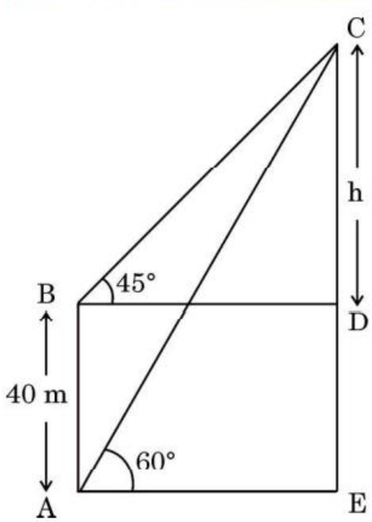
Arithmetic Progressions

(2025)

1. If the sum of first m terms of an AP is $2m^2 + 3m$, then its second term is :
(1 Mark) (2025)

(A) 10
(B) 9
(C) 12
(D) 4

2. Amrita stood near the base of a lighthouse, gazing up at its towering height. She measured the angle of elevation to the top and found it to be 60 . Then, she climbed a nearby observation deck, 40 metres higher than her original position and noticed the angle of elevation to the top of lighthouse to be 45° (4 Mark) (2025)



Based on the above given information, answer the following questions :

(i) If CD is h metres, find the distance BD in terms of 'h'.

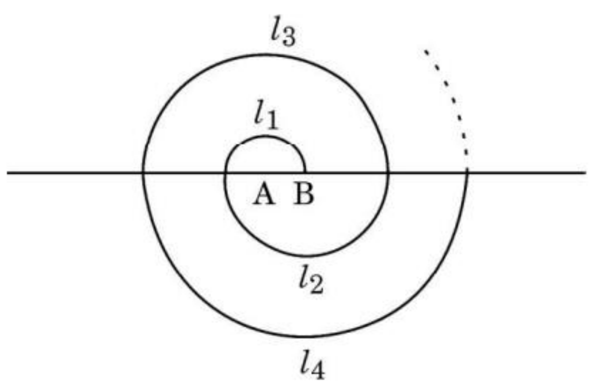
(ii) Find distance BC in terms of 'h'.

(iii) (a) Find the height CE of the lighthouse [Use $\sqrt{3} = 1.73$]

OR

(b) Find distance AE, if AC = 100 m.

3. In a garden, saplings of rose flowers were planted at equal intervals to form a spiral pattern. The spiral is made up of successive semicircles, with centres alternatively at A and B, starting with centre at A, of radii 50 cm, 100 cm, 150 cm, ... as shown in the figure given below. Spiral 1 has 10 flowers, Spiral 2 has 20 flowers, Spiral 3 has 30 flowers and so on. (6 Mark) (2025)



Based on the above information, answer the following questions :

(i) What is the radius of the 13th spiral ?
(ii) If the radius of the nth spiral is 500 cm, find the value of n.
(iii) (a) Find the total number of saplings till the 11th spiral.

OR

(b) Till which spiral, will there be a total of 450 saplings ?

Answer

1. (B) 9

2.

(i) $\frac{h}{BD} = \tan 45° = 1$

$\Rightarrow BD = h \text{ m}$

(ii) $\frac{h}{BC} = \sin 45° = \frac{1}{\sqrt{2}}$

$\Rightarrow BC = \sqrt{2}h \text{ m}$

(iii)(a) $\tan 60° = \frac{EC}{AE}$

$\Rightarrow \sqrt{3} = \frac{h+40}{h}$

$\Rightarrow \quad h = 20(\sqrt{3} + 1) = 20 \times 2.73 = 54.6 \text{ m}$

$\therefore CE = 54.6 + 40 = 94.6 \text{ m}$

OR

(iii)(b) $\cos 60° = \frac{AE}{AC}$

$\Rightarrow \frac{1}{2} = \frac{AE}{100}$

$\therefore \quad AE = 50 \text{ m}$

3. (i) $a_{13} = 650$ cm

(ii) $a_n = 500$

$50 + (n - 1)50 = 500$

$n = 10$

(iii) (a) $a = 10, d = 10$

$S_{11} = \frac{11}{2}[20 + 10 \times 10]$

$= 660$

(b) $a = 10, d = 10$

$450 = \frac{n}{2}[20 + (n - 1)\,10]$

$n_2 + n - 90 = 0$
$n = 9$

(2024)

1. In an A.P.; if a = 8 and a_{10} =-19, then value of d is: (2024)

(a) 3 (b) $-\frac{11}{9}$ (c) $-\frac{27}{10}$ (d) –3

Answer. (d) – 3

2. Two alarm clocks ring their alarms at regular intervals of 20 minutes and 25 minutes respectively. If they first beep together at 12 noon, at what time will they beep again together next time? (2024)

Answer. LCM (20, 25) = 100
∴ After 100 minutes from 12:00 noon
⇒ They will beep again together at 1:40 pm

3. In an A.P. if $S_n = 4n^2 - n$, then
(i) find the first term and common difference. (2024)

Answer. (i) Sn = 4n2 – n
S1 = 4 – 1 = 3 = a 1
S2 = 2a + d = 14 ⇒ d = 14 – 6 = 8

(ii) write the A.P. (2024)

Answer. A.P. is 3, 11, 19, 27, ...

(iii) which term of the A.P. is 107? (2024)

Answer. 107 = 3 + (n – 1)8 ⇒ n = 14

4. In an A.P., if the first term a = 7, nth term a_n = 84 and the sum of first (2024)

n terms $s_n = \frac{2093}{2}$, then n is equal to :

(a) 22
(b) 24
(c) 23

(d) 26

Answer. (c) 23

5. (A) The sum of first and eighth terms of an A.P. is 32 and their product is 60. Find the first term and common difference of the A.P. Hence, also find the sum of its first 20 terms. (2024)

Answer. $a + a_8 = 32 \Rightarrow 2a + 7d = 32$ ----- (i)
$a \times a_8 = 60 \Rightarrow a(a + 7d) = 60$ ----- (ii)
Solving (i) & (ii), we get
$a = 2$ or $a = 30$
and $d = 4$ or $d = -4$
First term and common difference of A.P. are 2 and 4 or 30 and – 4 respectively.
Now, for $a = 2$ & $d = 4$
$S_{20} = 10\ (4 + 76) = 800$
and for $a = 30$ & $d = -4$
$S_{20} = 10\ (60 - 76) = -160$

6. In an A.P. of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687. Determine the first term and common difference of A.P. Also, find the sum of all the terms of the A.P. (2024)

Answer. Here n = 40,
$S_9 = \frac{9}{2}\ [2a + 8d] = 153 \Rightarrow a + 4d = 17$ ---- (i)
and $S_{40} - S_{34} = 687$ or $a_{35} + a_{36} + a_{37} + a_{38} + a_{39} + a_{40} = 687$
$\Rightarrow 6a + 219d = 687$ or $2a + 73d = 229$ ---- (ii)
solving (i) and (ii) to get $a = 5, d = 3$
Also, $S_{40} = \frac{40}{2}(10 + 39 \times 3) = 2540$

7. Directions:
Assertion (A) is followed by a statement of Reason (R). Select the correct option from the following options: (2024)
(a) Both, Assertion (A) and Reason (R) are true. Reason (R) explains Assertion (A) completely.
(b) Both, Assertion (A) and Reason (R) are true. Reason (R) does not explain Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

Q. Assertion (A): The tangents drawn at the end points of a diameter of a circle, are parallel.
Reason (R): Diameter of a circle is the longest chord. **(2024)**

Answer. (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation for Assertion (A).

5.2 Arithmetic Progressions

MCQ

1. If a, b, c form an A.P. with common difference d, then the value of a 2b - cis equal to
(a) 2a + 4d
(b) 0
(c) -2a-4d
(d) -2a-3d (2023)

2. The next term of the A.P.: √7, √28, √63 is
(a) √70
(b) √80
(c) √97
(d) √112 (2023)

3. If k + 2, 4k - 6 and 3k - 2 are three consecutive terms of an A.P., then the value of k is
(a) 3
(b) -3
(c) 4
(d) -4 (2023)

4.

If $-\frac{5}{7}$, *a*, 2 are consecutive terms in an Arithmetic Progression, then the value of '*a*' is

(a) $\frac{9}{7}$ (b) $\frac{9}{14}$ (c) $\frac{19}{7}$ (d) $\frac{19}{14}$

(2020C) U

5. Which of the following is not an A.P.?
(a) -1.2, 0.8, 2.8,...

(b) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

(c) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$

(d) $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \ldots$ *(2020)*

6. The value of x for which 2x, (x + 10) and (3x + 2) are the three consecutive terms of an A.P., is
(a) 6
(b) -6
(c) 18
(d) -18 (2020)

7. The first three terms of an A.P. respectively are 3y-1, 3y+5 and 5y + 1. Then y equals
(a) -3
(b) 4
(c) 5
(d) 2 (Delhi 2014)

8. If k, 2k - 1 and 2k + 1 are three consecutive terms of an A.P., the value of k is
(a) 2
(b) 3
(c) -3
(d) 5 (Al 2014) VSA (1 mark)

9. Find the common difference of the Arithmetic
Progression (A.P.) $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}, \ldots (a \neq 0)$ *(2019)*

10. Write the common difference of A.P.
$\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$ (A/ 2019)

11. For what value of k will k+9, 2k - 1 and 2k + 7 are the consecutive terms of an A.P.? (AI 2016)

12. For what value of k will the consecutive terms 2k + 1, 3k+ 3 and 5k - 1 form an A.P.? (Foreign 2016) SAI (2 marks)

13. Find a and b so that the numbers a, 7, b, 23 are in A.P. (Term II, 2021-22)

14. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in A.P. (2020) U

5.3 n^{th} Term of an A.P.

MCQ

15. The first term of an A.P. is p and the common difference is q, then its 10^{th} term is
(a) q+9p
(b) p-9q
(c) p+9q
(d) 2p+9q (2020)

16. The next term of the A.P. $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$ is
(a) $\sqrt{70}$
(b) $\sqrt{84}$
(c) $\sqrt{97}$
(d) $\sqrt{112}$ (Foreign 2014)

VSA (1 mark)

17. If the nth term of an A.P. is pn + q, find its common difference. (2019C)

18. Which term of the A.P. 10, 7, 4, ... is - 41? (2019C)

19. If in an A.P., a = 15, d = - 3 and a, 0, then find the value of n. (2019)

20. How many two digit numbers are divisible by 3? (NCERT, Delhi 2019)

21. In an A.P., if the common difference (d) = -4, and the seventh term (a7) is 4, then find the first term. (2018)

22. What is the common difference of an A.P. in which
921-a7 = 84? (AI 2017)

23. Find the 9th term from the end (towards the first
term) of the A.P. 5, 9, 13, ..., 185. (Delhi 2016)

24.
Find the 25^{th} term of the A.P. $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$
(Foreign 2015)

SAI (2 marks)

For the A.P.; a_1, a_2, a_3, ... if $\frac{a_4}{a_7} = \frac{2}{3}$, then find $\frac{a_6}{a_8}$.

(Term II, 2021-22C)

26. Find the number of terms of the A.P. :
293, 285, 277,..., 53 (Term II, 2021-22C)

27. For what value of 'n', are the n^{th} terms of the A.P's:
9,7,5,.... and 15, 12, 9,.... the same? (Term II, 2021-22)

28.
Which term of the A.P. $-\frac{11}{2}, -3, -\frac{1}{2}$, ... is $\frac{49}{2}$?

(Term II, 2021-22)

29. Determine the A.P. whose third term is 5 and seventh term is 9. (Term II, 2021-22)

30. If the 9th term of an A.P. is zero, then show that its 29th term is double of its 19th term. (2019C)

31. Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21st term? (Delhi 2019)

32. If the 17th term of an A.P. exceeds its 10th term by 7, find the common difference. (AI 2019)

33. Find how many integers between 200 and 500 are
divisible by 8. (Delhi 2017)

34.
Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$, ... is the first negative term? *(AI 2017)* Ap

35. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term. (AI 2016)

36. Find the middle term of the A.P. 6, 13, 20, ..., 216. (Delhi 2015)

37. Find the middle term of the A.P.
213, 205, 197,....., 37. (Delhi 2015)

38. The fourth term of an A.P. is 11. The sum of the fifth and seventh terms of the A.P. is 34. Find its common difference. (Foreign 2015)

39. The fifth term of an A.P. is 20 and the sum of its seventh and eleventh terms is 64. Find the common difference of the A.P. (Foreign 2015)

40. The ninth term of an A.P. is -32 and the sum of its eleventh and thirteenth terms is -94. Find the common difference of the A.P. (Foreign 2015)

41. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5. (Al 2014)

SA II (3 marks)

42. How many terms are there in A.P. whose first and fifth term are -14 and 2, respectively and the last term is 62. (2023)

43. Which term of the A.P.: 65, 61, 57, 53, _______ is the first negative term? (2023)

44.

If the m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$ then show that its $(mn)^{th}$ term is 1. *(Delhi 2017)* Ap

45. The pth, qth and rth terms of an A.P. are a, b and c respectively. Show that $a(q-r)+b(r-p)+c(p-q)=0$. (Foreign 2016) Ev

46. Divide 56 in four parts in A.P. such that the ratio of the product of their extremes (1^{st} and 4^{th}) to the product of means (2^{nd} and 3^{rd}) is 5:6. (Foreign 2016)

47.

If the seventh term of an A.P. is $\frac{1}{9}$ and its ninth term is $\frac{1}{7}$, find its 63^{rd} term. *(Delhi 2014)*

48. The sum of the 5th and the 9th terms of an A.P. is 30. If its 25th term is three times its 8th term, find the A.P. (AI 2014)

49. The sum of the 2nd and the 7th term of an A.P. is 30. If its 15th term is 1 less than twice its 8th term, find the A.P. (Al 2014)

LA (4/5/6 marks)

50. In Mathematics, relations can be expressed in various ways. The matchstick patterns are based on linear relations. Different strategies can be used to calculate the number of matchsticks used in different figures. One such pattern is shown below. Observe the pattern and answer the following questions using Arithmetic Progression:

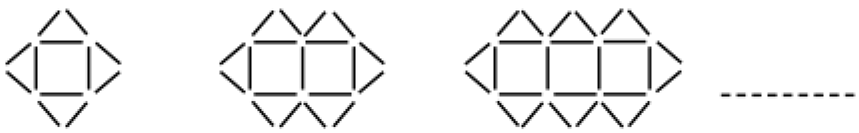


Figure 1 Figure 2 Figure 3

(a) Write the AP for the number of triangles used in the figures. Also, write the nth term of this AP.
(b) Which figure has 61 matchsticks?
(Term II, 2021-22)

51. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and lastterms to the product of two middle terms is 7: 15. Find the numbers. (2020, 2018)

52. Which term of the Arithmetic Progression -7, -12, -17,-22, ... will be -82 ? Is -100 any term of the A.P.? Give reason for your answer. (2019)

53. The sum of three numbers in A.P. is 12 and sum of their cubes is 288. Find the numbers. (Delhi 2016)

5.4 Sum of First n Terms of an A.P.

MCQ

54. Assertion (A): a, b, c are in A.P. if and only if $2b = a + c$.
Reason (R): The sum of first n odd natural numbers is n^2.
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true. (2023)

VSA (1 mark)

55. Find the sum of the first 100 natural numbers. (2020)

SAI (2 marks)

56. In an AP if $S_n = n(4n+1)$, then find the AP. (Term II, 2021-22)

57. Find the common difference 'd' of an A.P. whose first term is 10 and sum of the first 14 terms is 1505. (Term II, 2021-22)

58. Find the sum of first a_{20} terms of an AP in which d = 5 and a20 = 135. (Term II, 2021-22)

59. Find the sum of first 20 terms of an A.P. whose n^{th} term is given as $a_1 = 5 - 2n$. (Term II, 2021-22)

60. If S, the sum of first n terms of an A.P. is given by $S_1 = 3n^2 - 4n$, find the nth term. (Delhi 2019)

61. If S, the sum of the first n terms of an A.P. is given by $Sn = 2n^2+n$, then find its nth term. (A/ 2019)

62. Find the sum of first 8 multiples of 3. (2018)

63. How many terms of the A.P. 18, 16, 14, ... be taken so that their sum is zero? (Delhi 2016)

64. How many terms of the A.P. 27, 24, 21, ... should be taken so that their sum is zero? (Delhi 2016)

65. In an A.P., if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the A.P., where S_n denotes the sum of its first n terms. (AI 2015)

66. The first and the last terms of an A.P. are 7 and 49 respectively. If the sum of all its terms is 420, find its common difference. (Delhi 2014)

67. The first and the last terms of an A.P. are 8 and 65 respectively. If the sum of all its terms is 730, find its common difference. (Delhi 2014)

68. The sum of the first n terms of an A.P. is $3n^2 + 6n$. Find the nth term of this A.P. (Foreign 2014)

69. The sum of the first n terms of an A.P. is 5n - n^2. Find the nth term of this A.P. (Foreign 2014)

70. The sum of the first n terms of an A.P. is $4n^2 + 2n$. Find the nth term of this A.P. (Foreign 2014) SA II (3 marks)

71. The sum of first 15 terms of an A.P. is 750 and its first term is 15. Find its 20th term. (2023)

72. Rohan repays his total loan of 1,18,000 by paying every month starting with the first instalment of 1,000. If he increase the instalment by 100 every month, what amount will be paid by him in the 30th instalment? What amount of loan has he paid after 30th instalment? (2023)

73. Find the sum of first 16 terms of an Arithmetic Progression whose 4th and 9th terms are – 15 and - 30 respectively. (2020C)

74. In an A.P. given that the first term (a) = 54, the common difference (d) = -3 and the nth term
(a) = 0, find n and the sum of first n terms (S_1) of the A.P. (2020)

75. Find the sum : (-5)+(-8) + (-11) +...+ (-230) (2020) (Ap

76. For an A.P., it is given that the first term (a) = 5, common difference (d) = 3, and the nth term (a_n) = 50. Find n and sum of first n terms (S_n) of the A.P. (2020)

77.

If m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then find the sum of its first *mn* terms. *(2019, Delhi 2017)*

78. Find the sum of n terms of the series

$$\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+\ldots\ldots$$ *(Delhi 2017)*

79. The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P. (AI 2017, Delhi 2014)

80. If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first n terms of the A.P. (NCERT, Delhi 2016)

81. How many terms of the A.P. 65, 60, 55, ... be taken so that their sum is zero? (Delhi 2016)

82. If the ratio of the sum of first n terms of two A.P's is (7n + 1): (4n+27), find the ratio of their mth terms. (AI 2016)

83. The digits of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number. (AI 2016)

84. The sums of first n terms of three arithmetic progressions are S1, S2 and S3 respectively. The first term of each A.P. is 1 and their common differences are 1, 2 and 3 respectively. Prove that S_1 + S3 = 252. (Al 2016)

85. The sum of the first n terms of three A.P's are S_1, S_2 and S_3. The first term of each is 5 and their common difference are 2, 4 and 6 respectively. Prove that $S_1+S_3=2S_2$. (Foreign 2016)

86. If S, denotes the sum of first n terms of an A.P., prove that $S_{12} = 3(S_8 - S_4)$. (Delhi 2015)

87.

If the sum of the first *n* terms of an A.P. is $\frac{1}{2}(3n^2+7n)$, then find its n^{th} term. Hence write its 20^{th} term. *(Delhi 2015)*

88. If S, denotes the sum of first n terms of an A.P., prove that S303[S20-S10] (Delhi 2015, Foreign 2014)

89. The 14^{th} term of an A.P. is twice its 8th term. If its 6^{th} term is -8, then find the sum of its first 20 terms. (Al 2015)

90. The 16^{th} term of an A.P. is five times its third term. If its 10th term is 41, then find the sum of its first fifteen terms. (AI 2015)

91. The 13^{th} term of an A.P. is four times its 3rd term. If its fifth term is 16, then find the sum of its first ten terms. (AI 2015)

92. In an A.P., if the 12th term is -13 and the sum of its first four terms is 24, find the sum of its first ten terms. (Foreign 2015)

93. The tenth term of an A.P. is -37 and the sum of its first six terms is - 27. Find the sum of its first eight terms. (Foreign 2015)

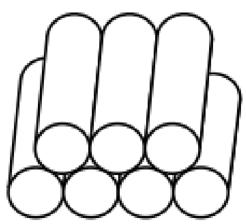
94. The sum of the first seven terms of an A.P. is 182. If its 4th and the 17th terms are in the ratio 1 : 5, find the A.P. (AI 2014)

95. The sum of the first 7 terms of an A.P. is 63 and the sum of its next 7 terms is 161. Find the 28th term of this A.P. (Foreign 2014)

LA (4/5/6 marks)

96. The ratio of the 11th term to 17th term of an A.P. is 3 4. Find the ratio of 5th term to 21st term of the same A.P. Also, find the ratio of the sum of first 5 terms to that of first 21 terms. (2023)

97. 250 logs are stacked in the following manner:
22 logs in the bottom row, 21 in the next row, 20 in the row next to it and so on (as shown by an example). In how many rows, are the 250 logs placed and how many logs are there in the top row? (2023)



(Example)

98. Solve: 1+4 +7 + 10 + ... + x = 287 (2020)

99. Find the sum of all odd numbers between 0 and 50. (2019C)

100. How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180 ? Explain the double answer. (2019C)

101. The first term of an A.P. is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the A.P. (Delhi 2019)

102. Find the sum of all the two digit numbers which leave the remainder 2 when divided by 5. (Al 2019)

103. The ratio of the sums of first m and first n terms of an A. P. is $m^2 : n^2$. Show that the ratio of its mth and nth terms is (2m - 1): (2n-1). (Delhi 2017, Foreign 2016)

104.If the sum of first m terms of an A.P. is the same as the sum of its first n terms, show that the sum of its first (m + n) terms is zero. (Delhi 2017)

105. If the ratio of the sum of the first n terms of two A.P's is (7n + 1): (4n+27), then find the ratio of their 9th terms. (AI 2017)

106. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief? (Delhi 2016)

107.A thief, after committing a theft, runs at a uniform speed of 50 m/minute. After 2 minutes, a policeman runs to catch him. He goes 60 m in first minute and increases his speed by 5 m/minute every succeeding minute. After how many minutes, the policeman will catch the thief? (Delhi 2016)

108. The houses in row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceeding the house numbered X is equal to sum of the numbers of houses following X. (AI 2016)

109. Reshma wanted to save at least 6500 for sending her daughter to school next year (after 12 months). She saved 450 in the first month and raised her savings by 20 every next month. How much will she be able to save in next 12 months? Will she be able to send her daughter to the school next year? What value is reflected in this question? (Foreign 2016)

110. Ramkali required ₹2500 after 12 weeks to send her daughter to school. She saved 100 in the first week and increased her weekly saving by *20 every week. Find whether she will be able to send her daughter to school after 12 weeks. What value is generated in the above situation? (Delhi 2015)

111. Find the 60th term of the A.P., 8, 10, 12, ..., if it has a total of 60 terms and hence find the sum of its last 10 terms. (AI 2015)

112. An arithmetic progression 5, 12, 19, ... has 50 terms. Find its last term. Hence find the sum of its last 15 terms. (Al 2015)

113. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 3, when divided by 4. Also find the sum of all numbers on both sides of the middle term separately. (Foreign 2015)

114. Find the middle term of the sequence formed by all numbers between 9 and 95, which leave a remainder 1 when divided by 3. Also find the sum of the numbers on both sides of the middle term separately. (Foreign 2015)

115. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 5 when divided by 7. Also find the sum of all numbers on both sides of the middle term separately. (Foreign 2015)

116. In an A.P. of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P. (Delhi 2014)

117.In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students. Which value is shown in this question? (Al 2014)

CBSE Sample Questions

5.3 nth Term of an A.P.

VSA (1 mark)

1. Which term of the A.P. 27, 24, 21,.. is zero? (2020-21)

2. In an Arithmetic Progression, if $d = 4$, $n = 7$, $a_1 = 4$, then find a. (2020-21)

SA I (2 marks)

3. Find the value of a25-a15 for the AP: 6, 9, 12, 15., ... (Term II, 2021-22)

4. If 7 times the seventh term of the AP is equal to 5 times the fifth term, then find the value of its 12th term. (Term II, 2021-22)

5.4 Sum of first n terms of an A.P.

LA (4/5/6 marks)

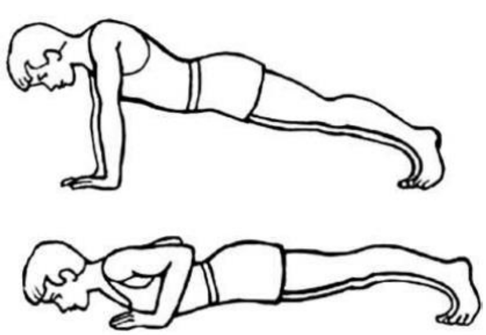
5. The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.

(i) If the first circular row has 30 seats, how many seats will be there in the 10th row?
(ii) For 1500 seats in the auditorium, how many rows need to be there?

OR

If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row?
(iii) If there were 17 rows in the auditorium, how many seats will be there in the middle row? (2022-23)

6. Push-ups are a fast and effective exercise for building strength. These are helpful in almost all sports including athletics. While the push-up primarily targets the muscles of the chest, arms, and shoulders, support required from other muscles helps in toning up the whole body.



Nitesh wants to participate in the push-up challenge. He can currently make 3000 push-ups in one hour. But he wants to achieve a target of 3900 push-ups in 1 hour for which he practices regularly. With each day of practice, he is able to make 5 more push- ups in one hour as compared to the previous day. If on first day of practice he makes 3000 push-ups and continues to practice regularly till his target is achieved. Keeping the above situation in mind answer the following questions:

(i) Form an A.P. representing the number of push- ups per day and hence find the minimum number of days he needs to practice before the day his goal is accomplished.

(ii) Find the total number of push-ups performed by Nitesh up to the day his goal is achieved. (Term II, 2021-22)

SOLUTIONS

Previous Years' CBSE Board Questions

1. (c): We have, a, b, c, are in A.P.

:- b=a+d, and c = a + 2d

Now, a -2b - c = a -2(a + d) - (a + 2d)

= a -2a - 2d-a-2d2a-4d

2. (d): We have, √7, √28, √63,... i.e., √7,2√7,3√7,...

Here, first term, a=√7 and common difference, d=√7

(:- d= a_2-a_1)

:- Next term, a_1 =a3 +d=3√7+√7=4√7=√112

3. (a): Since, k + 2, 4k - 6 and 3k - 2 are three consecutive terms of A.P.

:- a2-a1=A3-A2

= (4k - 6) - (k + 2) = (3k - 2) - (4k -6)

= 4k-6-k-2=3k-2-4k+6

= 3k-8=-k+4⇒ 4k = 12⇒k=3

4.

(b): Given, $\frac{-5}{7}$, a, 2 are in A.P. therefore common difference is same.

$\therefore \quad a_2 - a_1 = a_3 - a_2$

$$a-\left(\frac{-5}{7}\right)=2-a \Rightarrow a+\frac{5}{7}=2-a \Rightarrow 2a=\frac{9}{7} \Rightarrow a=\frac{9}{14}$$

5.

(c) : In option (c), We have

$a_2 - a_1 = \frac{7}{3} - \frac{4}{3} = \frac{3}{3} = 1$; $a_3 - a_2 = \frac{9}{3} - \frac{7}{3} = \frac{2}{3}$

As $a_2 - a_1 \neq a_3 - a_2$, the given list of numbers does not form an A.P.

6. (a): Given, 2x, (x + 10) and (3x+2) are in A.P.

:- (x+10) -2x=(3x+2) - (x + 10)

= -x+10=2x-8=-3x=-18⇒x=6

7. (c): Given, 3y - 1, 3y+5 and 5y + 1 are in A.P.

:- 3y+5-(3y-1)=5y + 1 - (3y+5)

= 3y+5-3y+1=5y+1-3y-5

$\Rightarrow \quad 6 = 2y - 4 \Rightarrow \quad y = \frac{10}{2} = 5$

8. (b): k, 2k - 1 and 2k + 1 are three consecutive terms of an A.P.

;- (2k-1) - (k) = (2k + 1) - (2k - 1)

= k=3 = k-1-2

9.

Given A.P. is $\frac{1}{a}, \frac{3-a}{3a}, \frac{3-2a}{3a}$

$\therefore \quad a_2 - a_1 = \frac{3-a}{3a} - \frac{1}{a} = \frac{3-a-3}{3a} = \frac{-a}{3a} = \frac{-1}{3}$

10. Given A.P. is, √3,√12,√27,√48,

or √3,2√3,3√3,4√3..........

:- d = common difference = 2√3-√3=√3

11. Given, k +9, 2k - 1 and 2k + 7 are in A.P.

;- (2k-1) - (k+9)= (2k+7) - (2k - 1)

= 2k-1-k-9=2k + 7 - 2k + 1⇒k-10-8 k = 18

12. Given, 2k + 1, 3k + 3 and 5k - 1 are in A.P.

:- 2(3k+3)=2k+1+5k - 1⇒ 6k + 6 = 7k ⇒ k=6

13. Since, a, 7, b, 23 are in A.P.
:- Common difference is same.
:- 7-a=b-7=23-b

Taking second and third terms, we get
b-7=23-b$\Rightarrow$ 2b = 30
$\Rightarrow$ b=15
Taking first and second terms, we get
7-a=b-7
= 7-a=15-7
= 7-a=8
= a = -1
Hence, a = -1, b = 15.

14. Let $a_1 = (a - b)^2$, $a_2 = (a^2 + b^2)$ and a3 = $(a + b)^2$
Now, a2-a_1 = $(a^2 + b^2)$ - (a - b)2
= $a^2 + b^2 - (a^2 + b^2 - 2ab)$
= $a^2 + b^2 - a^2 - b^2 + 2ab = 2ab$
Again a3-a2 = $(a + b)^2 - (a^2 + b^2)$
= $a^2 + b^2$+2ab-a2 - b^2 = 2ab
;- a2-a1a3-a2
So, $(a - b)^2$, (a2+ b^2) and (a + b)2 are in A.P.

15. (c): Given, first term, a = p and common difference,
d = q
:- 10th term, a10 = a + (10-1)d = p +9q

16. (d): First term, a = √7 and common difference,
d=√28-√7=2√7-17=√
:- Fourth term of the A.P. is (a4) = a + 3d
=√7+3√7=4√7=√112

17. Given, a, pn+q
= a + (n-1) d = pn+q
= (n - 1) d = pn+q-a

$$\Rightarrow d=\frac{pn+q-a}{n-1}$$

18. Let nth term of A.P. 10, 7, 4, ... is - 41.
:- a_n = a + (n-1)d
= -41=10+ (n-1)(-3) [:- d=7-10-3]
= -4110-3n+3
= -41=13-3n
= 3n=54⇒n=18
:- 18th term of given A.P. is -41.

19. Given, a 15, d = -3 and a_n = 0
:- a + (n-1)d=0
= 15+ (n-1)(-3)=0
= 15-3n+3=0⇒ 18-3n=0
= -3n-18⇒n=6

20. Two-digit numbers which are divisible by 3 are 12, 15, 18, ..., 99, which forms an A.P. with first term
(a) = 12, common difference (d) = 15-12= 3 and last term
(1) or n^{th} term (an) = 99
:- a+ (n-1)d = 99
= 12+ (n-1)3 = 99 - 3_n = 99-9

$$\Rightarrow \quad n=\frac{90}{3}=30$$

Thus, there are 30 two-digit numbers which are divisible by 3.

21. Let a be the first term of A.P.
Here, common difference (d) = − 4,
seventh term (az) = 4, n = 7
:- an = a + (n-1)d
⇒ a_1 = a+ (7-1) x (-4)=4
⇒ a+6x(-4)=4⇒a-24-4a=28

22. Leta be the first term and d be the common difference
of the A.P.
Given, a_{21}-a_7 = 84
...(i)
Now, a_n = a+ (n - 1)d
:- a_{21} = a+20d and a_7 = a + 6d

(a+20d) - (a+6d) = 84 [From (i)]
$\Rightarrow$ 14d=84$\Rightarrow$d=6

23. Given A.P. is 5, 9, 13, ..., 185.
Here, /= last term = 185
d = common difference = 9-5 = 4
:- 9th term from the end = 1-(9-1)d = 1-8d
= 185-8 x 4 = 185 - 32 = 153

24.

Given, $-5, -\frac{5}{2}, 0, \frac{5}{2}, \ldots$ are in A.P.

$\Rightarrow \quad a = -5, d = \left(\frac{-5}{2}\right) - (-5) = \frac{5}{2}$

We know that, $a_n = a + (n-1)d$

$\therefore \quad a_{25} = a + (25-1)d = (-5) + 24 \times \left(\frac{5}{2}\right) = -5 + 60 = 55$

25. Leta be the first term and d be the common difference of given A.P.

We have, $\frac{a_4}{a_7} = \frac{2}{3}$

$\Rightarrow \quad \frac{a+3d}{a+6d} = \frac{2}{3} \qquad [\because a_n = a + (n-1)d]$

$\Rightarrow \quad 3(a+3d) = 2(a+6d)$

$\Rightarrow \quad 3a + 9d = 2a + 12d$

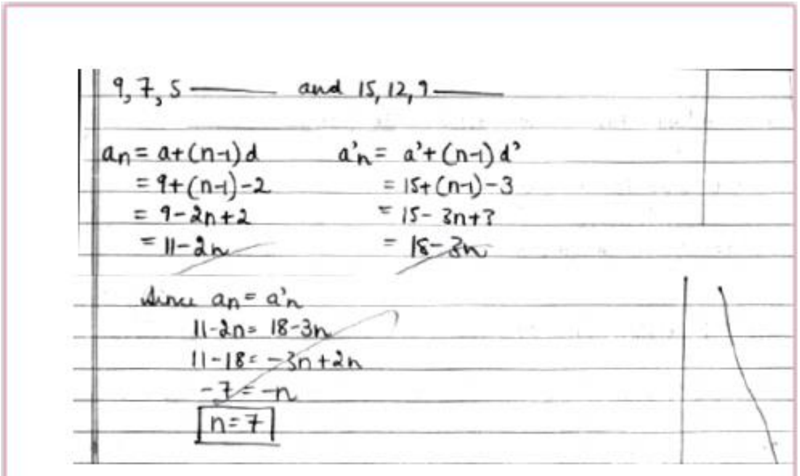
$\Rightarrow \quad 3a - 2a = 12d - 9d \Rightarrow a = 3d$

Now, $\frac{a_6}{a_8} = \frac{a+5d}{a+7d} = \frac{3d+5d}{3d+7d} = \frac{8d}{10d} = \frac{4}{5}$

$\therefore \quad \frac{a_6}{a_8} = \frac{4}{5}$

26. Given, 293, 285, 277,..., 53 be an A.P.
a = 293, d= 285-293=-8
We know, a_n = a + (n-1)d
= 53293+ (n-1)(-8)
= 53 293 (n - 1)(-8)
= -240 = (n - 1)(-8) $\Rightarrow$ 30= n − 1$\Rightarrow$n=31

27.



9, 7, 5 —— and 15, 12, 9 ——

$a_n = a+(n-1)d$ $a'_n = a'+(n-1)d'$

$= 9+(n-1)-2$ $= 15+(n-1)-3$

$= 9-2n+2$ $= 15-3n+3$

$= 11-2n$ $= 18-3n$

Since $a_n = a'_n$

$11-2n = 18-3n$

$11-18 = -3n+2n$

$-7 = -n$

$\boxed{n=7}$

[Topper's Answer, 2022]

28.

Let n^{th} term of the given A.P. be $\frac{49}{2}$.

Here, $a = -\frac{11}{2} \Rightarrow a_1 = \frac{-11}{2}$ and $a_2 = -3$

$\therefore \quad d = a_2 - a_1 = -3 - \left(\frac{-11}{2}\right) = -3 + \frac{11}{2} = \frac{-6+11}{2} = \frac{5}{2}$

And, $a_n = a + (n-1)d$

$\Rightarrow \frac{49}{2} = \frac{-11}{2} + (n-1) \times \frac{5}{2} \Rightarrow \frac{49}{2} + \frac{11}{2} = (n-1) \times \frac{5}{2}$

$\Rightarrow 30 = (n-1) \times \frac{5}{2} \Rightarrow n-1 = 12 \Rightarrow n = 13$

Thus, 13^{th} term of the given A.P. is $\frac{49}{2}$.

29. Let the first term and common difference of an A.P. be a and d, respectively.

Given, $a_3 = 5$ and $a_7 = 9$

$\Rightarrow a+(3-1)d=5$ and $a + (7-1)d = 9$ [$a_n = a + (n-1)d$]

$\Rightarrow a+2d=5$ ...(i)

and $a + 6d = 9$ ...(ii)

On subtracting (i) from (ii), we get

$4d=4 \Rightarrow d=1$

From (i), $a + 2(1) = 5 \Rightarrow a+2=5 \Rightarrow a=3$

So, required A.P. is a, a + d, a + 2d, a + 3d,....

i.e., 3, 3+1, 3+2(1), 3 + 3(1),..... i.e., 3, 4, 5, 6.,.....

30. Given, a, = 0, we have to show that a29 = 2919
= a+8d=0a=-8d
Now, a_{19} = a + 18d = -8d+ 18d = 10d [: a = -8d]
a_{29} = a + 28d=-8d+28d=20d = 2(10d) = $2a_{19}$
Hence, $a_{29} = 2a_{19}$

31. We have, first term, a = 3, common difference,
d = 15-3=12
n^{th} term of an A.P. is given by a_n = a + (n-1)d
:- a_{21}=3+ (20) x 12=3+240 243
Let the rth term of the A.P. be 120^{st} more than the 21st term.
= a+ (r-1)d=243 + 120
= 3+ (r-1) 12 363
= (r-1) 12 360⇒r-1=30⇒r=31

32. According to question, a_{17}-a_{10}=7
i.e., a + 16d - (a + 9d) = 7
where a = first term, d = common difference
7d=7. d = 1

33. Numbers divisible by 8 between 200 and 500 are
208, 216, 224, ..., 496 which forms an A.P.
First term (a) = 208, common difference (d) = 8
nth term of an A.P., $a_n = a + (n - 1)d$
= 496208+ (n-1)8
= 288 = (n − 1)8 ⇒ n-1=36⇒n=37

34. Given sequence is an A.P. in which a = 20 and

$$d = 19\frac{1}{4} - 20 = \frac{77}{4} - 20 = \frac{-3}{4}$$

Let n^{th} term of the given A.P. be the first negative term.

i.e., $a_n < 0 \Rightarrow a + (n-1)d < 0$

$$\Rightarrow \quad 20 + (n-1)\left(\frac{-3}{4}\right) < 0 \Rightarrow 80 - 3n + 3 < 0$$

$$\Rightarrow \quad 83 - 3n < 0$$

$$\Rightarrow \quad 3n > 83 \Rightarrow n > 27\frac{2}{3} \Rightarrow n = 28$$

35. Let the first term and common difference of the A.P. be a and d respectively.
Since, a_n = a + (n-1)d

:- $a_4=a+(4-1)\,d=0$
$\Rightarrow a+3d = 0 \Rightarrow a = -3d$ ...(i)
Now, $a_{25} = a + 24d$
$\Rightarrow a_{25}=-3d+24d$ [using (i)]
$\Rightarrow a_{25} = 21d$
Now, $a_{11} = a + 10d = -3d+ 10d$ [using (i)]
;- $9_{11}=7d$
Multiply both sides by 3, we get
$3a_{11} = 21d \Rightarrow 3a_{11} = a_{25}$

36. We have, first term, a = 6, common difference
d=13-6=7
Last term, 216= |
Now, $a_n = a + (n-1)d$
$= 216 = 6+ (n-1)7 \Rightarrow 216-6=7n-7 \Rightarrow 210+7=7n$

$$\Rightarrow \quad n=\frac{217}{7}=31 \text{, which is odd.}$$

$$\therefore \quad \text{Middle term} = \left(\frac{n+1}{2}\right)^{th} \text{term} = \left(\frac{31+1}{2}\right)^{th} \text{term}$$

$$= 16^{th} \text{ term}$$

$$a_{16} = a + (16 - 1)d = 6 + 15 \times 7 = 111$$

37. We have, first term (a) = 213,
common difference (d) = 205-213=-8

Last term (1) = 37
Now, I = a + (n-1)d
= 37=213+ (n-1) (-8)
= 37-2138(n - 1)
= -176-8(n-1)
= n-1=22⇒n= 23, which is odd.

$$\therefore \quad \text{Middle term} = \left(\frac{23+1}{2}\right)^{th} \text{term} = 12^{th} \text{ term}$$

$$\text{So, } a_{12} = a + 11d = 213 + 11\,(-8)$$
$$= 213 - 88 = 125$$

$\therefore$ Middle term $= \left(\frac{23+1}{2}\right)^{th}$ term = 12^{th} term

So, $a_{12} = a + 11d = 213 + 11(-8)$

$= 213 - 88 = 125$

38. Let the first term be a and d be the common difference of an A.P.

We have, $a_4 = 11 \Rightarrow a + 3d = 11$ ...(i)

:- According to question, $a_5 + a_7 = 34$

$= (a + 4d) + (a + 6d) = 34$

$= 2a+10d = 34 = a + 5d = 17$ ...(ii)

Subtracting (i) from (ii), we get

$2d=6 \Rightarrow d=3$

39. Let the first term bea and d be the common difference of the A.P.

Given, $a_5 = 20 \Rightarrow a + 4d = 20$ ...(i)

Also, $a_7+a_{11} = 64$

$= a+6d+a+ 10d = 642a+16d = 64$

$= a + 8d = 32$ ...(ii)

Subtracting (i) from (ii), we have

$4d = 12 = d = 3$

40. Let the first term be a and d be the common difference of the A.P.

Given, $a_9 = -32 \Rightarrow a+8d = -32$ ...(i)

Also, $a_{11}+ a_{13}=-94$

$\Rightarrow a + 10d + a + 12d=-94\ 2a + 22d = -94$

$\Rightarrow a+11d-47$ ...(ii)

Subtracting (ii) from (i), we have

$-3d\ 15 \Rightarrow d = -5$

41. Natural numbers between 101 and 999 which are divisible by both 2 and 5 are 110, 120,..., 990, which forms an A.P.

Here, $a = 110$, $d = 10$, $a_n = 990$

Now, $a + (n-1)d = a_n$

$\Rightarrow 110+ (n-1)10 = 990$

$\Rightarrow$ (n - 1)10-880 $\Rightarrow n - 1 = 88 \Rightarrow$ n=89
Hence, there are 89 numbers between 101 and 999 which are divisible by both 2 and 5.

42. We have
First term, a_1 = -14
Fifth term, a_5 = 2
Last term, a_n = 62
Let d be the common difference and n be the number of terms.
:- a_5=2
-14+(5-1)d=2
= 4d = 16
= d = 4
Now, a_n = 62
= -14+ (n-1)4 = 62
= 4n - 4 = 76
= 4n = 80
n = 20
:- There are 20 terms in A.P.

43. Given, A.P. is 65, 61, 57, 53, ...
Here, first term a = 65 and common difference, d = -4
Let the n^{th} term is negative.
Last term, $a_n = a + (n - 1)$ = 65+ (n-1)(-4)
= 65-4n+ 4
= 69 - 4n, which will be negative when n = 18
So, 18^{th} term is the first negative term.

44. Leta be the first term and d be the common difference of the given A.P.
rth term of A.P, $a_r = a + (r - 1)d$

According to question,

$$a_m = a+(m-1)d = \frac{1}{n} \quad \text{...(i)}$$

and $a_n = a+(n-1)d = \frac{1}{m}$ ...(ii)

Subtracting (ii) from (i), we get

$$(m-n)d = \frac{m-n}{mn} \Rightarrow d = \frac{1}{mn}$$

Putting $d = \frac{1}{mn}$ in (i), we get

$$a+(m-1)\frac{1}{mn} = \frac{1}{n} \Rightarrow a+\frac{1}{n}-\frac{1}{mn} = \frac{1}{n}$$

$$\Rightarrow \quad a = \frac{1}{mn}$$

$$\therefore \quad a_{mn} = a+(mn-1)d = \frac{1}{mn}+(mn-1)\frac{1}{mn} = \frac{1+mn-1}{mn} = 1$$

45. Let A be the first term and D be the common difference of given A.P.

:- Tp=A+ (p-1)D = a ...(i)

Tq=A+ (q-1) D=b ...(ii)

Tr=A+ (r - 1)D = c ...(iii)

Now, a(q-r)+b(r− p) + c(p − q)

= [A+ (p 1)D] (q - r) + [A+(q 1)D] (r− p) + [A+ (r-1)D] (p-q) [Using (i), (ii) and (iii)]

=A[q-r+r-p+ p - q] + D[(p − 1)(q − r) + (q − 1) (r− p) + (r - 1)(p − q)]

= 0 + D[pq − rp − q + r + qr - pq − r + p + rp - qr -p + q] = 0

46. Let the four parts that are in A.P. be a - 3d, a − d, a + d, a + 3d

:- a-3d+a-d+a+d+a+3d = 56

⇒ 4a=56⇒ a = 14

According to question,

$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{5}{6} \quad \Rightarrow \quad \frac{a^2-9d^2}{a^2-d^2} = \frac{5}{6}$$

$$\Rightarrow \quad 6(196 - 9d^2) = 5(196 - d^2) \quad [\because a = 14]$$

$= 1176\text{-}54d^2 = 980 - 5d^2$

$= 49d^2 = 196 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$

So, the four parts are 8, 12, 16 and 20 if d = 2 and 20, 16, 12 and 8 if d = -2.

47. Leta be the first term and d be the common difference of the A.P.

Given, $a_7 = \frac{1}{9}, a_9 = \frac{1}{7}$

$a_7 = a+(7-1)d = \frac{1}{9} \Rightarrow a+6d = \frac{1}{9}$ ...(i)

$a_9 = a+(9-1)d = \frac{1}{7} \Rightarrow a+8d = \frac{1}{7}$ ...(ii)

Subtracting (i) from (ii), we get

$2d = \frac{2}{63} \Rightarrow d = \frac{1}{63}$

Putting $d = \frac{1}{63}$ in (i), we get

$a + \frac{6}{63} = \frac{1}{9} \Rightarrow a = \frac{1}{63}$

So, $a_{63} = a+62d = \frac{1}{63} + \frac{62\times1}{63} = \frac{63}{63} = 1$

48. Leta be the first term and d be the common difference
of the A.P.
Now, an = a + (n-1)d
as+a9= 30 (Given)
a5 + 4d + a +8d = 30
2a + 12d = 30 ...(i)
Also, a25 = 3a8
a +24d = 3(a+7d) ⇒ a + 24d = 3a +21d
⇒2a-3d=0
On solving (i) and (ii), we get a = 3,d=2
..A.P. is 3, 5, 7,.....
49. Leta be the first term and d be the common difference
of the A.P.
Now, a_n = a + (n-1)d
⇒ a_2+a7=30⇒a+d+a+6d=30
⇒ 2a + 7d = 30 ...(i)
Also, a_{15} = $2a_8$-1
⇒ a + 14d = 2(a + 7d) - 1⇒ a + 14d = 2a + 14d 1
⇒ a=1 ...(ii)
Substituting (ii) in (i), we get

$2+7d=307d=28 \Rightarrow d=4$
Hence, the A.P. is formed as 1, 5, 9,....

50. (a) Number of triangles in figure 1 = 4
Number of triangles in figure 2 = 6
Number of triangles in figure 3 = 8
:- Required A.P. is 4, 6, 8 .....
$a=4, d=6-4=2$
:- $a_n = a + (n-1)d$
$= a_n=4+ (n-1)(2) = a_n=4+2n-2=2n+2$
$= a_n=2(n+1) = a_9 = 2(9+1) = 2(10) = 20$

(b) Numbers of matchsticks used in figure 1 = 12
Number of matchsticks used in figure 2 = 19
Number of matchsticks used in 3 = 26
Thus, required A.P. be 12, 19, 26......
$A = 12, D= 19-12=7$
:- $A_n= A+ (N-1) D$
$\Rightarrow 61=12+ (N-1)(7) \Rightarrow 61 = 12 + 7N-7$
$\Rightarrow 61=5+7N7N=56 \Rightarrow N=8$
Hence, figure 8 has 61 matchsticks.

51. Let the four consecutive numbers be (a - 3d), (a – d),
(a + d), (a + 3d).
Sum of four numbers = 32 [Given]
$\Rightarrow (a-3d) + (a-d) + (a+d) + (a+3d) = 32$
$\Rightarrow 4a= 32 \Rightarrow a = 8$

Also, $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\Rightarrow \quad \frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15} \Rightarrow 15a^2 - 135d^2 = 7a^2 - 7d^2$

$\Rightarrow \quad 8a^2 = 128d^2 \Rightarrow d^2 = \frac{8a^2}{128} = \frac{8 \times 64}{128} = 4 \quad \therefore d = \pm 2$

If d = 2, then the numbers are (8-6), (8 - 2), (8 + 2) and
(8+6) i.e., 2, 6, 10, 14.
If d = -2, then the numbers are (8 + 6), (8 + 2), (8 − 2), (8 − 6)
i.e., 14, 10, 6, 2.
Hence, the numbers are 2, 6, 10, 14 or 14, 10, 6, 2.

52. Given, A.P. is -7, -12, -17,-22,....
and nth term of given A.P. is -82.
:- $a_n = a + (n-1)d$
$= -82 = -7 + (n-1)(-5)$ [$\because d = -12 - (-7) = -12 + 7 = -5$]
= -82-7-5n+5-82-5n-2
= 5_n=82-2⇒ 5n=80⇒n=16
:- 16_{th} term of given A.P. is -82
:- 17_{th} term = -82-5=-87
18_{th} term = -87-5=-92
19_{th} term = -92-5=-97
20_{th} term = -97-5=-102
Hence, -100 is not any term of given A.P.

53. Let the three numbers in A.P. area - d, a, a + d.
According to question, a - d+a+a+d=12
→3a= 12a=4
Also, $(4-d)^3 + (4)^3 + (4+d)^3 = 288$
⇒ $64-48d+12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$
⇒ $24d2 + 192 = 288$ $d^2 = 4$ ⇒ $d = \pm 2$
The numbers will be a - d, a, a +d
4+2,4,4-2=6, 4, 2, if d = -2
or 4-2, 4, 4+2 = 2, 4, 6, if d = 2

54. (b): Since, a, b, c are in A. P., then b - a = c - b
⇒ 2b=a+c
First n odd natural number be 1, 3, 5,... (2n-1).

which form an A.P. with a = 1 and d = 2

$$\text{Sum of first n odd natural number } = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{n}{2}[2+(n-1)2] = n^2$$

Hence, assertion and reason are true but reason is not the correct explation of assertion.

55. First 100 natural numbers are 1, 2, 3,........ 100 which form an A.P. with a = 1, d= 1.

Sum of n terms, $S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{100}{2}[2\times1+(100-1)\times1] = 50\,[2 + 99] = 50 \times 101 = 5050$

56. Given S_1 = n(4n+ 1)

S_1 = 1 (4+1)=5=a1

S_2 = 2(8+1)=2(9) = 18

S_3 = 3(4(3)+1)=3(13) = 39

S_4 = 4(4(4)+1)=4(17) = 68

We know that an = Sn - Sn-1

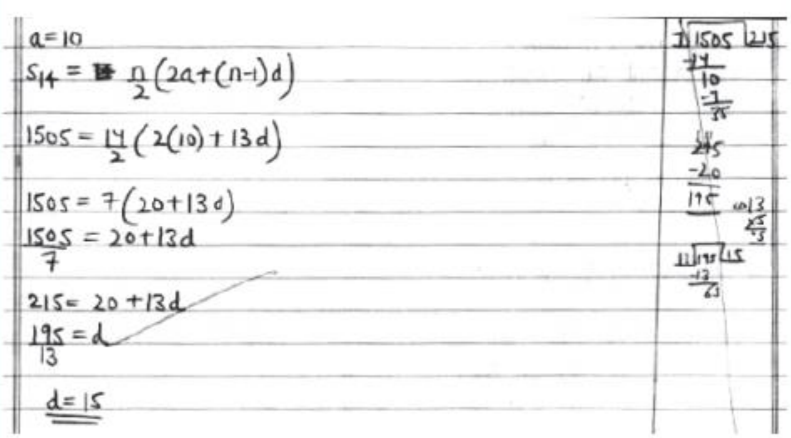
:- a_2=S_2-S_1 = 18-5=13

a_3=S_3-S_2=39-18=21

:- a4=S_4-S_3=68-39=29

The required A.P. is 5, 13, 21, 29, ...

27.



a = 10

$S_{14} = \frac{n}{2}(2a+(n-1)d)$

$1505 = \frac{14}{2}(2(10)+13d)$

$1505 = 7(20+13d)$

$\frac{1505}{7} = 20+13d$

$215 = 20+13d$

$\frac{195}{13} = d$

$d = 15$

[Topper's Answer, 2022]

58. Given, a_{20} = 135, d = 5

Let the first term of an A.P. be a and n^{th} term be an.

:- a_{20}=a+(201)d [:- a_n = a+ (n-1)d]

= 135 = a + 19(5)

= 135=a+95⇒ a = 135 - 95 = 40

Now, sum of first n terms of an A.P. is

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow \quad S_{20} = \frac{20}{2}[2(40)+19(5)] = 10[80+95]$$
$$= 10 \times 175 = 1750$$

Hence, the sum of first 20 terms of an A.P. is 1750.

59. Given, nth term of the A.P. series is
$a_n = 5-2n$ ... (i)
Put n = 1, $a_1 = 5-2(1)=5-2=3$
Put n=2, $a_2 = 5-2(2)=5-4=1$
Put n = 3, $a_3 = 5 - 2(3) = 5 - 6 = -1$
Put n = 4, $a_4 = 5-2(4)=5-8=-3$
So, the series becomes 3, 1, -1, -3, ...
Here, a = 3 and $d = a_2 - a_1 = 1 - 3 = -2$
We know that, sum of n terms of an A.P. is

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

∴ Sum of first 20 terms of an A.P. is

$$S_{20} = \frac{20}{2}[2(3)+(20-1)(-2)]$$
$$= 10[6 + 19 \times -2] = 10[6 - 38]$$
$$= 10 \times -32 = -320$$

Hence, the required sum of 20 terms of given A.P. is -320.

60. Given, $S_1 = 3n^2 - 4n$.
We know that $S_n - S_n - 1 = a_n$
$= 3n^2-4n-\{3(n-1)^2 - 4(n-1)\} = a_n$
$= 3n^2 - 4n - \{3(n^2 + 1-2n) - 4n+4\} = a_n$
$= 3n^2-4n-\{3n^2+3-6n-4n+4\} = an$
$= 3n^2-4n-3n^2-7+10n = an \Rightarrow 6n-7=a_n$

61. We have, $S_1 = 2n^2 + n$
:- $S_{n-1}=2(n-1)^2 + (n-1) = 2(n^2 + 1-2n) + n-1$
$= 2n^2+2-4n+ n - 1 = 2n^2 - 3n+1$
Now, n^{th} term of the A.P., $a_n = S_n - S_{n-1}$
$= (2n^2 + n) - (2n^2 - 3n+ 1) = 4n - 1$

62. Multiples of 3 are 3, 6, 9, 12, ________

These numbers are in A.P. such that a = 3, d = 6 − 3 = 3,

n = 8

$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_8 = \frac{8}{2}[2 \times 3 + (8 - 1) \times 3]$

$\Rightarrow \quad S_8 = 4[6 + 7 \times 3] = 4[6 + 21] = 4 \times 27 = 108$

$\therefore \quad S_8 = 108$

63. Leta be the first term and d be the common difference of A.P.

Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Here, $a = 18, d = 16 - 18 = -2$

$\therefore \ \frac{n}{2}[2(18) + (n-1)(-2)] = 0$

⇒ n[18-n+1]=0⇒n=19

:- Sum of 19 terms of the A.P. is zero.

(n+0)

64. Leta be the first term and d be the common difference of A.P.

Sum of n terms is given as $S_n = \frac{n}{2}[2a + (n-1)d]$

Here, $a = 27, d = 24 - 27 = -3$

According to question,

$0 = \frac{n}{2}[2(27) + (n-1)(-3)]$

$\Rightarrow \quad n[54 - 3n + 3] = 0$

$\Rightarrow \quad 3n = 57 \ \Rightarrow \ n = 19 \qquad (\because \ n \neq 0)$

65. Leta be the first term and d be the common difference of the A.P.

$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$

According to question, $S_5 + S_7 = 167$

$\Rightarrow \ \frac{5}{2}[2a + (5-1)d] + \frac{7}{2}[2a + (7-1)d] = 167$

$\Rightarrow \ \frac{5}{2}(2a + 4d) + \frac{7}{2}(2a + 6d) = 167$

$\Rightarrow \quad 5a + 10d + 7a + 21d = 167$

$\Rightarrow \quad 12a + 31d = 167 \qquad \text{...(i)}$

Also, $S_{10} = 235$

$\Rightarrow \ \frac{10}{2}[2a + (10-1)d] = 235$

$= 5(2a + 9d) = 235 \Rightarrow 2a + 9d = 47$
Multiplying both sides by 6, we get
$12a + 54d = 282$ ...(ii)
Subtracting (i) from (ii), we get
23d 115 d=5
Substituting the value of d in (i), we get
5 167
12a+31
$= 12a + 155 = 167$
$= 12a=12 \Rightarrow a=1$
Hence, the A.P. will be 1, 6, 11,...

66. Let a and d denote first term and common difference respectively of the A.P.
Given, a = 7 and /= 49 = a + (n-1) d
$= 49=7+ (n-1)d \Rightarrow (n - 1)d = 42$ ...(i)

$$S_n = \frac{n}{2}[7+49] = 420 \Rightarrow \frac{n}{2}(56) = 420 \Rightarrow n = \frac{420}{28} = 15$$

Putting $n = 15$ in (i), we get
$14d = 42 \Rightarrow d = 3$

67. Let a and d denote the first term and common difference respectively of the A.P.
Given, a = 8 and /= 65 = a + (n − 1)d
= 65=8+ (n-1)d 57 = (n - 1)d ...(i)

$$S_n = 730 \Rightarrow \frac{n}{2}(a+l) = 730$$

$$\Rightarrow \quad n[8+65] = 1460 \Rightarrow n = \frac{1460}{73} = 20$$

Putting value of n in (i), we get 57 = (20 - 1)d
$= 57 = 19d \Rightarrow d = 3$

68. We have, $S_1 = 3n^2 + 6n$
:- $S_{n-1}=3(n - 1)^2 + 6(n - 1) = 3(n^2 + 1\text{-}2n)+6n\text{-}6$
$=3n2+3-6n+6n-6=3n^2 - 3$
nth term of A.P., $a_n = S_n - S_{n-1}$
$= (3n^2+6n) - (3n^2 - 3) = 6n+3$

69. We have, $S_1 = 5n-n^2$

:- $S_{n-1}=5(n-1)-(n-1)^2$

$= 5n-5-(n^2 + 1-2n) = -n^2 + 7n - 6$

nth term of A.P., $a_n = S_n - S_{n-1}$

$=5n-n^2 - (-n^2 + 7n-6)$

$=5n-n^2+n^2-7n+6=-2n+6$

70. We have, $S_1 = 4n^2 + 2n$

:- $S_{n-1}=4(n-1)^2 + 2(n-1)$

$= 4(n^2 + 1 - 2n) +2n-2$

$= 4n^2+4 - 8n + 2n - 2 = 4n^2 - 6n+2$

n^{th} term of the A.P., an Sn - Sn-1

$= (4n^2 + 2n) - (4n^2 - 6n+2) = 8n - 2$

71. Here, a = 15 and S15 = 750

$\because \quad S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore \quad S_{15} = \frac{15}{2}[2\times15+(15-1)d]=750$

$\Rightarrow 15(15+7d) = 750$

$\Rightarrow 15+ 7d = 50$

$\Rightarrow 7d=35$

$\Rightarrow d=5$

$\Rightarrow d = 5$

Now, 20^{th} term = a + (n - 1)d

= 15+ (20-1) 5

= 15+ 95

= 110

72. Total amount of loan Rohan takes = ₹ 1,18,000

First instalment paid by Rohan = 1000

Second instalment paid by Rohan = 1000+ 100 = 1100

Third instalment paid by Rohan = 1100 + 100 = ₹ 1200 and so on.

Let its 30^{th} instalment be n.

Thus, we have 1000, 1100, 1200, ..., which forms an A.P.

with first term (a) = 1000

and common difference (d) = 1100 - 1000 = 100

n^{th} term of an A.P., $a_n = a + (n - 1)d$

For 30^{th} instalment, a_{30} = a + (30-1)d
= 1000+ (29)100 = 1000+ 2900 = 3900
So, 3900 will be paid by Rohan in the 30^{th} instalment.
Now, we have a = 1000, last term (/) = 3900

$$\therefore \quad \text{Sum of 30 instalments, } S_{30} = \frac{30}{2}[a+l]$$

S3015(1000+ 3900) = 73500
Total amount he still have to pay after the 30th instalment
= (Amount of loan) - (Sum of 30 instalments)
=1,18,000-73,500 = 44,500
Hence, 44,500 still have to pay after the 30th instalment.

73. Given, a4 = -15 and a, = -30
:= a + 3d = -15 ...(i) [ana+(n - 1)d]
a + 8d = -30 ...(ii)
On subtracting (ii) from (i), we have
-5d = 15 ⇒ d = -3
Put d = -3 in (i), we have
a+3(-3)=-15 ⇒ a − 9 = −15 ⇒ a= −6

$$\text{Now, } S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow S_{16} = \frac{16}{2}[2(-6)+(16-1)(-3)]$$

$$= 8[2(-6) + (15)(-3)] = 8[-12 - 45] = -456$$

74. Given, d= 3, a = 54 and a_n = 0
Since an = a_n + (n-1)d
:- 0=54+ (n-1) (-3)=0=54-3n+3=3n=57
⇒ n=19

$$\text{Now, } S_n = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{19}{2}[2\times 54+(19-1)(-3)]$$

$$= \frac{19}{2}[108-54] \quad = \frac{19}{2}\times 54 = 513$$

75. Let S_n = (-5) + (-8) + (- 11) + ... + (-230)
Here, a = - 5,

Now, - 8 -(-5)= - 8+5 = -3
-11-(-8)=-11+8=-3
So, common difference, d = -3 and last term, a_n = -230
Since, a_n = a + (n - 1)d
= -230-5+ (n-1) (-3)
= -230-5-3n+3-230= - 3n-2

$$\Rightarrow \quad 3n = 228 \quad \Rightarrow \quad n = \frac{228}{3} = 76$$

By using sum formula, we have

$$S_{76} = \frac{76}{2}[2(-5) + (76-1)(-3)]$$
$$= 38[-10 + 75(-3)] = 38[-10 - 225] = -8930$$

76. Given, a = 5, d = 3 and a_n = 50
Since, a_n = a + (n - 1)d
50=5+ (n-1)350=5+ 3n-3

$$\Rightarrow \quad 3n = 50 - 2 = 48 \Rightarrow n = \frac{48}{3} = 16$$

Now, sum of n terms $(S_n)=\frac{n}{2}[2a+(n-1)d]$

$$= \frac{16}{2}[2 \times 5 + (16-1)3] = 8[10 + 15 \times 3]$$
$$= 8[10 + 45] = 8 \times 55 = 440$$

77. Let a be first term and d be the common difference of an A.P. Since, wehave,

$$a_m = a+(m-1)d = \frac{1}{n} \qquad ...(i)$$

$$a_n = a+(n-1)d = \frac{1}{m} \qquad ...(ii)$$

Subtracting (ii) from (i), we get

$$(m-1)d-(n-1)d = \frac{1}{n} - \frac{1}{m}$$

$$\Rightarrow d(m-n) = \frac{m-n}{mn}$$

$$\Rightarrow d = \frac{1}{mn} \qquad ...(iii)$$

On substituting (iii) in (i), we get

$$a + (m-1) \times \frac{1}{mn} = \frac{1}{n} \Rightarrow a = \frac{1}{n} - \frac{m-1}{mn}$$

$\Rightarrow \quad a=\frac{1}{mn}$

Sum of first mn terms is

$$S_{mn}=\frac{mn}{2}\left[2\left(\frac{1}{mn}\right)+(mn-1)\frac{1}{mn}\right]$$

$$=\frac{mn}{2}\left[\frac{2}{mn}+1-\frac{1}{mn}\right]=\frac{mn}{2}\left[\frac{1}{mn}+1\right]=\frac{1+mn}{2}$$

78.

We have, $\left(4-\frac{1}{n}\right)+\left(4-\frac{2}{n}\right)+\left(4-\frac{3}{n}\right)+.....$ which forms an A. P. where first term $(a)=\left(4-\frac{1}{n}\right)$

Common difference $(d)=\left(4-\frac{2}{n}\right)-\left(4-\frac{1}{n}\right)=-\frac{1}{n}$

and last term $(l)=\left(4-\frac{n}{n}\right)=(4-1)=3$ ($\because$ Series has n terms)

$\therefore$ Sum of n terms $(S_n)=\frac{n}{2}(a+l)$

$=\frac{n}{2}\left(4-\frac{1}{n}+3\right)=\frac{n}{2}\left(7-\frac{1}{n}\right)=\frac{7n}{2}-\frac{1}{2}=\left(\frac{7n-1}{2}\right)$

79. Let a, n and d be first term, number of terms and common difference of the A.P. respectively.
We have, first term (a) = 5; last term (1) = a_n = 45
Sum of all terms (S_n) = 400

$\Rightarrow \quad \frac{n}{2}(a+l)=400 \Rightarrow \frac{n}{2}(5+45)=400 \Rightarrow n=\frac{800}{50}=16$

Now, $a_n = a + (n-1)d$

$\Rightarrow \quad 45 = 5 + (16-1)d$

$\Rightarrow \quad 40 = 15d \Rightarrow d=\frac{40}{15}=\frac{8}{3}$

80. Let a be the first term and d be the common difference of the A.P.

Sum of n terms, $S_n = \frac{n}{2}[2a+(n-1)d]$

We have, $S_7 = 49 \Rightarrow \frac{7}{2}[2a+6d]=49$

$\Rightarrow \quad 14a + 42d = 98 \Rightarrow a + 3d = 7$ ...(i)

and $S_{17} = 289 \Rightarrow \frac{17}{2}[2a+16d]=289$

$\Rightarrow \quad 34a + 272d = 578 \Rightarrow a + 8d = 17$ ...(ii)

On solving (i) and (ii), we get $a = 1, d = 2$

$\therefore \quad S_n = \frac{n}{2}[2+(n-1)2]=n^2$

81. Leta be the first term and d be the common difference of A.P.

Sum of n terms, $S_n = \frac{n}{2}[2a+(n-1)d]$

Here, $a = 65, d = 60 - 65 = -5$

$\therefore$ According to question,

$0=\frac{n}{2}[2(65)+(n-1)(-5)] \Rightarrow n[130 - 5n + 5] = 0$

$\Rightarrow \quad 5n = 135 \Rightarrow n = 27 \quad (\because n \neq 0)$

82. Let a_1, d_1 and a_2, d_2 be the first term and common difference of the two A.P.'s respectively.

$$\frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{7n+1}{4n+27} \quad \text{[Given]}$$

$$\Rightarrow \frac{a_1+\frac{(n-1)}{2}d_1}{a_2+\frac{(n-1)}{2}d_2} = \frac{7n+1}{4n+27}$$

Put $\frac{n-1}{2}=m-1 \Rightarrow n-1=2m-2$

$\Rightarrow \quad n = 2m - 2 + 1 = 2m - 1$

$$\therefore \quad \frac{a_1+(m-1)d_1}{a_2+(m-1)d_2} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-7+1}{8m-4+27} = \frac{14m-6}{8m+23}$$

83. Let the required three digit number be xyz.

:- Digits are in A.P. . x=y-d and z = y+d [where d is common difference]

According to question,

(y-d)+y+(y+d) = 15

$= 3y=15 \Rightarrow y=5$

Since, number obtained by reversing the digits (z y x) i.e., $100z + 10y + x$ is 594 less than original number.

$= (100x + 10y + z) - (100z + 10y+x) = 594$

$= (z - 100z) + (100x - x) = 594$

$= 99x99z=594 \Rightarrow x-z=6$

$= (y-d) - (y+d) = 6$

$= -2d=6d=-3$

So, $x=y-d=5-(-3)=8$ and $z =y+d=5-3=2$

:- The number is xyz = 852.

84.

We have, $a = 1, d_1 = 1, d_2 = 2$ and $d_3 = 3$

$$S_1 = \frac{n}{2}\{2(1)+(n-1)1\} = \frac{n}{2}\{2+n-1\} = \frac{n}{2}(n+1)$$

$$S_2 = \frac{n}{2}\{2(1)+(n-1)2\} = \frac{n}{2}(2+2n-2) = n^2$$

$$S_3 = \frac{n}{2}\{2(1)+(n-1)3\} = \frac{n}{2}\{2+3n-3\} = \frac{n(3n-1)}{2}$$

Now, $S_1 + S_3 = \frac{n}{2}(n+1) + \frac{n}{2}(3n-1)$

$$= \frac{n}{2}(n+1+3n-1) = \frac{n}{2} \times 4n = 2n^2 = 2S_2$$

85.

We have, $a = 5, d_1 = 2, d_2 = 4, d_3 = 6$

$$\therefore\ S_1 = \frac{n}{2}[10+(n-1)2] = n(5+n-1) = n(n+4)$$

$$S_2 = \frac{n}{2}[10+(n-1)4] = n(5+2n-2) = n(2n+3)$$

$$S_3 = \frac{n}{2}[10+(n-1)6] = n(5+3n-3) = n(3n+2)$$

Now, $S_1 + S_3 = n(n + 4) + n(3n + 2)$

$= n[n + 4 + 3n + 2] = n[4n + 6]$

$= 2n(2n + 3) = 2S_2$

Hence proved.

86. Leta be the first term and d be the common difference of the A.P.

$\therefore \quad S_{12} = \frac{12}{2}\{2a+(12-1)d\}$ $\qquad \left[\because S_n = \frac{n}{2}\{2a+(n-1)d\}\right]$

$= 6\{2a + 11d\} = 12a + 66d$

$S_8 = \frac{8}{2}\{2a+(8-1)d\} = 4\{2a + 7d\} = 8a + 28d$

$S_4 = \frac{4}{2}\{2a+(4-1)d\} = 2\{2a + 3d\} = 4a + 6d$

Now, $3(S_8 - S_4) = 3(8a + 28d - 4a - 6d)$

$= 3(4a + 22d) = 12a + 66d = S_{12}$

87.

We have, $S_n = \frac{1}{2}(3n^2 + 7n)$

$S_{n-1} = \frac{1}{2}\{3(n-1)^2 + 7(n-1)\} = \frac{1}{2}\{3(n^2 - 2n + 1) + 7n - 7\}$

$= \frac{1}{2}\{3n^2 - 6n + 3 + 7n - 7\} = \frac{1}{2}\{3n^2 + n - 4\}$

We know that, $a_n = S_n - S_{n-1}$

$= \frac{1}{2}(3n^2 + 7n) - \frac{1}{2}(3n^2 + n - 4)$

$= \frac{1}{2}(3n^2 + 7n - 3n^2 - n + 4) = \frac{1}{2}(6n + 4)$

$\Rightarrow \quad a_n = 3n + 2$

$\therefore \quad a_{20} = 3 \times 20 + 2 = 60 + 2 = 62$

88. Leta be the first term and d be the common difference of the A.P.

Now, sum of n terms, $S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore \quad S_{30} = \frac{30}{2}[2a+(30-1)d] = 15[2a+29d] = 30a + 435d$

$S_{20} = \frac{20}{2}[2a+(20-1)d] = 10[2a+19d] = 20a + 190d$

$S_{10} = \frac{10}{2}[2a+(10-1)d] = 5[2a+9d] = 10a + 45d$

$\therefore \quad 3[S_{20} - S_{10}] = 3[20a + 190d - 10a - 45d]$

$= 30a + 435d = S_{30}$

89. Leta be the first term and d be the common difference
of the A. P... $a_n = a + (n-1)d$

Here, $a_{14} = 2 \times a_8$
$\Rightarrow$ a+(14-1)d = 2[a + (8 − 1)d]
$\Rightarrow$ a + 13d = 2a + 14d = a+d=0 ...(i)
Also, a_6=-8=> a+5d = -8 ...(ii)
Subtracting (i) from (ii), we get
4d-8d=-2
From equation (i), we have a = 2

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore\ S_{20} = \frac{20}{2}[2\times2+19(-2)] = 10[4-38] = -340$

90. Leta be the first term and d be the common difference
of the A.P :- a_n = a + (n-1)d
Here, a_{16} = 5x a_3
= a+(16-1)d = 5[a + (3-1)d]
= a+15d5a+ 10d

$\Rightarrow\quad 4a = 5d \quad\Rightarrow\quad a = \frac{5d}{4}$

Also, $a_{10} = a + 9d = 41$

$\Rightarrow\ 41 = \frac{5d}{4} + 9d \Rightarrow 41 = \frac{5d+36d}{4} \ \Rightarrow\ d = 4$

$\therefore\ a = \frac{5\times4}{4} = 5$

Hence, A.P. is 5, 9, 13, ...

Now, sum of n terms, $S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore\ S_{15} = \frac{15}{2}[2\times5+14\times4] = \frac{15}{2}[10+56] = \frac{15}{2}\times66 = 495$

91. Leta be the first term and d be the common difference of the A.P.
nth term of an A.P., $a_1 = a + (n-1)d$

Given, $a_{13} = 4 \times a_3$

$\Rightarrow \quad a + 12d = 4(a + 2d) \Rightarrow a + 12d = 4a + 8d$

$\Rightarrow \quad 3a - 4d = 0$ ...(i)

Also, $a_5 = 16$

$\Rightarrow \quad a + 4d = 16$ ...(ii)

Adding (i) and (ii), we get

$4a = 16 \Rightarrow a = 4$

From (ii), we have

$4 + 4d = 16 \Rightarrow 4d = 12 \Rightarrow d = 3$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{10} = \frac{10}{2}[2 \times 4 + (10-1)3] = 5[8 + 27] = 5 \times 35 = 175$

92. Let the first term be a and d be the common difference of the A.P.

Given, a12-13 a + 11d=-13 ...(i)

Also, S4 = 24

$\Rightarrow \quad \frac{4}{2}[2a + 3d] = 24 \qquad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow \quad 2a + 3d = 12$ ...(ii)

Multiply (i) by 2 and then subtracting (ii) from it, we get

$19d = -38$

$\Rightarrow \quad d = -2$

$\therefore$ Put $d = -2$ in (i), we have

$a + 11(-2) = -13$

$\Rightarrow \quad a = -13 + 22 \quad \Rightarrow \quad a = 9$

$\therefore \quad S_{10} = \frac{10}{2}[2(9) + 9(-2)] = \frac{10}{2}[18 - 18] = 0$

93. Let first term be a and common difference be d. Given, a10 = -37

a + 9d = -37 ...(i)

Also, $S_6 = -27 \Rightarrow \frac{6}{2}(2a + 5d) = -27$

$\Rightarrow \quad 2a + 5d = -9$ ...(ii)

Multiply (i) by 2 and then subtracting (ii) from it, we get

$13d = -65 \Rightarrow d = -5$

Put $d = -5$ in (i), we get,

$a + 9(-5) = -37 \Rightarrow a - 45 = -37 \Rightarrow a = 8$

$\therefore$ Sum of first eight terms, $S_8 = \frac{8}{2}[2a + 7d]$

$= 4[2(8) + 7(-5)] = 4[16 - 35] = 4(-19) = -76$

94. Given, sum of first seven terms of an A.P.,. = , S7 = 182

$$i.e., 182=\frac{7}{2}[2a+(7-1)d]$$

$$\Rightarrow \quad 364 = 14a + 42d \Rightarrow 26 = a + 3d \qquad ...(i)$$

$$\text{Also, } \frac{a_4}{a_{17}}=\frac{1}{5} \Rightarrow \frac{a+3d}{a+16d}=\frac{1}{5}$$

= 5(a+3d) = a + 16d5a+ 15d = a + 16d

= 4a-d=0 d = 4a ...(ii)

Substituting (ii) in (i), we get

26 = a + 3(4a) ⇒ 13a=26 ⇒ a = 2

:- d=4(2) = 8

Hence, the A.P. is formed as 2, 10, 18, ...

95. Leta be the first term and d be the common difference of the A.P.

Given, S, 63 and S14 = 63 + 161 = 224

$$S_7=63 \Rightarrow \frac{7}{2}[2(a)+(7-1)d]=63 \qquad \left[\because S_n=\frac{n}{2}\{2a+(n-1)d\}\right]$$

$$\Rightarrow \frac{7}{2}(2a+6d)=63 \Rightarrow a+3d=9 \qquad ...(i)$$

$$\text{Also, } \frac{14}{2}[2a+(14-1)d]=224$$

$$\Rightarrow \quad 7(2a+13d)=224$$

$$\Rightarrow \quad 2a+13d=32 \qquad ...(ii)$$

Solving (i) and (ii), we get

$a = 3, d = 2$

Now, $a_{28} = a + 27d = 3 + 27(2) = 57$

96.

Given, $\frac{a_{11}}{a_{17}}=\frac{3}{4}$

$\Rightarrow \quad \frac{a+10d}{a+16d}=\frac{3}{4}$

$\Rightarrow \quad 4(a+10d)=3(a+16d)$

$\Rightarrow \quad 4a+40d=3a+48d$

$\Rightarrow \quad 4a+40d-3a-48d=0 \Rightarrow a=8d$

Also, $\frac{a_5}{a_{21}}=\frac{a+4d}{a+20d}$

$=\frac{8d+4d}{8d+20d} \qquad (\because a=8d)$

$=\frac{12d}{28d}=\frac{3}{7}$ *i.e.*, 3 : 7

Required ratio $=\frac{S_5}{S_{21}}$

$=\frac{\frac{5}{2}[2a+(5-1)d]}{\frac{21}{2}[2a+(21-1)d]} = \frac{5[2a+4d]}{21[2a+20d]}$

$=\frac{5[2(8d)+4d]}{21[2(8d)+20d]} \qquad (\because a=8d)$

$=\frac{5\times 20d}{21\times 36d} = \frac{25}{189}$ *i.e.*, 25 : 189

97. Number of logs in 1st row = 22
Number of logs in 2nd row = 21
Number of logs in 3rd row = 20
The number of logs i.e., 22, 21, 20,...., forms an A.P., where
a = 22,d=a_2-a_1 = 21-22=-1
Let the number of rows be n.

Now, $S_n = \frac{n}{2}[2(22)+(n-1)(-1)]$

$\Rightarrow \quad 250 = \frac{n}{2}[44-(n-1)]$ $\quad [\because \; S_n = 250 \text{ (Given)}]$

$\Rightarrow \quad 250 \times 2 = 44n - n(n-1)$

$\Rightarrow \quad 500 = 44n - n^2 + n \Rightarrow n^2 - 45n + 500 = 0$

$\Rightarrow \quad (n-20)(n-25) = 0 \Rightarrow n = 20$ or $n = 25$

$T_n = 0 \Rightarrow a + (n-1)d = 0$

$\Rightarrow \quad 22 + (n-1)(-1) = 0 \Rightarrow 22 - (n-1) = 0$

$\Rightarrow \quad 22 - n + 1 = 0 \Rightarrow n = 23$

$\Rightarrow \quad n = 23$ *i.e.*, 23^{rd} term becomes 0.

$\therefore \quad n = 25$ is not required.

$\therefore$ Number of rows = 20

Now, $T_{20} = a + (20-1)d$

$= 22 + 19(-1)$

$= 22 - 19 = 3$

$\therefore$ Number of logs in the 20^{th} (top) row is 3.

98. We have, 1 + 4 + 7 + 10 + ... + x = 287

It is an A.P. with a = 1, d=4-1=3

Let n be the number of terms.

$\therefore \quad \frac{n}{2}[2a + (n-1)d] = 287$

$\Rightarrow \quad \frac{n}{2}[2 + 3(n-1)] = 287 \Rightarrow n(2 + 3n - 3) = 574$

$\Rightarrow n(3n-1)=5743n^2-n-574=0$

$\Rightarrow 3n^2-42n+41n - 574 = 0$

$\Rightarrow 3n(n-14) + 41(n-14) = 0$

$\Rightarrow (n-14)(3n+41) = 0$

$\Rightarrow n = 14$

Now, x = a + (n-1)d = 1 + 13 x 3 = 40.

99. Odd numbers between 0 and 50 are 1, 3, 5, 7,....49

a = 1, d=3-1=2, $a_n = 49$

$a_n = a + (n-1)d$

$49=1+ (n-1)(2) \Rightarrow 49 = 1 + 2n - 2$

$\Rightarrow 492n-12n=50\Rightarrow n=25$

$$\text{Now, } S_n = \frac{n}{2}(a+l) \Rightarrow S_n = \frac{25}{2}(1+49) = \frac{25}{2} \times 50 = 625$$

100. Given, A.P. is 45, 39, 33.....

a = 45, d = 39-45=-6

S_1 = 180

$$\therefore S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow 180 = \frac{n}{2}[2\times45+(n-1)(-6)] \Rightarrow 180 = \frac{n}{2}[90-6n+6]$$

$$\Rightarrow 180 = \frac{n}{2}(96-6n) \Rightarrow 180 = \frac{n}{2}\times2(48-3n)$$

⇒ 180= n(48 - 3n) 180 = 48n - $3n^2$

⇒ $3n^2$ - 48n + 180 = 0

⇒ $3(n^2$ - 16n+60) = 0

⇒(n-6) (n-10)=0⇒n=6 or 10

The sum of given A.P. remains same either we are taking n = 6 or n = 10 as they are repeated with reverse sign.

101. Let a, n and d be the first term, number of terms and common difference of the A.P. respectively.

Given, first term, a = 3, last term, a_n = 83

Also, sum of all terms, S_n = 903 (Given)

$$\Rightarrow \frac{n}{2}[a+l] = 903 \Rightarrow \frac{n}{2}[3+83] = 903$$

$$\Rightarrow 43n = 903 \Rightarrow n = 21$$

$$\text{Now, } a_n = a + (n-1)d$$

$$\Rightarrow 83 = 3 + (21-1)d$$

$$\Rightarrow 80 = 20d \Rightarrow d = 4$$

102. Two digit numbers which leave the remainder 2, when divided by 5 are 12, 17, 22, 27, ..., 92, 97.

This is an A.P. with first term, a = 12, common difference, d = 5 and last term, /= 97

Here, a_n = 97a+ (n-1)d = 97

12+ (n-1)5 = 97 ⇒ 12+5n-5=97

⇒ 5n=90⇒ n = 18

Now, $S_n = \frac{n}{2}(a+l)$

$\therefore \quad S_{18} = \frac{18}{2}(12+97) = 981$

103. Let a be the first term and d be the common difference of the given A.P. Then, the sums of m terms and n terms are respectively given by

$$S_m = \frac{m}{2}\{2a+(m-1)d\} \text{ and } S_n = \frac{n}{2}\{2a+(n-1)d\}$$

Also, $\frac{S_m}{S_n} = \frac{m^2}{n^2}$ (Given)

$$\Rightarrow \frac{\frac{m}{2}\{2a+(m-1)d\}}{\frac{n}{2}\{2a+(n-1)d\}} = \frac{m^2}{n^2} \Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$$

$$\Rightarrow \{2a + (m - 1)d\}n = \{2a + (n - 1)d\}m$$

$$\Rightarrow 2a(n - m) = d\{(n - 1)m - (m - 1)n\}$$

$$\Rightarrow 2a(n - m) = d(n - m) \Rightarrow d = 2a$$

$$\therefore \frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)2a}{a+(n-1)2a} = \frac{2m-1}{2n-1}$$

104. Let a be the first term and d be the common difference of the A.P. Sum of m and n terms of A.P. are

$S_m = \frac{m}{2}[2a+(m-1)d]$

and $S_n = \frac{n}{2}[2a+(n-1)d]$

Given that $S_m = S_n$

$$\therefore \quad \frac{m}{2}[2a+(m-1)d] = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow 2a(m - n) + \{m(m - 1) - n(n - 1)\}d = 0$$

$$\Rightarrow 2a(m - n) + \{(m^2 - n^2) - (m - n)\}d = 0$$

$$\Rightarrow (m - n)\{2a + (m + n - 1)d\} = 0$$

$$\Rightarrow 2a + (m + n - 1)d = 0 \qquad [\because m - n \neq 0] \quad ...(i)$$

$$\text{Now, } S_{m+n} = \frac{m+n}{2}[2a+(m+n-1)d]$$

$$= \frac{m+n}{2}[0] = 0 \quad \text{[Using (i)]}$$

105. Let a1, a2 be the first terms and d_1, d_2 be common differences of the two A.P.'s respectively.

Given, ratio of sum of first n terms of two A.P. = $\frac{7n+1}{4n+27}$

$$\therefore \quad \frac{\frac{n}{2}\{2a_1+(n-1)d_1\}}{\frac{n}{2}\{2a_2+(n-1)d_2\}} = \frac{7n+1}{4n+27} \qquad \text{...(i)}$$

$$\Rightarrow \quad \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \quad \frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_2+\left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+27}$$

Putting $\frac{n-1}{2} = 8$, we get

$$\frac{a_1+8d_1}{a_2+8d_2} = \frac{7(17)+1}{4(17)+27} \qquad \left\{\because \frac{n-1}{2} = 8 \Rightarrow n = 17\right\}$$

$$\Rightarrow \quad \frac{a_1+8d_1}{a_2+8d_2} = \frac{120}{95} = \frac{24}{19}$$

$$\therefore \quad \text{Ratio of } 9^{th} \text{ terms} = \frac{a_1+8d_1}{a_2+8d_2} = \frac{24}{19}$$

106. Let the total number of minutes be n.
:- Distance covered (in m) by thief = 100n
Distance covered (in m) by policeman
= 100 + 110 + 120+ ... + (n - 1) terms
According to question,

$$100n = \frac{n-1}{2}[200+(n-2)10]$$

$100n = 5n^2 + 105n - 10n - 210$
5n25n-210 = 0

n^2-n-42=0
(n-7)(n+6)=0⇒n=7 [∴ n≠-6]
The policeman will catch the thief after 5 minutes.

108.

$$\underbrace{1,2,3,....,X-1}_{S}, X, \underbrace{X+1,.....,49}_{S'}$$

$$S = 1 + 2 + 3 + ... + (X - 1)$$

$$= \left(\frac{X-1}{2}\right)[2+X-2] = \left(\frac{X-1}{2}\right)(X)$$

$$S' = (X + 1) + (X + 2) + ... + 49$$

$$= \left(\frac{49-X}{2}\right)(X+1+49) = \frac{49-X}{2}(X+50)$$

For $S = S'$, we have

$$X^2 - X = 49X + 49 \times 50 - X^2 - 50X$$

$$\Rightarrow \quad 2X^2 = 49 \times 50 \Rightarrow X^2 = 49 \times 25$$

$$\therefore \quad X = 35$$

109. Amount saved by Reshma in first month = *450
Amount saved by her in second month =(450+20) = 470
Continuing in this manner, we have following A.P. as 450,
470, 490,.....
Here, a = 450, d = 20

$$\therefore S_{12} = \frac{12}{2}[2\times450+(12-1)20]$$

$$= 6[900 + 220] = 6 \times 1120 = 6720$$

Hence, Reshma will save ₹6720 in next 12 months. Yes, she will be able to send her daughter to the school next year.
So, we observe that small and regular savings can minimize
the problems in our daily life.

110. Saving of first week=100
Saving of second week = 100+₹20 = ₹120
Saving of third week = 120+20=*140
So, 100, 120, 140,........ [forms an A.P.]
Here, a = 100 and d = 120-100 = 20, n = 12

$\therefore\ S_{12} = \frac{12}{2}\{2\times100+(12-1)20\}$ $\left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$

$= 6\{200 + 220\} = 2520$

Since 2520 > 2500

So, she would be able to send her daughter to school after 12 weeks. Values generated are awareness, responsibility and we learnt that small savings can fulfill our big desires.

111. Here, a = 8, d= 10-8=2, n = 60

$a_n = a + (n-1)d$

:- 9_{60}=8+(601) × 2=8+59×2=126

10^{th} term from the last = $(60\ 10 + 1)^{th}$ term from the beginning = 51^{st} term from the beginning

9_{51}=a+(511)d = 8+ 50 x 2 = 108

Sum of n term, $S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore$ Sum of last 10 terms = $\frac{10}{2}[2a_{51}+(10-1)d]$

$= 5[2 \times 108 + 9 \times 2] = 5 \times 234 = 1170$

112. Here, a = 5, d = 12-5=7, $a_n = a + (n-1)d$

:- Last term = a_{50} = 5+ (50-1)x7=5+49×7

=5+343 348

15^{th} term form the last = $(50 - 15+ 1)^{th}$ term from the beginning = 36^{th} term from the beginning

:- a_{36} = a+ (36-1)d = 5 + 35 x 7 = 250

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore$ Sum of last 15 terms $= \frac{15}{2}[2a_{36}+(15-1)d]$

$= \frac{15}{2}[2\times250+14\times7] = \frac{15}{2}\times598 = 4485$

113. The three digit numbers which leave a remainder 3, when divided by 4 are

103, 107, 111,....,999

It forms an A. P. with a = 103 and d = 107 - 103 = 4

Last term, a_n, 999
= 999 103+ (n-1)4 (:- $a_n = a + (n-1)d$)
= 999 = 103 + 4n – 4

$$\Rightarrow \quad 999 = 99 + 4n \Rightarrow n = \frac{900}{4}$$

$$\Rightarrow \quad n = 225, \text{ an odd number}$$

$$\therefore \quad \text{Middle term} = \left(\frac{n+1}{2}\right)^{th} \text{term} = \left(\frac{225+1}{2}\right)^{th} \text{term}$$

= 113th term
:- a113 = a + 112d = 103 + 112 x 4
= 103 +448 = 551

$$\text{We know that, } S_n = \frac{n}{2}\{2a+(n-1)d\}$$

$$\therefore \quad S_{112} = \frac{112}{2}\{2\times103+(112-1)4\}$$

= 56{1110 +444} = 56 × 1554 = 87024

114. Numbers between 9 and 95 which leaves a remainder 1 when divided by 3 are 10, 13, 16, 19,...., 94.
It forms an A.P. with first term, a = 10 and common difference, d = 13-10=3
Now, a_n=94 = a+ (n − 1)d = 94
= 10+ (n-1)3 = 94 = (n-1)3 = 84

$$\Rightarrow \quad n-1 = \frac{84}{3} \quad \Rightarrow \quad n = 29 \text{ (odd number)}$$

$$\text{So, middle term} = \left(\frac{29+1}{2}\right)^{th} \text{term} = 15^{th} \text{ term}$$

$$\therefore \quad a_{15} = a + 14d = 10 + 14 \times 3 = 52$$

$$\text{We know that, } S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\therefore \quad \text{Sum of first 14 terms} = \frac{14}{2}[2\times10+(14-1)3]$$

$$= 7[20 + (13 \times 3)] = 7 \times 59 = 413$$

For last 14 terms, first term is $a_{16} = a + 15d$
$= 10 + 15(3) = 55$

$$\therefore \quad \text{Sum of last 14 terms} = \frac{14}{2}[2\times55+(14-1)3]$$

$$= 7[110 + (13 \times 3)] = 7 \times 149 = 1043$$

115. All three-digit numbers which leave a remainder 5, when divided by 7 are 103, 110, 117,...., 999.
It forms an A.P. with first term, a = 103 and
d=110-103 = 7
Now, an = 999
= 103+ (n-1)7 = 999
= (n - 1)7999 - 103 = 896

$\Rightarrow \quad n-1=\frac{896}{7}=128$

$\Rightarrow \quad n = 129$, which is an odd number

$\therefore \quad \text{Middle term}=\left(\frac{n+1}{2}\right)^{th} \text{term} =\left(\frac{129+1}{2}\right)^{th} \text{term}$

$= 65^{th}$ term

$a_{65} = a + 64d = 103 + 64 \times 7 = 103 + 448 = 551$

We know that, $S_n=\frac{n}{2}\{2a+(n-1)d\}$

$\therefore$ Sum of first 64 terms $=\frac{64}{2}[2\times103+(63\times7)]$

$= 32(206 + 441) = 32 \times 647 = 20704$

For sum of last 64 terms, *i.e.*, first term = a_{66}

$\Rightarrow \quad a_{66} = 103 + 65 \times 7 = 558$

$\therefore$ Sum of last 64 terms $=\frac{64}{2}[2\times558+63\times7]$

$= 32[1116 + 441] = 32\times 1557 = 49824$

116. Let a and d be the first term and the common difference of the A. P., respectively.

Given, sum of the first 10 terms $=\frac{10}{2}[2a+9d]$

⇒ 210=5[2a + 9d] => 42 = 2a + 9d ...(i)
15^{th} term from the last = $(50 - 15+ 1)^{th}$ term from the beginning = 36^{th} term from the beginning
Now, a36 = a + 35d

$\therefore \quad$ Sum of the last 15 terms $=\frac{15}{2}[2(a+35d)+14d]$

$= 2565 = 15[a + 42d] \Rightarrow 171 = a + 42d$ ...(ii)
Solving (i) and (ii), we get d = 4, a = 3
Thus, A.P. is 3, 7, 11, 15, ...

117. Number of trees planted by 1st class
= 1x2+1x2=4
Number of trees planted by 2nd class = 2×2+2×2=8
Number of trees planted by 3rd class = 3x2+3x2 = 12
Number of trees planted by 12th class = 12×2 + 12×2=48
This will form an A.P. 4, 8, 12,...., 48 with first term a = 4,
common difference, d = 8-4 = 4
:- Number of plants planted are

$$S_{12} = \frac{12}{2}[2(4)+(11)4] = 6[8+44] = 6(52) = 312$$

Hence, 312 plants were planted by the students. Value shown is that we should take care of our environment.

CBSE Sample Questions

1. We have, a = 27, d = -3 and $a_n = 0$.
Since, $a_n = a + (n - 1)d$
$= 0=27+ (n-1) (-3)$ (1/2)
$\Rightarrow 30=3n$
n = 10th Thus, 10th term of A.P. will be zero. (1/2)

2. Since $a_n = a + (n-1)d$
$\Rightarrow 4=a+6\times(-4)$ [:d=-4, n = 7, $a_n = 4$ (Given)] (1/2)
$\Rightarrow a = 28$ (1/2)

3. Here, a (first term) = 6,
d (common difference) = 9 - 6=3
Using $a_n = a + (n - 1)d$
So, $a_{25} = 6+24(3) = 78$ (1)
$\Rightarrow a_{15} = 6+14(3) = 48$
$\Rightarrow a_25-a_{15} = 78-48=30$ (1)

4. Given that, $7a7=5a_5$
$\Rightarrow 7(a + 6d)=5(a + 4d)$
$\Rightarrow 2a +22d=0 \Rightarrow a+11d=0 \Rightarrow a_{12}=0$

5. (i) Since each row is increasing by 10 seats, so it is an
A.P. with first term a = 30, and common difference d = 10.
In an A.P. n^{th} term a_n is
$a_n = a + (n-1)d$,
where, a is first term, n is number of terms, d is common difference.
So, number of seats in 10^{th} row = $a_{10} = a + 9d$
$= 30 + 9 \times 10 = 120$ (1)

(ii) $S_n = \frac{n}{2}(2a + (n-1)d)$ **(1/2)**

$\Rightarrow 1500 = \frac{n}{2}(2 \times 30 + (n-1)10)$

$\Rightarrow 3000 = 50n + 10n^2$
$\Rightarrow n^2 + 5n - 300 = 0$ (1/2)
$\Rightarrow n^2 + 20n - 15n - 300 = 0$
$\Rightarrow (n+20)(n = 15) = 0$
$\Rightarrow n = -20, 15$
:- n = 15
Hence, required number of rows is 15. (1)

OR

No. of seats already put up to the 10^{th} row = S_{10} (1/2)

$S_{10} = \frac{10}{2}\{2 \times 30 + (10-1)10)\}$
$= 5(60 + 90) = 750$

$= 5(60+90) = 750$ (1)
So, the number of seats still required to be put are
1500-750 = 750 (1/2)
(iii) If no. of rows =17
then the middle row is the 9^{th} row
$a_9 = a + 8d$
= 30+80= 110 seats (1)

6. (i) Let n be the number of days to reach his goal.
;- Required A.P. will be: 3000, 3005, 3010, ..., 3900 (1)
Here, a = 3000, d = 3005 - 3000 = 5,1 = 3900
:- $a_n = a + (n-1)d$
$\Rightarrow 3900 = 3000 + (n-1)5$
$900 = 5n - 5 \Rightarrow 5n = 905 \Rightarrow n = 181$

Minimum number of days of practice = n - 1 = 180 days. (1)

(ii) $S_n = \frac{n}{2}(a+l)$ **(1)**

$= \frac{181}{2} \times (3000+3900) = 624450$ push-ups **(1)**