# 4. Quadratic Equations

## (2025)

**1. The discriminant of the quadratic equation x2 3x 2 = 0 is : (1 Mark) (2025)**

(A) 1 (B) 17

(C) $\sqrt{17}$ (D) $-\sqrt{17}$

**2. The equation $x + \frac{1}{x} = 3$ (x 0) is expressed as a quadratic equation in the**

**form of $ax^2 + bx + c = 0$. The value of a b + c is : (1 Mark) (2025)**

(A) 5
(B) 2
(C) 1
(D) 1

**3. The difference of the squares of two positive numbers is 180. The square of the smaller number is 8 times the greater number. Find the two numbers. (5 Mark) (2025)**

**4. Find the value(s) of k for which the equation $2x^2 + kx + 3 = 0$ has real and equal roots. Hence, find the roots of the equations so obtained. (5 Mark) (2025)**

## Answers

**1.** (B) 17

**2.** (A) 5

**3.** Let the smaller number be y and greater number be x.

A.T.Q.

$x^2 - y^2 = 180$
$y^2 = 8x$
$\Rightarrow x^2 - 8x = 180$
$x^2 - 8x - 180 = 0$
$(x - 18)(x + 10) = 0$
$x = 18, x = -10$ (rejected)
∴ The numbers are 18 and 12

**4.** For equal roots; $b^2 - 4ac = 0$
$k^2 - 24 = 0$
$\Rightarrow k = \pm 2\sqrt{6}$
Equations are

$2x^2 + 2\sqrt{6}x + 3 = 0;$ $\quad$ $2x^2 - 2\sqrt{6}x + 3 = 0$

Roots are $x = -\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}};$ $\quad$ $x = \sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}$

## (2024)

1. If the roots of quadratic equation $4x^2-5x+k=0$ are real and equal, then value of k is : (2024)

(a) $\frac{5}{4}$ (b) $\frac{25}{16}$ (c) $-\frac{5}{4}$ (d) $-\frac{25}{16}$

**Answer.** (b) $\frac{16}{25}$

2. A rectangular floor area can be completely tiled with 200 square tiles. If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor.

(i) Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information. (2024)

**Answer.** (i) 200 x 2 = 128 (x + 1)2

(ii) Write the corresponding quadratic equation in standard form. (2024)

**Answer.** $25x^2 = 16x^2 + 32x + 16$

$\Rightarrow 9x^2 - 32x - 16 = 0$

(iii) (a) Find the value of x, the length of side of a tile by factorisation. (2024)

**Answer.** (a) $9x^2 - 32x - 16 = 0$

$\Rightarrow (9x + 4)(x - 4) = 0$

$x \neq \frac{-4}{9}$ so, $x = 4$

3. Solve the quadratic equation for x, using quadratic formula. (2024)

**Answer.**

(b) $x = \frac{32 \pm \sqrt{1024+576}}{18} = \frac{32 \pm 40}{18}$

$x \neq \frac{-4}{9}$ so, $x = 4$

**4. If the roots of equation $ax^2 + bx + c = 0$, a 0 are real and equal, then which of the following relation is true? (2024)**

(a) $a = \frac{b^2}{c}$ (b) $b^2 = ac$ (c) $ac = \frac{b^2}{4}$ (d) $c = \frac{b^2}{a}$

**Answer.**

(c) $ac = \frac{b^2}{4}$

## 4.2 Quadratic Equations

### VSA (1 mark)

1. If the sum of the roots of the quadratic equation $ky^2 - 11y + (k - 23) = 0$ is $\frac{13}{21}$ more than the product of the roots, then find the value of $k$.

*(Term II, 2021-22)*

2. Write the quadratic equation in x whose roots are 2 and -5. (2021 C)

3. If one root of the quadratic equation $2x^2 + 2x + k = 0$ is $-\frac{1}{3}$, then find the value of $k$. *(2019 C)*

4. Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other. (Delhi 2019)

5. Find the value of k for which $x = 2$ is a solution of the equation $kx^2 + 2x - 3=0$.
(A/ 2019)

6. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx-6= 0$, then find the value of k.(2018)

### SAI (2 marks)

8. Find the sum and product of the roots of the quadratic equation $2x^2 - 9x+4=0$. (2023)

9. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other. (AI 2017)

10. If $x = 2/3$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x+ b = 0$, find the values of a and b. (Delhi 2016)

**SA II (3 marks)**

11. Find the value of 'p' for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other. (2023)

12. One root of the quadratic equation $2x^2 - 8x - k = 0$

is $\frac{5}{2}$. Find the value of $k$. Also, find the other root.
*(2021 C)*

## 4.3 Solution of a Quadratic Equation by Factorisation

**MCQ**

13. The roots of the equation $x^2 + 3x - 10 = 0$ are
(a) 2,-5
(b) -2,5
(c) 2,5
(d) -2,-5 (2023)

**SAI (2 marks)**

14. Solve the quadratic equation for x:
$x^2-2ax - (4b^2 - a^2)=0$ (Term II, 2021-22, AI 2015)

15. Solve for x:

$\frac{x+1}{x-1}+\frac{x-2}{x+2}=4-\frac{2x+3}{x-2};\ x\neq 1,-2,2$ *(Delhi 2016)*

16. Solve for x:$\sqrt{2x+9}+x=13$ (AI 2016)

17. Solve for x: $\sqrt{6x+7}-(2x-7)=0$ (AI 2016)

18. A two digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number. (Foreign 2016)

19.

Solve for $x: \frac{1}{x-3}-\frac{1}{x+5}=\frac{1}{6}, x\neq 3,-5$
*(Foreign 2016)*

20.

Solve for $x$ (in terms of $a$ and $b$) :

$$\frac{a}{x-b}+\frac{b}{x-a}=2, x \neq a,b$$ *(Foreign 2016)*

21. Solve the following quadratic equation for x:
$4x^2-4a^2x+(a-b)=0$ (Delhi 2015)

22. Solve the following quadratic equation for x:
$9x^2-6b2x-(a-b)=0$ (Delhi 2015)

23. Solve the following quadratic equation for x:
$4x^2+4bx-(a^2-b^2)=0$ (AI 2015)

24. Solve for x:
$x^2-(\sqrt{3}+1)x+\sqrt{3}=0$ (Foreign 2015)

25. Solve the quadratic equation $2x^2+ax-a^2=0$ for x. (Delhi 2014) (Ev)

SA II (3 marks)

26. Sum of the areas of two squares is 157 $m^2$. If the sum of their perimeters is 68 m, find the sides of the two squares. (2019)

27. A plane left 30 minutes later than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed. (2018)

28. Solve for x:

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0, x \neq 3, -3/2$$ *(Delhi 2016)*

29. Solve the following quadratic equation for x:

$$x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$$ *(Delhi 2016)*

30. Solve for x:

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}, x \neq 1,2,3$$ *(AI 2016)*

31. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers. (Al 2016)

32.

Solve for $x: \frac{1}{x}+\frac{2}{2x-3}=\frac{1}{x-2}, x\neq 0,\frac{3}{2},2.$

*(Foreign 2016)*

33. Solve for x:
$2x^2+6\sqrt{3}x-60=0$ (AI 2015)

34. Solve for x:
$x^2+5x-(a^2+a-6)=0$ (Foreign 2015)

35. Solve the equation

$\frac{4}{x}-3=\frac{5}{2x+3}; x\neq 0,-\frac{3}{2}$, for x. *(Delhi 2014)*

36.

Solve the equation $\frac{3}{x+1}-\frac{1}{2}=\frac{2}{3x-1}; x\neq -1, x\neq \frac{1}{3}$, for x.

*(Delhi 2014)*

37. Solve the equation

$\frac{14}{x+3}-1=\frac{5}{x+1}; x\neq -3,-1$, for x. *(Delhi 2014)*

38. Solve for x:

$\frac{16}{x}-1=\frac{15}{x+1}; x\neq 0,-1$ *(AI 2014)*

## LA (4/5/6 marks)

39. In the picture given below, one can see a rectangular in-ground swimming pool installed by a family in their backyard. There is a concrete sidewalk around the pool of width x m. The outside edges of the sidewalk measure 7 m and 12 m. The area of the pool is 36 sq.m.

Based on the information given above, form a quadratic equation in terms of x. Find the width of the sidewalk around the pool. (Term II, 2021-22)

40. The sum of two numbers is 34. If 3 is subtracted from one number and 2 is added to another, the product of these two numbers becomes 260. Find the numbers. (Term II, 2021-22)

41. The hypotenuse (in cm) of a right angled triangle is 6 cm more than twice the length of the shortest side. If the length of third side is 6 cm less than thrice the length of shortest side, then find the dimensions of the triangle. (Term II, 2021-22)

42. A 2-digit number is such that the product of its digits is 24. If 18 is subtracted from the number, the digits interchange their places. Find the number. (Term II, 2021-22)

43. Sum of the areas of two squares is 544 $m^2$. If the difference of their perimeters is 32 m, find the sides of the two squares. (2020)

44. A motorboat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. (NCERT, 2020, 2018, AI 2014)

45. Solve the following equation for x:

$$\frac{1}{x+1}+\frac{2}{x+2}=\frac{7}{x+5}, x \neq -1, -2, -5$$ *(2019 C)*

46.

Two water taps together can fill a tank in $1\frac{7}{8}$ hours.

The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately. (Delhi 2019)

47. A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken
1 hr less for the same journey. Find the speed of the train. (NCERT, AI 2019)

48. Solve for x:

$$\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}; a\neq b\neq 0, x\neq 0, x\neq -(a+b)$$

(AI 2019)

49. A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed? (2018)

50. Speed of a boat in still water is 15 km/h. It goes 30 km upstream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream. (Delhi 2017)

51. Solve for x:

$$\frac{1}{x+1}+\frac{3}{5x+1}=\frac{5}{x+4}, x\neq -1, -\frac{1}{5}, -4$$

(AI 2017)

52. Two taps running together can fill a tank in

$3\frac{1}{13}$ hours. If one tap takes 3 hours more than the

other to fill the tank, then how much time will each tap take to fill the tank? (AI 2017)

53. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/hour than the usual speed. Find the usual speed of the plane. What value is depicted in this question? (Delhi 2016)

54. Find x in terms of a, b and c :

$\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c},\ x\neq a,b,c$ *(Delhi 2016)*

55. The time taken by a person to cover 150 km was $2\frac{1}{2}$ hours more than the time taken in the return journey. If he returned at a speed of 10 km/hour more than the speed while going, find the speed per hour in each direction. (Delhi 2016)

56. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream. (AI 2016)

57. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park. (NCERT, AI 2016)

58. Two water taps together can fill a tank in 9 hours 36 minutes. The tap of larger diameter takes 8 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank. (Foreign 2016)

59. The denominator of a fraction is one more than twice its numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction. *(Foreign 2016)*

60. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\frac{29}{20}$. Find the original fraction. *(Delhi 2015)*

61. To fill a swimming pool two pipes are to be used. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. Find how long it would take for each pipe to fill the

pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool. (Delhi 2015)

62. Solve for x:

$$\frac{3}{x+1}+\frac{4}{x-1}=\frac{29}{4x-1}; x \neq 1,-1,\frac{1}{4}$$ *(Delhi 2015)*

63. The diagonal of a rectangular field is 16 metres more than the shorter side. If the longer side is 14 metres more than the shorter side, then find the lengths of the sides of the field. (AI 2015)

64. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed? (AI 2015)

65. A bus travels at a certain average speed for a distance of 75 km and then travels a distance of 90 km at an average speed of 10 km/h more than the first speed. If it takes 3 hours to complete the total journey, find its first speed. (AI 2015)

66. A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at an average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck. (Al 2015) Ap

67. The total cost of a certain length of a piece of cloth is

68. If the piece was 5 m longer and each metre of cloth cost 2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre? (Foreign 2015)

69. The difference of two natural numbers is 5 and

the difference of their reciprocals is $\frac{3}{28}$. Find the numbers. *(Delhi 2014)*

70. The difference of two natural numbers is 5 and the difference of their reciprocals is $\frac{5}{14}$. Find the numbers. *(Delhi 2014)* Ap

71.

Solve for $x$: $\frac{x-2}{x-3}+\frac{x-4}{x-5}=\frac{10}{3}$; $x\neq 3,5$ *(AI 2014)*

72.

Solve for $x$: $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5; x\neq -3,\frac{1}{2}$

*(Foreign 2014)*

73. The sum of the squares of two consecutive even numbers is 340. Find the numbers. (Foreign 2014)

74.

Solve for $x$: $3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5; x\neq \frac{1}{3},-\frac{3}{2}.$

*(Foreign 2014)*

75. The sum of the squares of two consecutive multiples of 7 is 637. Find the multiples. (Foreign 2014)

76.

Solve for $x$: $3\left(\frac{7x+1}{5x-3}\right)-4\left(\frac{5x-3}{7x+1}\right)=11; x\neq \frac{3}{5},-\frac{1}{7}.$

*(Foreign 2014)*

## Solution of a Quadratic Equation by Quadratic Formula

### MCQ

77. The discriminant of the quadratic equation $3\sqrt{3}x^2+10x+\sqrt{3}=0$ is

(a) +8

(b) 8

(c) 100-4√3

(d) 64 (2020 C)

### VSA (1 marks)

78. Write the discriminant of the quadratic equation

(x + 5)2 = 2 (5x – 3). (2019)

### SAI (2 marks)

79. Solve the quadratic equation for x:
$x^2 - 2ax - (4b^2 - a^2) = 0$ (Term II, 2021-22)

80. Solve for x: $2x^2-2\sqrt{2}x+1=0$ (Term II, 2021-22 C)

81.

Solve for $y$: $y^2+\frac{3\sqrt{5}}{2}y-5=0$ *(Term II, 2021-22C)*

82. Solve the quadratic equation:
$x^2 - 2ax + (a^2 - b^2) = 0$ for x. (Term II, 2021-22)

83. Solve the quadratic equation $x^2+2\sqrt{2}x-6=0$ for x. (NCERT Exemplar, Term II, 2021-22)

84. Find the roots of the quadratic equation
$\sqrt{2}x^2+7x+5\sqrt{2}=0$. (Delhi 2017)

85. Solve for x: $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$ (Foreign 2016, AI 2015, 2014)

SA II (3 marks)

86. Using quadratic formula, solve the following
equation for x: $abx^2 + (b^2 - ac)x - bc = 0$ (2021 C)

87. Solve for x:
$x^2 - (2b-1)x+(b^2 - b - 20) = 0$ (Foreign 2015)

88. Solve for x:
$x^2+6x-(a^2+2a-8)=0$ (Foreign 2015)
(4/5/6 marks)

89. The difference of the squares of two numbers is

90. The square of the smaller number is 8 times the greater number. Find the two numbers. (Term II, 2021-22)

91. Solve for x:

$\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4}, x\neq-1,-2,-4$ *(AI 2016)*

91.

Two pipes running together can fill a tank in $11\frac{1}{9}$

minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately. (Al 2016)

92. Solve for x:

$$\frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}, x\neq 0,-1,2 \qquad \textit{(Delhi 2015)}$$

93.

Solve for $x$: $\frac{x-3}{x-4}+\frac{x-5}{x-6}=\frac{10}{3};\ x\neq 4,6$ *(AI 2014)*

94.

Solve for $x$: $\frac{x-4}{x-5}+\frac{x-6}{x-7}=\frac{10}{3};\ x\neq 5,7$ *(AI 2014)*

## 4.4 Nature of Roots

**MCQ**

95. The least positive value of k, for which the quadratic equation $2x^2 + kx - 4 = 0$ has rational roots, is

(a) $+2\sqrt{2}$

(b) 2

(c) +2

(d) $\sqrt{2}$ (2023)

96. The value(s) of k for which the quadratic equation $2x^2+kx+2=0$ has equal roots, is

(a) 4

(b) +4

(c) -4

(d) O (2020)

**VSA (1 mark)**

97. Find the nature of roots of the quadratic equation
$2x^2 - 4x + 3 = 0$. (2019)

98. For what values of 'a' the quadratic equation
$9x^2 - 3ax + 1 = 0$ has equal roots? (2019 C)

99. For what values of k, the roots of the equation
$x^2 + 4x + k = 0$ are real? (Delhi 2019) Ev

100. Find the value of k for which the quadratic equation
$3x^2 + kx + 3 = 0$ has real and equal roots. (Al 2019)

101. If the quadratic equation $px^2 - 2\sqrt{5}\, px + 15 = 0$ has
two equal roots, then find the value of p.(Al 2015) Ev

**SAI (2 marks)**

102. Find the discriminant of the quadratic equation
$4x^2 - 5 = 0$ and hence comment on the nature of roots of the equation. (2023)

103. Find the value of m for which the quadratic equation
$(m-1)\, x^2+2\, (m-1)\, x + 1 = 0$ has two real and equal roots. (Term II, 2021-22)

104. If the quadratic equation
$(1 + a^2)x^2 + 2abx+ (b^2 - c^2) = 0$
has equal and real roots, then prove that
$b^2 = c^2(1 + a^2)$. (Term II, 2021-22)

105. Find the nature of roots of the quadratic equation
$3x^2 - 4\sqrt{\sqrt{3}}x+4= 0$. If the roots are real, find them. (2020C)

106. Find the value of k for which the equation
$x^2+k(2x + k - 1) + 2 = 0$ has real and equal roots. (Delhi 2017)

107. Find the values of p for which the quadratic equation
$4x^2 + px + 3 = 0$ has equal roots. (AI 2014)

108. Find the values of k for which the quadratic equation
$9x^2 - 3kx + k = 0$ has equal roots. (AI 2014)

109. Find the value of p so that the quadratic equation
$px(x-3)+9=0$ has equal roots. (AI 2014)

**SA II (3 marks)**

110. Find the value of 'p' for which the quadratic equation px(x-2)+6=0 has two equal real roots. (2023)

111. Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained. (2019)

112.If the roots of the quadratic equation in x : $(a^2 + b^2) x^2 - 2(ac + bd)x+(c^2 + d^2) = 0$ are equal, prove that ad = bc. (2019 C)

113. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots then show that $c^2 = a^2(1+m^2)$. (Delhi 2017)

114. If ad bc, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd) x + (c^2 + d^2) = 0$ has no real roots. (AI 2017)

115. If the roots of the quadratic equation $(a - b)x^2 + (b - c)x + (ca) = 0$ are equal, prove that 2a = b + c. (Al 2016)

116. Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k-1)x + x2 = 0$ has equal roots. Hence, find the roots of the equation. (Delhi 2015)

117. Find that value of p for which the quadratic equation $(p + 1)x^2 - 6(p + 1)x + 3(p+9) = 0$, p + -1 has equal roots. Hence, find the roots of the equation. (Delhi 2015)

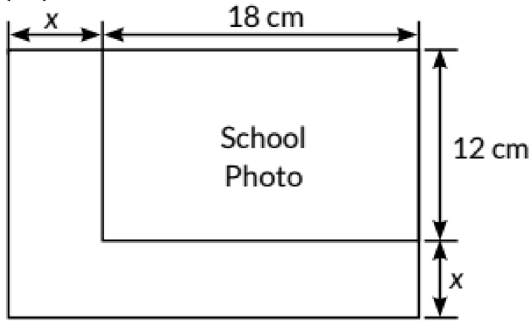
118. If 2 is a root of the quadratic equation $3x^2 + px-8=0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find the value of k. (Foreign 2014)

119.If 1 is a root of the quadratic equation $3x^2 + ax-2=0$ and the quadratic equation $a(x^2 + 6x) - b = 0$ has equal roots, find the value of b. (Foreign 2014)

## LA (4/5/6 marks)

120. Case Study: While designing the school year book, a teacher asked the student that the length and width of a particular photo is increased by x units each to double the area of the photo. The original photo is 18 cm long and 12 cm wide.
Based on the above information, answer the following questions:
(i) Write an algebraic equation depicting the above information.

(ii) Write the corresponding quadratic equation in standard form.
(iii) What should be the new dimensions of the enlarged photo?




**OR**

Can any rational value of x make the new area equal to 220 $cm^2$? (2023)

121. Find the positive values (s) of k for which quadratic equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ both will have real roots. (Foreign 2016)

122. If roots of quadratic equation $x^2 + 2px + mn = 0$ are real and equal, show that the roots of the quadratic equation $x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$ are also (Foreign 2016)

123. If x = -2 is a root of the equation $3x^2 + 7x + p = 0$, find the values of k so that the roots of the equation
$x^2+k(4x+k-1) + p = 0$ are equal. (Foreign 2015)

124. If x = 3 is root of the equation $x^2 - x + k = 0$, find
the value of p so that the roots of the equation
$x^2+k(2x+k+2)+ p = 0$ are equal. (Foreign 2015)

125. If $x = -4$ is a root of the equation $x^2 + 2x + 4p=0$,
find the values of k for which the equation
$x^2+px(1+3k) +7(3 + 2k) = 0$ has equal roots. (Foreign 2015)

126. Find the values of k for which the quadratic equation
$(k + 4)x^2 + (k + 1)x + 1 = 0$ has equal roots. Also find these roots. (Delhi 2014)

127. Find the values of k for which the quadratic equation
$(3k+1)x^2 + 2(k+1)x + 1 = 0$ has equal roots. Also find the roots. (Delhi 2014)

128. Find the value of p for which the quadratic equation
$(2p+1)x^2 - (7p+2)x + (7p - 3) = 0$ has equal roots.
Also find these roots. (Delhi 2014)

## CBSE Sample Questions

### 4.3 Solution of a Quadratic Equation by Factorisation

**MCQ**

1. Let p be a prime number. The quadratic equation having its roots as factors of p is
(a) $x^2 - px + p = 0$
(b) $x^2 - (p+1)x + p = 0$
(c) $x^2 + (p + 1)x + p = 0$
(d) $x^2 - px + p + 1 = 0$ (2022-23)

**VSA (1 mark)**

2. Find the roots of the equation $x^2 + 7x + 10 = 0$. (2020-21)

**SAI (2 marks)**

3. If Ritu were younger by 5 years than what she really is, then the square of her age would have been 11 more than five times her present age. What is her present age? (Term II, 2021-22)

**SA II (3 marks)**

4. If one root of the quadratic equation $3x^2 + px + 4 = 0$ is 2/3, then find the value of p and the other root of the equation. (2020-21)

**LA (4/5/6 marks)**

5. To fill a swimming pool two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool? (2022-23)

6. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr from its usual speed and

the time of the flight increased by 30 min. Find the scheduled duration of the flight. (2022-23)

## Solution of a Quadratic Equation by Quadratic Formula

**SAI (2 marks)**

7. Solve for x: $9x^2 - 6px + (p^2 - q^2) = 0$ (Term II, 2021-22)

## 4.4 Nature of Roots

**VSA (1 mark)**

8. For what values of k, the equation $9x^2 + 6kx + 4 = 0$ has equal roots? (2020-21)

9. For what value(s) of 'a' quadratic equation
$30ax^2 - 6x + 1 = 0$ has no real roots? (2020-21)

**SAI (2 marks)**

10. Find the value of m so that the quadratic equation $mx(5x-6)+9 = 0$ has two equal roots. (Term II, 2021-22)

# SOLUTIONS

## Previous Years' CBSE Board Questions

1. Given quadratic equation is
$ky^2 - 11y + (k-23) = 0$ ...(i)
On comparing with $ay^2 + bx + c = 0$, we have $a = k$, $b = -11$,
$c = (k-23)$

If a and ß are the roots of equation (i), then

$$\alpha+\beta=\frac{-b}{a}=\frac{-(-11)}{k}=\frac{11}{k} \quad ...(ii)$$

$$\text{and } \alpha\beta=\frac{c}{a}=\frac{k-23}{k} \quad ...(iii)$$

According to the question,

$$(\alpha+\beta)=\alpha\beta+\frac{13}{21}$$

Now, from (ii) and (iii), we have

$$\frac{11}{k}=\left(\frac{k-23}{k}\right)+\frac{13}{21} \Rightarrow \frac{11}{k}+\frac{23}{k}=1+\frac{13}{21}$$

$$\Rightarrow \frac{34}{k}=\frac{34}{21} \Rightarrow k=21$$

2. Roots of quadratic equation are given as 2 and -5.
Sum of roots=2+(-5)=-3
Product of roots = 2-(-5)=-10
Quadratic equation can be written as
$x^2$ - (sum of roots)x + Product of roots = 0
= $x^2+3x-10=0$

3.

As $x=\frac{-1}{3}$ is one of the root of $2x^2+2x+k=0$

$$2\left(\frac{-1}{3}\right)^2+2\left(\frac{-1}{3}\right)+k=0$$

$$\Rightarrow 2\times\frac{1}{9}-\frac{2}{3}+k=0 \Rightarrow \frac{-4}{9}+k=0 \Rightarrow k=\frac{4}{9}$$

4. The given equation is $3x^2 - 10x + k = 0$.
Let the roots be a and 1/a.

$$\text{Now, } a\times\frac{1}{a}=\frac{k}{3} \Rightarrow 1=\frac{k}{3} \Rightarrow k=3$$

5. Since, x = 2 is a solution of $kx^2+2x-3=0$.
:- $k(2)^2+2(2)-3=0$

$$\Rightarrow 4k+4-3=0 \Rightarrow k=-\frac{1}{4}$$

6. As, 3 is the root of quadratic equation
$x^2$-2kx-6=0
:- (3)2-2k(3)-6=0
⇒ 9-6k-603-6k=0

$$\Rightarrow \quad 6k = 3 \Rightarrow k = \frac{3}{6} = \frac{1}{2} \quad \therefore \quad k = \frac{1}{2}$$

7.

$$\because x = -\frac{1}{2} \text{ is a solution of } 3x^2 + 2kx - 3 = 0$$

$$\therefore 3\left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$

$$\Rightarrow \frac{3}{4} - k - 3 = 0 \Rightarrow k = \frac{3}{4} - \frac{3}{1} = \frac{-9}{4}$$

8. Let a and ß be the roots of given quadratic equation
$2x^2$-9x+4=0

$$\therefore \quad \text{Sum of roots, } \alpha + \beta = \frac{-b}{a} = -\frac{(-9)}{2} = \frac{9}{2}$$

$$\text{and product of roots, } \alpha\beta = \frac{c}{a} = \frac{4}{2} = 2$$

9. Given equation is $px^2$ - 14x+8=0
Let roots of the equation be a, B.
Then, B = 6α = 6-B=0 ...(i)

Also, sum of roots = $\alpha + \beta = -\left(\frac{-14}{p}\right) = \frac{14}{p}$ ...(ii)

and product of roots = $\alpha\beta = \frac{8}{p}$ ...(iii)

On solving (i) and (ii), we get

$\alpha = \frac{2}{p}$ and $\beta = \frac{12}{p}$

Putting these values in (iii), we get

$$\left(\frac{2}{p}\right) \times \left(\frac{12}{p}\right) = \frac{8}{p}$$

$$\Rightarrow \quad 8p^2 = 24p \Rightarrow 8p(p-3) = 0$$

$$\Rightarrow \quad p = 3 \qquad (\because p \neq 0)$$

10. Given, roots of quadratic equation

$ax^2 + 7x + b = 0$ are $\frac{2}{3}$ and $-3$

$\therefore$ Sum of roots = $\frac{-7}{a}$

$\Rightarrow \frac{2}{3}+(-3)=\frac{-7}{a} \Rightarrow \frac{-7}{3}=\frac{-7}{a} \Rightarrow a=3$

Product of roots $=\frac{b}{a}$

$\Rightarrow \left(\frac{2}{3}\right)(-3)=\frac{b}{3} \Rightarrow b=-6 \therefore a = 3, b = -6$

11. Given, $px^2 - 14x+8=0$

Let one root of given quadratic equation is a, then other root is 6α.

Sum of roots, $\alpha+6\alpha=-\frac{(-14)}{p}=\frac{14}{p}$

$\Rightarrow 7\alpha=\frac{14}{p}\Rightarrow \alpha=\frac{2}{p}$ ...(i)

Product of roots, $\alpha(6\alpha)=\frac{8}{p}$

$\Rightarrow 6\alpha^2=\frac{8}{p} \Rightarrow \alpha^2=\frac{4}{3p}$ ...(ii)

From (i) and (ii), we get

$\frac{4}{p^2}=\frac{4}{3p} \Rightarrow \frac{1}{p}=\frac{1}{3}\Rightarrow p=3$

12. Given quadratic equation is $2x^2 - 8x - k = 0$ ...(i)

Let us consider two roots a and ß of eqn. (i)

$\therefore \alpha+\beta=\frac{-b}{a}$

From eqn. (i), we have $a = 2, b = -8, c = -k$

$\frac{5}{2}+\beta=-\frac{(-8)}{2}=4$ $\left[\because \alpha=\frac{5}{2}\text{(given)}\right]$

$\beta=\left(4-\frac{5}{2}\right)$

$\therefore \beta=\frac{3}{2}$ (other root)

It α is a root of eqn. (i) then α will satisfy the given eqn. (i).

$\therefore 2\times\left(\frac{5}{2}\right)^2-8\times\frac{5}{2}-k=0 \Rightarrow \frac{25}{2}-20=k \Rightarrow k=\frac{-15}{2}$

13. (a): We have, $x^2+3x-10=0$

$\Rightarrow x^2+5x-2x-10=0$

$= x(x+5)-2(x+5)=0$

$= (x-2)(x+5)=0 \Rightarrow x=2,-5$

14. We have, $x^2 - 2ax - (4b2 - a^2)=0$

$= (x^2 - 2ax + a^2) - 4b2 = 0$

$= (x-a)^2 - (2b)2 = 0$

$= (x-a-2b)(x-a + 2b) = 0$ [$\because A^2 - B^2 = (A - B)(A + B)$]

$\Rightarrow$ x-a-2b=0 or x-a+2b=0

$\Rightarrow$ x=2b+a or x = a - 2b

Thus, the roots of the given quadratic equation are a + 2b and a - 2b.

15.

We have, $\frac{x+1}{x-1}+\frac{x-2}{x+2}=4-\frac{2x+3}{x-2}$

$\Rightarrow \frac{(x+1)(x+2)+(x-2)(x-1)}{(x-1)(x+2)}=\frac{4(x-2)-(2x+3)}{(x-2)}$

$\Rightarrow \frac{x^2+3x+2+x^2-3x+2}{x^2+x-2}=\frac{2x-11}{(x-2)}$

$= (2x^2+4)(x-2)=(2x-11)(x^2+x-2)$

$= 2x^3- 4x^2+4x-8= 2x3 + 2x^2 - 4x - 11x^2 - 11x+22$

$= -4x^2+4x-8=-9x^2 - 15x+22$

$= 5x^2+19x-30=0 \Rightarrow 5x^2 + 25x-6x-30 = 0$

$= 5x(x + 5) - 6(x + 5) = 0 = (5x - 6)(x+5) = 0$

$\Rightarrow x=\frac{6}{5}, -5$

16.

We have, $\sqrt{2x+9}+x=13$

or $\sqrt{2x+9}=13-x$

On squaring both sides, we get $2x+9= (13-x)^2$

$= 2x+9=169+x^2 - 26x$

$= x^2 - 28x + 160 = 0 \Rightarrow x^2- 20x - 8x + 160 = 0$

$= x(x-20)-8(x - 20) = 0$
$= (x-20)(x-8)=0$.. $x = 20$ or $8$

17. We have, $\sqrt{6x+7}=(2x-7)$
On squaring both sides, we get
$(6x+7)=(2x-7)^2$
$= 6x+7=4x^2 + 49 - 28x$
$= 4x^2 - 34x+42=0$ $2x^2 - 17x+21=0$
$= 2x^2-14x-3x+21=0$
$= 2x(x-7)-3(x-7)=0 \Rightarrow (2x-3)\ (x - 7) = 0$

$$\Rightarrow \quad x=\frac{3}{2},\ x=7$$

18. Let ten's place digit of the number be x and one's place digit be y.
:- Number $= 10x + y$
Now, $10x + y = 4(x + y) \Rightarrow 10x+y=4x+4y$
$\Rightarrow 6x=3y = y = 2x$ ...(i)
Also, $10x + y = 3(xxy)$
$\Rightarrow 10x+2x=3(xx\ 2x)$ [Using (i)]
$\Rightarrow 12x=6x^2-6x2 - 12x=0$ $6x\ (x-2)=0$
$\Rightarrow x=2$ [x 0] . $y=2x2=4$
:- Required number = 24

19.

$$\text{We have, } \frac{1}{x-3}-\frac{1}{x+5}=\frac{1}{6}$$

$$\Rightarrow \quad \frac{(x+5)-(x-3)}{(x-3)(x+5)}=\frac{1}{6} \Rightarrow 6(8) = (x-3)(x+5)$$

$$\Rightarrow \quad 48 = x^2 + 2x - 15$$

$$\Rightarrow \quad x^2 + 2x - 63 = 0 \Rightarrow x^2 + 9x - 7x - 63 = 0$$

$$\Rightarrow \quad x(x+9) - 7(x+9) = 0 \Rightarrow (x+9)(x-7) = 0$$

$$\Rightarrow \quad x = -9, 7.$$

20.

$$\text{We have, } \frac{a}{x-b} + \frac{b}{x-a} = 2$$

$$\Rightarrow \frac{a(x-a)+b(x-b)}{(x-b)(x-a)} = 2$$

$$\Rightarrow \quad ax - a^2 + bx - b^2 = 2(x^2 - bx - ax + ab)$$
$$\Rightarrow \quad 2x^2 - 3bx - 3ax + 2ab + a^2 + b^2 = 0$$
$$\Rightarrow \quad 2x^2 - 3(a+b)x + (a+b)^2 = 0$$
$$\Rightarrow \quad 2x^2 - 2(a+b)x - (a+b)x + (a+b)^2 = 0$$
$$\Rightarrow \quad 2x[x - (a+b)] - (a+b)[x - (a+b)] = 0$$
$$\Rightarrow \quad [x - (a+b)][2x - (a+b)] = 0$$
$$\Rightarrow \quad x = a+b \quad \text{or} \quad x = \frac{a+b}{2}$$

21. We have, $4x^2 - 4a^2x + (a - b) = 0$
$\Rightarrow (2x)^2 - 2(2x)a^2 + (a^2)^2 - (b^2)^2 = 0$
$\Rightarrow (2x-a^2)^2 - (b^2)^2 = 0$
$\Rightarrow (2x-a^2 + b^2)(2x - a^2 - b^2) = 0$
$\Rightarrow 2x\text{-}a2+ b^2 = 0$ or $2x - a^2 - b^2 = 0$
$\Rightarrow 2x = a^2 - b^2$ or $2x = a^2 + b^2$

$$\Rightarrow \quad x = \frac{a^2 - b^2}{2} \quad \text{or} \quad x = \frac{a^2 + b^2}{2}$$

22. We have, $9x2 - 6b^2x - (a^2 - b^1) = 0$
$\Rightarrow 9x^2\text{-} 6b^2x\text{-}a^1+b^2=0$
$= \{(3x)^2 - 2(3x)b^2 + (b^2)^2\} - (a^2)^2 = 0$
$= (3x\text{-}b^2)^2 - (a2)^2 = 0$
$= (3x\text{-}b^2 + a^2)(3x - b^2 - a^2) = 0$
$= 3x\text{-}b2+ a2 = 0$ or $3x - b^2 - a^2 = 0$
$= 3x=b2\text{-}a2$ or $3x = b^2 + a^2$

$$\Rightarrow \quad x = \frac{b^2 - a^2}{3} \text{ or } x = \frac{a^2 + b^2}{3}$$

23. We have, $4x^2+4bx - (a^2 - b^2) = 0$
$= 4x^2+4bx - a^2 + b^2 = 0$
$= (2x)^2+2(2x)(b) + b^2 - a^2 = 0$

$= (2x+b)^2 - a^2 = 0$
$= (2x+b+a)(2x+b-a) = 0$
$= 2x+b+a=0$ or $2x + b - a=0$
$\Rightarrow \quad x=-\frac{(a+b)}{2}$ or $x=\frac{a-b}{2}$

24. We have, $x^2-(\sqrt{3}+1)x+\sqrt{3}=0$
$\Rightarrow x^2-\sqrt{3}x-x+\sqrt{3}=0 \Rightarrow x(x-\sqrt{3})-1(x-\sqrt{3})=0$
$=> (x-\sqrt{3})(x-1)=0$
$\Rightarrow x-\sqrt{\sqrt{3}}=0$ or $x-1=0 \Rightarrow x=\sqrt{\sqrt{3}}$ or $x=1$

25. We have, $2x^2 + ax-a^2 = 0$
$2x^2+2ax-ax-a^2=0 \Rightarrow 2x(x + a) - a(x+a) = 0$
$\Rightarrow (2x-a)(x+a)=0 \Rightarrow x=-a$ or $x=\frac{a}{2}$.

26. Let the length of the side of one square be x m and the length of the side of another square be y m.
Given, $x^2 + y^2 = 157$ ...(i)
and $4x+4y=68$ ...(ii)
$\Rightarrow x+y=17$
$\Rightarrow y = 17-x$ ...(iii)
On putting the value of y in (i), we get
$x^2+(17-x)^2 = 157$
$= x^2+289+x^2 - 34x = 157$
$= 2x^2 - 34x+132 = 0$
$= x^2-17x+66=0 \Rightarrow x^2 - 11x-6x+66=0$
$= x(x-11)-6(x-1)=0$
$= (x-11)(x-6)=0 \Rightarrow x=6$ or $x = 11$
On putting the value of x in (iii), we get
$y = 17-6=11$ or $y = 17-11=6$
Hence, the sides of the squares be 11 m and 6 m.

27. Let the usual speed of the plane be x km/h.
Then, the increased speed of the plane = (x + 100) km/h
and distance covered = 1500 km
According to question,

$\frac{1500}{x} - \frac{1500}{x+100} = \frac{1}{2}$ $\left[\because 30 \text{ minutes} = \frac{30}{60} = \frac{1}{2} \text{ hours}\right]$

$\Rightarrow \frac{1500(x+100) - 1500x}{x(x+100)} = \frac{1}{2}$

$\Rightarrow \frac{1500x + 150000 - 1500x}{x(x+100)} = \frac{1}{2}$

$\Rightarrow (150000)2 = x(x + 100)$

$\Rightarrow x^2 + 100x - 300000 = 0$

$\Rightarrow x^2 + 600x - 500x - 300000 = 0$

$\Rightarrow (x + 600)(x - 500) = 0$

$\Rightarrow$ Either $x + 600 = 0$ or $x - 500 = 0$

$\Rightarrow x = 500$ ($\because$ Speed can never be negative)

Hence, usual speed = 500 km/hr

28.

We have, $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$

$\Rightarrow \frac{2x(2x+3) + (x-3) + 3x + 9}{(x-3)(2x+3)} = 0$

$= 4x^2 + 6x + x - 3 + 3x + 9 = 0$

$= 4x^2 + 10x + 6 = 0 \Rightarrow 2x^2 + 5x + 3 = 0$

$= 2x^2 + 2x + 3x + 3 = 0 \Rightarrow 2x(x+1) + 3(x+1) = 0$

$\Rightarrow (x + 1)(2x + 3) = 0 \Rightarrow x = -1$ or $x = \frac{-3}{2}$

$\therefore x = -1$ $\left[\because x \neq \frac{-3}{2}\right]$

29.

We have, $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$

$\Rightarrow x^2 + \frac{a}{a+b}x + \frac{a+b}{a}x + 1 = 0$

$\Rightarrow x\left(x + \frac{a}{a+b}\right) + \frac{a+b}{a}\left(x + \frac{a}{a+b}\right) = 0$

$\Rightarrow \left(x + \frac{a+b}{a}\right)\left(x + \frac{a}{a+b}\right) = 0 \Rightarrow x = \frac{-(a+b)}{a}, \frac{-a}{(a+b)}$

30.

Let $x - 2 = t$. So, given equation becomes

$$\frac{1}{t(t+1)} + \frac{1}{t(t-1)} = \frac{2}{3} \Rightarrow \frac{t-1+t+1}{t(t+1)(t-1)} = \frac{2}{3}$$

$\Rightarrow \quad 3t = t(t+1)(t-1) \Rightarrow 3t = t(t^2 - 1)$

$\Rightarrow \quad 3t = t^3 - t \Rightarrow t^3 - 4t = 0$

$\Rightarrow \quad t(t^2 - 4) = 0 \Rightarrow t = 0$ or $t^2 - 4 = 0$

$\Rightarrow \quad t^2 - 4 = 0$ ($t \neq 0$ as $x \neq 2$)

$\Rightarrow \quad t = \pm 2$ or $x - 2 = \pm 2 \quad \therefore \quad x = 0, 4$

31. Let three consecutive natural numbers be x, x + 1,x+2. According to question,

$(x+1)^2 - [(x+2)^2 - (x)^2] = 60$

$= (x+1)^2 - [x^2+4+4x-x^2] = 60$

$= x^2+1+2x-4-4x=60$

$= x^2-2x-63=0 \Rightarrow x^2-9x+7x-63=0$

$= x(x-9)+7(x-9)=0 \Rightarrow (x+7)(x-9) = 0$

= x=9 (x-7 as x is a natural number)

:- The required numbers are 9, 10, 11.

32.

We have, $\frac{1}{x} + \frac{2}{2x-3} = \frac{1}{x-2}$

$\Rightarrow \quad \frac{2x-3+2x}{x(2x-3)} = \frac{1}{x-2}$

$= (x-2)(4x-3)= x(2x-3)$

$= 4x^2-3x-8x+6=2x^2 - 3x$

$= 2x^2-8x+6=0x^2-4x+3=0$

$= (x-3)\ (x-1)=0 \Rightarrow x=3$ or $x = 1$.

33. We have, $2x^2+6\sqrt{3}x-60=0$

$\Rightarrow x^2+3\sqrt{3}x-30=0$

$= x^2+5\sqrt{3}x-2\sqrt{3}x-30-0$

$\Rightarrow x(x+5\sqrt{3})-2\sqrt{3}(x+5\sqrt{3})=0$

$\Rightarrow (x+5\sqrt{3})(x-2\sqrt{3})=0$

$\Rightarrow x+5\sqrt{3}=0$ or $x-2\sqrt{3}=0$

$\Rightarrow x=-5\sqrt{\sqrt{3}}$ or $x=2\sqrt{3}$.

34. We have, $x^2 + 5x - (a^2 + a - 6) = 0$

$= x^2+5x-(a-2)(a+3) = 0$

$= x^2+(a+3)x-(a-2)x - (a-2)(a+3) = 0$

$= x\{x+(a+3)\} - (a-2)\{x+(a+3)\} = 0$

$= \{x+(a+3)\}\{x-(a-2)\} = 0$

$= x = -(a+3)$ or $x = (a-2)$

35.

We have, $\frac{4}{x} - 3 = \frac{5}{2x+3}; x \neq 0, \frac{-3}{2}$

$\Rightarrow \frac{4}{x} - \frac{5}{2x+3} = 3 \Rightarrow \frac{4(2x+3)-5(x)}{x(2x+3)} = 3$

$\Rightarrow 8x - 5x + 12 = 3x(2x+3)$

$= 3x+12=6x^2+9x$

$= 6x^2+6x-12=0 \Rightarrow x^2+x-2=0$

$= x^2+2x-x-2=0 \Rightarrow x(x+2)-1(x+2)=0$

$= (x+2)(x-1)=0 \Rightarrow x=1$ or $x=-2$

36.

We have, $\frac{3}{x+1} - \frac{1}{2} = \frac{2}{3x-1}; x \neq -1, \frac{1}{3}$

$\Rightarrow \frac{3}{x+1} - \frac{2}{3x-1} = \frac{1}{2} \Rightarrow \frac{3(3x-1)-2(x+1)}{(x+1)(3x-1)} = \frac{1}{2}$

$= 2[9x-2x-32] = (x+1)(3x-1)$

$= 2[7x5] = 3x^2+2x-1 \Rightarrow 14x-10=3x^2+2x-1$

$= 3x^2-12x+9=0 \Rightarrow x^2-4x+3=0$

$= x^2-x-3x+3=0x(x-1)-3(x-1)=0$

$= (x-1)(x-3)=0 \Rightarrow x=1orx=3$

37.

We have, $\frac{14}{x+3} - 1 = \frac{5}{x+1}; x \neq -3, -1$

$\Rightarrow \frac{14}{x+3} - \frac{5}{x+1} = 1 \Rightarrow \frac{14(x+1)-5(x+3)}{(x+3)(x+1)} = 1$

$= 14x+14-5x-15= x^2+4x+3$
$= x^2-5x+4=0 \Rightarrow x^2-4x-x+4=0$
$= x(x-4)-1(x-4)=0$
$= (x-4)(x-1)=0 \Rightarrow x=1$ or $x=4$

38.

We have, $\frac{16}{x}-1=\frac{15}{x+1}$; $x \neq 0, -1$

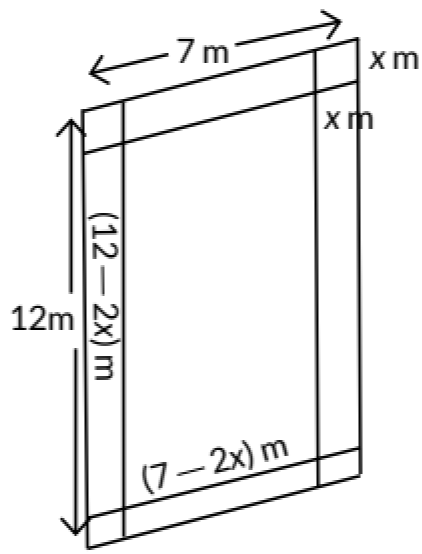
$\Rightarrow \frac{16-x}{x}=\frac{15}{(x+1)} \Rightarrow (16-x)(x+1)=15x$

$\Rightarrow 16x+16-x^2-x=15x$

$\Rightarrow 16-x^2=0 \Rightarrow (4)^2-(x)^2=0 \Rightarrow (4-x)(4+x)=0$

$\Rightarrow x=4$ or $x=-4$

39. Given, width of the sidewalk = xm,
Area of the pool = 36 sq.m
:- Inner length of the pool
= (12-2x)m
Inner width of the pool
= (7-2x) m
:- Area of the pool, A = 1 x b
= 36(12-2x) x (7-2x)
= 3684-24x14x + $4x^2$
= $4x^2$ - 38x+48=0
= $2x^2$ - 19x + 24 = 0, is the
required quadratic equation.




Area of the pool given by quadratic equation is
$2x^2-19x+24=0 \Rightarrow 2x^2 - 16x-3x + 24 = 0$
$= 2x(x-8)-3(x-8) = 0 = (x-8)(2x-3)=0$

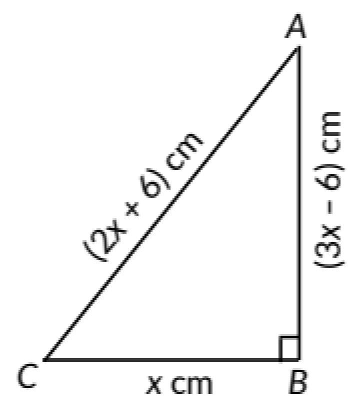
$\Rightarrow x=8$ (not possible) or $x=\frac{3}{2}=1.5$

$\therefore$ Width of the sidewalk $=1.5$ m

40. Let one number be x and another number be y.
Since, $x + y = 34 \Rightarrow y=34-x$ ... (i)
Now, according to the question,
$(x-3)(y+2)=260$ ... (ii)

Putting the value of y from (i) in (ii), we get
$(x-3)(34-x+2)=260$
$= (x-3)(36-x) = 260$
$= 36x-x^2-108+ 3x = 260$
$= x^2 - 39x+368=0 \Rightarrow x^2-23x-16x+368 = 0$
$= x(x-23)-16(x-23) = 0$
$= (x-23)(x-16)=0 \Rightarrow x=23$ or 16
Hence, when x = 23 from (i), y = 34-23=11
When x = 16, then y = 34 - 16 = 18
Hence the required numbers are 23 and 11 or 16 and 18.

41. Let the shortest side be x cm.
Then, hypotenuse be (2x + 6) cm
and third side be (3x-6) cm
In a right angled AABC, $AB^2 + BC2 = AC^2$
by Pythagoras Theorem,
$(3x-6)^2+x^2= (2x+6)^2$
$= 9x^2+36-36x+x^2 = 4x^2+36 +24x$
$= 6x^2- 60x=0$
$= 6x(x-10)=0 \Rightarrow 6x=0$ or $x-10=0$
= x = 0 (rejected) or x = 10
Hence, AB = 3x-6= 3 x 10-6 = 24 cm,
BC= x = 10 cm and
AC=2x+6=2x 10+ 6 = 26 cm.




42. Let digit at unit's place be x and digit at ten's place be y.
Then, number = 10y + x
:- Number obtained when digits interchange their places = 10x+y
According to the question, xy = 24 ...(i)
Also, 10y + x - 18 = 10x + y ...(ii)
$9y-9x=18 \Rightarrow y-x=2$ or $y=2+x$
Put y = 2 + xin (i), we get
$x(2+x) = 24$
$= 2x+x2=24x^2+2x-24=0$
$= x^2+6x-4x-24=0$
$= x(x+6) - 4(x+6)=0 \Rightarrow (x+6)(x-4)=0$
= X=4 (x-6)

From (i), $y = \frac{24}{x} = \frac{24}{4} = 6$

Hence, required number = $10y + x$

$= 10(6) + 4 = 60 + 4 = 64.$

43. Let the sides of the two squares be x m and y m, where x>y.
Then, their areas are $x^2$ and $y^2$ and their perimeters are
4x and 4y respectively.
By the given condition, $x^2 + y^2 = 544$ ...(i)
and $4x - 4y = 32$
$\Rightarrow x - y = 8$
$\Rightarrow x = y + 8$ ...(ii)
Substituting the value of x from (ii) in (i) we get
$(y+8)^2 + y^2 = 544$
$\Rightarrow y^2 + 64 + 16y + y^2 = 544$
$\Rightarrow 2y^2 + 16y - 4800 \Rightarrow y^2 + 8y - 240 = 0$
$\Rightarrow y^2 + 20y - 12y - 240 = 0$
$\Rightarrow y(y+20) - 12(y + 20) = 0 \Rightarrow (y - 12)(y + 20) = 0$
$\Rightarrow y = 12$ (..y * -20 as length cannot be negative)
From (ii), $x = 12 + 8 = 20$
Thus, the sides of the two squares are 20 m and 12 m.

44. Let the speed of the stream be x km/hr.
:- Speed of the boat upstream = (18 - x) km/hr
Speed of the boat downstream = (18+ x) km/hr
According to question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1 \Rightarrow \frac{24(18+x) - 24(18-x)}{(18+x)(18-x)} = 1$$

$$\Rightarrow \frac{24(18+x-18+x)}{324-x^2} = 1$$

$\Rightarrow 24(2x) = 324 - x^2 \Rightarrow x^2 + 48x - 324 = 0$

$\Rightarrow x^2 + 54x - 6x - 324 = 0 \Rightarrow x(x + 54) - 6(x + 54) = 0$

$\Rightarrow (x - 6)(x + 54) = 0$

$\Rightarrow$ Either $x - 6 = 0$ or $x + 54 = 0$

$\Rightarrow x = 6$ [$\because x \neq -54$ as speed can't be negative]

Hence, the speed of the stream is 6 km/hr.

45.

We have, $\frac{1}{x+1}+\frac{2}{x+2}=\frac{7}{x+5}$

$\Rightarrow \frac{x+2+2(x+1)}{(x+1)(x+2)}=\frac{7}{x+5}$

$\Rightarrow \frac{3x+4}{x^2+3x+2}=\frac{7}{x+5}$

$= (3x+4)(x+5)=7(x^2+3x+2)$

$= 3x^2+19x+20=7x^2+21x+14$

$= 4x^2+2x-6=0 \Rightarrow 2x^2+x-3=0$

$= 2x^2+3x-2x-3=0$

$\Rightarrow x(2x+3)-1(2x+3) \Rightarrow (x-1)(2x+3)=0$

$\Rightarrow x=1$ or $x=\frac{-3}{2}$

46. Let smaller tap fill the tank in x hours. Then, larger tap will fill the tank in (x-2) hours.

Since, both the taps can fill the tank in $1\frac{7}{8}$ hours *i.e.*; $\frac{15}{8}$hours

$\therefore \frac{1}{x}+\frac{1}{x-2}=\frac{8}{15} \Rightarrow \frac{x-2+x}{x(x-2)}=\frac{8}{15}$

$\Rightarrow \frac{2x-2}{x^2-2x}=\frac{8}{15} \Rightarrow 30x-30=8x^2-16x$

$\Rightarrow 8x^2-46x+30=0 \Rightarrow 4x^2-23x+15=0$

$\Rightarrow 4x^2-20x-3x+15=0 \Rightarrow 4x(x-5)-3(x-5)=0$

$\Rightarrow (4x-3)(x-5)=0 \Rightarrow x=\frac{3}{4}$ or $x=5$

But $x \neq \frac{3}{4} \quad \therefore \quad x=5$

When x = 5, x-2=5-2=3

:- Smaller tap will fill the tank in 5 hours and larger tap will fill the tank in 3 hours.

47. Let the speed of the train be x km/hr. According to question,

$$\frac{360}{x}-\frac{360}{x+5}=1 \Rightarrow \frac{(x+5-x)360}{x(x+5)}=1$$

$\Rightarrow \quad 1800 = x^2 + 5x \Rightarrow x^2 + 5x - 1800 = 0$

$\Rightarrow \quad x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow \quad x(x + 45) - 40(x + 45) = 0 \Rightarrow (x - 40)(x + 45) = 0$

$\Rightarrow \quad x = 40$ ($\because$ Speed can't be negative)

Hence, the speed of the train is 40 km/hr.

48.

Given, $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}; a\neq b\neq 0, x\neq 0, x\neq -(a+b)$

$\Rightarrow \frac{1}{a+b+x}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b}$

$\Rightarrow \frac{x-a-b-x}{x(a+b+x)}=\frac{b+a}{ab} \Rightarrow \frac{-(a+b)}{x(a+b+x)}=\frac{a+b}{ab}$

$\Rightarrow \frac{-1}{x(a+b+x)}=\frac{1}{ab}$

$\Rightarrow \quad x^2 + ax + bx + ab = 0 \Rightarrow x(x + a) + b(x + a) = 0$

$\Rightarrow \quad (x + b)(x + a) = 0 \Rightarrow x = -a, -b$

49. Let original average speed of the train be x km/hr. According to question,

$\frac{63}{x}+\frac{72}{x+6}=3$

$\Rightarrow \quad 9\left(\frac{7}{x}+\frac{8}{x+6}\right)=3$

$\Rightarrow \quad \frac{7}{x}+\frac{8}{x+6}=\frac{1}{3} \Rightarrow \frac{7(x+6)+8x}{x(x+6)}=\frac{1}{3}$

$= 3(7x+42+8x) = x^2+6x$

$= 3(15x+42) = x^2+6x$

$= 45x+126 = x^2+6x= x^2 - 39x - 126 = 0$

$= x^2 - 42x + 3x - 126 = 0 \Rightarrow (x - 42)(x + 3) = 0$

= Either $x - 42 = 0$ or $x+3=0 \Rightarrow x=42$ ($\because x > 0$)

Hence, the original speed of the train is 42 km/h.

50. Speed of the boat in still water = 15 km/h

Let speed of the stream be s km/h.

:- Speed of boat in upstream = (15 - s) km/h

Speed of boat in downstream = (15+ s) km/h
According to question,

51.

We have, $\frac{1}{x+1}+\frac{3}{5x+1}=\frac{5}{x+4}$

$\Rightarrow \frac{(5x+1)+3(x+1)}{(x+1)(5x+1)}=\frac{5}{x+4} \Rightarrow \frac{8x+4}{5x^2+5x+x+1}=\frac{5}{x+4}$

$\Rightarrow 8x^2+4x+32x+16=25x^2+25x+5x+5$

$\Rightarrow 17x^2-6x-11=0 \Rightarrow 17x^2-17x+11x-11=0$

$\Rightarrow 17x(x-1)+11(x-1)=0$

$\Rightarrow (x-1)(17x+11)=0$

$\Rightarrow x=1 \text{ or } x=\frac{-11}{17}.$

52. Let first tap fill the tank in x hours then second tap will fill the tank in (x-3) hours.

Since, both taps can fill the tank in $3\frac{1}{13}$ hours i.e.; $\frac{40}{13}$hours

$\therefore \frac{1}{x}+\frac{1}{x-3}=\frac{13}{40} \Rightarrow \frac{(x-3)+x}{(x)(x-3)}=\frac{13}{40}$

$\Rightarrow \frac{2x-3}{x^2-3x}=\frac{13}{40} \Rightarrow 40(2x-3)=13(x^2-3x)$

$\Rightarrow 80x-120=13x^2-39x$

$\Rightarrow 13x^2-119x+120=0$

$\Rightarrow 13x^2-104x-15x+120=0 \Rightarrow 13x(x-8)-15(x-8)$

$\Rightarrow (13x-15)(x-8)=0$

$\Rightarrow x=8 \text{ or } x=\frac{15}{13} \Rightarrow x=8 \quad \left(\because x\neq\frac{15}{13}\right)$

:- First tap will fill the tank in 8 hours.
and second tap will fill the tank in (8 - 3) = 5 hours.

53. Let the usual speed of plane be x km/hr. According to question,

$\frac{1500}{x}-\frac{1500}{x+250}=\frac{1}{2} \Rightarrow \frac{x+250-x}{x(x+250)}=\frac{1}{2\times1500}$

$\Rightarrow \frac{250}{x^2+250x}=\frac{1}{3000}$

$x^2+250x-750000 = 0$
$x^2 + 1000x - 750x - 750000 = 0$
$x(x+1000) - 750(x + 1000) = 0$
$(x + 1000)(x-750) = 0$
x = 750 (:- Speed cannot be negative)
Speed of plane = 750 km/hr
The value depicted is that we should be helpful in every condition and sincere towards the duty.

We have, $\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$

$\Rightarrow \frac{a(x-b)+b(x-a)}{(x-a)(x-b)}=\frac{2c}{x-c}$

$\Rightarrow a(x - b)(x - c) + b(x - a)(x - c) = 2c(x - a)(x - b)$
$\Rightarrow ax^2 - acx - abx+ abc + bx^2 - bcx - abx+abc$
$=2cx^2 - 2bcx - 2acx+2abc$
$\Rightarrow x^2(a+b-2c) = x(-2bc-2ac + ac+ab+bc+ab)$
$x^2(a+b-2c) + x(bc + ac - 2ab) = 0$
$\Rightarrow x[x(a+b-2c) + (bc + ac - 2ab)] = 0$

$\Rightarrow x=\frac{-(bc+ac-2ab)}{(a+b-2c)}, 0.$

55. Let the speed of person while going be x km/hr.
:- According to question,

$\frac{150}{x}-\frac{150}{x+10}=\frac{5}{2}$

$= 2(150 (x+10) - 150x) = 5x(x + 10)$
$= 2(150x + 1500-150x)=5(x^2+10x)$
$= 3000=5x^2+50x = x^2+10x - 600 = 0$
$= x^2+30x - 20x - 600 = 0$
$= (x-20)(x+30)=0 \Rightarrow x=20, -30$
Since, speed cannot be negative.
:- Speed of person while going = 20 km/hr
and speed of person while returning = 30 km/hr

56. Speed of boat in still water = 24 km/hr
Let the speed of stream be x km/hr.

Speed in upstream = (24 - x) km/hr

Time taken to go upstream $= \frac{\text{Distance}}{\text{speed}} = \frac{32}{24-x}$

Speed in downstream = (24 + x) km/hr

Time taken to go downstream $= \frac{\text{Distance}}{\text{speed}} = \frac{32}{24+x}$

According to question,

$$\frac{32}{24-x} - \frac{32}{24+x} = 1 \Rightarrow 32\left[\frac{24+x-(24-x)}{(24-x)(24+x)}\right] = 1$$

⇒ 32[24+x-24+x] = (24 - x)(24+x)

⇒ $64x = (24)^2 - x^2 = x^2 + 64x - 576 = 0$

⇒ $x^2 + 72x - 8x - 516 = 0$

⇒ x(x+72) - 8(x + 72) = 0

⇒ (x-8)(x + 72)=0⇒ x=8 or x=-72

:- Speed of stream = 8 km/hr (x-72)

57. Let x and y be length and breadth of the rectangular park respectively.

:- y=x -3

:- Area of rectangular park = x(x-3)

Also, area of isosceles triangular park

$$= \frac{1}{2} \times (x-3) \times 12 = 6(x-3)$$

Now, according to question,

x(x-3)-6(x-3)=4⇒ $x^2 - 3x - 6x + 18 = 4$

⇒ $x^2 - 9x + 14 = 0$ ⇒ $x^2 - 7x - 2x + 14 = 0$

⇒ x(x-7)-2(x-2)=0⇒ (x-7)(x-2)=0

x=7 (x+2)

Length of rectangular park = x = 7m

Breadth of rectangular park =x-3=7-3=4m

58. Let tap of smaller diameter fill the tank in x hours.

:- Tap of larger diameter fill the tank in (x - 8) hours

Both taps together can fill the tank in 9 hours 36 minutes *i.e.*, $\frac{48}{5}$ hours .

$\therefore$ According to question,

$$\frac{1}{x}+\frac{1}{x-8}=\frac{5}{48} \quad \Rightarrow \quad \frac{x-8+x}{x(x-8)}=\frac{5}{48}$$

$\Rightarrow \quad 48(2x - 8) = 5(x^2 - 8x)$

$\Rightarrow \quad 96x - 384 = 5x^2 - 40x$

$\Rightarrow \quad 5x^2 - 136x + 384 = 0$

$\Rightarrow \quad 5x^2 - 120x - 16x + 384 = 0$

$\Rightarrow \quad 5x(x - 24) - 16(x - 24) = 0$

$\Rightarrow \quad (x - 24)(5x - 16) = 0 \quad \Rightarrow \quad x = 24 \text{ or } x = \frac{16}{5}$

When $x = 24, x - 8 = 16$

When, $x=\frac{16}{5}, x-8=\frac{-24}{5}$, which is not possible

So, smaller tap can fill the tank in 24 hours and larger tap will fill the tank in 16 hours.

59.

Let the required fraction be $\frac{x}{y}$.

$\therefore \quad y = 2x + 1$ ...(i)

According to question,

$$\frac{x}{y}+\frac{y}{x}=2\frac{16}{21} \quad \Rightarrow \quad \frac{x^2+y^2}{xy}=\frac{58}{21}$$

$= 21[x^2 + (2x+1)^2] = 58x\,(2x+1)$ [Using (i)]

$= 21[x^2+4x^2 + 1 + 4x] = 58(2x^2+x)$

$= 105x^2 + 84x+21=116x^2 +58x$

$= 11x^2-26x-21=0 \Rightarrow 11x^2 - 33x+7x-21=0$

$= 11x(x-3)+7(x-3)=0$

$\Rightarrow \quad (11x + 7)(x - 3) = 0 \quad \Rightarrow \quad x = 3 \quad \left(\because x \neq \frac{-7}{11}\right)$

$\therefore \quad x = 3, y = 2 \times 3 + 1 = 7$

$\therefore$ Required fraction is 3/7.

$\therefore$ Required fraction $= \frac{x-3}{x}$

According to the given condition,

New fraction $= \frac{x-3+2}{x+2} = \frac{x-1}{x+2}$

According to question,

$$\frac{x-3}{x} + \frac{x-1}{x+2} = \frac{29}{20}$$

$$\Rightarrow \quad \frac{(x+2)(x-3)+x(x-1)}{x(x+2)} = \frac{29}{20}$$

$$\Rightarrow \quad \frac{x^2-3x+2x-6+x^2-x}{x^2+2x} = \frac{29}{20}$$

$$\Rightarrow \quad \frac{2x^2-2x-6}{x^2+2x} = \frac{29}{20}$$

$$\Rightarrow \quad 40x^2 - 40x - 120 = 29x^2 + 58x$$

$$\Rightarrow \quad 11x^2 - 98x - 120 = 0$$

$$\Rightarrow \quad 11x^2 - 110x + 12x - 120 = 0$$

60. Let denominator of the fraction be x, so, numerator of the fraction = x-3

$$\Rightarrow \quad 11x(x-10) + 12(x-10) = 0 \Rightarrow (x-10)(11x+12) = 0$$

$$\Rightarrow \quad x - 10 = 0 \text{ or } 11x + 12 = 0 \Rightarrow x = 10 \left(\because x \neq -\frac{12}{11}\right)$$

$\therefore$ Required fraction $= \frac{x-3}{x} = \frac{10-3}{10} = \frac{7}{10}$.

61. Let volume of the swimming pool be V.

Time taken to fill the pool by the pipe of larger diameter be x hours

:- Time taken to fill the pool by the pipe of smaller diameter = (x + 10)hours

Part of the pool filled by the pipe of larger diameter in

$1 \text{ hour} = \frac{V}{x}$

Part of the pool filled by the pipe of smaller diameter in

$1 \text{ hour} = \frac{V}{x+10}$

According to question,

$$\frac{4V}{x} + \frac{9V}{x+10} = \frac{1}{2}V$$

$$\Rightarrow \frac{4x+40+9x}{x(x+10)} = \frac{1}{2} \Rightarrow \frac{13x+40}{x^2+10x} = \frac{1}{2}$$

$= x^2+10x = 26x+80$

$= x^2+10x - 26x-800 \Rightarrow x^2 - 16x-80 = 0$

$= x^2 - 20x+4x-80 = 0 \Rightarrow x(x - 20)+4(x-20) = 0$

$= (x+4)(x - 20) = 0 \Rightarrow x=-4 \text{ or } x=20$

$= x = 20$ (.x cannot be negative)

Hence, the pipe with larger diameter fills the tank in 20 hours and the pipe with smaller diameter fills the tank in 30 hours.

62

$$\text{We have, } \frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}$$

$$\Rightarrow \frac{3(x-1)+4(x+1)}{(x+1)(x-1)} = \frac{29}{4x-1}$$

$$\Rightarrow \frac{3x-3+4x+4}{x^2-1} = \frac{29}{4x-1} \Rightarrow \frac{7x+1}{x^2-1} = \frac{29}{4x-1}$$

$= 29(x^2-1) = (7x+1)(4x-1)$

$= 29x2-2928x^2 - 7x+4x-1$

$= x^2+3x-28=0$

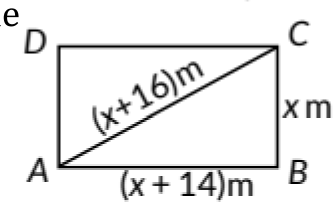
$= x^2+7x-4x-28=0 \Rightarrow x(x+7) - 4(x+7)=0$

$= (x+7)\ (x-4)=0$

$= x = -7 \text{ or } x = 4.$

63.Let ABCD be the given rectangular field. Let the shorter side of field be x m.

:- Longer side = (x + 14) m



and diagonal = (x + 16) m

Using Pythagoras theorem in AABC,

$AB^2 + BC^2 = AC^2$

$= (x+14)^2 + x^2 = (x + 16)^2$

$\Rightarrow x^2+28x+196+x^2 = x^2 + 32x + 256$

$= x^2-4x-60=0 \Rightarrow x^2-10x+6x-60=0$

$= x(x-10)+6(x - 10) = 0 \Rightarrow (x+6)(x - 10) = 0$

$= x = -6$ or $x = 10$

$= x = 10$ (. Side cannot be negative)

:- Sides of the field are 10 m and 10 + 14 = 24 m.

64. Let first speed of train be x km/h.

Time taken to travel 54 km $= \frac{54}{x}$ h

Time taken to travel 63 km $= \frac{63}{x+6}$ h

According to question, $\frac{54}{x} + \frac{63}{x+6} = 3$

$\Rightarrow 9\left(\frac{6}{x} + \frac{7}{x+6}\right) = 3 \Rightarrow \frac{6x+36+7x}{x(x+6)} = \frac{3}{9}$

$\Rightarrow \frac{13x+36}{x^2+6x} = \frac{1}{3} \Rightarrow x^2+6x = 39x+108$

$\Rightarrow x^2 - 33x-1080 \Rightarrow x^2 - 36x + 3x - 108 = 0$

$\Rightarrow x(x-36)+3(x - 36) = 0 \Rightarrow (x+3)(x - 36) = 0$

$\Rightarrow x=-3$ or $x = 36$

$\Rightarrow x=36$ (. Speed cannot be negative)

:- First speed of train = 36 km/h.

65. Let first speed of the bus be x km/h

Time taken to travel 75 km = $\frac{75}{x}$ hrs

Time taken to travel 90 km = $\left(\frac{90}{x+10}\right)$ hrs

According to question, we have

$\frac{75}{x} + \frac{90}{x+10} = 3$

$= 75x+750+ 90x = 3(x^2+10x)$
$= 165x+750 = 3(x^2+10x)$
$= 55x+250 = x^2+10x \Rightarrow x^2 - 45x-250 = 0$
$= x^2-50x+5x-2500$ $(x -50) (x+5)=0$
$= x=50$ or $x=-5$ (Neglected, as speed can't be negative)
:- Speed of bus at first = 50 km/h.

66. Let first speed of the truck be x km/h.

$$\text{Time taken to cover 150 km} = \frac{150}{x} \text{h}$$

$$\text{Time taken to cover 200 km} = \frac{200}{x+20} \text{h}$$

According to question,

$$\frac{150}{x}+\frac{200}{x+20}=5 \Rightarrow \frac{30}{x}+\frac{40}{x+20}=1 \Rightarrow \frac{30x+600+40x}{x(x+20)}=1$$

$= x^2+20x = 70x+600 \Rightarrow x^2-50x-600 = 0$
$= x^2- 60x + 10x - 6000 \Rightarrow x(x-60) + 10 (x-60) = 0$
$= (x + 10)(x-60) = 0 \Rightarrow x=-10$ or $x = 60$
$= x = 60$ (.Speed cannot be negative)
So, the first speed of the truck = 60 km/h

67. Let the length of cloth be x m.

$$\text{Cost per metre} = ₹\,\frac{200}{x}$$

New length of cloth = $(x + 5)$m

$$\text{New cost per metre} = ₹\,\frac{200}{x+5}$$

According to given condition, $\frac{200}{x}-\frac{200}{x+5}=2$

$$\Rightarrow 200\left(\frac{1}{x}-\frac{1}{x+5}\right)=2 \Rightarrow 100\left(\frac{x+5-x}{x^2+5x}\right)=1$$

$= 500 = x + 5x= x + 5x - 500 = 0$
$= x^2+25x-20x - 500 = 0$
$= x(x+25) - 20 (x+25) = 0 \Rightarrow (x+25)(x - 20) = 0$
$= x+25=0$ or $x-20=0 \Rightarrow x=-25$ or $x = 20$
$= x=20$ :- [:: Length cannot be negative]
Length of cloth = 20 m

Original cost per metre $= ₹\ \frac{200}{20} = ₹\ 10.$

68. Let one number be x.
Then, another number = x + 5
According to question,

$$\frac{1}{x} - \frac{1}{x+5} = \frac{1}{10} \Rightarrow \frac{x+5-x}{x(x+5)} = \frac{1}{10}$$

$\Rightarrow \quad x^2 + 5x - 50 = 0 \Rightarrow x^2 + 10x - 5x - 50 = 0$

$\Rightarrow \quad x(x+10) - 5(x+10) = 0$

$\Rightarrow \quad (x+10)(x-5) = 0 \Rightarrow x = 5 \ (\because x \neq -10)$

$\therefore$ Required numbers are 5 and 10.

69. Let one number be x.
:- Another number = x+3
According to question,

$$\frac{1}{x} - \frac{1}{x+3} = \frac{3}{28} \Rightarrow \frac{x+3-x}{x(x+3)} = \frac{3}{28}$$

$\Rightarrow \quad 3 \times 28 = 3x(x+3) \Rightarrow 28 = x^2 + 3x$

$\Rightarrow \quad x^2 + 3x - 28 = 0 \Rightarrow x^2 + 7x - 4x - 28 = 0$

$\Rightarrow \quad x(x+7) - 4(x+7) = 0$

$\Rightarrow \quad (x+7)(x-4) = 0 \Rightarrow x = 4 \quad (\because x \neq -7)$

$\therefore$ Required numbers are 4 and 7.

70. Let one number be x.
Then, another number = x + 5
According to question,

$$\frac{1}{x} - \frac{1}{x+5} = \frac{5}{14} \Rightarrow \frac{x+5-x}{(x)(x+5)} = \frac{5}{14}$$

$$\Rightarrow \frac{5}{x(x+5)} = \frac{5}{14}$$

$\Rightarrow \quad x^2 + 5x = 14 \Rightarrow x^2 + 5x - 14 = 0$

$\Rightarrow \quad x^2 + 7x - 2x - 14 = 0$

$\Rightarrow \quad x(x+7) - 2(x+7) = 0$

$\Rightarrow \quad (x+7)(x-2) = 0 \Rightarrow x = 2 \quad (\because x \neq -7)$

$\therefore$ Required natural numbers are 2 and 7.

71.

We have, $\frac{x-2}{x-3}+\frac{x-4}{x-5}=\frac{10}{3}$ ; $x \neq 3, 5$

$\Rightarrow \quad \frac{(x-2)(x-5)+(x-4)(x-3)}{(x-3)(x-5)}=\frac{10}{3}$

$\Rightarrow \quad 3\,(x^2-7x+10+x^2-7x+12)=10(x^2-8x+15)$

$\Rightarrow \quad 3[2x^2-14x+22]=10[x^2-8x+15]$

$\Rightarrow \quad 6x^2-42x+66=10x^2-80x+150$

$\Rightarrow \quad 4x^2-38x+84=0 \Rightarrow 2x^2-19x+42=0$

$\Rightarrow \quad 2x^2-12x-7x+42=0$

$\Rightarrow \quad 2x\,(x-6)-7(x-6)=0$

$\Rightarrow \quad (2x-7)(x-6)=0 \Rightarrow x=\frac{7}{2},6$

72

We have, $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5, x\neq-3,\frac{1}{2}$

Let $\frac{2x-1}{x+3}=y$

$\therefore$ Given equation becomes $\frac{2y}{1}-\frac{3}{y}=5$

$\Rightarrow \quad \frac{2y^2-3}{y}=5 \quad \Rightarrow \quad 2y^2-3=5y$

$\Rightarrow \quad 2y^2-5y-3=0 \Rightarrow 2y^2+y-6y-3=0$

$\Rightarrow \quad y(2y+1)-3(2y+1)=0 \Rightarrow (y-3)(2y+1)=0$

$\Rightarrow y=3 \quad \text{or} \quad y=\frac{-1}{2}$

$\therefore \quad \frac{2x-1}{x+3}=3 \quad \text{or} \quad \frac{2x-1}{x+3}=\frac{-1}{2}$

$\Rightarrow \quad 2x-1=3x+9 \quad \text{or} \quad 4x-2=-x-3$

$\Rightarrow \quad x=-10 \quad \text{or} \quad x=\frac{-1}{5}\cdot$

73. Let two consecutive even numbers be x and x + 2.

According to question, $(x)^2+(x+2)^2=340$

$\Rightarrow x^2+x^2+4x+4=340$

$\Rightarrow 2x^2+4x-336=0 \Rightarrow x^2+2x-168=0$

$\Rightarrow x^2+14x-12x-168=0 \Rightarrow x(x+14)-12(x+14)=0$

$\Rightarrow$ (x+14) (x - 12)=0$\Rightarrow$ x= 12 or x=-14
Hence, two consecutive even numbers are 12, 14

74.

$$\text{We have, } 3\left(\frac{3x-1}{2x+3}\right)-2\left(\frac{2x+3}{3x-1}\right)=5, x\neq\frac{1}{3},\frac{-3}{2} \quad ...(i)$$

Let $\frac{3x-1}{2x+3}=y$ , then (i) becomes $3y-\frac{2}{y}=5$

$\Rightarrow 3y^2-2=5y = 3y^2-5y-2=0$
$\Rightarrow 3y^2+y-6y-2=0$
$\Rightarrow y(3y+1)-2(3y+ 1) = 0$
$\Rightarrow (y-2)(3y+ 1) = 0$

$$\Rightarrow y=2 \text{ or } y=\frac{-1}{3}$$
$$\therefore \frac{3x-1}{2x+3}=2 \text{ or } \frac{3x-1}{2x+3}=\frac{-1}{3}$$

3x-1=4x+6 or 9x-3=-2x-3
x= -7 or 11x=0$\Rightarrow$x=-7 or x=0

75. Let two consecutive multiples of 7 be k, k+7. According to question,
$(k)^2+(k+7)^2 = 637$
$= k^2 + k^2 +49 +14k = 637$
$= 2k^2 + 14k - 5880$
$= k^2+21k - 14k - 294 = 0$
$= k(k +21) - 14(k +21) = 0$
$= k^2+7k - 294 = 0$
$= (k +21)(k-14) = 0$ k = 14 or k=-21
Hence, required multiples of 7 are 14, 21 or -14, -21.

76.

We have,

$$3\left(\frac{7x+1}{5x-3}\right)-4\left(\frac{5x-3}{7x+1}\right)=11\ ,\ x\neq\frac{3}{5},\frac{-1}{7} \quad \text{...(i)}$$

Let $\frac{7x+1}{5x-3}=y$,

then (i) becomes $3y-\frac{4}{y}=11$

$\Rightarrow\ 3y^2-4=11y \Rightarrow 3y^2-11y-4=0$

$\Rightarrow\ 3y^2+y-12y-4=0$

$\Rightarrow\ y(3y+1)-4(3y+1)=0$

$\Rightarrow\ (y-4)(3y+1)=0 \Rightarrow y=4$ or $y=\frac{-1}{3}$

$\therefore\ \frac{7x+1}{5x-3}=4$ or $\frac{7x+1}{5x-3}=\frac{-1}{3}$

$\Rightarrow\ 7x+1=20x-12$ or $21x+3=-5x+3$

$\Rightarrow\ 13=13x$ or $26x=3-3$

$\Rightarrow\ x=1$ or $x=0$

77. (d): Given quadratic equation is $3\sqrt{3}x^2+10x+\sqrt{3}=0$ ...(i)
Comparing (i) with $ax^2+bx+c=0$, we have
$a=3\sqrt{3}, b=10, c=\sqrt{3}$
So, discriminant, D= b2-4ac = $(10)^2 - 4\times 3\sqrt{3}x\sqrt{3}$
= 100-36 = 64

78. Given quadratic equation is
(x + 5) = 2 (5x – 3)
$= x^2+10x+25=10x-6 \Rightarrow x^2+31=0$
On comparing with $ax^2+bx+c=0$,
we have a = 1, b = 0, c = 31
Now, discriminant, $D = b^2 - 4ac$
$=0-4\times1\times31 = -124$.

79.

Ques 5

(a) $x^2 - 2ax - (4b^2 - a^2) = 0$

$b^2 - 4ac = 4a^2 - 4[-(4b^2 - a^2)](1)$

$= 4a^2 - 4[-4b^2 + a^2]$

$= 4a^2 + 16b^2 - 4a^2$

$= 16b^2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2a \pm \sqrt{16b^2}}{2(1)} = \frac{2a \pm 4b}{2}$

$\Rightarrow x = \frac{2a + 4b}{2}$ $x = \frac{2a - 4b}{2}$

$x = a + 2b$ $x = a - 2b$

[Topper's Answer, 2022]

80. $2x^2 - 2\sqrt{2}x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 2$, $b = -2\sqrt{2}$, $c = 1$

:- $D = b2 - 4ac = (-2\sqrt{2})^2 - 4 \times 2 \times 1 = 8 - 8 = 0$

Since, $D = 0$, so we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} - = \frac{-(-2\sqrt{2}) \pm 0}{2 \times 2} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

So, $x = \frac{\sqrt{2}}{2}$ and $x = \frac{\sqrt{2}}{2}$

83. Given, quadratic equation is $x^2 + 2\sqrt{2}x - 6 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

$a = 1$, $b = 2\sqrt{2}$ and $c = -6$

By quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-(2\sqrt{2}) \pm \sqrt{(2\sqrt{2})^2 - 4(1)(-6)}}{2(1)} = \frac{-2\sqrt{2} \pm \sqrt{8 + 24}}{2}$$

$$= \frac{-2\sqrt{2} \pm \sqrt{32}}{2} = \frac{-2\sqrt{2} \pm 4\sqrt{2}}{2} = -\sqrt{2} \pm 2\sqrt{2}$$

$\Rightarrow x=-\sqrt{2}+2\sqrt{2}$ or $x=-\sqrt{\sqrt{2}}-2\sqrt{2}$
$\Rightarrow x=\sqrt{2}$ or $x=-3\sqrt{2}$
Hence, $\sqrt{2}$ and $-3\sqrt{2}$ are the roots of the given quadratic equation.

84. Given, quadratic equation is $\sqrt{2}x^2+7x+5\sqrt{2}=0$ On comparing it with ax2 + bx + c = 0, we get
$a=\sqrt{2}, b=7, c=5\sqrt{2}$
By using quadratic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(7)\pm\sqrt{(7)^2-4(\sqrt{2})5(\sqrt{2})}}{2\sqrt{2}}$$

$$=\frac{-7\pm\sqrt{49-40}}{2\sqrt{2}}=\frac{-7\pm\sqrt{9}}{2\sqrt{2}}=\frac{-7\pm 3}{2\sqrt{2}}$$

Hence, $x=\frac{-4}{2\sqrt{2}}=-\sqrt{2}$ or $x=\frac{-10}{2\sqrt{2}}=\frac{-5\sqrt{2}}{2}$.

85. We have, $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}-0$
On comparing it with ax2 + bx + c = 0, we get
$a=\sqrt{3}, b=-2\sqrt{2}c=-2\sqrt{3}$
Using quadratic formula,

$$x=\frac{-(-2\sqrt{2})\pm\sqrt{(-2\sqrt{2})^2-4(\sqrt{3})(-2\sqrt{3})}}{2\sqrt{3}}$$

$$=\frac{2\sqrt{2}\pm\sqrt{8+24}}{2\sqrt{3}}=\frac{2\sqrt{2}\pm\sqrt{32}}{2\sqrt{3}}=\frac{2\sqrt{2}\pm 4\sqrt{2}}{2\sqrt{3}}$$

$$\Rightarrow x=\frac{\sqrt{2}+2\sqrt{2}}{\sqrt{3}} \text{ or } x=\frac{\sqrt{2}-2\sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow x=\frac{3\sqrt{2}}{\sqrt{3}}=\sqrt{6} \text{ or } x=-\sqrt{\frac{2}{3}}$$

86. $abx^2 + (b^2 - ac)x - bc = 0$
On comparing with $Ax^2 + Bx + C = 0$,
We have, $A = ab$, $B = b^2 - ac$, $c = -bc$
:- $D = B^2 - 4AC$
$= (b^2 - ac)^2 - 4x\, ab \times (-bc)$
$= b^2 + a^2c^2 - 2b^2ac + 4ab2c$

$= b^2 + a^2c^2 + 2ab2c$

$D = (b^2 + ac)^2$

Now, $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$x = \frac{-(b^2 - ac) \pm \sqrt{(b^2 + ac)^2}}{2 \times ab}$

$x = \frac{-b^2 + ac \pm (b^2 + ac)}{2ab}$

So, $x = \frac{-b^2 + ac + b^2 + ac}{2ab}$ or $x = \frac{-b^2 + ac - b^2 - ac}{2ab}$

$x = \frac{2ac}{2ab}$ or $x = \frac{-2b^2}{2ab}$

$\Rightarrow \quad x = \frac{c}{b}$ or $x = \frac{-b}{a}$

87. We have, $x^2 - (2b - 1)x + (b^2 - b - 20) = 0$

Discriminant, $D = (2b - 1)^2 - 4(1)(b^2 - b - 20)$

$= 4b2+1-4b-4b2+4b+80=81$

Using quadratic formula,

$x = \frac{(2b-1) \pm \sqrt{81}}{2(1)} = \frac{2b - 1 \pm 9}{2}$

$\Rightarrow \; x = \frac{(2b-1)+9}{2}$ or $x = \frac{(2b-1)-9}{2}$

$\Rightarrow \; x = \frac{2b+8}{2} = b + 4$ or $x = \frac{2b-10}{2} = b - 5$

88. We have, $x^2 + 6x - (a^2 + 2a - 8) = 0$

Discriminant, $D = (6)^2 - 4\ (1)\ (-\ (a^2 + 2a - 8))$

$= 36 + 4(a^2 + 2a - 8) = 4a^2 + 8a + 4 = 4(a + 1)^2$

Using quadratic formula,

$x = \frac{-6 \pm \sqrt{4(a+1)^2}}{2(1)} \;\Rightarrow\; x = \frac{-6 \pm 2(a+1)}{2}$

$\Rightarrow \; x = \frac{-6 + 2a + 2}{2}$ or $x = \frac{-6 - 2a - 2}{2}$

$\Rightarrow \; x = \frac{2a-4}{2} = a - 2$ or $x = \frac{-2a-8}{2} = -(a+4)$

89. Let the greater number be x.
Given, $(\text{smaller number})^2 = 8(\text{greater number})$
$\Rightarrow (\text{smaller number})^2 = 8x$
$\Rightarrow$ smaller number $= \sqrt{8x}$
Now, according to the given condition,
$x^2-(\sqrt{8x})^2=180$
$\Rightarrow x^2-8x=180 \Rightarrow x^2 - 8x-180 = 0$
Comparing it with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = -8$ and $c = -180$
:- $b2-4ac = (-8)^2 - 4(1)(-180) = 64+ 720 = 784$

By quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\therefore \quad x=\frac{-(-8)\pm\sqrt{784}}{2(1)} \Rightarrow x=\frac{8\pm28}{2}$

Taking positive sign, $x=\frac{8+28}{2}=\frac{36}{2}=18$

Taking negative sign, $x=\frac{8-28}{2}=\frac{-20}{2}=-10$ (not possible)

$\therefore$ The greater number, $x = 18$
And smaller number $=\sqrt{8x}=\sqrt{8\times18}=\sqrt{144}=\pm12$
Thus, smaller number = 12 or −12
Hence, the two numbers are 18 and 12, 18 and −12.

90.

We have, $\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4} \Rightarrow \frac{x+2+2(x+1)}{x^2+3x+2}=\frac{4}{x+4}$

$= (3x+4)(x+4)=4(x^2+3x+2)$
$= 3x^2+16x+16=4x^2 + 12x+8= x^2-4x-8=0$
Comparing it with $ax^2 + bx + c = 0$,
we have $a = 1$, $b = -4$, $c = -8$
Using quadratic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{4\pm\sqrt{16+32}}{2}=\frac{4\pm4\sqrt{3}}{2}=2\pm2\sqrt{3}$$

$\Rightarrow \quad x=2+2\sqrt{3}$ or $2-2\sqrt{3}$

91. Let one pipe fill the tank in x minutes, then the second pipe fill the tank in (x-5) minutes.
According to question,

$$\frac{1}{x}+\frac{1}{x-5}=\frac{9}{100}$$

$$\Rightarrow \frac{x-5+x}{x(x-5)}=\frac{9}{100} \Rightarrow \frac{2x-5}{x(x-5)}=\frac{9}{100}$$

$$\Rightarrow 100(2x-5)=9x(x-5)$$

$$\Rightarrow 200x-500=9x^2-45x \Rightarrow 9x^2-245x+500=0$$

By using quadratic formula

$$x=\frac{245\pm\sqrt{(245)^2-(4)(9)(500)}}{18}$$

$$\Rightarrow x=\frac{245\pm\sqrt{42025}}{18} \Rightarrow x=\frac{245+205}{18} \text{ or } \frac{245-205}{18}$$

x = 25 or 2.2
(When x = 2.2, then (x-5)= 2.25 2.8, which is not possible, as minutes are not negative)
:- x = 25
So, first pipe fill the tank in 25 minutes and second pipe fill the same tank in 20 minutes.

92.

$$\text{We have, } \frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}$$

$$\Rightarrow \frac{4(x-2)+3(x+1)}{2(x+1)(x-2)}=\frac{23}{5x}$$

$$\Rightarrow \frac{4x-8+3x+3}{2(x^2-2x+x-2)}=\frac{23}{5x} \Rightarrow \frac{7x-5}{2x^2-2x-4}=\frac{23}{5x}$$

$23(2x^2-2x-4)= 5x\ (7x-5)$
$46x^2-46x-92=35x^2 - 25x11x^2 - 21x-92=0$

Using quadratic formula

$$x=\frac{-(-21)\pm\sqrt{(-21)^2-4\times11\times(-92)}}{2\times11}$$

$$=\frac{21\pm\sqrt{441+4048}}{22}=\frac{21\pm67}{22}$$

$$\Rightarrow\ x=\frac{21+67}{22}=4\ \text{or}\ \frac{21-67}{22}=-\frac{23}{11}$$

93.

We have, $\frac{x-3}{x-4}+\frac{x-5}{x-6}=\frac{10}{3}, x\neq4,6$

$$\Rightarrow\quad \frac{(x-3)(x-6)+(x-5)(x-4)}{(x-4)(x-6)}=\frac{10}{3}$$

$$\Rightarrow\quad \frac{x^2-9x+18+x^2-9x+20}{(x-4)(x-6)}=\frac{10}{3}$$

$$\Rightarrow\quad 3(2x^2-18x+38)=10(x^2-10x+24)$$

$$\Rightarrow\quad 6x^2-54x+114=10x^2-100x+240$$

$$\Rightarrow\quad 4x^2-46x+126=0\ \Rightarrow\ 2x^2-23x+63=0$$

Using quadratic formula

$$x=\frac{23\pm\sqrt{(23)^2-4(2)(63)}}{2(2)}=\frac{23\pm\sqrt{529-504}}{4}$$

$$\Rightarrow x=\frac{23\pm\sqrt{25}}{4}=\frac{23\pm5}{4}\quad\Rightarrow\ x=\frac{28}{4}\ \text{or}\ \frac{18}{4}\quad\Rightarrow\ x=7,\frac{9}{2}$$

94.

We have,

$$\frac{x-4}{x-5}+\frac{x-6}{x-7}=\frac{10}{3}, x\neq5,7$$

$$\Rightarrow\quad \frac{(x-4)(x-7)+(x-6)(x-5)}{(x-5)(x-7)}=\frac{10}{3}$$

$= 3(x^2 - 11x+28+x^2 - 11x+30) = 10(x^2 - 12x+35)$

$= 3(2x^2 - 22x+58) = 10(x^2 - 12x+35)$

$= 3(x^2-11x+29)=5(x^2 - 12x+35)$

$= 3x^2 - 33x+87 = 5x^2 - 60x + 175$

$= 2x^2-27x+88=0$

By quadratic Formula, we have

$$x=\frac{-(-27)\pm\sqrt{(-27)^2-4(88)(2)}}{2(2)}$$

$$=\frac{27\pm\sqrt{729-704}}{4}=\frac{27\pm\sqrt{25}}{4}=\frac{27\pm5}{4}$$

$$\Rightarrow \quad x=\frac{27+5}{4} \text{ or } x=\frac{27-5}{4} \Rightarrow x=8 \text{ or } x=5.5$$

95. (b): For rational roots, D is a perfect square

$D=k^2+32$

If k = 2 then,

$D=2^2+32=36$, which is a perfect square

96. (b): Given quadratic equation is

$2x^2+kx+2=0$

Since, the equation has equal roots.

:- Discriminant = 0

$= k^2-4\times2\times2=0\Rightarrow k^2 - 16 = 0$

$= k^2 = 16$ k=+4

97. Given quadratic equation is

$2x^2-4x+3=0$

Here, D = b2-4ac

=(-4)2-4×2×3 = 16-24-8 <0

:- Given quadratic equation has no real roots.

98. Given quadratic equation is

$9x^2 - 3ax+1=0$

Since, equation has equal roots.

:- D=0

= b2-4ac = 0

$= (-3a)^2 - 4x9x1=0$

$= 9a^2 - 36=0$

$= a^2=4a= +2$

99. Given, $x^2+4x+k=0$

For real roots, discriminant, $D\geq0$

:- $b2$-4ac $\geq 0$
= 16-4(1)(k) $\geq 0$16-4k$\geq 0 \Rightarrow$ k$\leq 4$

100. Given quadratic equation is, $3x^2 + kx + 3 = 0$.
Since, equation has real and equal roots.
:- Discriminant, D=0
$\Rightarrow (k)^2 - 4(3)(3) = 0$ $k^2 - 36=0$
$\Rightarrow k^2=36 \Rightarrow k=\pm 6$

101. Given quadratic equation has equal roots.
$px^2-2\sqrt{5}px+15=0$
:- Discriminant, D=0
i.e., $(-2\sqrt{5}p)^2$-4xpx15=0$\Rightarrow$ $20p^2$ -60p=0
= p2-3p=0 = p(p-3)=0
= p=0 or p=3p=3 (.p cannot be zero)

102. Given quadratic equation is $4x^2 - 5 = 0$
Discriminant, $D = b^2 - 4ac = 0^2 - 4(4)(-5) = 80 > 0$
Hence, the roots of the given quadratic equation are real and distinct.

103. We have, $(m-1)\,x^2+2\,(m-1)x+1=0$ ...(i)
On comparing the given equation with $ax^2 + bx + c = 0$, we have a = (m-1), b = 2 (m-1), c = 1
Discriminant, D = 0
= $b^2$-4ac = 0$\Rightarrow$ $4m^2 + 4 - 8m - 4m+4=0$
= $4m^2-12m+8=0$
= $m^2-3m+2=0$
= $m^2-2m-m+2=0$
= m(m-2)-1 (m-2) = 0
= (m-1)(m-2) = 0$\Rightarrow$ m=1,2

104. Here, the given quadratic equation is
$(1 + a^2)x^2+2abx+ (b^2 - c^2) = 0$ ...(i)
On comparing (i) with $Ax^2 + Bx + C = 0$, we get
$A = (1 + a^2)$, B = 2ab, $C = (b^2 - c^2)$
Discriminant, $D = B^2 - 4AC = (2ab)^2 - 4(1 + a^2)(b^2 - c^2)$
$\Rightarrow D=4a^2b^2 - 4(b^2 - c^2 + a^2b^2 - a^2c^2)$
$D = 4a^2b^2 - 4b^2 + 4c^2 - 4a^2b^2+4a^2c^2$

$= 4[a^2b^2 - b^2 + c^2 - a^2b^2 + a^2c^2] = 4(-b^2 + c^2 + a^2c^2)$
:- Given, equation (i) has equal and real roots.
:- D=0
$\Rightarrow 4(-b^2+c^2+a^2c^2) = 0 = -b^2 + c^2+a^2c^2 = 0$
$\Rightarrow c^2+a^2c^2 = b^2 \Rightarrow c^2(1+a^2) = b^2$
or $b^2 = c^2(1+a^2)$
Hence proved.

105. The given equation is $3x^2 - 4\sqrt{3}x+4=0$ ...(i)
On comparing the given equation with $ax^2 + bx + c = 0$,
we have a = 3, b = $-4\sqrt{3}$,c=4
b2-4ac = $(-4\sqrt{3})^2 - 4\times3\times4$
= 48-48=0
Since, $b^2 - 4ac = 0$
The given equation has real and equal roots which are given by

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \Rightarrow x=\frac{-(-4\sqrt{3})\pm0}{2\times3} \Rightarrow x=\frac{4\sqrt{3}}{6}=\frac{2\sqrt{3}}{3}$$

106. Given, $x^2 + k(2x+ k - 1)+2=0$
or $x^2+2xk+k^2-k+2=0$
Since, equation has real and equal roots.
:- Discriminant, D=0
$= (2k)^2 - 4(1)(k^2 - k+2)=0$
$= 4k^2-4k^2 + 4k -8-0$ 4k-8=0
= 4k 8k 2

107. Given, $4x^2 + px+3=0$
Since, the roots are equal.
:- D=0= $p^2 - 4(4)(3) = 0$
$\Rightarrow p^2-48=0 \Rightarrow p^2 = 48$ $p=\pm4\sqrt{3}$

108. Given, 9x2-3kx+k=0
Since, equation has equal roots.
D=0 ⇒ (3k)2-4(9) (k) = 0
⇒ 9k2-36k=0
⇒ 9k(k-4)=0⇒ k=0,4

109. Given, $px(x-3)+9=0 \Rightarrow px^2 - 3px + 9 = 0$
Since, roots are equal.
:- $D=0 \Rightarrow (-3p)^2 - 4(p)(9) = 0$
9p2-36p-09p (p-4)=0⇒ p=4 [.p# 0]

110. The given quadratic equation is $px(x - 2) + 6 = 0$
$\Rightarrow px^2 - 2xp + 6 = 0$
On comparing with ax2 + bx + c = 0, we get
a = p, b = -2p and c = 6
Since, the quadratic equations has two equal real roots.
:- Discriminant, D=0
⇒ b2-4ac = 0
$\Rightarrow (-2p)^2 - 4x\ p\times6 = 0$
⇒ 4p2-24p=0⇒ $p^2 - 6p = 0$
⇒ p(p-6)=0
But p#0
p=0orp=6
Hence, the value of p is 6.

111. We have, $x^2 + px + 16 = 0$
Since, roots are qual.
:- D=0
$\Rightarrow p^2 - 4x16 = 0 \Rightarrow p^2 - 64 = 0$
$\Rightarrow p^2 = 64 \Rightarrow p = +8$
Now, put p = 8 in (i), then we get
$x^2+8x+16=0$
$\Rightarrow x^2+4x+4x+16=0$
$\Rightarrow (x+4)^2=0 \Rightarrow x=-4,-4$
If we put p = -8 in (i), then we get
$x^2-8x+16=0$
$\Rightarrow x^2-4x-4x+16=0$
$\Rightarrow (x-4)^2=0 \Rightarrow x=4,4$

112. We have, $(a^2+b^2x^2 - 2(ac + bd)x+(c^2 + d^2) = 0$
Since, the given quadratic equation has equal roots.
:- $D=0 \Rightarrow B^2 - 4AC = 0$
$\Rightarrow (-2\ (ac+bd))^2 - 4\ x\ (a^2 + b^2)\ x\ (c^2 + d^2) = 0$ ...(i)
$\Rightarrow 4(a^2c^2+b^2d^2+2abcd) - 4\ (a^2c^2 + a^2d^2 + b^2c^2+b^2d^2) = 0$

$\Rightarrow 4[a^2c^2 + b^2d^2+2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2] = 0$
$\Rightarrow 4[2abcd - a^2d^2 - b^2c^2]=0$
$\Rightarrow 4(ad - bc)^2=0\Rightarrow ad- bc=0\Rightarrow ad = bc$

113. We have, $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$
Since, equation has equal roots.
:- Discriminant, D=0
$\Rightarrow (2mc)^2 - 4(1+m^2)(c^2 - a^2) = 0$
$\Rightarrow 4m^2c^2 - 4c^2+4a^2 - 4m^2c^2+4a^2m^2 = 0$
$\Rightarrow c^2 = a^2 + a^2m^2 = a^2(1+m^2)$

114. We have, $(a^2 + b^2)x^2 + 2(ac + bd)x+(c^2 + d^2) = 0$
Discriminant, $D = 4(ac + bd)^2 - 4(a^2 + b^2) (c^2 + d^2)$ [ad#bc]
$\Rightarrow 4(a^2c^2 + b^2d^2+2acbd) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$
$\Rightarrow 4(a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2)$
$\Rightarrow 4(2abcd - a^2d^2 - b^2c^2) = -4(ad - bc)2 < 0$
Thus, given equation has no real roots.

115. Given, $(a - b)x^2 + (b - c) x + (c - a) = 0$
:- Roots are equal. Discriminant, D = 0
i.e., $(bc)^2 - 4(c-a)(a - b) = 0$
$\Rightarrow (b^2 + c^2 - 2bc) -4(ac - a^2 + ab - bc) = 0$
$\Rightarrow b2+ c^2-2bc-4ac + 4bc-4ab+4a^2 = 0$
$\Rightarrow 4a^2 + b^2 + c^2-4ac + 2bc-4ab=0$
$\Rightarrow [(-2a) +b+c]^2 = 0$
$\Rightarrow -2a+b+c=0\Rightarrow 2a = b+c$

116. We have, $kx^2 + 1-2(k - 1)x + x^2=0$
or $(k+1)x^2 - 2(k - 1)x + 1 = 0$ ...(i)
Since, roots are equal. .. D=0
$\Rightarrow \{-2(k-1)\}2-4x (k+1) x1 = 0$
$\Rightarrow 4k^2 - 8k + 4-4k-4=0$
$\Rightarrow 4k^2 - 12k 0 \Rightarrow 4k(k-3)=0$
$\Rightarrow k=0$ or $k-3=0\Rightarrow k=3$ (.'k+0)
Substituting the value of k in (i), we get
$(3 + 1)x^2 - 2(3-1)x+1 = 0$
$\Rightarrow 4x^2-4x+1=0$
$\Rightarrow 4x^2-2x-2x+1=0 \Rightarrow 2x(2x-1) -1(2x 1) = 0\Rightarrow (2x-1) (2x 1) = 0$

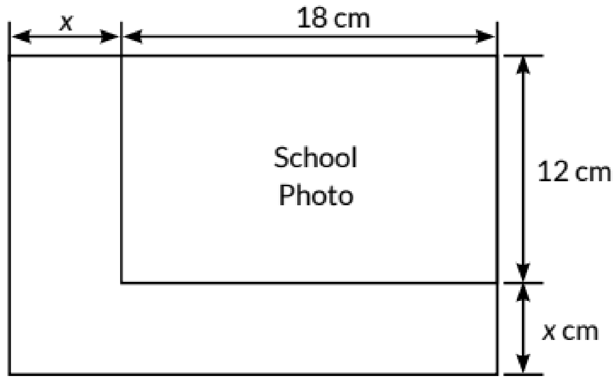
$\Rightarrow \quad 2x - 1 = 0$ or $2x - 1 = 0 \Rightarrow x = \frac{1}{2}, \frac{1}{2}$

117. We have, $(p + 1)x^2 - 6(p + 1)x+3(p+9)=0$
Since, roots are equal.
. D=0
$\Rightarrow \{-6(p + 1)\}^2 - 4(p + 1) \times 3(p+9)=0$
$\Rightarrow 36(p+1)^2 - 12(p + 1)(p+9)=0$
$\Rightarrow 12(p + 1) [3(p+ 1) = (p+9)] = 0$
$\Rightarrow (p + 1)(3p+ 3-p-9)=0$
$\Rightarrow (p + 1)(2p-6)=0 \Rightarrow (p+1)=0$ or $(2p-6)=0$
$\Rightarrow p=3$ (As p+-1)
$\Rightarrow p=-1$ or $p=3$
For p = 3, the given equation becomes
(3+1)x2 - 6(3+1)x + 3(3+9)=0
$\Rightarrow 4x^2 - 24x+36=0$ or $x^2-6x+9=0$
$\Rightarrow (x-3)(x-3)=0 \Rightarrow x=3,3$

118. Since, 2 is a root of the equation $3x^2 + px-8=0$
:- $3(2)^2+p(2)-8=0$
$2p+4=0 \Rightarrow p=-2$
Putting p = -2 in 4x2 - 2px + k = 0, we get
$4x^2+4x+k=0$ ...(i)
Since, (i) has equal roots.
:- D=0⇒42-4(4)(k) = 0 ⇒ 16 = 16k k=1

119. Since, 1 is a root of $3x^2+ax-2=0$
;- $3(1)^2+a(1)$ 20⇒a+1=0⇒ a = -1
Putting a = -1 in $a(x^2 + 6x) - b = 0$, we get
$x^2+6x+b=0$ ...(i)
Since, (i) has equal roots. .. D=0⇒62-4(b) = 0
⇒ 36=4bb=9

120. Area = 18 x 12 cm
Length (/) is increased by x cm
So, new length = (18+ x) cm
New width = (12+x) cm




(i) Area of photo after increasing the length and width

$= (18+x)(12+x) = 2 \times 18 \times 12$

i.e., $(18+x)(12+x) = 432$ is the required algebric equation.

(ii) From part (i) we get, $(18 + x)(12+x) = 432$

$= 216 + 18x + 12x+x^2 = 432 \Rightarrow x^2+30x - 216=0$

(iii) $x^2+30x-216=0$

$= x^2+36x-6x-216=0$

$= x(x+36) - 6(x+36)=0 \Rightarrow x=6,-36$

-36 is not possible.

So, new length = (18+6) cm = 24 cm

New width = (12+6) cm = 18 cm

So, new dimension = 24 cm x 18 cm

**OR**

According to question $(18+x)(12+x) = 220$

$= 216+30x+x^2 = 220$

$= x^2+30x+216-220 = 0 \Rightarrow x^2 + 30x-4=0$

For rational value of x, discriminant (D) must be perfect square.

So, $D = b^2 - 4ac$

$= (30)^2 - 4(1)(-4) = 900+ 16 = 916$

:- 916 is not a perfect square.

So, no rational value of x is possible.

121. We have, $x^2 + kx+64 = 0$ ...(i)

and $x^2 - 8x + k=0$ ...(ii)

Both equations (i) and (ii) will have real roots if

Discriminant, $D \geq 0$

i.e., $k^2$ - 4(64) > 0 and $(-8)^2$ - 4k≥0
⇒$k^2$ 256 and 64≥4k
= k≥16;k<-16
and 16>k⇒ k = 16.
So, possible value of k is 16.

122. We have, $x^2 + 2px + mn = 0$ ...(i)
and $x^2 - 2(m + n)x+(m^2 + n^2 + 2p^2) = 0$ ...(ii)
Since, equation (i) has equal roots.
:- D=0⇒ $(2p)^2 - 4mn = 0$
$4p^2 = 4mn = p^2 = mn$ ...(iii)
Now, for equation (ii), discriminant, D
= (-2(m + n))2 - 4 (m2 tr + 2)
= $4(m^2 + n^2 + 2mn) - 4(m^2 + n^2 + 2mn)$ (Using (iii))
= 0
Thus, equation (ii) also has equal roots.

123. Given, x = -2 is a root of the equation, $3x^2+7x+p=0$
:- $3(-2)^2+7(-2)+p=0$
= 12-14+ p=0⇒p=2
Also, we have, $x^2 + k(4x + k - 1) + p=0$
= $x^2 + 4kx + k^2 - k+2=0$ (p=2)
:- Roots are equal... D = 0
= $(4k)^2 - 4(1)(k^2 - k+2)=0$
= $16k^2 - 4k^2 + 4k - 8 = 0 = 12k^2+4k-8=0$
= $3k^2+k-2=0$⇒ (k + 1)(3k - 2) = 0

$$\Rightarrow \quad k+1=0 \text{ or } 3k-2=0 \Rightarrow \quad k=-1 \text{ or } k=\frac{2}{3}$$

124. Since, x = 3 is the root of the equation
$x^2-x+k=0$
:- $(3)^2-3+k=0$⇒ 9-3+k=0⇒ k=-6
We have, $x^2 + k(2x+k+2)+p=0$
= $x^2+(-6)[2x+(-6)+2]+p=0$
= $x^2 - 12x + 24+ p=0$
;- Roots are equal. .. D=0
= $(-12)^2 - 4(1)(24+p) = 0$

$= 144-96-4p=048-4p = 0$
$= 4p=48\ p=12$

125. Since, - 4 is a root of the equation $x^2 + 2x + 4p = 0$
:- $(-4)^2+2(-4)+4p = 0$
$= 16-8+4p=0$
$4p=-8p=-2$
:- The given equation becomes
$x^2-2(1+3k)x+7(3+2k) = 0$
:- Roots are equal... D=0
$= [-2(1+3k)]^2 - 4 \times 1 \times 7(3+2k) = 0$
$= 1+6k+9k^2 - 21-14k = 0$
$= 9k^2-8k-20=0$
$= 9k^2 - 18k + 10k-20=0$
$= 9k\ (k-2)+10\ (k-2) = 0$
$= (9k+10)(k-2) = 0$
$= 9k+ 100$ or $k-2=0$
$$\Rightarrow \quad k=\frac{-10}{9} \text{ or } k=2$$

126. We have, $(k + 4)x^2 + (k + 1)x+1=0$
Since, roots are equal.
:- D=0
$\Rightarrow (k+1)^2 - 4(k + 4) = 0$
$\Rightarrow k^2+2k+1-4k-16=0$
$\Rightarrow k^2-2k-15=0$
$\Rightarrow (k-5)(k + 3) = 0$
$\Rightarrow k-5\ 0$ or $k+3=0$
$\Rightarrow k = 5$ or $k = -3$
For $k = 5$, the given equation becomes
$9x^2+6x+1=0$
$= (3x)^2+2(3x) + 1 = 0 = (3x+1)^2=0$
$$\Rightarrow \quad x=\frac{-1}{3}, \frac{-1}{3}$$

For $k = -3$, the given equation becomes
$x^2-2x+1=0 \Rightarrow (x -1)^2 = 0 \Rightarrow x= 1,1$

Thus, the equal roots of the given quadratic equation are either 1 or -1/3.

127. We have, $(3k + 1)x^2 + 2(k + 1)x+1 = 0$ ...(i)
Since, roots are equal... D = 0
$\Rightarrow [2(k+1)]^2 - 4 (3k+ 1) = 0$
$\Rightarrow [4(k^2+1+2k)] - 4(3k+ 1) = 0$
$\Rightarrow k^2+2k+1-3k-1=0$
$\Rightarrow k^2-k=0k (k-1)=0$ $k=0$ or $k = 1$
When $k = 0$, (i) becomes $x^2 + 2x+1=0$
$\Rightarrow (x+1)^2=0 \Rightarrow x=-1,-1$
When $k = 1$, (i) becomes $4x^2+4x+1=0$

$\Rightarrow \quad (2x+1)^2 = 0 \;\Rightarrow\; x = \frac{-1}{2}, \frac{-1}{2}$
Thus, equal roots of given equation are either $-1$ or $\frac{-1}{2}$.

128. Given, $(2p+ 1)x^2 - (7p+2)x+(7p - 3)=0$ ...(i)
:- Roots are equal.
:- D=0
$\Rightarrow (7p+ 2)^2 - 4(2p+ 1)(7p - 3) = 0$
$\Rightarrow 49p^2+4+28p - 4(14p^2 + 7p - 6p - 3) = 0$
$\Rightarrow 49p^2+28p +4-56p^2 - 4p+12=0$
$\Rightarrow 7p^2 - 24p-16=0$
$\Rightarrow 7p^2+4p-28p-16=0$
$\Rightarrow p(7p+4)-4(7p+4)=0$
$\Rightarrow (p-4) (7p+4)=0$

$\Rightarrow p = 4$ or $p = \frac{-4}{7}$
When $p = 4$, (i) becomes $9x^2 - 30x + 25 = 0$
$\Rightarrow \quad (3x)^2 - 2(3x)(5) + (5)^2 = 0$
$\Rightarrow \quad (3x-5)^2 = 0 \;\Rightarrow\; x = \frac{5}{3}, \frac{5}{3}$
When $p = \frac{-4}{7}$, (i) becomes
$\frac{-x^2}{7} + 2x - 7 = 0 \;\Rightarrow\; x^2 - 14x + 49 = 0$
$\Rightarrow \quad (x-7)^2 = 0 \Rightarrow x = 7, 7$
Thus, equal roots of given equation are either 5/3 or 7.

# CBSE Sample Questions

1. (b): Factors of prime number p is p and 1.
So, quadratic equation is:
$x^2 - (p+1)x + p=0$ (1)

2. We have $x^2 + 7x+10=0$
$\Rightarrow x^2+5x+2x+10=0$ (1/2)
$\Rightarrow (x+5)(x+2)=0$
$\Rightarrow x=-5, x=-2$ (1/2)

3. Let the present age of Ritu be x years.
According to question, we have
$(x - 5) = 5x+11$ (1)
$\Rightarrow x^2 - 10x+25=5x+11 \Rightarrow x^2-15x+14=0$
$\Rightarrow (x-14)(x-1)=0$
$\Rightarrow x = 1$ or $14$ (1/2)
$\Rightarrow x=14$ (rejecting $x = 1$)
Hence, present age of Ritu is 14 years. (1/2)

4. Given equation is $3x^2 + px+4=0$ ...(i) (1/2)

As, $\frac{2}{3}$ is the root of the equation (i).

$\therefore \quad 3\left(\frac{2}{3}\right)^2 + p\left(\frac{2}{3}\right) + 4 = 0$ **(1/2)**

$\Rightarrow \quad \frac{4}{3} + \frac{2p}{3} + 4 = 0$

$\Rightarrow \quad 2p + 16 = 0 \Rightarrow p = -8$ **(1/2)**

When $p = -8$, (i) becomes

$3x^2 - 8x + 4 = 0$

$\Rightarrow \quad 3x^2 - 6x - 2x + 4 = 0$ **(1/2)**

$\Rightarrow \quad 3x(x-2) - 2(x-2) = 0 \Rightarrow x = \frac{2}{3}$ or $x = 2$ **(1/2)**

Hence, other root is 2. **(1/2)**

5. Let the time taken by larger pipe alone to fill the tank = t hours.

Therefore, the time taken by the smaller pipe = t + 10 hours (1/2)

Water filled by larger pipe running for 4 hours = $\frac{4}{t}$ part **(1/2)**

Water filled by smaller pipe running for 9 hours = $\frac{9}{t+10}$ part **(1/2)**

We know that

$$\frac{4}{t}+\frac{9}{t+10}=\frac{1}{2}$$ **(1)**

Which on simplification gives:
= $t^2$-16t-80=0
= t2-20t+4t-80=0
= t(t-20)+4(t-20) = 0
= (t+4)(t - 20) = 0
= t = -4,20 (1/$^1/_2$)
As, t cannot be negative.
Thus, t = 20, t + 10 = 30
Larger pipe would alone fill the tank in 20 hours and smaller pipe would fill the tank alone in 30 hours. (1)

6. Suppose the usual speed of aircraft be s km/hr
and the reduced speed of the aircraft be (s - 200) km/hr
Distance = 600 km [Given] (1/2)
According to the question,
(time taken at reduced speed) - (Schedule time)
= 30 minutes = 0.5 hours. (1/2)

$$\Rightarrow \quad \frac{600}{s-200}-\frac{600}{s}=\frac{1}{2}$$

Which on simplification gives: (1)
⇒ s2-200s-240000 = 0
⇒ s2-600s + 400s - 240000 = 0
⇒ s(s-600) + 400(s - 600) = 0
⇒ (s-600) (s+400) = 0
s = 600 or s = -400 (1/2)
But speed cannot be negative.

:. The usual speed is 600 km/hr and (1/2)
the scheduled duration of the flight is 600/600 = 1 hour

7. We have, 9x2 - 6px + ($p^2$ - $q^2$) = 0
Here, a = 9, b = - 6p and c = $p^2$ - $q^2$ (1/2)
= D = $b^2$ - 4ac = $(-6p)^2$ - 4(9) $(p^2 - q^2)$ = $36q^2$
By quadratic formula, roots of the given equation are (1/2)

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{6p \pm 6q}{18} = \frac{p+q}{3} \text{ or } \frac{p-q}{3}. \quad \textbf{(1)}$$

8. Given, $9x^2$+6kx + 4 = 0 has equal roots
:- D=0
= $(6k)^2$ - 4×9×4=0 (1/2)
= $36k^2$ = 36×$4k^2$ = 4 ⇒ k=±2 (1/2)

9. The equation $30ax^2$-6x+1 = 0 will have no real roots if
:- $(-6)^2$ - 4(30a) (1) < 0
= 36-120a < 0 (1/2)
= 120a > 36⇒ a > 0.3 (1/2)

10. The given equation is mx (5x-6)+9=0
= 5mx2-6mx+9=0
Since, the roots are real and equal.
;- D=0
= b2-4ac = 0 (1)
= $(-6m)^2$ - 4(5m)(9) = 0
= $36m^2$ - 180m=0
= 36m(m-5)=0 m=0,5 m = 5 ( :- m can't be zero) (1)