and QR .



Ans :

[Board Term-1 2012]

Since $XY \parallel OR$, by BPT we have

$$\frac{PX}{XQ} = \frac{PY}{YR}$$

$$\frac{1}{2} = \frac{PY}{PR - PY}$$

$$= \frac{4}{PR - 4}$$

$$PR - 4 = 8 \Rightarrow PR = 12 \text{ cm}$$

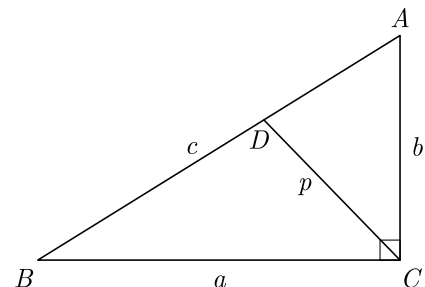
In right ΔPQR we have

$$QR^2 = PR^2 - PQ^2$$

$$= 12^2 - 6^2 = 144 - 36 = 108$$

Thus $QR = 6\sqrt{3}$ cm

56. ABC is a right triangle right angled at C . Let $BC = a$, $CA = b$, $AB = c$. PQR , $ST \parallel QR$ and p be the length of perpendicular from C to AB . Prove that $cp = ab$.



Ans :

[Board Term-1 2012]

In the given figure $CD \perp AB$, and $CD = p$

$$\text{Area, } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times AB \times CD = \frac{1}{2} cp$$

$$\text{Also, Area of } \triangle ABC = \frac{1}{2} \times BC \times AC = \frac{1}{2} ab$$

$$\text{Thus } \frac{1}{2} cp = \frac{1}{2} ab$$

$$cp = ab$$

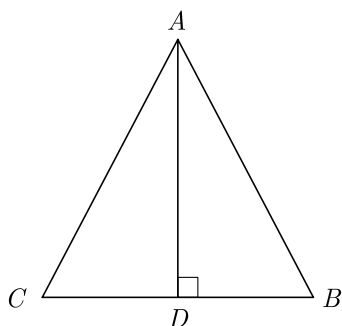
Proved

57. In an equilateral triangle ABC , AD is drawn perpendicular to BC meeting BC in D . Prove that $AD^2 = 3BD^2$.

Ans :

[Board Term-1 2012]

In $\triangle ABD$, from Pythagoras theorem,



$$AB^2 = AD^2 + BD^2$$

Since $AB = BC = CA$, we get

$$BC^2 = AD^2 + BD^2,$$

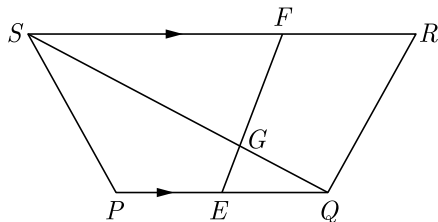
Since $\perp$ is the median in an equilateral $\triangle$, $BC = 2BD$

$$(2BD)^2 = AD^2 + BD^2$$

$$4BD^2 - BD^2 = AD^2$$

$$3BD^2 = AD^2$$

58. In the figure, $PQRS$ is a trapezium in which $PQ \parallel RS$. On PQ and RS , there are points E and F respectively such that EF intersects SQ at G . Prove that $EQ \times GS = GQ \times FS$.



Ans :

[Board Term-1 2016]

In $\triangle GEQ$ and $\triangle GFS$,

Due to vertical opposite angle,

$$\angle EGQ = \angle FGS$$

Due to alternate angle,

$$\angle EQG = \angle FSG$$

Thus by AA similarity we have

$$\triangle GEQ \sim \triangle GFS$$

$$\frac{EQ}{FS} = \frac{GQ}{GS}$$

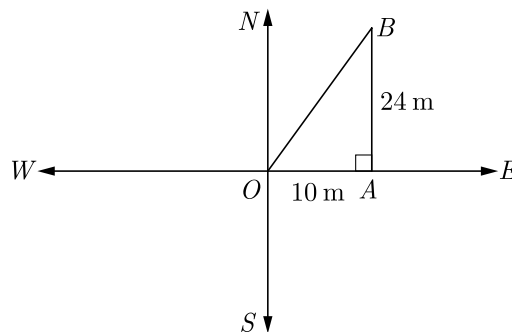
$$EQ \times GS = GQ \times FS$$

59. A man steadily goes 10 m due east and then 24 m due north.

- (1) Find the distance from the starting point.
- (2) Which mathematical concept is used in this problem?

Ans :

(1) Let the initial position of the man be at O and his final position be B . The man goes to 10 m due east and then 24 m due north. Therefore, $\triangle AOB$ is a right triangle right angled at A such that $OA = 10$ m and $AB = 24$ m. We have shown this condition in figure below.



By Pythagoras theorem,

$$\begin{aligned} OB^2 &= OA^2 + AB^2 \\ &= (10)^2 + (24)^2 \\ &= 100 + 576 = 676 \end{aligned}$$

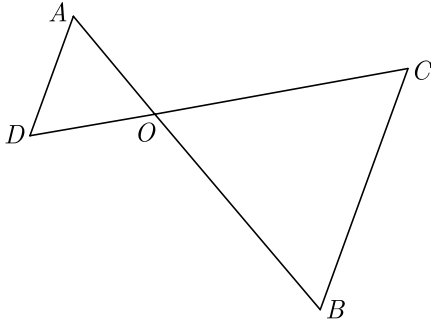
$$\text{or, } OB = \sqrt{676} = 26 \text{ m}$$

Hence, the man is at a distance of 26 m from the starting point.

- (2) Pythagoras Theorem

60. In the given figure, $OA \times OB = OC \times OD$, show that

$$\angle A = \angle C \text{ and } \angle B = \angle D.$$



$$3x - 10 = 2x - 3$$

$$3x - 2x = 10 - 3 \Rightarrow x = 7$$

Thus $x = 7$.

Ans :

[Board Term-1 2012]

We have $OA \times OB = OC \times OD$

$$\frac{OA}{OD} = \frac{OC}{OB}$$

Due to the vertically opposite angles,

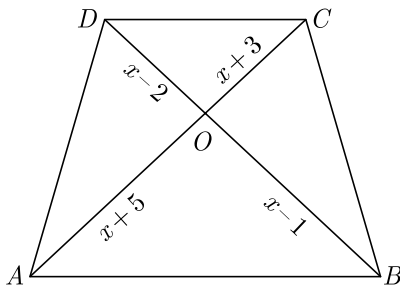
$$\angle AOD = \angle COB$$

Thus by SAS similarity we have

$$\Delta AOD \sim \Delta COB$$

Thus $\angle A = \angle C$ and $\angle B = \angle D$. because of corresponding angles of similar triangles.

61. In the given figure, if $AB \parallel DC$, find the value of x .



Ans :

[Board Term-1 2012]

We know that diagonals of a trapezium divide each other proportionally. Therefore

$$\frac{OA}{OC} = \frac{BO}{OD}$$

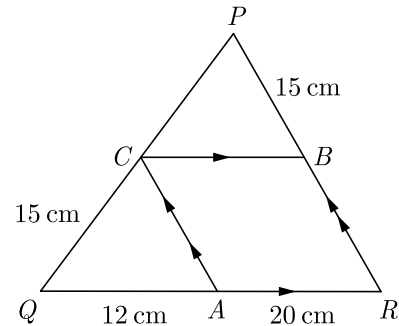
$$\frac{x+5}{x+3} = \frac{x-1}{x-2}$$

$$(x+5)(x-2) = (x-1)(x+3)$$

$$x^2 - 2x + 5x - 10 = x^2 + 3x - x - 3$$

$$x^2 + 3x - 10 = x^2 + 2x - 3$$

62. In the given figure, $CB \parallel QR$ and $CA \parallel PR$. If $AQ = 12$ cm, $AR = 20$ cm, $PB = CQ = 15$ cm, calculate PC and BR .



Ans :

[Board Term-1 2012]

In ΔPQR , $CA \parallel PR$

By BPT similarity we have

$$\frac{PC}{CQ} = \frac{RA}{AQ}$$

$$\frac{PC}{15} = \frac{20}{12}$$

$$PC = \frac{15 \times 20}{12} = 25 \text{ cm}$$

In $\triangle PQR$, $CB \parallel QR$

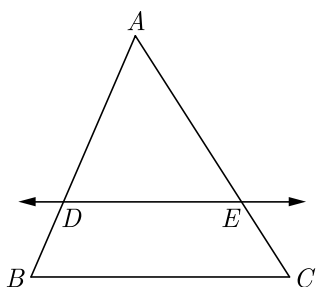
Thus $\frac{PC}{CQ} = \frac{PR}{BR}$

$$\frac{25}{15} = \frac{15}{BR}$$

$$BR = \frac{15 \times 15}{25} = 9 \text{ cm}$$

THREE MARKS QUESTIONS

63. In Figure, in $\triangle ABC$, $DE \parallel BC$ such that $AD = 2.4$ cm, $AB = 3.2$ cm and $AC = 8$ cm, then what is the length of AE ?



Ans :

[Board 2020 Delhi Basic]

We have $DE \parallel BC$

By BPT, $\frac{AD}{DB} = \frac{AE}{EC}$

$$\frac{2.4}{AB - AD} = \frac{AE}{AC - AE}$$

$$\frac{2.4}{3.2 - 2.4} = \frac{AE}{8 - AE}$$

$$\frac{2.4}{0.8} = \frac{AE}{8 - AE}$$

$$3 = \frac{AE}{8 - AE}$$

$$\frac{3}{1 + 3} = \frac{AE}{8 - AE + AE}$$

$$\frac{3}{4} = \frac{AE}{8} \Rightarrow AE = 6 \text{ cm}$$

64. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC . If AC and BD intersect at P , prove that $AP \times PC = BP \times DP$.

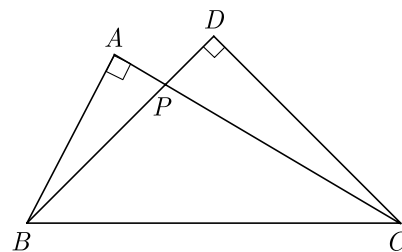
Ans :

[Board 2019 OD]

Let $\triangle ABC$, and $\triangle DBC$ be right angled at A and D respectively.

As per given information in question we have drawn

the figure given below.



In $\triangle BAP$ and $\triangle CDP$ we have

$$\angle BAP = \angle CDP = 90^\circ$$

and due to vertical opposite angle

$$\angle BPA = \angle CPD$$

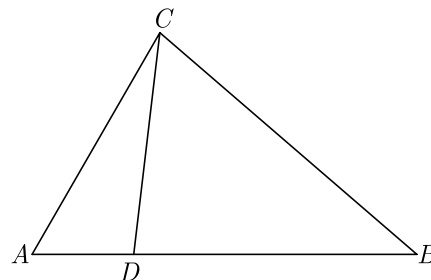
By AA similarity we have

$$\triangle BAP \sim \triangle CDP$$

Therefore $\frac{BP}{PC} = \frac{AP}{PD}$

$$AP \times PC = BP \times PD \quad \text{Hence Proved}$$

65. In the given figure, if $\angle ACB = \angle CDA$, $AC = 6$ cm and $AD = 3$ cm, then find the length of AB .



Ans :

[Board 2020 SQP Standard]

In $\triangle ABC$ and $\triangle ACD$ we have

$$\angle ACB = \angle CDA \quad \text{[given]}$$

$$\angle CAB = \angle CAD \quad \text{[common]}$$

By AA similarity criterion we get

$$\triangle ABC \sim \triangle ACD$$

Thus $\frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$

Now $\frac{AB}{AC} = \frac{AC}{AD}$

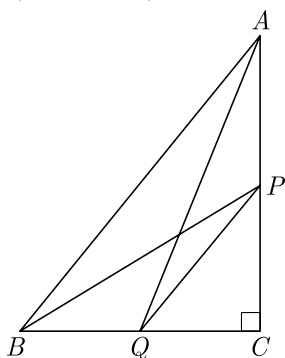
$$AC^2 = AB \times AD$$

$$6^2 = AB \times 3$$

$$AB = \frac{36}{3} = 12 \text{ cm}$$

66. If P and Q are the points on side CA and CB

respectively of $\triangle ABC$, right angled at C , prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$



Ans :

[Board 2019 Delhi]

In right angled triangles ACQ and PCB

$$AQ^2 = AC^2 + CQ^2 \quad \dots(1)$$

and $BP^2 = PC^2 + CB^2 \quad \dots(2)$

Adding eq (1) and eq (2), we get

$$\begin{aligned} AQ^2 + BP^2 &= (AC^2 + CQ^2) + (PC^2 + CB^2) \\ &= (AC^2 + CB^2) + (PC^2 + CQ^2) \end{aligned}$$

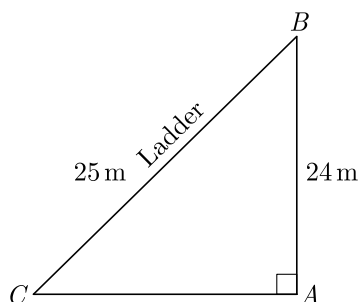
Thus $AQ^2 + BP^2 = AB^2 + PQ^2$ Hence Proved

- 67.** A ladder 25 m long just reaches the top of a building 24 m high from the ground. What is the distance of the foot of ladder from the base of the building?

Ans :

[Board 2020 OD Basic]

Let AB be the building and CB be the ladder. As per information given we have drawn figure below.



Here $AB = 24$ m

$CB = 25$ m

and $\angle CAB = 90^\circ$

By Pythagoras Theorem,

$$CB^2 = AB^2 + CA^2$$

or, $CA^2 = CB^2 - AB^2$
 $= 25^2 - 24^2$

$$= 625 - 576 = 49$$

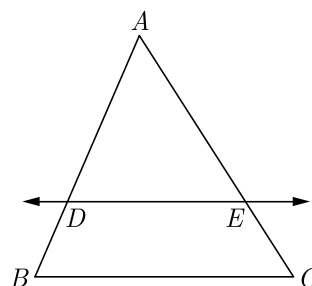
Thus

$$CA = 7 \text{ m}$$

Hence, the distance of the foot of ladder from the building is 7 m.

THREE MARKS QUESTIONS

- 68.** In Figure, in $\triangle ABC$, $DE \parallel BC$ such that $AD = 2.4$ cm, $AB = 3.2$ cm and $AC = 8$ cm, then what is the length of AE ?



Ans :

[Board 2020 Delhi Basic]

We have

$$DE \parallel BC$$

By BPT,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{2.4}{AB - AD} = \frac{AE}{AC - AE}$$

$$\frac{2.4}{3.2 - 2.4} = \frac{AE}{8 - AE}$$

$$\frac{2.4}{0.8} = \frac{AE}{8 - AE}$$

$$3 = \frac{AE}{8 - AE}$$

$$\frac{3}{1 + 3} = \frac{AE}{8 - AE + AE}$$

$$\frac{3}{4} = \frac{AE}{8} \Rightarrow AE = 6 \text{ cm}$$

- 69.** Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC . If AC and BD intersect at P , prove that

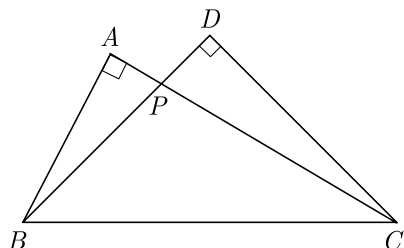
$$AP \times PC = BP \times DP.$$

Ans :

[Board 2019 OD]

Let $\triangle ABC$, and $\triangle DBC$ be right angled at A and D respectively.

As per given information in question we have drawn the figure given below.



In $\triangle BAP$ and $\triangle CDP$ we have

$$\angle BAP = \angle CDP = 90^\circ$$

and due to vertical opposite angle

$$\angle BPA = \angle CPD$$

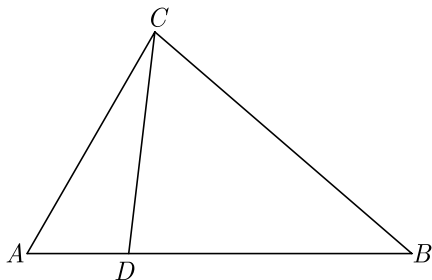
By AA similarity we have

$$\triangle BAP \sim \triangle CDP$$

Therefore $\frac{BP}{PC} = \frac{AP}{PD}$

$$AP \times PC = BP \times PD \quad \text{Hence Proved}$$

70. In the given figure, if $\angle ACB = \angle CDA$, $AC = 6$ cm and $AD = 3$ cm, then find the length of AB .



Ans :

[Board 2020 SQP Standard]

In $\triangle ABC$ and $\triangle ACD$ we have

$$\angle ACB = \angle CDA \quad \text{[given]}$$

$$\angle CAB = \angle CAD \quad \text{[common]}$$

By AA similarity criterion we get

$$\triangle ABC \sim \triangle ACD$$

Thus $\frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$

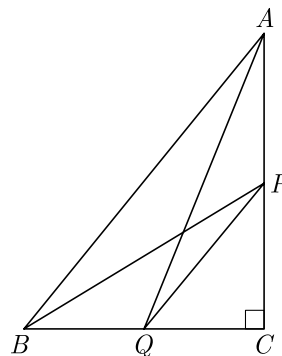
Now $\frac{AB}{AC} = \frac{AC}{AD}$

$$AC^2 = AB \times AD$$

$$6^2 = AB \times 3$$

$$AB = \frac{36}{3} = 12 \text{ cm}$$

71. If P and Q are the points on side CA and CB respectively of $\triangle ABC$, right angled at C , prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$



Ans :

[Board 2019 Delhi]

In right angled triangles ACQ and PCB

$$AQ^2 = AC^2 + CQ^2 \quad \dots(1)$$

and $BP^2 = PC^2 + CB^2 \quad \dots(2)$

Adding eq (1) and eq (2), we get

$$\begin{aligned} AQ^2 + BP^2 &= (AC^2 + CQ^2) + (PC^2 + CB^2) \\ &= (AC^2 + CB^2) + (PC^2 + CQ^2) \end{aligned}$$

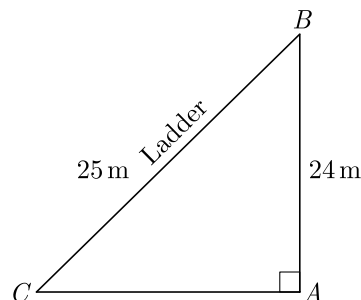
Thus $AQ^2 + BP^2 = AB^2 + PQ^2 \quad \text{Hence Proved}$

72. A ladder 25 m long just reaches the top of a building 24 m high from the ground. What is the distance of the foot of ladder from the base of the building?

Ans :

[Board 2020 OD Basic]

Let AB be the building and CB be the ladder. As per information given we have drawn figure below.



Here $AB = 24$ m

$$CB = 25 \text{ m}$$

and $\angle CAB = 90^\circ$

By Pythagoras Theorem,

$$\begin{aligned}
 CB^2 &= AB^2 + CA^2 \\
 \text{or, } CA^2 &= CB^2 - AB^2 \\
 &= 25^2 - 24^2 \\
 &= 625 - 576 = 49
 \end{aligned}$$

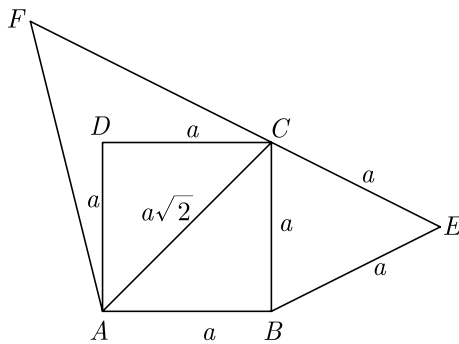
Thus $CA = 7$ m

Hence, the distance of the foot of ladder from the building is 7 m.

- 73.** Prove that area of the equilateral triangle described on the side of a square is half of this area of the equilateral triangle described on its diagonal.

Ans : [Board 2018, 2015]

As per given condition we have drawn the figure below. Let a be the side of square.



By Pythagoras theorem,

$$\begin{aligned}
 AC^2 &= AB^2 + BC^2 \\
 &= a^2 + a^2 = 2a^2
 \end{aligned}$$

$$AC = \sqrt{2} a$$

Area of equilateral triangle $\triangle BCE$,

$$\text{area}(\triangle BCE) = \frac{\sqrt{3}}{4} a^2$$

Area of equilateral triangle $\triangle ACF$,

$$\text{area}(\triangle ACF) = \frac{\sqrt{3}}{4} (\sqrt{2} a)^2 = \frac{\sqrt{3}}{2} a^2$$

$$\text{Now, } \frac{\text{area}(\triangle ACF)}{\text{area}(\triangle BCE)} = 2$$

$$\text{area}(\triangle ACF) = 2\text{area}(\triangle BCE)$$

$$\text{area}(\triangle BCE) = \frac{1}{2}\text{area}(\triangle ACF) \text{ Hence Proved.}$$

74.

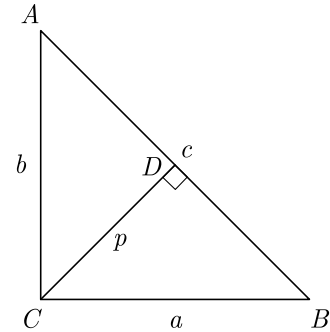
- 75.** $\triangle ABC$ is right angled at C . If p is the length of the perpendicular from C to AB and a, b, c are the lengths of the sides opposite $\angle A, \angle B$ and $\angle C$ respectively,

then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Ans :

[Board Term-1 2016]

As per given condition we have drawn the figure below.



In $\triangle ACB$ and $\triangle CDB$, $\angle B$ is common and

$$\angle ABC = \angle CDB = 90^\circ$$

Because of AA similarity we have

$$\triangle ABC \sim \triangle CDB$$

$$\text{Now } \frac{b}{p} = \frac{c}{a}$$

$$\frac{1}{p} = \frac{c}{ab}$$

$$\frac{1}{p^2} = \frac{c^2}{a^2 b^2}$$

$$\frac{1}{p^2} = \frac{a^2 + b^2}{a^2 b^2} \quad (c^2 = a^2 + b^2)$$

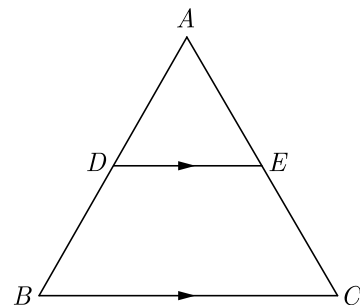
$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} \quad \text{Hence Proved}$$

- 76.** In $\triangle ABC$, $DE \parallel BC$. If $AD = x + 2$, $DB = 3x + 16$, $AE = x$ and $EC = 3x + 5$, then find x .

Ans :

[Board Term-1 2015]

As per given condition we have drawn the figure below.



In the give figure

$$DE \parallel BC$$

By BPT we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x+2}{3x+16} = \frac{x}{x3+5}$$

$$(x+2)(3x+5) = x(3x+16)$$

$$3x^2 + 5x + 6x + 10 = 3x^2 + 16x$$

$$11x + 10 = 16x$$

$$11x + 10 = 10$$

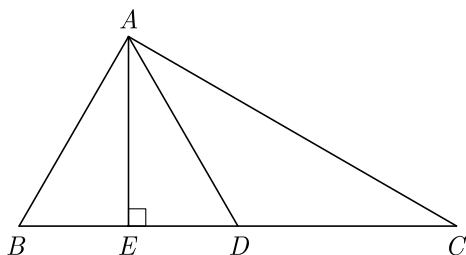
$$5x = 10 \Rightarrow x = 2$$

- 77.** If in $\triangle ABC$, AD is median and $AE \perp BC$, then prove that $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$.

Ans :

[Board Term-1 2015]

As per given condition we have drawn the figure below.



In $\triangle ABE$, using Pythagoras theorem we have

$$\begin{aligned} AB^2 &= AE^2 + BE^2 \\ &= AD^2 - DE^2 + (BD - DE)^2 \\ &= AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE \\ &= AD^2 + BD^2 - 2BD \times DE \quad \dots(1) \end{aligned}$$

In $\triangle AEC$, we have

$$\begin{aligned} AC^2 &= AE^2 + EC^2 \\ &= (AD^2 - ED^2) + (ED + DC)^2 \\ &= AD^2 - ED^2 + ED^2 + DC^2 + 2ED \times DC \\ &= AD^2 + CD^2 + 2ED \times CD \\ &= AD^2 + DC^2 + 2DC \times DE \quad \dots(2) \end{aligned}$$

Adding equation (1) and (2) we have

$$AB^2 + AC^2 = 2(AD^2 + BD^2) \quad (BD = DC)$$

$$= 2AD^2 + 2\left(\frac{1}{2}BC\right)^2 \quad (BD = \frac{1}{2}BC)$$

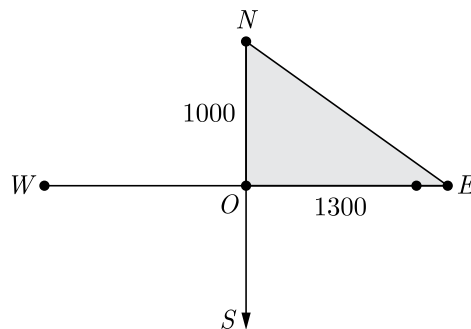
$$= 2AD^2 + \frac{1}{2}BC^2 \quad \text{Hence Proves}$$

- 78.** From an airport, two aeroplanes start at the same time. If speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h then find the distance between the two aeroplanes after 2 hours.

Ans :

[Board Term-1 2015]

As per given condition we have drawn the figure below.



Distance covered by first aeroplane due North after two hours,

$$y = 500 \times 2 = 1,000 \text{ km.}$$

Distance covered by second aeroplane due East after two hours,

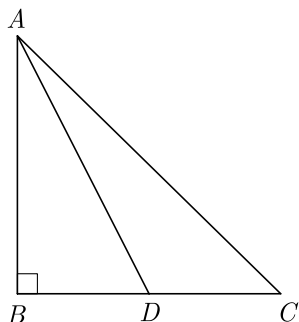
$$x = 650 \times 2 = 1,300 \text{ km.}$$

Distance between two aeroplane after 2 hours

$$\begin{aligned} NE &= \sqrt{ON^2 + OE^2} \\ &= \sqrt{(1000)^2 + (1300)^2} \\ &= \sqrt{1000000 + 1690000} \\ &= \sqrt{2690000} \\ &= 1640.12 \text{ km} \end{aligned}$$

- 79.** In the given figure, ABC is a right angled triangle, $\angle B = 90^\circ$. D is the mid-point of BC . Show that

$$AC^2 = AD^2 + 3CD^2.$$



Ans :

[Board Term-1 2016]

We have $BD = CD = \frac{BC}{2}$

$$BC = 2BD$$

Using Pythagoras theorem in the right $\triangle ABC$, we have

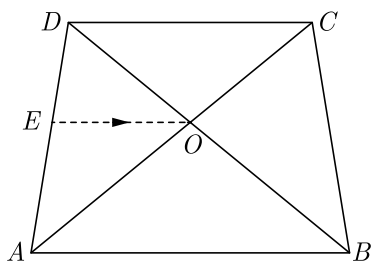
$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= AB^2 + (2BD)^2 \\ &= AB^2 + 4BD^2 \\ &= (AB^2 + BD^2) + 3BD^2 \\ AC^2 &= AD^2 + 3CD^2 \end{aligned}$$

- 80.** If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

Ans :

[Board Term-1 2011]

As per given condition we have drawn quadrilateral $ABCD$, as shown below.



We have drawn $EO \parallel AB$ on DA .

In quadrilateral $ABCD$, we have

$$\begin{aligned} \frac{AO}{BO} &= \frac{CO}{DO} \\ \frac{AO}{CO} &= \frac{BO}{DO} \end{aligned} \quad \dots(1)$$

In $\triangle ABD$, $EO \parallel AB$

By BPT we have

$$\frac{AE}{ED} = \frac{BO}{DO} \quad \dots(2)$$

From equation (1) and (2), we get

$$\frac{AE}{ED} = \frac{AO}{CO}$$

In $\triangle ADC$, $\frac{AE}{ED} = \frac{AO}{CO}$

$EO \parallel DC$ (Converse of BPT)

$EO \parallel AB$ (Construction)

$AB \parallel DC$

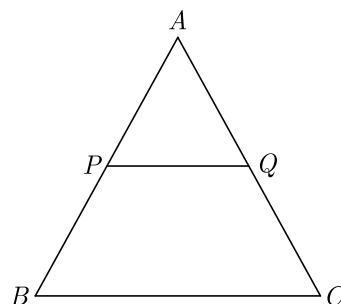
Thus in quadrilateral $ABCD$ we have

$AB \parallel CD$

Thus $ABCD$ is a trapezium.

Hence Proved

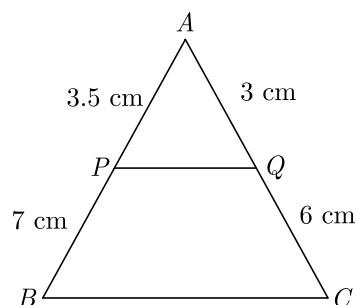
- 81.** In the given figure, P and Q are the points on the sides AB and AC respectively of $\triangle ABC$, such that $AP = 3.5$ cm, $PB = 7$ cm, $AQ = 3$ cm and $QC = 6$ cm. If $PQ = 4.5$ cm, find BC .



Ans :

[Board Term-1 2011]

We have redrawn the given figure as below.



We have

$$\frac{AP}{AB} = \frac{3.5}{10.5} = \frac{1}{3}$$

and $\frac{AQ}{AC} = \frac{3}{9} = \frac{1}{3}$

In $\triangle ABC$, $\frac{AP}{AB} = \frac{AQ}{AC}$ and $\angle A$ is common.

Thus due to SAS we have

$$\triangle APQ \sim \triangle ABC$$

$$\frac{AP}{AB} = \frac{PQ}{BC}$$

$$\frac{1}{3} = \frac{4.5}{BC}$$

$$BC = 13.5 \text{ cm.}$$

$$\angle A = \angle D \quad (\text{Corresponding angles})$$

$$2\angle 1 = 2\angle 2$$

Also $\angle B = \angle E$ (Corresponding angles)

$$\frac{AP}{DQ} = \frac{AB}{DE} \quad \text{Hence Proved}$$

(2) Since $\triangle ABC \sim \triangle DEF$

$$\angle A = \angle D$$

and $\angle C = \angle F$

$$2\angle 3 = 2\angle 4$$

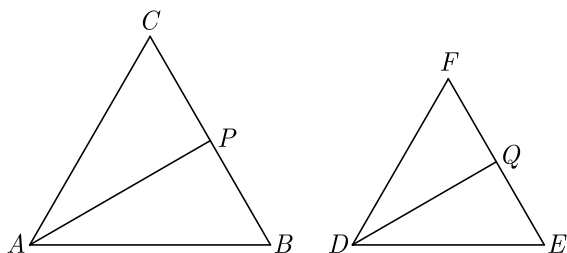
$$\angle 3 = \angle 4$$

By AA similarity we have

$$\triangle CAP \sim \triangle FDQ$$

83. In the given figure, $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$.

82. In given figure $\triangle ABC \sim \triangle DEF$. AP bisects $\angle CAB$ and DQ bisects $\angle FDE$.



Prove that :

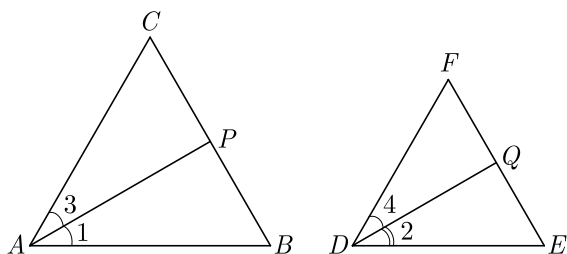
$$(1) \frac{AP}{DQ} = \frac{AB}{DE}$$

$$(2) \triangle CAP \sim \triangle FDQ.$$

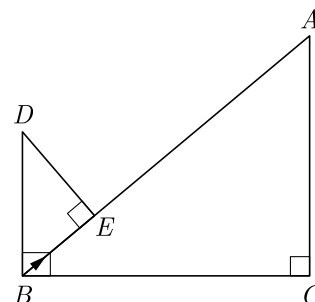
Ans :

[Board Term-1 2016]

As per given condition we have redrawn the figure below.



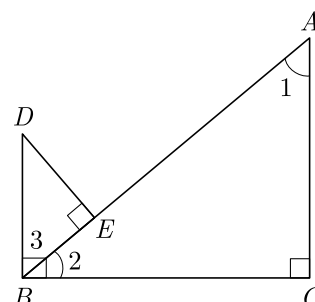
(1) Since $\triangle ABC \sim \triangle DEF$



Ans :

[Board Term-1 2011]

As per given condition we have redrawn the figure below.



We have $DB \perp BC$, $DE \perp AB$ and $AC \perp BC$.

In $\triangle ABC$, $\angle C = 90^\circ$, thus

$$\angle 1 + \angle 2 = 90^\circ$$

But we have been given,

$$\angle 2 + \angle 3 = 90^\circ$$

Hence $\angle 1 = \angle 3$

In $\triangle ABC$ and $\triangle BDE$,

$$\angle 1 = \angle 3$$

and $\angle ACB = \angle DEB = 90^\circ$

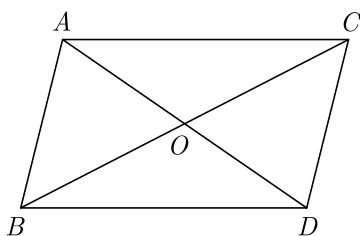
Thus by AA similarity we have

$$\triangle ABC \sim \triangle BDE$$

Thus $\frac{AC}{BC} = \frac{BE}{DE}$. Hence Proved

84. In the given figure, $\triangle ABC$ and $\triangle DBC$ are on the same base BC . AD and BC intersect at O .

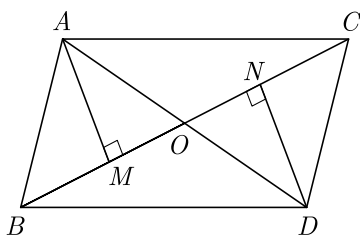
Prove that $\frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{AO}{DO}$.



Ans :

[Board 2020 OD Std, 2016, 2011]

As per given condition we have redrawn the figure below. Here we have drawn $AM \perp BC$ and $DN \perp BC$.



In $\triangle AOM$ and $\triangle DON$,

$$\angle AOM = \angle DON$$

(Vertically opposite angles)

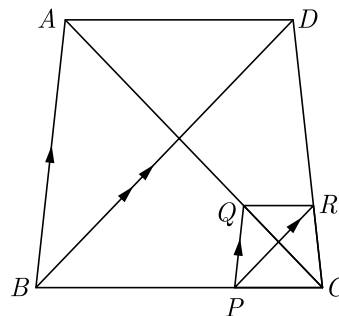
$$\angle AMO = \angle DNO = 90^\circ \text{ (Construction)}$$

or, $\triangle AOM \sim \triangle DON$ (By AA similarity)

Thus $\frac{AO}{DO} = \frac{AM}{DN}$... (1)

Now, $\frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN} = \frac{AM}{DN} = \frac{AO}{DO}$ From equation (1)

85. In the given figure, two triangles ABC and DBC lie on the same side of BC such that $PQ \parallel BA$ and $PR \parallel BD$. Prove that $QR \parallel AD$.



Ans :

[Board Term-1 2011]

In $\triangle ABC$, we have $PQ \parallel AB$ and $PR \parallel BD$.

By BPT we have

$$\frac{BP}{PC} = \frac{AQ}{QC} \quad \dots (1)$$

Again in $\triangle BCD$, we have

$$PR \parallel BD$$

By BPT we have

$$\frac{BP}{PC} = \frac{DR}{RC} \quad (\text{by BPT}) \dots(2)$$

$$\frac{AQ}{QC} = \frac{DR}{RC}$$

By converse of BPT,

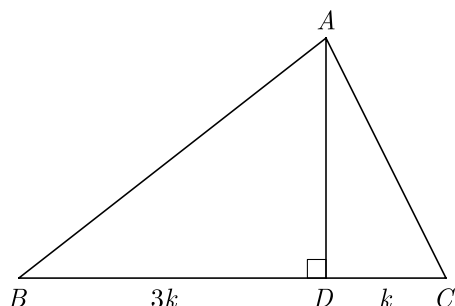
$$PR \parallel AD \quad \text{Hence proved}$$

86. The perpendicular AD on the base BC of a $\triangle ABC$ intersects BC at D so that $DB = 3CD$. Prove that $2(AB)^2 = 2(AC)^2 + BC^2$.

Ans :

[Board Term-1 2011, 2012, 2016]

As per given condition we have drawn the figure below.



Here

$$DB = 3CD$$

$$BD = \frac{3}{4}BC$$

$$DC = \frac{1}{4}BC$$

In $\triangle ADB$, we have

$$AB^2 = AD^2 + BD^2 \quad \dots(1)$$

$$\text{In } \triangle ADC, \quad AC^2 = AD^2 + CD^2 \quad \dots(2)$$

Subtracting equation (2) from (1), we get

$$AB^2 - AC^2 = BD^2 - CD^2$$

Since $DB = 3CD$ we get

$$\begin{aligned} AB^2 - AC^2 &= \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 \\ &= \frac{9}{16}BC^2 - \frac{1}{16}BC^2 = \frac{BC^2}{2} \end{aligned}$$

$$2(AB^2 - AC^2) = BC^2$$

$$2(AB)^2 = 2AC^2 + BC^2 \quad \text{Hence Proved}$$

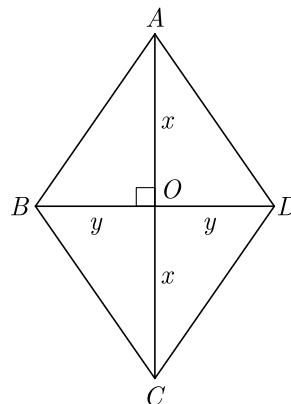
87. Prove that the sum of squares on the sides of a

rhombus is equal to sum of squares of its diagonals.

Ans :

[Board Term-1 2011]

Let, $ABCD$ is a rhombus and we know that diagonals of a rhombus bisect each other at 90° .



Now

$$AO = OC \Rightarrow AO^2 = OC^2$$

$$BO = OD \Rightarrow BO^2 = OD^2$$

and

$$\angle AOB = 90^\circ$$

$$AB^2 = OA^2 + BO^2 = x^2 + y^2$$

Similarly,

$$AD^2 = OA^2 + OD^2 = x^2 + y^2$$

$$CD^2 = OC^2 + OD^2 = x^2 + y^2$$

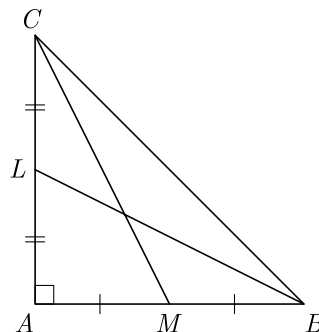
$$CB^2 = OC^2 + OB^2 = x^2 + y^2$$

$$\begin{aligned} AB^2 + BC^2 + CD^2 + DA^2 &= 4x^2 + 4y^2 \\ &= (2x)^2 + (2y)^2 \end{aligned}$$

$$AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$$

Hence Proved

88. In the given figure, BL and CM are medians of $\triangle ABC$, right angled at A . Prove that $4(BL^2 + CM^2) = 5BC^2$.



Ans :

[Board T

We have a right angled triangle $\triangle ABC$ at A where BL and CM are medians.

$$\begin{aligned}\text{In } \triangle ABL, \quad BL^2 &= AB^2 + AL^2 \\ &= AB^2 + \left(\frac{AC}{2}\right)^2 \quad (BL \text{ is median})\end{aligned}$$

$$\begin{aligned}\text{In } \triangle ACM, \quad CM^2 &= AC^2 + AM^2 \\ &= AC^2 + \left(\frac{AB}{2}\right)^2 \quad (CM \text{ is median})\end{aligned}$$

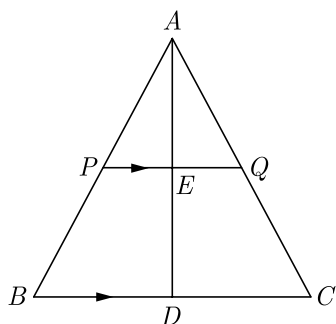
$$\begin{aligned}\text{Now} \quad BL^2 + CM^2 &= AB^2 + AC^2 + \frac{AC^2}{4} + \frac{AB^2}{4} \\ 4(BL^2 + CM^2) &= 5AB^2 + 5AC^2 \\ &= 5(AB^2 + AC^2) \\ &= 5BC^2 \quad \text{Hence Proved}\end{aligned}$$

89. In a $\triangle ABC$, let P and Q be points on AB and AC respectively such that $PQ \parallel BC$. Prove that the median AD bisects PQ .

Ans :

[Board Term-1 2011]

As per given condition we have drawn the figure below.



The median AD intersects PQ at E .

$$\begin{aligned}\text{We have,} \quad PQ &\parallel BE \\ \angle APE &= \angle B \quad \text{and} \quad \angle AQE \\ &= \angle C\end{aligned}$$

(Corresponding angles)

Thus in $\triangle APE$ and $\triangle ABD$ we have

$$\begin{aligned}\angle APE &= \angle ABD \\ \angle PAE &= \angle BAD \quad (\text{common})\end{aligned}$$

Thus $\triangle APE \sim \triangle ABD$

$$\frac{PE}{BD} = \frac{AE}{AD} \quad \dots(1)$$

Similarly, $\triangle AQE \sim \triangle ACD$

$$\text{or,} \quad \frac{QE}{CD} = \frac{AE}{AD} \quad \dots(2)$$

From equation (1) and (2) we have

$$\frac{PE}{BD} = \frac{QE}{CD}$$

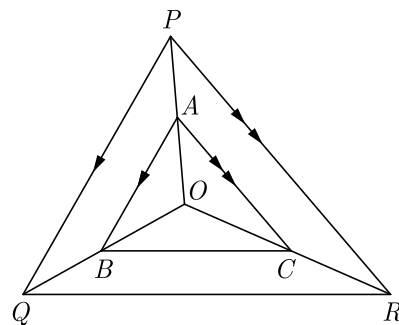
As $CD = BD$, we get

$$\frac{PE}{BD} = \frac{QE}{BD}$$

$$PE = QE$$

Hence, AD bisects PQ .

90. In the given figure A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Prove that $BC \parallel QR$.



Ans :

[Board Term-1 2012]

$$\begin{aligned}\text{In } \triangle POQ, \quad AB &\parallel PQ \\ \text{By BPT} \quad \frac{AO}{AP} &= \frac{OB}{BQ} \quad \dots(1)\end{aligned}$$

$$\begin{aligned}\text{In } \triangle POR, \quad AC &\parallel PR, \\ \text{By BPT} \quad \frac{OA}{AP} &= \frac{OC}{CR} \quad (2)\end{aligned}$$

From equations (1) and (2), we have

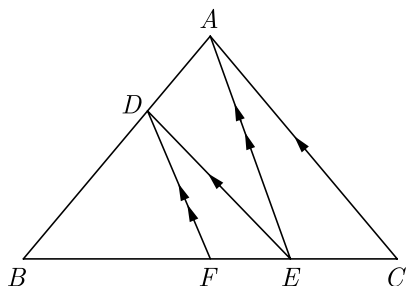
$$\frac{OB}{BQ} = \frac{OC}{CR}$$

By converse of BPT we have

$$BC \parallel QR$$

Hence Proved

91. In the given figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\frac{BE}{FE} = \frac{BE}{EC}$.



Ans :

[Board 2020 Delhi Std, 2012]

In $\triangle ABC$, $DE \parallel AC$, (Given)

By BPT $\frac{BD}{DA} = \frac{BE}{EC}$... (1)

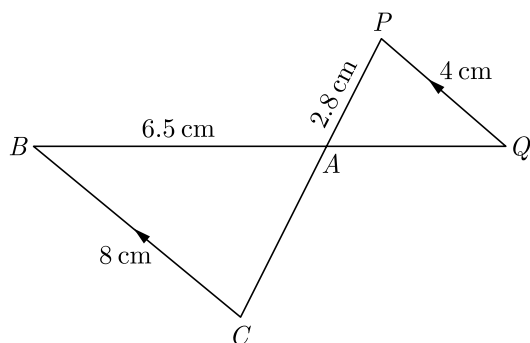
In $\triangle ABE$, $DF \parallel AE$, (Given)

By BPT $\frac{BD}{DA} = \frac{BF}{FE}$... (2)

From (1) and (2), we have

$$\frac{BF}{FE} = \frac{BE}{EC}.$$

92. In the given figure, $BC \parallel PQ$ and $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm $AP = 2.8$ cm Find CA and AQ .



Ans :

[Board Term-1 2012]

In $\triangle ABC$ and $\triangle APQ$, $AB = 6.5$ cm, $BC = 8$ cm,

$PQ = 4$ cm and $AP = 2.8$ cm.

We have $BC \parallel PQ$

Due to alternate angles

$$\angle CBA = \angle AQP$$

Due to vertically opposite angles,

$$\angle BAC = \angle PAQ$$

Due to AA similarity,

$$\triangle ABC \sim \triangle AQP$$

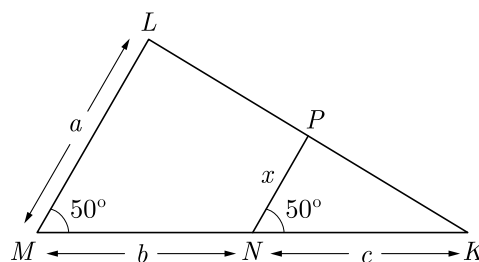
$$\frac{AB}{AQ} = \frac{BC}{QP} = \frac{AC}{AP}$$

$$\frac{6.5}{AQ} = \frac{8}{4} = \frac{AC}{AP}$$

$$AQ = \frac{6.5}{2} = 3.25 \text{ cm}$$

$$AC = 2 \times 2.5 = 5.6 \text{ cm}$$

93. In the given figure, find the value of x in terms of a, b and c .



Ans :

[Board Term-1 2012]

In triangles LMK and PNK , $\angle K$ is common and

$$\angle M = \angle N = 50^\circ$$

Due to AA similarity,

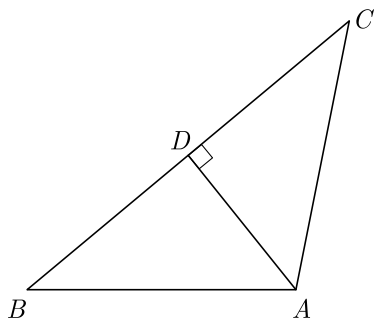
$$\triangle LMK \sim \triangle PNK$$

$$\frac{LM}{PN} = \frac{KM}{KN}$$

$$\frac{a}{x} = \frac{b+c}{c}$$

$$x = \frac{ac}{b+c}$$

94. In the given figure, if $AD \perp BC$, prove that $AB^2 + CD^2 = BD^2 + AC^2$.



Ans :

[Board 2020 OD Standard]

In right $\triangle ADC$,

$$AC^2 = AD^2 + CD^2 \quad \dots(1)$$

In right $\triangle ADB$,

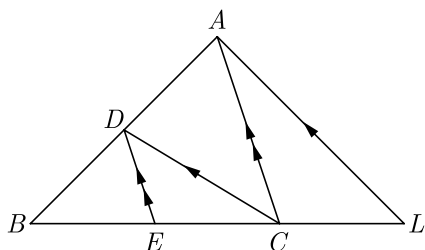
$$AB^2 = AD^2 + BD^2 \quad \dots(2)$$

Subtracting equation (1) from (2) we have

$$AB^2 - AC^2 = BD^2 - CD^2$$

$$AB^2 + CD^2 = AC^2 + BD^2.$$

95. In the given figure, $CD \parallel LA$ and $DE \parallel AC$. Find the length of CL , if $BE = 4$ cm and $EC = 2$ cm.



Ans :

[Board Term-1 2012]

In $\triangle ABC$, $DE \parallel AC$, $BE = 4$ cm and $EC = 2$ cm

By BPT $\frac{BD}{DA} = \frac{BE}{EC} \quad \dots(1)$

In $\triangle ABL$, $DC \parallel AL$

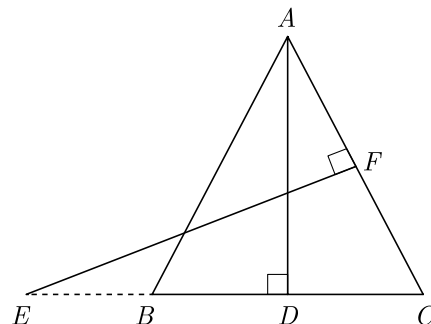
By BPT $\frac{BD}{DA} = \frac{BC}{CL} \quad \dots(2)$

From equations (1) and (2),

$$\frac{BE}{EC} = \frac{BC}{CL}$$

$$\frac{4}{2} = \frac{6}{CL} \Rightarrow CL = 3 \text{ cm}$$

96. In the given figure, $AB = AC$. E is a point on CB produced. If AD is perpendicular to BC and EF perpendicular to AC , prove that $\triangle ABD$ is similar to $\triangle CEF$.



Ans :

[Board Term-1 2012]

In $\triangle ABD$ and $\triangle CEF$, we have

$$AB = AC$$

Thus $\angle ABC = \angle ACB$

$$\angle ABD = \angle ECF$$

$$\angle ADB = \angle EFC \quad (\text{each } 90^\circ)$$

Due to AA similarity

$$\triangle ABD \sim \triangle ECF$$

Hence proved

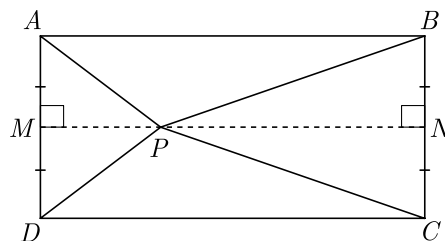
FOUR MARKS QUESTIONS

97. In a rectangle $ABCD$, P is any interior point. Then prove that $PA^2 + PC^2 = PB^2 + PD^2$.

Ans :

[Board 2020 OD Basic]

As per information given we have drawn figure below.



Here P is any point in the interior of rectangle $ABCD$. We have drawn a line MN through point P and parallel to AB and CD .

We have to prove $PA^2 + PC^2 = PB^2 + PD^2$

Since $AB \parallel MN$, $AM \parallel BN$ and $\angle A = 90^\circ$, thus $ABNM$ is rectangle. $MNCD$ is also a rectangle.

Here, $PM \perp AD$ and $PN \perp BC$,

$$AM = BN \text{ and } MD = NC \quad \dots(1)$$

$$\text{Now, in } \triangle AMP, \quad PA^2 = AM^2 + MP^2 \quad \dots(2)$$

$$\text{In } \triangle PMD, \quad PD^2 = MP^2 + MD^2 \quad \dots(3)$$

$$\text{In } \triangle PNB, \quad PB^2 = PN^2 + BN^2 \quad \dots(4)$$

$$\text{In } \triangle PNC, \quad PC^2 = PN^2 + NC^2 \quad \dots(5)$$

From equation (2) and (5) we obtain,

$$PA^2 + PC^2 = AM^2 + MP^2 + PN^2 + NC^2$$

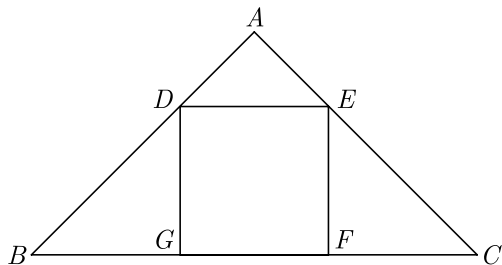
Using equation (1) we have

$$\begin{aligned} PA^2 + PC^2 &= BN^2 + MP^2 + PN^2 + MD^2 \\ &= (BN^2 + PN^2) + (MP^2 + MD^2) \end{aligned}$$

Using equation (3) and (4) we have

$$PA^2 + PC^2 = PB^2 + PD^2$$

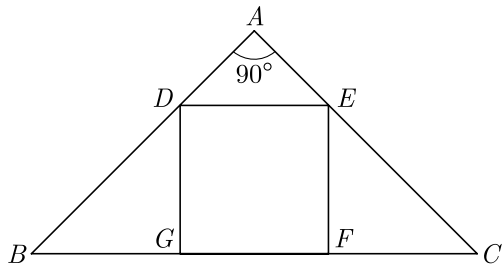
98. In the given figure, $DEFG$ is a square and $\angle BAC = 90^\circ$. Show that $FG^2 = BG \times FC$.



Ans :

[Board 2020 SQP Standard]

We have redrawn the given figure as shown below.



In $\triangle ADE$ and $\triangle GBD$, we have

$$\angle DAE = \angle BGD \quad [\text{each } 90^\circ]$$

Due to corresponding angles we have

$$\angle ADE = \angle GDB$$

Thus by AA similarity criterion,

$$\triangle ADE \sim \triangle GBD$$

Now, in $\triangle ADE$ and $\triangle FEC$,

$$\angle EAD = \angle CFE \quad [\text{each } 90^\circ]$$

Due to corresponding angles we have

$$\angle AED = \angle FCE$$

Thus by AA similarity criterion,

$$\triangle ADE \sim \triangle FEC$$

Since $\triangle ADE \sim \triangle GBD$ and $\triangle ADE \sim \triangle FEC$ we have

$$\triangle GBD \sim \triangle FEC$$

$$\text{Thus} \quad \frac{GB}{FE} = \frac{GD}{FC}$$

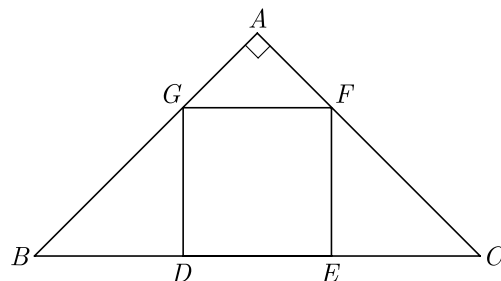
Since $DEFG$ is square, we obtain,

$$\frac{BG}{FG} = \frac{FG}{FC}$$

Therefore $FG^2 = BG \times FC$ Hence Proved

99. In Figure $DEFG$ is a square in a triangle ABC right angled at A . Prove that

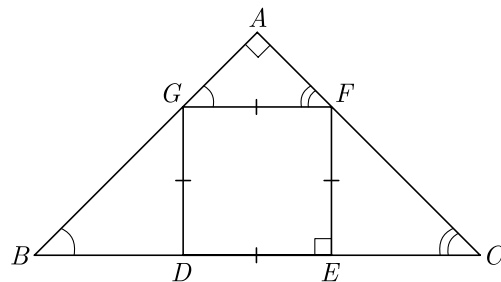
$$(i) \triangle AGF \sim \triangle DBG \quad (ii) \triangle AGF \sim \triangle EFC$$



Ans :

[Board 2020 Delhi, OD Basic]

We have redrawn the given figure as shown below.



Here ABC is a triangle in which $\angle BAC = 90^\circ$ and $DEFG$ is a square.

(i) In $\triangle AGF$ and $\triangle DBG$

$$\angle GAF = \angle BDG \quad (\text{each } 90^\circ)$$

Due to corresponding angles,

$$\angle AGF = \angle GBD$$

Thus by AA similarity criterion,

$$\triangle AGF \sim \triangle DBG$$

Hence Proved

(ii) In $\triangle AGF$ and $\triangle EFC$,

$$\angle GAF = \angle CEF \quad (\text{each } 90^\circ)$$

Due to corresponding angles,

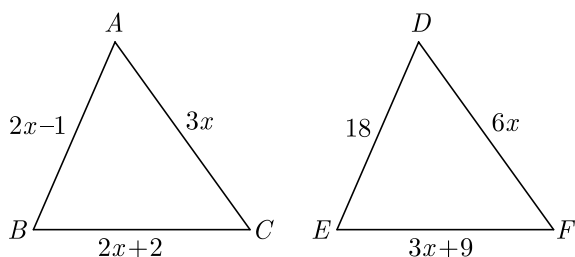
$$\angle AFG = \angle FCE$$

Thus by AA similarity criterion,

$$\triangle AGF \sim \triangle EFC$$

Hence Proved

100. In Figure, if $\triangle ABC \sim \triangle DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle.

**Ans :**

[Board 2020 OD Standard]

Since $\triangle ABC \sim \triangle DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{2x-1}{2x+2} = \frac{18}{3x+9}$$

$$(2x-1)(3x+9) = 18(2x+2)$$

$$(2x-1)(x+3) = 6(2x+2)$$

$$2x^2 - x + 6x - 3 = 12x + 12$$

$$2x^2 + 5x - 12x - 15 = 0$$

$$2x^2 - 7x - 15 = 0$$

$$2x^2 - 10x + 3x - 15 = 0$$

$$2x(x-5) + 3(x-5) = 0$$

$$(x-5)(2x+3) = 0 \Rightarrow x = 5 \text{ or } x = -\frac{3}{2}$$

But $x = -\frac{3}{2}$ is not possible, thus $x = 5$.

Now in $\triangle ABC$, we get

$$AB = 2x - 1 = 2 \times 5 - 1 = 9$$

$$BC = 2x + 2 = 2 \times 5 + 2 = 12$$

$$AC = 3x = 3 \times 5 = 15$$

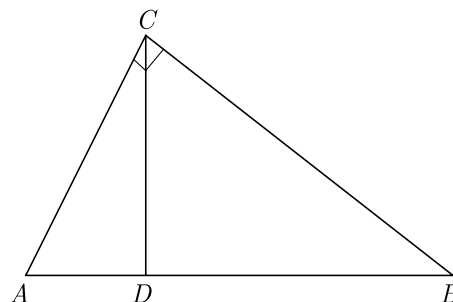
and in $\triangle DEF$, we get

$$DE = 18$$

$$EF = 3x + 9 = 3 \times 5 + 9 = 24$$

$$DF = 6x = 6 \times 5 = 30.$$

101. In Figure, $\angle ACB = 90^\circ$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.

**Ans :**

[Board 2019 Delhi]

In $\triangle ACB$ we have

$$\angle ACB = 90^\circ$$

and $CD \perp AB$

$$\text{Thus } AB^2 = CA^2 + CB^2 \quad \dots(1)$$

In $\triangle CAD$, $\angle ADC = 90^\circ$, thus we have

$$CA^2 = CD^2 + AD^2 \quad \dots(2)$$

and in $\triangle CDB$, $\angle CDB = 90^\circ$, thus we have

$$CB^2 = CD^2 + BD^2 \quad \dots(3)$$

Adding equation (2) and (3), we get

$$CA^2 + CB^2 = 2CD^2 + AD^2 + BD^2$$

Substituting AB^2 from equation (1) we have

$$AB^2 = 2CD^2 + AD^2 + BD^2$$

$$AB^2 - AD^2 = BD^2 + 2CD^2$$

$$(AB + AD)(AB - AD) = BD^2 + 2CD^2$$

$$(AB + AD)BD - BD^2 = 2CD^2$$

$$BD[(AB + AD) - BD] = 2CD^2$$

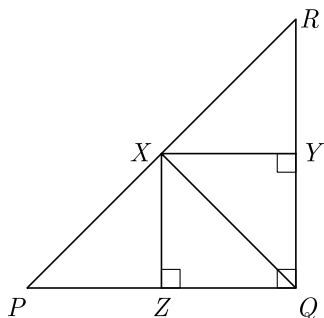
$$BD[AD + (AB - BD)] = 2CD^2$$

$$BD[AD + AD] = 2CD^2$$

$$BD \times 2AD = 2CD^2$$

$$CD^2 = BD \times AD \quad \text{Hence Proved}$$

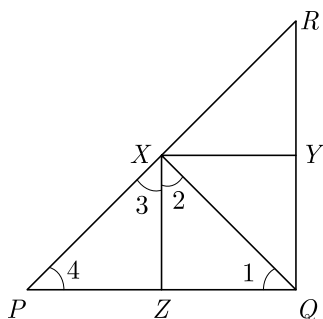
102. ΔPQR is right angled at Q . $QX \perp PR$, $XY \perp RQ$ and $XZ \perp PQ$ are drawn. Prove that $XZ^2 = PZ \times ZQ$.



Ans :

[Board Term-1 2015]

We have redrawn the given figure as below.



It may be easily seen that $RQ \perp PQ$ and $XZ \perp PQ$ or $XZ \parallel YQ$.

Similarly $XY \parallel ZQ$

Since $\angle PQR = 90^\circ$, thus $XYQZ$ is a rectangle.

$$\text{In } \Delta XZQ, \quad \angle 1 + \angle 2 = 90^\circ \quad \dots(1)$$

$$\text{and in } \Delta PZX, \quad \angle 3 + \angle 4 = 90^\circ \quad \dots(2)$$

$$XQ \perp PR \text{ or, } \angle 2 + \angle 3 = 90^\circ \quad \dots(3)$$

$$\text{From eq. (1) and (3), } \angle 1 = \angle 3$$

$$\text{From eq. (2) and (3), } \angle 2 = \angle 4$$

Due to AA similarity,

$$\Delta PZX \sim \Delta XZQ$$

$$\frac{PZ}{XZ} = \frac{XZ}{ZQ}$$

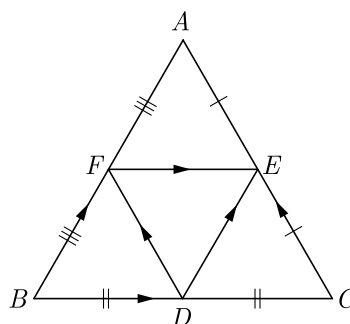
$$XZ^2 = PZ \times ZQ \quad \text{Hence proved}$$

103. In ΔABC , the mid-points of sides BC , CA and AB are D , E and F respectively. Find ratio of $ar(\Delta DEF)$ to $ar(\Delta ABC)$.

Ans :

[Board Term-1 2015]

As per given condition we have given the figure below. Here F, E and D are the mid-points of AB, AC and BC respectively.



Hence, $FE \parallel BC$, $DE \parallel AB$ and $DF \parallel AC$

By mid-point theorem,

If $DE \parallel BA$ then $DE \parallel BF$

and if $FE \parallel BC$ then $FE \parallel BD$

Therefore $FEDB$ is a parallelogram in which DF is diagonal and a diagonal of parallelogram divides it into two equal Areas.

$$\text{Hence } ar(\Delta BDF) = ar(\Delta DEF) \quad \dots(1)$$

$$\text{Similarly } ar(\Delta CDE) = ar(\Delta DEF) \quad \dots(2)$$

$$(\Delta AFE) = ar(\Delta DEF) \quad \dots(3)$$

$$(\Delta DEF) = ar(\Delta DEF) \quad \dots(4)$$

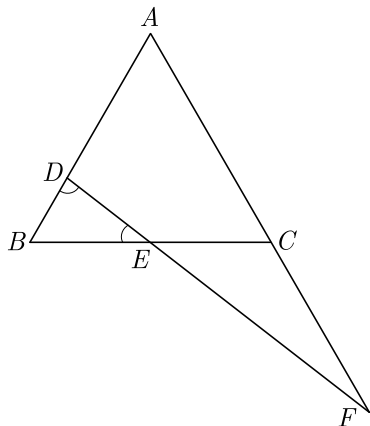
Adding equation (1), (2), (3) and (4), we have

$$\begin{aligned} ar(\Delta BDF) + ar(\Delta CDE) + ar(\Delta AFE) + ar(\Delta DEF) \\ = 4ar(\Delta DEF) \end{aligned}$$

$$ar(\Delta ABC) = 4ar(\Delta DEF)$$

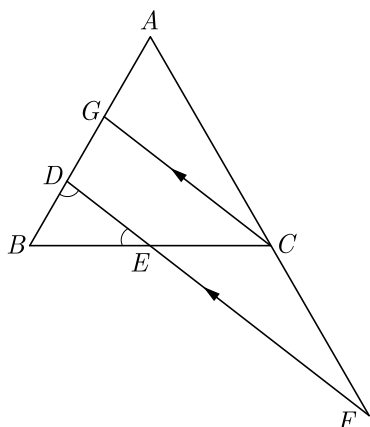
$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

- 104.** In the figure, $\angle BED = \angle BDE$ and E is the mid-point of BC . Prove that $\frac{AF}{CF} = \frac{AD}{BE}$.



Ans :

We have redrawn the given figure as below. Here $CG \parallel FD$.



We have $\angle BED = \angle BDE$

Since E is mid-point of BC ,

$$BE = BD = EC \quad \dots(1)$$

In $\triangle BCG$, $DE \parallel FG$

From (1) we have

$$\frac{BD}{DG} = \frac{BE}{EC} = 1$$

$$BD = DG = EC = BE$$

In $\triangle ADF$, $CG \parallel FD$

By BPT $\frac{AG}{GD} = \frac{AC}{CF}$

$$\frac{AG + GD}{GD} = \frac{AF + CF}{CF}$$

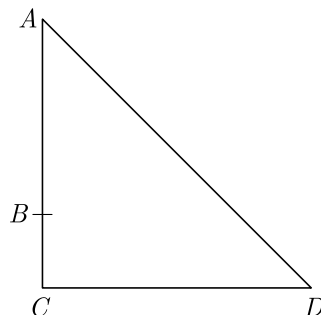
$$, \quad \frac{AD}{GD} = \frac{AF}{CF}$$

$$\text{Thus} \quad \frac{AF}{CF} = \frac{AD}{BE}$$

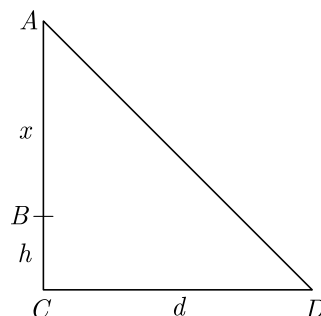
- 105.** In the right triangle, B is a point on AC such that $AB + AD = BC + CD$. If $AB = x$, $BC = h$ and $CD = d$, then find x (in term of h and d).

Ans :

[Board Term-1 2015]



We have redrawn the given figure as below.



We have $AB + AD = BC + CD$

$$AD = BC + CD - AB$$

$$AD = h + d - x$$

In right $\triangle ACD$, we have

$$AD^2 = AC^2 + DC^2$$

$$(h + d - x)^2 = (x + h)^2 + d^2$$

$$(h + d - x)^2 - (x + h)^2 = d^2$$

$$(h + d - x - x - h)(h + d - x + x + h) = d^2$$

$$(d - 2x)(2h + d) = d^2$$

$$2hd + d^2 - 4hx - 2xd = d^2$$

$$2hd = 4hx + 2xd$$

$$= 2(2h + d)x$$

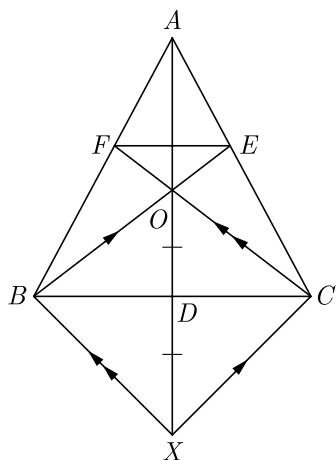
or,

$$x = \frac{hd}{2h+d}$$

- 106.** In $\triangle ABC$, AD is a median and O is any point on AD . BO and CO on producing meet AC and AB at E and F respectively. Now AD is produced to X such that $OD = DX$ as shown in figure.

Prove that :

- (1) $EF \parallel BC$
 (2) $AO : AX = AF : AB$

**Ans :**

[Board Term-1 2015]

Since BC and OX bisect each other, $BXCO$ is a parallelogram. Therefore $BE \parallel XC$ and $BX \parallel CF$.

In $\triangle ABX$, by BPT we get,

$$\frac{AF}{FB} = \frac{AO}{OX} \quad \dots(1)$$

$$\text{In } \triangle AXC, \quad \frac{AE}{EC} = \frac{AO}{OX} \quad \dots(2)$$

From (1) and (2) we get

$$\frac{AF}{FB} = \frac{AE}{EC}$$

By converse of BPT we have

$$EF \parallel BC$$

$$\text{From (1) we get } \frac{OX}{OA} = \frac{FB}{AF}$$

$$\frac{OX+OA}{OA} = \frac{FB+AF}{AF}$$

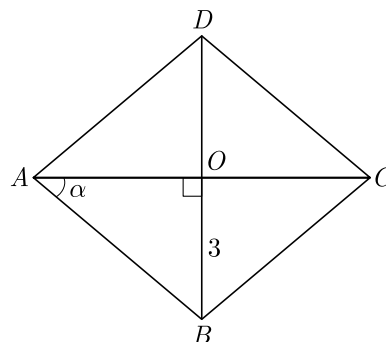
$$\frac{AX}{OA} = \frac{AB}{AF}$$

$$\frac{AO}{AX} = \frac{AF}{AB}$$

Thus $AO : AX = AF : AB$

Hence Proved

- 107.** $ABCD$ is a rhombus whose diagonal AC makes an angle α with AB . If $\cos \alpha = \frac{2}{3}$ and $OB = 3$ cm, find the length of its diagonals AC and BD .

**Ans :**

[Board Term-1 2013]

$$\text{We have } \cos \alpha = \frac{2}{3} \text{ and } OB = 3 \text{ cm}$$

$$\text{In } \triangle AOB, \quad \cos \alpha = \frac{2}{3} = \frac{AO}{AB}$$

$$\text{Let } OA = 2x \text{ then } AB = 3x$$

Now in right angled triangle $\triangle AOB$ we have

$$AB^2 = AO^2 + OB^2$$

$$(3x)^2 = (2x)^2 + (3)^2$$

$$9x^2 = 4x^2 + 9$$

$$5x^2 = 9$$

$$x = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}}$$

$$\text{Hence, } OA = 2x = 2\left(\frac{3}{\sqrt{5}}\right) = \frac{6}{\sqrt{5}} \text{ cm}$$

$$\text{and } AB = 3x = 3\left(\frac{3}{\sqrt{5}}\right) = \frac{9}{\sqrt{5}} \text{ cm}$$

$$\text{Diagonal } BD = 2 \times OB = 2 \times 3 = 6 \text{ cm}$$

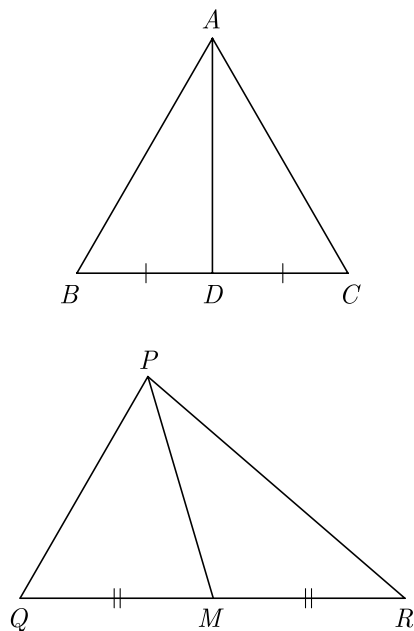
$$\begin{aligned} \text{and } AC &= 2AO \\ &= 2 \times \frac{6}{\sqrt{5}} = \frac{12}{\sqrt{5}} \text{ cm} \end{aligned}$$

- 108.** In $\triangle ABC$, AD is the median to BC and in $\triangle PQR$, PM is the median to QR . If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$. Prove that $\triangle ABC \sim \triangle PQR$.

Ans :

[Board Term-1 2012, 2013]

As per given condition we have drawn the figure below.



In $\triangle ABC$ AD is the median, therefore

$$BC = 2BD$$

and in $\triangle PQR$, PM is the median,

$$QR = 2QM$$

Given,
$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$$

or,
$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2BD}{2QM}$$

In triangles ABD and PQM ,

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$$

By SSS similarity we have

$$\triangle ABD \sim \triangle PQM$$

By CPST we have

$$\angle B = \angle Q,$$

In $\triangle ABC$ and $\triangle PQR$,

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

By SAS similarity we have

$$\angle B = \angle Q,$$

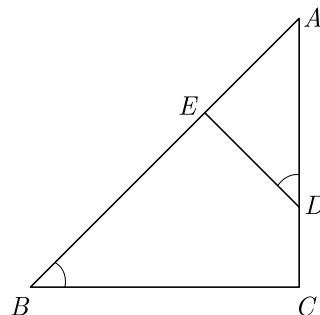
Thus

$$\triangle ABC \sim \triangle PQR.$$

Hence Proved.

- 109.** In $\triangle ABC$, if $\angle ADE = \angle B$, then prove that $\triangle ADE \sim \triangle ABC$.

Also, if $AD = 7.6$ cm, $AE = 7.2$ cm, $BE = 4.2$ cm and $BC = 8.4$ cm, then find DE .



Ans :

[Board Term-1 2015]

In $\triangle ADE$ and $\triangle ABC$, $\angle A$ is common.

and we have $\angle ADE = \angle ABC$

Due to AA similarity,

$$\triangle ADE \sim \triangle ABC$$

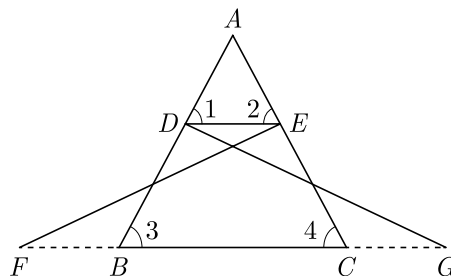
$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{AD}{AE + BE} = \frac{DE}{BC}$$

$$\frac{7.6}{4.2 + 4.2} = \frac{DE}{8.4}$$

$$DE = \frac{7.6 \times 8.4}{11.4} = 5.6 \text{ cm}$$

- 110.** In the following figure, $\triangle FEC \cong \triangle GBD$ and $\angle 1 = \angle 2$. Prove that $\triangle ADE \cong \triangle ABC$.



Ans :

[Board Term-1 2012]

Since $\triangle FEC \cong \triangle GBD$

$$EC = BD \quad \dots(1)$$

Since $\angle 1 = \angle 2$, using isosceles triangle property

$$AE = AD \quad \dots(2)$$

From equation (1) and (2), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$$DE \parallel BC, \quad (\text{Converse of BPT})$$

Due to corresponding angles we have

$$\angle 1 = \angle 3 \text{ and } \angle 2 =$$

Thus in $\triangle ADE$ and $\triangle ABC$,

$$\angle A = \angle A$$

$$\angle 1 = \angle 3$$

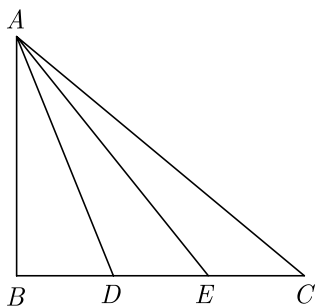
$$\angle 2 = \angle 4$$

Sy by AAA criterion of similarity,

$$\triangle ADE \sim \triangle ABC$$

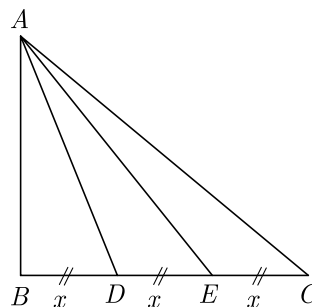
Hence proved

- 111.** In the given figure, D and E trisect BC . Prove that $8AE^2 = 3AC^2 + 5AD^2$.

**Ans :**

[Board Term-1 2013]

As per given condition we have drawn the figure below.

Since D and E trisect BC , let $BD = DE = EC$ be x .Then $BE = 2x$ and $BC = 3x$

$$\text{In } \triangle ABE, \quad AE^2 = AB^2 + BE^2 = AB^2 + 4x^2 \quad \dots(1)$$

$$\text{In } \triangle ABC, \quad AC^2 = AB^2 + BC^2 = AB^2 + 9x^2 \quad \dots(2)$$

$$\text{In } \triangle ADB, \quad AD^2 = AB^2 + BD^2 = AB^2 + x^2 \quad \dots(3)$$

Multiplying (2) by 3 and (3) by 5 and adding we have

$$\begin{aligned} 3AC^2 + 5AD^2 &= 3(AB^2 + 9x^2) + (AB^2 + x^2) \\ &= 3AB^2 + 27x^2 + AB^2 + x^2 \\ &= 4AB^2 + 28x^2 \\ &= 4AB^2 + 32x^2 \\ &= 8(AB^2 + 4x^2) = 8AE^2 \end{aligned}$$

$$\text{Thus } 3AC^2 + 5AD^2 = 8AE^2$$

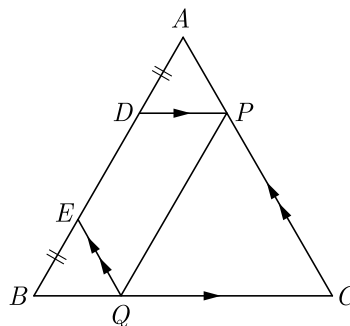
Hence Proved

- 112.** Let ABC be a triangle D and E be two points on side AB such that $AD = BE$. If $DP \parallel BC$ and $EQ \parallel AC$, then prove that $PQ \parallel AB$.

Ans :

[Board Term-1 2012]

As per given condition we have drawn the figure below.



$$\text{In } \triangle ABC, \quad DP \parallel BC$$

$$\text{By BPT we have } \frac{AD}{DB} = \frac{AP}{PC}, \quad \dots(1)$$

$$\text{Similarly, in } \triangle ABC, \quad EQ \parallel AC$$

$$\frac{BQ}{QC} = \frac{BE}{EA} \quad \dots(2)$$

From figure, $EA = AD + DE$
 $= BE + ED \quad (\text{BE} = AD)$
 $= BD$

Therefore equation (2) becomes,

$$\frac{BQ}{QC} = \frac{AD}{BD} \quad \dots(3)$$

From (1) and (3), we have

$$\frac{AP}{PC} = \frac{BQ}{QC}$$

By converse of BPT,

$$PQ \parallel AB \quad \text{Hence Proved}$$

113. Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides.

[Board 2020 Delhi Basic, 2019 Delhi, 2018]

or

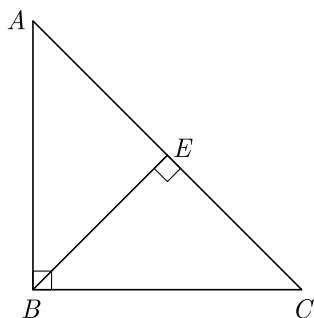
Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. Using the above result, prove that, in rhombus $ABCD$, $4AB^2 = AC^2 + BD^2$.

Ans :

[Board Term -2 SQP 2017, 2015]

(1) As per given condition we have drawn the figure below. Here $AB \perp BC$.

We have drawn $BE \perp AC$



In $\triangle AEB$ and $\triangle ABC$ $\angle A$ common and

$$\angle E = \angle B \quad (\text{each } 90^\circ)$$

By AA similarity we have

$$\triangle AEB \sim \triangle ABC$$

$$\frac{AE}{AB} = \frac{AB}{AC}$$

$$AB^2 = AE \times AC$$

Now, in $\triangle CEB$ and $\triangle CBA$, $\angle C$ is common and

$$\angle E = \angle B \quad (\text{each } 90^\circ)$$

By AA similarity we have

$$\triangle AEB \sim \triangle CBA$$

$$\frac{CE}{BC} = \frac{BC}{AC}$$

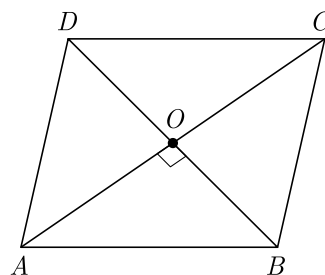
$$BC^2 = CE \times AC \quad \dots(2)$$

Adding equation (1) and (2) we have

$$\begin{aligned} AB^2 + BC^2 &= AE \times AC + CE \times AC \\ &= AC(AE + CE) \\ &= AC \times AC \end{aligned}$$

Thus $AB^2 + BC^2 = AC^2$ Hence proved

(2) As per given condition we have drawn the figure below. Here $ABCD$ is a rhombus.



We have drawn diagonal AC and BD .

$$AO = OC = \frac{1}{2}AC$$

and

$$BO = OD = \frac{1}{2}BD$$

$$AC \perp BD$$

Since diagonal of rhombus bisect each other at right angle,

$$\angle AOB = 90^\circ$$

$$AB^2 = OA^2 + OB^2$$

$$= \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$$= \frac{AC^2}{4} + \frac{BD^2}{4}$$

or

$$4AB^2 = AC^2 + BD^2 \quad \text{Hence proved}$$

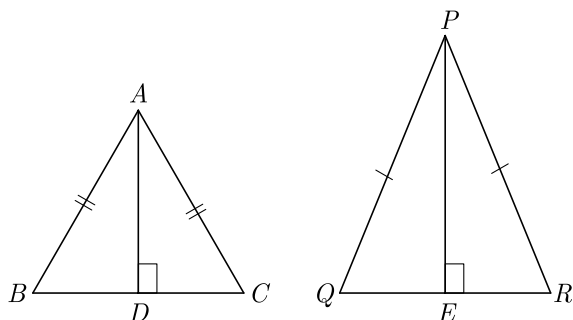
114. Vertical angles of two isosceles triangles are equal. If their areas are in the ratio 16:25, then find the ratio

of their altitudes drawn from vertex to the opposite side.

Ans :

[Board Term-1 2015]

As per given condition we have drawn the figure below.



Here $\angle A = \angle P$, $\angle B = \angle C$ and $\angle Q = \angle R$

Let $\angle A = \angle P$ be x .

In $\triangle ABC$, $\angle A + \angle B + \angle C = 180^\circ$

$$x + \angle B + \angle B = 180^\circ \quad (\angle B = \angle C)$$

$$2\angle B = 180^\circ - x$$

$$\angle B = \frac{180^\circ - x}{2} \quad \dots(1)$$

Now, in $\triangle PQR$,

$$\angle P + \angle Q + \angle R = 180^\circ \quad (\angle Q = \angle R)$$

$$x + \angle Q + \angle Q = 180^\circ$$

$$2\angle Q = 180^\circ - x$$

$$\angle Q = \frac{180^\circ - x}{2}$$

In $\triangle ABC$ and $\triangle PQR$,

$$\angle A = \angle P \quad \text{[Given]}$$

$$\angle B = \angle Q \quad \text{[From eq. (1) and (2)]}$$

Due to AA similarity,

$$\triangle ABC \sim \triangle PQR$$

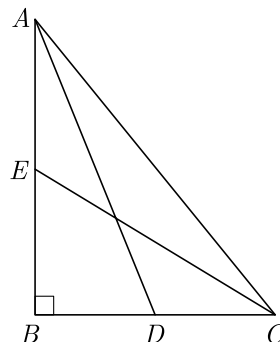
$$\text{Now } \frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \frac{AD^2}{PE^2}$$

$$\frac{16}{25} = \frac{AD^2}{PE^2}$$

$$\frac{4}{5} = \frac{AD}{PE}$$

$$\text{Thus } \frac{AD}{PE} = \frac{4}{5}$$

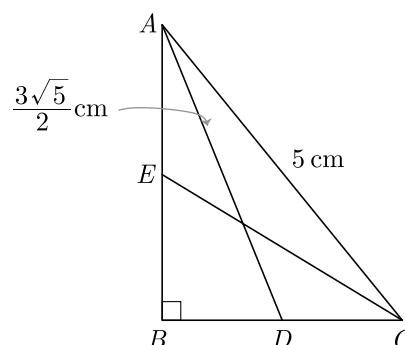
115. In the figure, ABC is a right triangle, right angled at B . AD and CE are two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE .



Ans :

[Board Term-1 2013]

We have redrawn the given figure as below.



Here in $\triangle ABC$, $\angle B = 90^\circ$, AD and CE are two medians.

Also we have $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$.

By Pythagoras theorem we get

$$AC^2 = AB^2 + BC^2 = (5)^2 = 25 \quad \dots(1)$$

$$\text{In } \triangle ABD, \quad AD^2 = AB^2 + BD^2$$

$$\left(\frac{3\sqrt{5}}{2}\right)^2 = AB^2 + \frac{BC^2}{4}$$

$$\frac{45}{4} = AB^2 + \frac{BC^2}{4} \quad \dots(2)$$

$$\text{In } \triangle EBC, \quad CE^2 = BC^2 + \frac{AB^2}{4} \quad \dots(3)$$

Subtracting equation (2) from equation (1),

$$\frac{3BC^2}{4} = 25 - \frac{45}{4} = \frac{55}{4}$$

$$BC^2 = \frac{55}{3} \quad \dots(4)$$

From equation (2) we have

$$AB^2 + \frac{55}{12} = \frac{45}{4}$$

$$AB^2 = \frac{45}{4} - \frac{55}{12} = \frac{20}{3}$$

From equation (3) we get

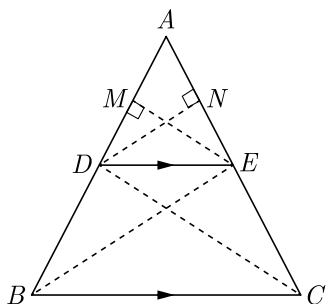
$$CE^2 = \frac{55}{3} + \frac{20}{3 \times 4} = \frac{240}{12} = 20$$

Thus $CE = \sqrt{20} = 2\sqrt{5} \text{ cm.}$

- 116.** If a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove it.

Ans : [Board 2019 OD, SQP 2020 STD, 2012]

A triangle ABC is given in which $DE \parallel BC$. We have drawn $DN \perp AE$ and $EM \perp AD$ as shown below. We have joined BE and CD .



In $\triangle ADE$,

$$\text{area}(\triangle ADE) = \frac{1}{2} \times AE \times DN \quad \dots(1)$$

In $\triangle DEC$,

$$\text{area}(\triangle DCE) = \frac{1}{2} \times CE \times DN \quad \dots(2)$$

Dividing equation (1) by (2) we have,

$$\frac{\text{area}(\triangle ADE)}{\text{area}(\triangle DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$$

or, $\frac{\text{area}(\triangle ADE)}{\text{area}(\triangle DEC)} = \frac{AE}{CE} \quad \dots(3)$

Now in $\triangle ADE$,

$$\text{area}(\triangle ADE) = \frac{1}{2} \times AD \times EM \quad \dots(4)$$

and in $\triangle DEB$,

$$\text{area}(\triangle DEB) = \frac{1}{2} \times EM \times BD \quad \dots(5)$$

Dividing eqn. (4) by eqn. (5),

$$\frac{\text{area}(\triangle ADE)}{\text{area}(\triangle DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$$

or, $\frac{\text{area}(\triangle ADE)}{\text{area}(\triangle DEB)} = \frac{AD}{BD} \quad \dots(6)$

Since $\triangle DEB$ and $\triangle DEC$ lie on the same base DE and between two parallel lines DE and BC .

$$\text{area}(\triangle DEB) = \text{area}(\triangle DEC)$$

From equation (3) we have

$$\frac{\text{area}(\triangle ADE)}{\text{area}(\triangle DEB)} = \frac{AE}{CE} \quad \dots(7)$$

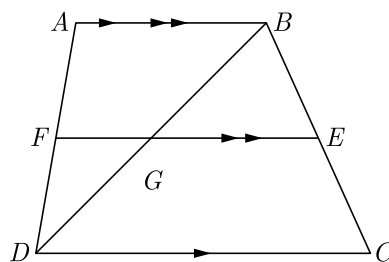
From equations (6) and (7) we get

$$\frac{AE}{CE} = \frac{AD}{BD}. \quad \text{Hence proved.}$$

- 117.** In a trapezium $ABCD$, $AB \parallel DC$ and $DC = 2AB$. $EF = AB$, where E and F lie on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$. Diagonal DB intersects EF at G . Prove that, $7EF = 11AB$.

Ans : [Board Term-1 2012]

As per given condition we have drawn the figure below.



In trapezium $ABCD$,

$$AB \parallel DC \text{ and } DC = 2AB.$$

Also, $\frac{BE}{EC} = \frac{4}{3}$

Thus $EF \parallel AB \parallel CD$

$$\frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$$

In $\triangle BGE$ and $\triangle BDC$, $\angle B$ is common and due to corresponding angles,

$$\angle BEG = \angle BCD$$

Due to AA similarity we get

$$\triangle BGE \sim \triangle BDC$$

$$, \quad \frac{EG}{CD} = \frac{BE}{BC} \quad \dots(1)$$

$$\text{As,} \quad \frac{BE}{EC} = \frac{4}{3}$$

$$\frac{BE}{BE+EC} = \frac{4}{4+3} = \frac{4}{7}$$

$$\frac{BE}{BC} = \frac{4}{7} \quad \dots(2)$$

From (1) and (2) we have

$$\frac{EG}{CD} = \frac{4}{7}$$

$$, \quad EG = \frac{4}{7} CD \quad \dots(3)$$

Similarly, $\triangle DGF \sim \triangle DBA$

$$\frac{DF}{DA} = \frac{FG}{AB}$$

$$\frac{FG}{AB} = \frac{3}{7}$$

$$FG = \frac{3}{7} AB \quad \dots(4)$$

$$\left[\frac{AF}{AD} = \frac{4}{7} = \frac{BE}{BC} \Rightarrow \frac{EC}{BC} = \frac{3}{7} = \frac{DE}{DA} \right]$$

Adding equation (3) and (4) we have

$$EG + FG = \frac{4}{7} DC + \frac{3}{7} AB$$

$$EF = \frac{4}{7} \times (2AB) + \frac{3}{7} AB$$

$$= \frac{8}{7} AB + \frac{3}{7} AB = \frac{11}{7} AB$$

$$7EF = 11AB \quad \text{Hence proved.}$$

118. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR . Show that $\triangle ABC \sim \triangle PQR$.

Ans :

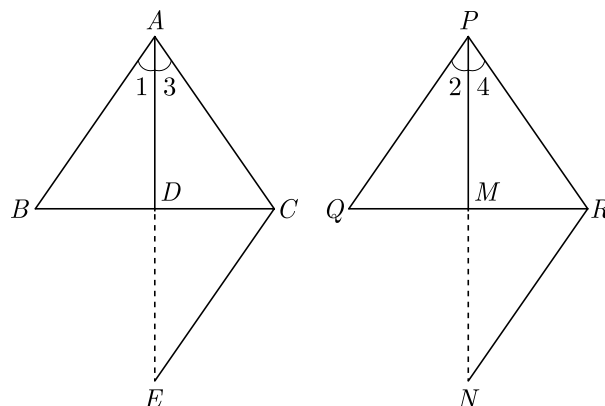
[Board Term-1 2012]

It is given that in $\triangle ABC$ and $\triangle PQR$, AD and PM

are their medians,

$$\text{such that} \quad \frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$$

We have produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$. We join CE and RN . As per given condition we have drawn the figure below.



In $\triangle ABD$ and $\triangle EDC$,

$$AD = DE \quad (\text{By construction})$$

$$\angle ADB = \angle EDC \quad (\text{VOA})$$

$$BD = DC \quad (AD \text{ is a median})$$

By SAS congruency

$$\triangle ABD \cong \triangle EDC$$

$$AB = CE \quad (\text{By CPCT})$$

Similarly, $PQ = RN$ and $\angle A = \angle 2$

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR} \quad (\text{Given})$$

$$, \quad \frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$$

$$\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$$

By SSS similarity, we have

$$\triangle AEC \sim \triangle PNR$$

$$\angle 3 = \angle 4$$

$$\angle 1 = \angle 2$$

$$\angle 1 + \angle 3 = \angle 2 + \angle 4$$

By SAS similarity, we have

$$\triangle ABC \sim \triangle PQR$$

Hence Proved

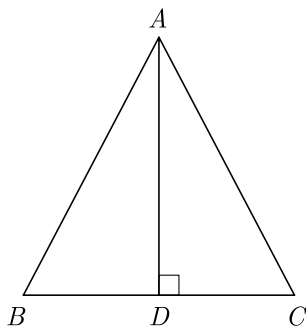
- 119.** In $\triangle ABC$, $AD \perp BC$ and point D lies on BC such that $2DB = 3CD$. Prove that $5AB^2 = 5AC^2 + BC^2$.

Ans :

[Board Term-1 2015]

It is given in a triangle $\triangle ABC$, $AD \perp BC$ and point D lies on BC such that $2DB = 3CD$.

As per given condition we have drawn the figure below.



Since $2DB = 3CD$

$$\frac{DB}{CD} = \frac{3}{2}$$

Let DB be $3x$, then CD will be $2x$ so $BC = 5x$.

Since $\angle D = 90^\circ$ in $\triangle ADB$, we have

$$\begin{aligned} AB^2 &= AD^2 + DB^2 = AD^2 + (3x)^2 \\ &= AD^2 + 9x^2 \end{aligned}$$

$$5AB^2 = 5AD^2 + 45x^2$$

$$5AD^2 = 5AB^2 - 45x^2 \quad \dots(1)$$

and

$$\begin{aligned} AC^2 &= AD^2 + CD^2 = AD^2 + (2x)^2 \\ &= AD^2 + 4x^2 \end{aligned}$$

$$5AC^2 = 5AD^2 + 20x^2$$

$$5AD^2 = 5AC^2 - 20x^2 \quad \dots(2)$$

Comparing equation (1) and (2) we have

$$5AB^2 - 45x^2 = 5AC^2 - 20x^2$$

$$5AB^2 = 5AC^2 - 20x^2 + 45x^2$$

$$= 5AC^2 + 25x^2$$

$$= 5AC^2 + (5x)^2$$

$$= 5AC^2 + BC^2 \quad [BC = 5x]$$

Therefore $5AB^2 = 5AC^2 + BC^2$ Hence proved

- 120.** In a right triangle ABC , right angled at C . P and Q are points of the sides CA and CB respectively, which

divide these sides in the ratio $2:1$.

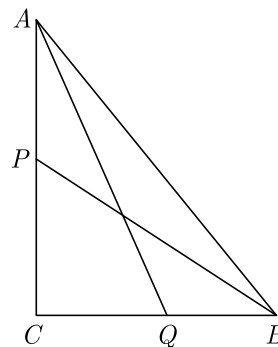
Prove that : $9AQ^2 = 9AC^2 + 4BC^2$

$$9BP^2 = 9BC^2 + 4AC^2$$

$$9(AQ^2 + BP^2) = 13AB^2$$

Ans :

As per given condition we have drawn the figure below.



Since P divides AC in the ratio $2:1$

$$CP = \frac{2}{3}AC$$

and Q divides CB in the ratio $2:1$

$$QC = \frac{2}{3}BC$$

$$AQ^2 = QC^2 + AC^2$$

$$= \frac{4}{9}BC^2 + AC^2$$

$$\text{or, } 9AQ^2 = 4BC^2 + 9AC^2 \quad \dots(1)$$

Similarly, we get

$$9BP^2 = 9BC^2 + 4AC^2 \quad \dots(2)$$

Adding equation (1) and (2), we get

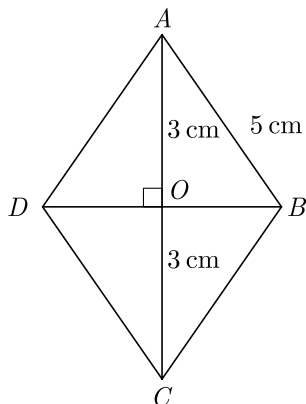
$$9(AQ^2 + BP^2) = 13AB^2$$

- 121.** Find the length of the second diagonal of a rhombus, whose side is 5 cm and one of the diagonals is 6 cm.

Ans :

As per given condition we have drawn the figure

below.



We have $AB = BC = CD = AD = 5$ cm and $AC = 6$ cm

Since $AO = OC$, $AO = 3$ cm

Here $\triangle AOB$ is right angled triangle as diagonals of rhombus intersect at right angle.

By Pythagoras theorem,

$$OB = 4 \text{ cm.}$$

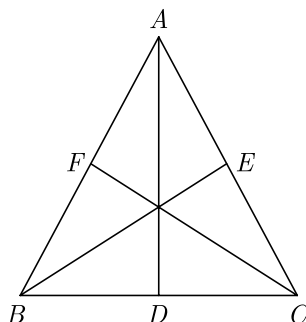
Since $DO = OB$, $BD = 8$ cm, length of the other diagonal $= 2(BO)$ where $BO = 4$ cm

Hence $BD = 2 \times BO = 2 \times 4 = 8$ cm

- 122.** Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

Ans :

As per given condition we have drawn the figure below.



In triangle sum of squares of any two sides is equal to twice the square of half of the third side, together with twice the square of median bisecting it.

If AD is the median,

$$AB^2 + AC^2 = 2\left\{AD^2 + \frac{BC^2}{4}\right\}$$

$$2(AB^2 + AC^2) = 4AD^2 + BC^2 \quad \dots(1)$$

Similarly by taking BE and CF as medians,

$$2(AB^2 + BC^2) = 4BE^2 + AC^2 \quad \dots(2)$$

$$\text{and } 2(AC^2 + BC^2) = 4CF^2 + AB^2 \quad \dots(3)$$

Adding, (1), (2) and (iii), we get

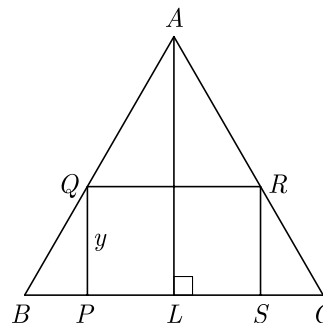
$$3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$$

Hence proved

- 123.** ABC is an isosceles triangle in which $AB = AC = 10$ cm $BC = 12$ cm $PQRS$ is a rectangle inside the isosceles triangle. Given $PQ = SR = y$, $PS = PR = 2x$. Prove that $x = 6 - \frac{3y}{4}$.

Ans :

As per given condition we have drawn the figure below.



Here we have drawn $AL \perp BC$.

Since it is isosceles triangle, AL is median of BC ,

$$BL = LC = 6 \text{ cm.}$$

In right $\triangle ALB$, by Pythagoras theorem,

$$\begin{aligned} AL^2 &= AB^2 - BL^2 \\ &= 10^2 - 6^2 = 64 = 8^2 \end{aligned}$$

Thus $AL = 8$ cm.

In $\triangle BPQ$ and $\triangle BLA$, angle $\angle B$ is common and

$$\angle BPQ = \angle BLA = 90^\circ$$

Thus by AA similarity we get

$$\triangle BPQ \sim \triangle BLA$$

$$\frac{PB}{PQ} = \frac{BL}{AL}$$

$$\frac{6-x}{y} = \frac{6}{8}$$

$$x = 6 - \frac{3y}{4} \quad \text{Hence proved.}$$

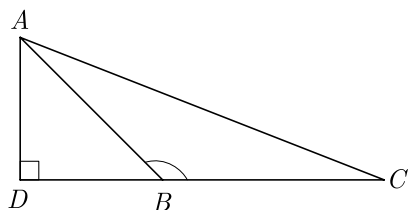
124. If $\triangle ABC$ is an obtuse angled triangle, obtuse angled at B and if $AD \perp CB$. Prove that :

$$AC^2 = AB^2 + BC^2 + 2BC \times BD$$

Ans :

[Board 2020 Delhi Basic]

As per given condition we have drawn the figure below.



In $\triangle ADB$, by Pythagoras theorem

$$AB^2 = AD^2 + BD^2 \quad \dots(1)$$

In $\triangle ADC$, By Pythagoras theorem,

$$\begin{aligned} AC^2 &= AD^2 + CD^2 \\ &= AD^2 + (BC + BD)^2 \\ &= AD^2 + BC^2 + 2BC \times BD + BD^2 \\ &= (AD^2 + BD^2) + 2BC \times BD \end{aligned}$$

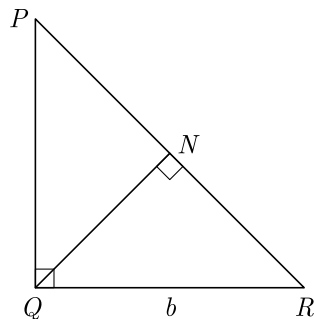
Substituting $(AD^2 + BD^2) = AB^2$ we have

$$AC^2 = AB^2 + BC^2 + 2BC \times BD$$

125. If A be the area of a right triangle and b be one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4 + 4A^2}}$.

Ans :

As per given condition we have drawn the figure below.



Let $QR = b$, then we have

$$\begin{aligned} A &= ar(\triangle PQR) \\ &= \frac{1}{2} \times b \times PQ \end{aligned}$$

$$PQ = \frac{2 \cdot A}{b} \quad \dots(1)$$

Due to AA similarity we have

$$\triangle PNQ \sim \triangle PQR$$

$$\frac{PQ}{PR} = \frac{NQ}{QR} \quad \dots(2)$$

From $\triangle PQR$

$$PQ^2 + QR^2 = PR^2$$

$$\frac{4A^2}{b^2} + b^2 = PR^2$$

$$PR = \sqrt{\frac{4A^2 + b^4}{b^2}}$$

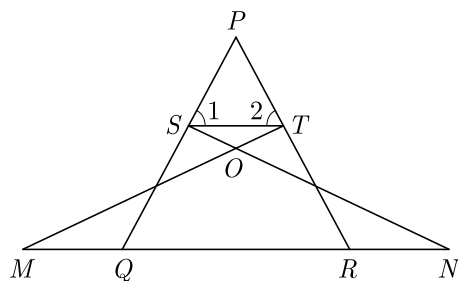
Equation (2) becomes

$$\frac{2A}{b \times PR} = \frac{NQ}{b}$$

$$NQ = \frac{2A}{PR}$$

Altitude, $NQ = \frac{2Ab}{\sqrt{4A^2 + b^4}}$ Hence Proved.

126. In given figure $\angle 1 = \angle 2$ and $\triangle NSQ \sim \triangle MTR$, then prove that $\triangle PTS \sim \triangle PRO$.



Ans :

[Board Term-1 SQP 2017]

We have $\triangle NSQ \cong \triangle MTR$

$$9AD^2 = 7AB^2$$

Hence Proved

By CPCT we have

$$\angle SQN = \angle TRM$$

From angle sum property we get

$$\angle P + \angle 1 + \angle 2 = \angle P + \angle PQR + \angle PRQ$$

$$\angle 1 + \angle 2 = \angle PQR + \angle PRQ$$

Since $\angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$ we get

$$2\angle 1 = 2\angle PQR$$

$$\angle 1 = \angle PQR$$

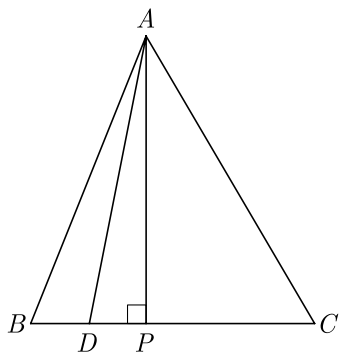
Also $\angle 2 = \angle QPR$ (common)

Thus by AAA similarity,

$$\triangle PTS \sim \triangle PRQ$$

127. In an equilateral triangle ABC , D is a point on the side BC such the $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.**Ans :**

[Board 2018, SQP 2017]

As per given condition we have shown the figure below. Here we have drawn $AP \perp BC$.Here $AB = BC = CA$ and $BD = \frac{1}{3}BC$.In $\triangle ADP$,

$$\begin{aligned}
 AD^2 &= AP^2 + DP^2 \\
 &= AP^2 + (BP - BD)^2 \\
 &= AP^2 + BP^2 + BD^2 + 2BP \cdot BD
 \end{aligned}$$

From $\triangle APB$ using $AP^2 + BP^2 = AB^2$ we have

$$\begin{aligned}
 AD^2 &= AB^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right) \\
 &= AB^2 + \frac{AB^2}{9} - \frac{AB^2}{3} = \frac{7}{9}AB^2
 \end{aligned}$$