# CHAPTER 4

# QUADRATIC EQUATIONS

## ONE MARK QUESTIONS

### Multiple Choice Questions

**1.** The sum and product of the zeroes of a quadratic polynomial are 3 and $-10$ respectively. The quadratic polynomial is

(a) $x^2 - 3x + 10$ (b) $x^2 + 3x - 10$

(c) $x^2 - 3x - 10$ (d) $x^2 + 3x + 10$

**Ans :** [Board 2020 Delhi Basic]

Sum of zeroes, $\alpha + \beta = 3$

and product of zeroes, $\alpha\beta = -10$

Quadratic polynomial,

$$p(x) = x^2 - (\alpha + \beta) + \alpha\beta$$

$$= x^2 - 3x - 10$$

Thus (c) is correct option.

**2.** If the sum of the zeroes of the quadratic polynomial $kx^2 + 2x + 3k$ is equal to their product, then $k$ equals

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

**Ans :** [Board 2020 OD Basic]

We have $p(x) = kx^2 + 2x + 3k$

Comparing it by $ax^2 + bx + c$, we get $a = k$, $b = 2$ and $c = 3k$.

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = -\frac{2}{k}$

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{3k}{k} = 3$

According to question, we have

$$\alpha + \beta = \alpha\beta$$

$$-\frac{2}{k} = 3 \Rightarrow k = -\frac{2}{3}$$

Thus (d) is correct option.

**3.** If $\alpha$ and $\beta$ are the zeroes of the polynomial $x^2 + 2x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to

(a) $-2$ (b) 2

(c) 0

**Ans :** [Board 2020 Delhi Basic]

Since $\alpha$ and $\beta$ are the zeros of polynomial $x^2 + 2x + 1$,

Sum of zeroes, $\alpha + \beta = -\frac{2}{1} = -2$

and product of zeroes, $\alpha\beta = \frac{1}{1} = 1$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = -\frac{2}{1} = -2$

Thus (a) is correct option.

**4.** If $\alpha$ and $\beta$ are the zeroes of the polynomial $2x^2 - 13x + 6$, then $\alpha + \beta$ is equal to

(a) $-3$ (b) 3

(c) $\frac{13}{2}$ (d) $-\frac{13}{2}$

**Ans :** [Board 2020 Delhi Basic]

We have $p(x) = 2x^2 - 13x + 6$

Comparing it with $ax^2 + bx + c$ we get $a = 2$, $b = -13$ and $c = 6$

Sum of zeroes $\alpha + \beta = -\frac{b}{a} = -\frac{(-13)}{2} = \frac{13}{2}$

Thus (c) is correct option.

**5.** The roots of the quadratic equation $x^2 - 0.04 = 0$ are

(a) $\pm 0.2$ (b) $\pm 0.02$

(c) 0.4 (d) 2

**Ans :** [Board 2020 OD Standard]

We have $x^2 - 0.04 = 0$

$$x^2 = 0.04$$

$$x = \pm\sqrt{0.04}$$

$$x = \pm 0.2.$$

Thus (a) is correct option.

**6.** If ½ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of $k$ is

(a) 2 (b) $-2$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

**Ans :**

We have $x^2 + kx - \frac{5}{4} = 0$

Since, ½ is a root of the given quadratic equation, it must satisfy it.

$$\text{Thus} \quad \left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$

$$\frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$

$$\frac{1 + 2k - 5}{4} = 0$$

$$2k - 4 = 0 \Rightarrow k = 2$$

Thus (a) is correct option.

**7.** Each root of $x^2 - bx + c = 0$ is decreased by 2. The resulting equation is $x^2 - 2x + 1 = 0$, then

(a) $b = 6, c = 9$ (b) $b = 3, c = 5$

(c) $b = 2, c = -1$ (d) $b = -4, c = 3$

**Ans :**

For $x^2 - bx + c = 0$ we have

$$\alpha + \beta = b$$

$$\alpha\beta = c$$

$$\text{Now} \quad \alpha - 2 + \beta - 2 = \alpha + \beta - 4 = b - 4$$

$$(\alpha - 2)(\beta - 2) = \alpha\beta - 2(\alpha + \beta) +$$

$$= c - 2b + 4$$

For $x^2 - 2x + 1 = 0$ we have

$$2 = b - 4 \Rightarrow b = 6$$

$$\text{and} \quad 1 = c - 2b + 4$$

$$= c - 2 \times 6 + 4$$

$$= c - 8$$

$$c = 1 + 8 = 9$$

Thus (a) is correct option.

**8.** Value(s) of $k$ for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is/are

(a) 0 (b) 4

(c) 8 (d) 0, 8

**Ans :**

We have $2x^2 - kx + k = 0$

Comparing with $ax^2 + bx + c = 0$ we $a = 2$, $b = -k$ and $c = k$.

For equal roots, the discriminant must be zero.

$$\text{Thus} \quad b^2 - 4ac = 0$$

$$(-k)^2 - 4(2)k = 0$$

$$k^2 - 8k = 0$$

$$k(k - 8) = 0 \Rightarrow k = 0,\ 8$$

Hence, the required values of $k$ are 0 and 8.

Thus (d) is correct option.

**9.** If the equation $(m^2 + n^2)x^2 - 2(mp + nq)x + p^2 + q^2 = 0$ has equal roots, then

(a) $mp = nq$ (b) $mq = np$

(c) $mn = pq$ (d) $mq = \sqrt{np}$

**Ans :**

For equal roots, $b^2 = 4ac$

$$4(mp + nq)^2 = 4(m^2 + n^2)(p^2 \quad q^2$$

$$m^2q^2 + n^2p^2 - 2mnpq = 0$$

$$(mq - np)^2 = 0$$

$$mq - np = 0$$

$$mq = np$$

Thus (b) is correct option.

**10.** The linear factors of the quadratic equation $x^2 + kx + 1 = 0$ are

(a) $k \geq 2$ (b) $k \leq 2$

(c) $k \geq -2$ (d) $2 \leq k \leq -2$

**Ans :**

We have, $x^2 + kx + 1 = 0$

Comparing with $ax^2 + bx + c = 0$ we get $a = 1$, $b = k$ and $c = 1$.

$$\text{For linear factors,} \quad b^2 - 4ac \geq 0$$

$$k^2 - 4 \times 1 \times 1 \geq 0$$

$$(k^2 - 2^2) \geq 0$$

$$(k - 2)(k + 2) \geq 0$$

$$k \geq 2 \text{ and } k \leq -2$$

Thus (d) is correct option.

**11.** If one root of the quadratic equation $ax^2 + bx + c = 0$ is the reciprocal of the other, then

(a) $b = c$ (b) $a = b$

(c) $ac = 1$ (d) $a = c$

**Ans :**

If one root is $\alpha$, then the other $\frac{1}{\alpha}$.

Product of roots, $\alpha \cdot \frac{1}{\alpha} = \frac{c}{a}$

$$1 = \frac{c}{a} \Rightarrow a = c$$

Thus (d) is correct option.

**12.** The quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

**Ans :**

We have $2x^2 - \sqrt{5x} + 1 = 0$

Comparing with $ax^2 + bx + c = 0$ we get $a = 2$, $b = -\sqrt{5}$ and $c = 1$,

Now $b^2 - 4ac = (-\sqrt{5})^2 - 4 \times (2) \times (1)$

$$= 5 - 8 = -3 < 0$$

Since, discriminant is negative, therefore quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has no real roots i.e., imaginary roots.

Thus (c) is correct option.

**13.** The real roots of the equation $x^{2/3} + x^{1/3} - 2 = 0$ are

(a) 1, 8 (b) $-1, -8$

(c) $-1, 8$ (d) $1, -8$

**Ans :**

We have $x^{2/3} + x^{1/3} - 2 = 0$

Substituting $x^{1/3} = y$ we obtain,

$$y^2 + y - 2 = 0$$

$$(y-1)(y+2) = 0 \Rightarrow y = 1 \text{ or } y = -2$$

Thus $x^{1/3} = 1 \Rightarrow x = (1)^3 = 1$

or $x^{1/3} = -2 \Rightarrow x = (-2)^3 = -8$

Hence, the real roots of the given equations are 1, $-8$.

Thus (d) is correct option.

**14.** $(x^2+1)^2 - x^2 = 0$ has

(a) four real roots (b) two real roots

(c) no real roots (d) one real root

**Ans :**

We have $(x^2+1)^2 - x^2 = 0$

$$x^4 + 1 + 2x^2 - x^2 = 0$$

$$x^4 + x^2 + 1 = 0$$

$$(x^2)^2 + x^2 + 1 = 0$$

Let $x^2 = y$ then we have

$$y^2 + y + 1 = 0$$

Comparing with $ay^2 + by + c = 0$ we get $a = 1$, $b = 1$ and $c = 1$

Discriminant, $D = b^2 - 4ac$

$$= (1)^2 - 4(1)(1)$$

$$= 1 - 4 = -3$$

Since, $D < 0$, $y^2 + y + 1 = 0$ has no real roots.

i.e. $x^4 + x^2 + 1 = 0$ or $(x^2+1)^2 - x^2 = 0$ has no real roots.

Thus (c) is correct option.

**15.** The equation $2x^2 + 2(p+1)x + p = 0$, where $p$ is real, always has roots that are

(a) Equal

(b) Equal in magnitude but opposite in sign

(c) Irrational

(d) Real

**Ans :**

We have $2x^2 + 2(p+1)x + p = 0$,

Comparing with $ax^2 + bx + c = 0$ we get $a = 2$, $b = 2(p+1)$ and $c = p$.

Now $b^2 - 4ac = [2(p+1)]^2 - 4(2\quad)$

$$= 4(p+1)^2 - 8p$$

$$= 4p^2 + 8p + 4 - 8p$$

$$= 4(p^2+1)$$

For any real value of $p$, $4(p^2+1)$ will always be positive as $p^2$ cannot be negative for real $p$. Hence, the discriminant $b^2 - 4ac$ will always be positive.

When the discriminant is greater than 0 or is positive, then the roots of a quadratic equation are real.

Thus (d) is correct option.

**16.** The condition for one root of the quadratic equation

$ax^2 + bx + c = 0$ to be twice the other, is

(a) $b^2 = 4ac$ (b) $2b^2 = 9ac$

(c) $c^2 = 4a + b^2$ (d)

**Ans :**

Sum of zeroes $\alpha + 2\alpha = -\frac{b}{a}$

$$3\alpha = -\frac{b}{a} \Rightarrow \alpha = -\frac{b}{3a}$$

Product of zeroes $\alpha \times 2\alpha = \frac{c}{a}$

$$2\alpha^2 = \frac{c}{a}$$

$$2\left(-\frac{b}{3a}\right)^2 = \frac{c}{a}$$

$$\frac{2b^2}{9a^2} = \frac{c}{a}$$

$$2ab^2 - 9a^2c = 0$$

$$a(2b^2 - 9ac) = 0$$

Since, $a \neq 0$, $2b^2 = 9ac$

Hence, the required condition is $2b^2 = 9ac$.

Thus (b) is correct option.

**17.** If $x^2 + y^2 = 25$, $xy = 12$, then $x$ is

(a) $(3, 4)$ (b) $(3, -3)$

(c) $(3, 4, -3, -4)$ (d) $(3, -3)$

**Ans :**

We have $x^2 + y^2 = 25$

and $xy = 12$

$$x^2 + \left(\frac{12}{x}\right)^2 = 25$$

$$x^4 + 144 - 25x^2 = 0$$

$$(x^2 - 16)(x^2 - 9) = 0$$

Hence, $x^2 = 16 \Rightarrow x = \pm 4$

and $x^2 = 9 \Rightarrow x = \pm 3$

Thus (c) is correct option.

**18.** The quadratic equation $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

**Ans :**

We have $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

Here $a = 2$, $b = -3\sqrt{2}$, $c = \frac{9}{4}$

Discriminant $D = b^2 - 4ac$

$$= (-3\sqrt{2})^2 - 4 \times 2 \times \frac{9}{4}$$

$$= 18 - 18 = 0$$

Thus, $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ has real and equal roots.

Thus (b) is correct option.

**19.** The quadratic equation $x^2 + x - 5 = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

**Ans :**

We have $x^2 + x - 5 = 0$

Here, $a = 1$, $b = 1$, $c = -5$

Now, $D = b^2 - 4ac$

$$= (1)^2 - 4 \times 1 \times (-5)$$

$$= 21 > 0$$

So $x^2 + x - 5 = 0$ has two distinct real roots.

Thus (a) is correct option.

**20.** The quadratic equation $x^2 + 3x + 2\sqrt{2} = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

**Ans :**

We have $x^2 + 3x + 2\sqrt{2} = 0$

Here, $a = 1$, $b = 3$ and $c = 2\sqrt{2}$

Now, $D = b^2 - 4ac$

$$= (3)^2 - 4(1)(2\sqrt{2})$$

$$= 9 - 8\sqrt{2} < 0$$

Hence, roots of the equation are not real.
Thus (c) is correct option.

**21.** The quadratic equation $5x^2-3x+1=0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

**Ans :**

We have $5x^2-3x+1=0$

Here $a=5,\ b=-3,\ c=1$

Now, $D=b^2-4ac=(-3)^2-4(5)(1)$

$=9-20<0$

Hence, roots of the equation are not real.
Thus (c) is correct option.

**22.** The quadratic equation $x^2-4x+3\sqrt{2}=0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

**Ans :**

We have $x^2-4x+3\sqrt{2}=0$

Here $a=1,\ b=-4$ and $c=3\sqrt{2}$

Now $D=b^2-4ac=(-4)^2-4(1)(3\sqrt{2})$

$=16-12\sqrt{2}$

$=16-12\times(1.41)$

$=16-16.92=-0.92$

$b^2-4ac<0$

Hence, the given equation has no real roots.
Thus (c) is correct option.

**23.** The quadratic equation $x^2+4x-3\sqrt{2}=0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

**Ans :**

We have $x^2+4x-3\sqrt{2}=0$

Here $a=1,\ b=4$ and $c=-3\sqrt{2}$

Now $D=b^2-4ac=(4)^2-4(1)(-3\sqrt{2})$

$=16+12\sqrt{2}>0$

Hence, the given equation has two distinct real roots,
Thus (a) is correct option.

**24.** The quadratic equation $x^2-4x-3\sqrt{2}=0$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

**Ans :**

We have $x^2-4x-3\sqrt{2}=0$

Here $a=1,\ b=-4$ and $c=-3\sqrt{2}$

Now $D=b^2-4ac$

$=(-4)^2-4(1)(-3\sqrt{2})$

$=16+12\sqrt{2}>0$

Hence, the given equation has two distinct real roots.
Thus (a) is correct option.

**25.** The quadratic equation $3x^2+4\sqrt{3}x+4$ has
(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than 2 real roots

**Ans :**

We have $3x^2+4\sqrt{3}x+4=0$

Here, $a=3,\ b=4\sqrt{3}$ and $c=4$

Now $D=b^2-4ac=(4\sqrt{3})^2-4(3)(4)$

$=48-48=0$

Hence, the equation has real and equal roots.
Thus (b) is correct option.

**26.** Which of the following equations has 2 as a root?
(a) $x^2-4x+5=0$ (b) $x^2+3x-12=0$
(c) $2x^2-7x+6=0$ (d) $3x^2-6x-2=0$

**Ans :**

(a) Substituting, $x=2$ in $x^2-4x+5$, we get

$$(2)^2 - 4(2) + 5 = 4 - 8 + 5 = 1 \neq 0$$

So, $x = 2$ is not a root of

$$x^2 - 4x + 5 = 0$$

(b) Substituting, $x = 2$ in $x^2 + 3x - 12$, we get

$$(2)^2 + 3(2) - 12 = 4 + 6 - 12 = -2 \neq 0$$

So, $x = 2$ is not a root of $x^2 + 3x - 12 = 0$.

(c) Substituting, $x = 2$ in $2x^2 - 7x + 6$, we get

$$2(2)^2 - 7(2) + 6 = 2(4) - 14 + 6$$
$$= 8 - 14 + 6$$
$$= 14 - 14 = 0.$$

So, $x = 2$ is a root of the equation $2x^2 - 7x + 6 = 0$.

(d) Substituting, $x = 2$ in $3x^2 - 6x - 2$, we get

$$3(2)^2 - 6(2) - 2 = 12 - 12 - 2 = -2 \neq 0$$

So, $x = 2$ is not a root of

$$3x^2 - 6x - 2 = 0.$$

Thus (c) is correct option.

**27.** Which of the following equations has the sum of its roots as 3 ?

(a) $2x^2 - 3x + 6 = 0$ (b) $-x^2 + 3x - 3 = 0$

(c) $\sqrt{2}\,x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ (d) $3x^2 - 3x + 3 = 0$

**Ans :**

Sum of the roots,

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\frac{b}{a}$$

Option a : $\alpha + \beta = -\left(\frac{-3}{2}\right) = \frac{3}{2} \neq 3$

Option b : $\alpha + \beta = -\left(\frac{3}{-1}\right) = 3$

Option c : $\alpha + \beta = -\left(\frac{\frac{3}{\sqrt{2}}}{\sqrt{2}}\right) = \frac{3}{2} \neq 3$

Option d $\alpha + \beta = -\left(\frac{-3}{3}\right) = 1 \neq 3$

Thus (b) is correct option.

**28. Assertion :** $4x^2 - 12x + 9 = 0$ has repeated roots.

**Reason :** The quadratic equation $ax^2 + bx + c = 0$ have repeated roots if discriminant $D > 0$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

**Ans :**

Reason is false because if $D = 0$, equation has repeated roots.

Assertion $4x^2 - 12x + 9 = 0$

$$D = b^2 - 4ac$$
$$= (-12)^2 - 4(4)(9)$$
$$= 144 - 144 = 0$$

Roots are repeated.

Assertion (A) is true but reason (R) is false.

Thus (c) is correct option.

**29. Assertion :** The equation $x^2 + 3x + 1 = (x - 2)^2$ is a quadratic equation.

**Reason :** Any equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$, is called a quadratic equation.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

**Ans :**

We have, $x^2 + 3x + 1 = (x - 2)^2 = x^2 - 4x + 4$

$$x^2 + 3x + 1 = x^2 - 4x + 4$$
$$7x - 3 = 0$$

It is not of the form $ax^2 + bx + c = 0$

(d) Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

**30. Assertion :** The values of $x$ are $-\frac{a}{2}, a$ for a quadratic equation $2x^2 + ax - a^2 = 0$.

**Reason :** For quadratic equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

**Ans :**

We have $2x^2 + ax - a^2 = 0$

$$x = \frac{-a \pm \sqrt{a^2 \quad 8a}}{4}$$

$$= \frac{-a + 3a}{4} = \frac{2a}{4}, \frac{-4a}{4}$$

$$x = \frac{a}{2}, -a$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

**31. Assertion :** The equation $8x^2 + 3kx + 2 = 0$ has equal roots then the value of $k$ is $\pm\frac{8}{3}$.

**Reason :** The equation $ax^2 + bx + c = 0$ has equal roots if $D = b^2 - 4ac = 0$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

**Ans :**

We have $8x^2 + 3kx + 2 = 0$

Discriminant, $D = b^2 - 4ac$

$$= (3k)^2 - 4 \times 8 \times 2 = 9k^2 - 64$$

For equal roots, $D = 0$

$$9k^2 - 64 = 0$$

$$9k^2 = 64$$

$$k^2 = \frac{64}{9} \Rightarrow k = \pm\frac{8}{3}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

**32. Assertion :** The roots of the quadratic equation $x^2 + 2x + 2 = 0$ are imaginary.

**Reason :** If discriminant $D = b^2 - 4ac < 0$ then the roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

**Ans :**

We have $x^2 + 2x + 2 = 0$

Discriminant, $D = b^2 - 4ac$

$$= (2)^2 - 4 \times 1 \times 2$$

$$= 4 - 8 = -4 < 0$$

Roots are imaginary.

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

## Fill in the Blank Questions

**33.** A real number $\alpha$ is said to be .......... of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

**Ans :**

root

**34.** For any quadratic equation $ax^2 + bx + c = 0$, $b^2 - 4ac$, is called the .......... of the equation.

**Ans :**

discriminant

**35.** If the discriminant of a quadratic equation is zero, then its roots are .......... and ..........

**Ans :**

real, equal

**36.** If the discriminant of a quadratic equation is greater than zero, then its roots are .......... and ..........

**Ans :**

real, distinct

**37.** A polynomial of degree 2 is called the .......... polynomial.

**Ans :**

quadratic

**38.** A quadratic equation cannot have more than ......... roots.

**Ans :**

two

**39.** Let $ax^2+bx+c=0$, where $a,b,c$ are real numbers, $a\neq 0$, be a quadratic equation, then this equation has no real roots if and only if .........

**Ans :**

$b^2<4ac$

**40.** If the product $ac$ in the quadratic equation $ax^2+bx+c$ is negative, then the equation cannot have .......... roots.

**Ans :**

Non-real

**41.** The equation of the form $ax^2+bx=0$ will always have .......... roots.

**Ans :**

real

**42.** A quadratic equation in the variable $x$ is of the form

$$ax^2+bx+c=0,$$

where $a,b,c$ are real numbers and $a$ ..........

**Ans :**

$\neq 0$

**43.** The roots of a quadratic equation is same as the .......... of the corresponding quadratic polynomial.

**Ans :**

zero

**44.** Value of the roots of the quadratic equation, $x^2-x-6=0$ are ......... .

**Ans :** [Board 2020 OD Basic]

$$x^2-x-6=0$$

$$x^2-3x+2x-6=0$$

$$x(x-3)+2(x-3)=0$$

$$(x-3)(x+2)=0\Rightarrow x=3 \text{ and } x=-2$$

**45.** If quadratic equation $3x^2-4x+k=0$ has equal roots, then the value of $k$ is .......... .

**Ans :** [Board 2020 Delhi Basic]

Given, quadratic equation is $3x^2-4x+k=0$

Comparing with $ax^2+bx+c=0$, we get $a=3$, $b=-4$ and $c=k$

For equal roots, $b^2-4ac=0$

$$(-4)^2-4(3)(k)=0$$

$$16-12k=0$$

$$k=\frac{16}{12}=\frac{4}{3}$$

## Very Short Answer Questions

**46.** Find the positive root of $\sqrt{3x^2+6}=9$.

**Ans :** [Board Term-2, 2015]

We have $\sqrt{3x^2+6}=9$

$$3x^2+6=81$$

$$3x^2=81-6=75$$

$$x^2=\frac{75}{3}=25$$

Thus $x=\pm 5$

Hence 5 is positive root.

**47.** If $x=-\frac{1}{2}$, is a solution of the quadratic equation $3x^2+2kx-3=0$, find the value of $k$.

**Ans :** [Board Term-2, Delhi 2015]

We have $3x^2+2kx-3=0$

Substituting $x=-\frac{1}{2}$ in given equation we get

$$3\left(-\frac{1}{2}\right)^2+2k\left(-\frac{1}{2}\right)-3=0$$

$$\frac{3}{4}-k-3=0$$

$$k=\frac{3}{4}-3$$

$$=\frac{3-12}{4}=\frac{-9}{4}$$

Hence $k=\frac{-9}{4}$

**48.** Find the roots of the quadratic equation $\sqrt{3}\,x^2-2x-\sqrt{3}=0$

**Ans :** [Board Term-2, 2012, 2011]

We have $\sqrt{3}\,x^2-2x-\sqrt{3}=0$

$$\sqrt{3}\,x^2-3x+x-\sqrt{3}=0$$

$$\sqrt{3}\,x(x-\sqrt{3})+1(x-\sqrt{3}=0$$

$$(x-\sqrt{3})(\sqrt{3}\,x+1) = 0$$

Thus $$x = \sqrt{3}, \frac{-1}{\sqrt{3}}$$

**49.** Find the value of $k$, for which one root of the quadratic equation $kx^2 - 14x + 8 = 0$ is six times the other.

**Ans :** [Board Term-2, 2016]

We have $$kx^2 - 14x + 8 = 0$$

Let one root be $\alpha$ and other root be $6\alpha$.

Sum of roots, $$\alpha + 6\alpha = \frac{14}{k}$$

$$7\alpha = \frac{14}{k} \text{ or } \alpha = \frac{2}{k} \quad ...(1)$$

Product of roots , $$\alpha(6\alpha) = \frac{8}{k} \text{ or } 6\alpha^2 = \frac{8}{k} \quad ...(2)$$

Solving (1) and (2), we obtain

$$6\left(\frac{2}{k}\right)^2 = \frac{8}{k}$$

$$6 \times \frac{4}{k^2} = \frac{8}{k}$$

$$\frac{3}{k^2} = \frac{1}{k}$$

$$3k = k^2$$

$$3k - k^2 = 0$$

$$k[3-k] = 0$$

$$k = 0 \text{ or } k = 3$$

Since $k = 0$ is not possible, therefore $k = 3$.

**50.** If one root of the quadratic equation $6x^2 - x - k = 0$ is $\frac{2}{3}$, then find the value of $k$.

**Ans :** [Board Term-2 Foreign-2, 2017]

We have $$6x^2 - x - k = 0$$

Substituting $x = \frac{2}{3}$, we get

$$6\left(\frac{2}{3}\right)^2 - \frac{2}{3} - k = 0$$

$$6 \times \frac{4}{9} - \frac{2}{3} - k = 0$$

$$\frac{8}{3} - \frac{2}{3} - k = 0$$

$$\frac{8-2}{3} - k = 0$$

$$2 - k = 0$$

Thus $k = 2$.

**51.** Find the value(s) of $k$ if the quadratic equation $3x^2 - k\sqrt{3}\,x + 4 = 0$ has real roots.

**Ans :** [SQP 2017]

If discriminant $D = b^2 - 4ac$ of quadratic equation is equal to zero, or more than zero, then roots are real.

We have $$3x^2 - k\sqrt{3}\,x + 4 = 0$$

Comparing with $ax^2 + bx + c = 0 = 0$ we get

$$a = 3, b = -k\sqrt{3} \text{ and } c = 4$$

For real roots $$b^2 - 4ac \geq 0$$

$$(-k\sqrt{3})^2 - 4 \times 3 \times 4 \geq 0$$

$$3k^2 - 48 \geq 0$$

$$k^2 - 16 \geq 0$$

$$(k-4)(k+4) \geq 0$$

Thus $k \leq -4$ and $k \geq 4$

## TWO MARKS QUESTIONS

**52.** For what values of $k$, the roots of the equation $x^2 + 4x + k = 0$ are real?

**Ans :** [Board 2019 Delhi]

We have $$x^2 + 4x + k = 0.$$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 1$, $b = 4, c = k$.

Since, given the equation has real roots,

$$D \geqslant 0$$

$$b^2 - 4ac \geqslant 0$$

$$4^2 - 4 \times 1 \times k \geqslant 0$$

$$4k \leqslant 16$$

$$k \leqslant 4$$

**53.** Find the value of $k$ for which the roots of the equations $3x^2 - 10x + k = 0$ are reciprocal of each other.

**Ans :** [Board 2019 Delhi]

We have $$3x^2 - 10x + k = 0$$

Comparing the given equation with $ax^2 + bx + c = 0$

we get $a = 3,\ b = -10, c = k$

Let one root be α so other root is $\frac{1}{\alpha}$.

Now product of roots $\alpha \times \frac{1}{\alpha} = \frac{c}{a}$

$$1 = \frac{k}{3} \Rightarrow k = 3$$

Hence, value of $k$ is 3.

**54.** Find the value of $k$ such that the polynomial $x^2 - (k+6)x + 2(2k+1)$ has sum of its zeros equal to half of their product.

**Ans :** [Board 2019 Delhi]

Let α and β be the roots of given quadratic equation

$$x^2 - (k+6)x + 2(2k+1) = 0$$

Now sum of roots,. $\alpha + \beta = -\frac{-(k+6)}{1} = k+6$

Product of roots, $\alpha\beta = \frac{2(2k+1)}{1} = 2(2k+1)$

According to given condition,

$$\alpha + \beta = \frac{1}{2}\alpha\beta$$

$$k+6 = \frac{1}{2}[2(2k+1)]$$

$$k+6 = 2k+1 \Rightarrow k = 5$$

Hence, the value of $k$ is 5.

**55.** Find the nature of roots of the quadratic equation $2x^2 - 4x + 3 = 0$.

**Ans :** [Board 2019 OD]

We have $2x^2 - 4x + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 2\ b = -4,\ c = 3$

Now $D = b^2 - 4ac$

$= (-4)^2 - 4(2) \times (3)$

$= -8 < 0$ or $(-\text{ve})$

Hence, the given equation has no real roots.

**56.** Find the roots of the quadratic equation $6x^2 - x - 2 = 0$ .

**Ans :** [Board Term-2, 2012]

We have $6x^2 - x - 2 = 0$

$$6x^2 + 3x - 4x - 2 = 0 \qquad (3 \times 4 = 2 \times 6)$$

$$3x(2x+1) - 2(2x+1) = 0$$

$$(2x+1)(3x-2) = 0$$

$$3x - 2 = 0 \text{ or } 2x + 1 = 0$$

$$x = \frac{2}{3} \text{or } x = -\frac{1}{2}$$

Hence roots of equation are $\frac{2}{3}$ and $-\frac{1}{2}$.

**57.** Find the roots of the following quadratic equation :
$15x^2 - 10\sqrt{6}\,x + 10 = 0$

**Ans :** [Board Term-2, 2012]

We have $15x^2 - 10\sqrt{6}\,x + 10 = 0$

$$3x^2 - 2\sqrt{6}\,x + 2 = 0$$

$$3x^2 - \sqrt{6}\,x - \sqrt{6}\,x + 2 = 0$$

$$\sqrt{3}\,x(\sqrt{3}\,x - \sqrt{2}) - \sqrt{2}(\sqrt{3}\,x - \sqrt{2}) = 0$$

$$(\sqrt{3}\,x - \sqrt{2})(\sqrt{3}\,x - \sqrt{2}) = 0$$

Thus $x = \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$

**58.** Solve the following quadratic equation for $x$ :
$4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$

**Ans :** [Board Term-2, 2013, 2012]

We have $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$

$$4\sqrt{3}\,x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$$4x(\sqrt{3}\,x + 2) - \sqrt{3}(\sqrt{3}\,x + 2 = 0$$

$$(\sqrt{3}\,x + 2)(4x - \sqrt{3}) = 0$$

Thus $x = -\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$

**59.** Solve for $x$ : $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

**Ans :** [Board Term-2 Foreign 2015]

We have

$$x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$$

$$x^2 - \sqrt{3}\,x - 1x + \sqrt{3} = 0$$

$$x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0$$

$$(x - \sqrt{3})(x - 1) = 0$$

Thus $x = \sqrt{3}, x = 1$

**60.** Find the roots of the following quadratic equation :
$(x+3)(x-1) = 3\left(x - \frac{1}{3}\right)$

**Ans :** [Board Term-2 2012]

We have $(x+3)(x-1) = 3\left(x-\frac{1}{3}\right)$

$$x^2+3x-x-3 = 3x-1$$
$$x^2-x-2 = 0$$
$$x^2-2x+x-2 = 0$$
$$x(x-2)+1(x-2) = 0$$
$$(x-2)(x+1) = 0$$

Thus $x = 2, -1$

**61.** Find the roots of the following quadratic equation :
$$\frac{2}{5}x^2-x-\frac{3}{5} = 0$$

**Ans :** [Board Term-2, 2012 ]

We have $\frac{2}{5}x^2-x-\frac{3}{5} = 0$

$$\frac{2x^2-5x-3}{5} = 0$$
$$2x^2-5x-3 = 0$$
$$2x^2-6x+x-3 = 0$$
$$2x(x-3)+1(x-3) = 0$$
$$(2x+1)(x-3) = 0$$

Thus $x = -\frac{1}{2},\ 3$

**62.** Solve the following quadratic equation for $x$ :
$$4x^2-4a^2x+(a^4-b^4) = 0$$

**Ans :** [Delhi Term-2, 2015]

We have $4x^2-4a^2x+(a^4-b^4) = 0$

Comparing with $Ax^2+Bx+C = 0$ we have

$$A = 4, B = -4a^2, C = (a^4-b^4$$
$$x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$
$$= \frac{4a^2\pm\sqrt{(-4a^2)^2-4\times4(a^4-b^4)}}{2\times4}$$
$$= \frac{4a^2\pm\sqrt{16a^2-16a^4+16b^4}}{8}$$
$$= \frac{4a^2\pm\sqrt{16b^4}}{8}$$

or, $x = \frac{4a^2\pm4b^2}{8} = \frac{a^2\pm b^2}{2}$

Thus either $x = \frac{a^2+b^2}{2}$ or $x = \frac{a^2-b^2}{2}$

**63.** Solve the following quadratic equation for $x$ :
$$9x^2-6b^2x-(a^4-b^4) = 0$$

**Ans :** [Delhi Term-2, 2015]

We have $9x^2-6b^2x-(a^4-b^4) = 0$

Comparing with $Ax^2+Bx+C = 0$ we have

$$A = 9, B = -6b^2, C = -(a^4-b^4)$$
$$x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$
$$x = \frac{6b^2\pm\sqrt{(-6b^2)^2-4\times9\times\{(a^4-b^4)\}}}{2\times9}$$
$$= \frac{6b^2\pm\sqrt{36b^4+36a^4-36b^4}}{18}$$
$$= \frac{6b^2\pm\sqrt{36a^4}}{18} = \frac{6b^2\pm6a^2}{18}$$

Thus $x = \frac{a^2+b^2}{3}, \frac{b^2-a^2}{3}$

**64.** Solve the following equation for $x$ :
$$4x^2+4bx-(a^2-b^2) = 0$$

**Ans :** [Board Term-2, OD 2012]

We have $4x^2+4bx+b^2-a^2 = 0$

$$(2x+b)^2-a^2 = 0$$
$$(2x+b+a)(2x+b-a) = 0$$

Thus $x = \frac{-(a+b)}{2}$ and $x = \frac{a-b}{2}$

**65.** Solve the following quadratic equation for $x$ :
$$x^2-2ax-(4b^2-a^2) = 0$$

**Ans :** [Board Term-2, 2015]

We have $x^2-2ax-(4b^2-a^2) = 0$

$$x^2-2ax+a^2-4b^2 = 0$$
$$(x-a)^2-(2b)^2 = 0$$

$$(x-a+2b)(x-a-2b)=0$$

Thus $x=a-2b, x=a+2b$

**66.** Solve the quadratic equation, $2x^2+ax-a^2=0$ for $x$.

**Ans :** [Board Term-2 Delhi 2014]

We have $$2x^2+ax-a^2=0$$

Comparing with $Ax^2+Bx+C=0$ we have

$$A=2, B=a, C=-a^2$$

Now $$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$=\frac{-a\pm\sqrt{a^2-4\times2\times(-a^2}}{2\times2}$$

$$=\frac{-a\pm\sqrt{a^2+8a^2}}{4}$$

$$=\frac{-a\pm\sqrt{9a^2}}{4}=\frac{-a\pm3a}{4}$$

$$x=\frac{-a+3a}{4},\frac{-a-3a}{4}$$

Thus $x=\frac{a}{2},-a$

**67.** Find the roots of the quadratic equation $4x^2-4px+(p^2-q^2)=0$

**Ans :** [Board Term-2, 2014]

We have $$4x^2-4px+(p^2-q^2)=0$$

Comparing with $ax^2+bx+c=0$ we get

$$a=4, b=-4p, c=(p^2-q^2)$$

The roots are given by the quadratic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{4p\pm\sqrt{16p^2-4\times4\times(p^2-q^2)}}{2\times4}$$

$$=\frac{4p\pm\sqrt{16p^2-16p^2+16q^2}}{8}$$

$$=\frac{4p\pm4q}{8}$$

Thus roots are $\frac{p+q}{2},\frac{p-q}{2}$.

**68.** Solve for $x$ (in terms of $a$ and $b$) :

$$\frac{a}{x-b}+\frac{b}{x-a}=2, x\neq a, b$$

**Ans :**

We have $$\frac{a(x-a)+b(x-b}{(x-b)(x-a)}=2$$

$$a(x-a)+b(x-b)=2[x^2-(a+b)x+ab$$

$$ax-a^2+bx-b^2=2x^2-2(a+b)x+2ab$$

$$2x^2-3(a+b)x+(a+b)^2=0$$

$$2x^2-2(a+b)x-(a-b)x+(a+b)^2=0$$

$$[2x-(a+b)][x-(a+b)]=0$$

Thus $$x=a+b,\frac{a+b}{2}$$

**69.** Solve for $x$ : $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$

**Ans :**

We have

$\sqrt{3}x^2-3\sqrt{2}x+\sqrt{2}x-2\sqrt{3}$

$=0$

$$\sqrt{3}x[x-\sqrt{6}]+\sqrt{2}[x-\sqrt{6}]=0$$

$$(x-\sqrt{6})(\sqrt{3}x+\sqrt{2})=0$$

Thus $$x\ \sqrt{6},-\sqrt{\frac{2}{3}}$$

**70.** If $x=\frac{2}{3}$ and $x=-3$ are roots of the quadratic equation $ax^2+7x+b=0$, find the values of $a$ and $b$.

**Ans :** [Board Term-2 Delhi 2016]

We have $$ax^2+7x+b=0 \quad (1)$$

Substituting $x=\frac{2}{3}$ in above equation we obtain

$$\frac{4}{9}a+\frac{14}{3}+b=0$$

$$4a+42+9b=0$$

$$4a+9b=-42 \quad (2)$$

and substituting $x=-3$ in (1) we obtain

$$9a-21+b=0$$

$$9a+b=21 \quad (3)$$

Solving (2) and (3), we get $a = 3$ and $b = -6$

**71.** Solve for $x$ : $\sqrt{6x+7} - (2x-7) = 0$
Ans : [Board Term-2 OD 2016]

We have $\sqrt{6x+7} - (2x-7) = 0$

or, $\sqrt{6x+7} = (2x-7$

Squaring both sides we get

$$6x+7 = (2x-7^{2}$$
$$6x+7 = 4x^2 - 28x + 49$$
$$4x^2 - 34x + 42 = 0$$
$$2x^2 - 17x + 21 = 0$$
$$2x^2 - 14x - 3x + 21 = 0$$
$$2x(x-7) - 3(x-7) = 0$$
$$(x-7)(2x-3) = 0$$

Thus $x = 7$ and $x = \frac{2}{3}$.

**72.** Find the roots of $x^2 - 4x - 8 = 0$ by the method of completing square.
Ans : [Board Term-2, 2015]

We have $x^2 - 4x - 8 = 0$

$$x^2 - 4x + 4 - 4 - 8 = 0$$
$$(x-2)^2 - 12 = 0$$
$$(x-2)^2 = 12$$
$$(x-2)^2 = (2\sqrt{3})^2$$
$$x - 2 = \pm 2\sqrt{3}$$
$$x = 2 \pm 2\sqrt{3}$$

Thus $x = 2 + 2\sqrt{3},\ 2 - 2\sqrt{3}$

**73.** Solve for $x$ : $\sqrt{2x+9} + x = 13$
Ans : [Board Term-2 OD 2016]

We have $\sqrt{2x+9} + x = 13$

$$\sqrt{2x+9} = 13 - x$$

Squaring both side we have

$$2x + 9 = (13-x)^2$$
$$2x + 9 = 169 + x^2 - 26x$$
$$0 = x^2 + 169 - 26x - 9 - 2x$$
$$x^2 - 28x + 160 = 0$$
$$x^2 - 20x - 8x + 160 = 0$$
$$x(x-20) - 8(x-20) = 0$$
$$(x-8)(x-20) = 0$$

Thus $x = 8$ and $x = 20$.

**74.** Find the roots of the quadratic equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
Ans : [Board Term-2 OD 2017]

We have $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$$\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$$
$$\sqrt{2}x(x+\sqrt{2}) + 5(x+\sqrt{2}) = 0$$
$$(x+\sqrt{2})(\sqrt{2}x+5) = 0$$

Thus $x = -\sqrt{2}$ and $= -\frac{5}{\sqrt{2}} = -\frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{5}{2}\sqrt{2}$

**75.** Find the value of $k$ for which the roots of the quadratic equation $2x^2 + kx + 8 = 0$ will have the equal roots ?
Ans : [Board Term-2 OD Compt., 2017]

We have $2x^2 + kx + 8 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 2, b = k, \text{ and } c = 8$$

For equal roots, $D = 0$

$$b^2 - 4ac = 0$$
$$k^2 - 4 \times 2 \times 8 = 0$$
$$k^2 = 64$$
$$k = \pm\sqrt{64}$$

Thus $k = \pm 8$

**76.** Solve for $x$ : $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$
Ans : [Board Term-II Foreign 2017 Set-2]

We have $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$$\sqrt{3}x^2 + 3x + 7x + 7\sqrt{3} = 0$$
$$\sqrt{3x}(x+\sqrt{3}) + 7(x+\sqrt{3} = 0$$
$$(x+\sqrt{3})(\sqrt{3}x+7) = 0$$

Thus $x = -\sqrt{3}$ and $x = -\frac{7}{\sqrt{3}}$

**77.** Find $k$ so that the quadratic equation $(k+1)x^2 - 2(k+1)x + 1 = 0$ has equal roots.
Ans : [Board Term-2, 2016]

We have $(k+1)x^2-2(k+1)x+1=0$

Comparing with $Ax^2+Bx+C=0$ we get

$A=(k+1), B=-2(k+1), C=1$

If roots are equal, then $D=0$, i.e.

$$B^2=4AC$$

$$4(k+1)^2=4(k+$$

$$k^2+2k+1=k+1$$

$$k^2+k=0$$

$$k(k+1)=0$$

$$k=0,-1$$

$k=-1$ does not satisfy the equation, thus $k=0$

**78.** If 2 is a root of the equation $x^2+kx+12=0$ and the equation $x^2+kx+q=0$ has equal roots, find the value of $q$.

**Ans :** [Board Term 2 SQP 2016]

We have $x^2+kx+12=0$

If 2 is the root of above equation, it must satisfy it.

$$(2)^2+2k+12=0$$

, $2k+16=0$

$$k=-8$$

Substituting $k=-8$ in $x^2+kx+q=0$ we have

$$x^2-8x+q=0$$

For equal roots,

$$(-8)^2-4(1)q=0$$

$$64-4q=0$$

$$4q=64 \Rightarrow q=16$$

**79.** Find the values of $k$ for which the quadratic equation $9x^2-3kx+k=0$ has equal roots.

**Ans :** [Board Term-2 Delhi, OD 2014]

We have $9x^2-3kx+k=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=9, b=-3k, c=k$$

Since roots of the equation are equal, $b^2-4ac=0$

$$(-3k)^2-(4\times 9\times k)=0$$

$$9k^2-36k=0$$

$$k^2-4k=0$$

$$k(k-4)=0 \Rightarrow k=0 \text{ or } k=4$$

Hence, $k=4$.

**80.** If the equation $kx^2-2kx+6=0$ has equal roots, then find the value of $k$.

**Ans :** [Board Term-2, 2012]

We have $kx^2-2kx+6=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=k, b=-2k, c=6$$

Since roots of the equation are equal, $b^2-4ac=0$

$$(-2k)^2-4(k)(6)=0$$

$$4k^2-24k=0$$

$$4k(k-6)=0$$

$$k=0,6$$

But $k\neq 0$, as coefficient of $x^2$ can't be zero.

Thus $k=6$

**81.** Find the positive value of $k$ for which $x^2-8x+k=0$ , will have real roots.

**Ans :** [Board Term-2, 2014]

We have $x^2-8x+k=0$

Comparing with $Ax^2+Bx+C=0$ we get

$$A=1, B=-8, C=k$$

Since the given equation has real roots, $B^2-4AC>0$

$$(-8)^2-4(1)(k)\geq 0$$

$$64-4k\geq 0$$

$$16-k\geq 0$$

$$16\geq k$$

Thus $k\leq 16$

**82.** Find the values of $p$ for which the quadratic equation $4x^2+px+3=0$ has equal roots.

**Ans :** [Board Term-2, 2014]

We have $4x^2+px+3=0$

Comparing with $ax^2+bx+c=0$ we get

$$a=4, b=p, c=3$$

Since roots of the equation are equal,

$$b^2 - 4ac = 0$$
$$p^2 - 4 \times 4 \times 3 = 0$$
$$p^2 - 48 = 0$$
$$p^2 = 48$$
$$p = \pm 4\sqrt{3}$$

**83.** Find the nature of the roots of the quadratic equation : $13\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

**Ans :** [Board Term-2, 2012]

We have $13\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 13\sqrt{3}, b = 10, c = \sqrt{3}$$
$$b^2 - 4ac = (10)^2 - 4(13\sqrt{3})\sqrt{3}$$
$$= 100 - 156$$
$$= -56$$

As $D < 0$, the equation has not real roots.

## THREE MARKS QUESTIONS

**84.** Solve the following equation: $\frac{1}{x} - \frac{1}{x-2} = 3,\ x \neq 0, 2$

**Ans :** [Board 2020 SQP Standard]

We have $\frac{1}{x} - \frac{1}{x-2} = 3 \qquad (x \neq 0, 2)$

$$\frac{x-2-x}{x(x-2)} = 3$$
$$\frac{-2}{x(x-2)} = 3$$
$$3x(x-2) = -2$$
$$3x^2 - 6x + 2 = 0$$

Comparing it by $ax^2 + bx + c$, we get $a = 3,\ b = -6$ and $c = 2$.

Now,
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$$
$$= \frac{6 \pm \sqrt{36-24}}{6} = \frac{6 \pm \sqrt{12}}{6}$$
$$= \frac{6 \pm 2\sqrt{3}}{6}$$
$$= \frac{3+\sqrt{3}}{3}, \frac{3-\sqrt{3}}{3}$$

**85.** Find the values of $k$ for which the quadratic equation $x^2 + 2\sqrt{2k}\,x + 18 = 0$ has equal roots.

**Ans :** [Board 2020 SQP Standard]

We have $x^2 + 2\sqrt{2k}\,x + 18 = 0$

Comparing it by $ax^2 + bx + c$, we get $a = 1,\ b = 2\sqrt{2k}$ and $c = 18$.

Given that, equation $x^2 + 2\sqrt{2}\,kx + 18 = 0$ has equal roots.

$$b^2 - 4ac = 0$$
$$(2\sqrt{2}\,k)^2 - 4 \times 1 \times 18 = 0$$
$$8k^2 - 72 = 0$$
$$8k^2 = 72$$
$$k^2 = \frac{72}{8} = 9$$
$$k = \pm 3$$

**86.** If $\alpha$ and $\beta$ are the zeroes of the polynomial $f(x) = x^2 - 4x - 5$ then find the value of $\alpha^2 + \beta^2$

**Ans :** [Board 2020 Delhi Basic]

We have $p(x) = x^2 - 4x - 5$

Comparing it by $ax^2 + bx + c$, we get $a = 1,\ b = -4$ and $c = -5$

Since, given $\alpha$ and $\beta$ are the zeroes of the polynomial,

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{1} = 4$

and product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{-5}{1} = -5$

Now,
$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$
$$= (4)^2 - 2(-5)$$
$$= 16 + 10 = 26$$

**87.** Find the quadratic polynomial, the sum and product of whose zeroes are $-3$ and 2 respectively. Hence find the zeroes.

**Ans :** [Board 2020 OD Basic]

Sum of zeroes $\alpha+\beta = -3$ ...(1)

and product of zeroes $\alpha\beta = 2$

Thus quadratic equation is

$$x^2-(\alpha+\beta)x+\alpha\beta = 0$$

$$x^2-(-3)x+2 = 0$$

$$x^2+3x+2 = 0$$

Thus quadratic equation is $x^2+3x+2=0$.

Now above equation can be written as

$$x^2+2x+x+2 = 0$$

$$x(x+2)+(x+2) = 0$$

$$(x+2)(x+1) = 0$$

Hence, zeroes are $-2$ and $-1$.

**88.** If $\alpha$ and $\beta$ are the zeroes of the polynomial $f(x)=5x^2-7x+1$ then find the value of $\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$

**Ans :** [Board 2020 OD Basic]

Since, $\alpha$ and $\beta$ are the zeroes of the quadratic polynomial $f(x)=5x^2-7x+1$,

Sum of zeros, $\alpha+\beta = -\left(\frac{-7}{5}\right)=\frac{7}{5}$ ...(1)

Product of zeros, $\alpha\beta = \frac{1}{5}$ ...(2)

Now, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

$$= \frac{\left(\frac{7}{5}\right)^2-2\times\frac{1}{5}}{\frac{1}{5}}$$

$$= \frac{49-2\times5}{5} = \frac{39}{5}$$

**89.** Find the zeroes of the quadratic polynomial $6x^2-3-7x$ and verify the relationship between the zeroes and the coefficients.

**Ans :** [Board 2020 Delhi Basic]

We have $p(x) = 6x^2-3-7x$

For zeroes of polynomial, $p(x)=0$,

$$6x^2-7x-3 = 0$$

$$6x^2-9x+2x-3 = 0$$

$$3x(2x-3)+1(2x-3) = 0$$

$$(2x-3)(3x+1) = 0$$

Thus $2x-3 = 0$ and $3x+1 = 0$

Hence $x = \frac{3}{2}$ and $x = -\frac{1}{3}$

Therefore $\alpha=\frac{3}{2}$ and $\beta=-\frac{1}{3}$ are the zeroes of the given polynomial.

**Verification :**

Sum of zeroes, $\alpha+\beta = \frac{3}{2}+\left(-\frac{1}{3}\right)$

$$= \frac{3}{2}-\frac{1}{3} = \frac{7}{6}$$

$$= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

and product of zeroes $\alpha\beta = \left(\frac{3}{2}\right)\left(-\frac{1}{3}\right) = -\frac{1}{2}$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

**90.** Find the zeroes of the quadratic polynomial $x^2+7x+10$, and verify the relationship between the zeroes and the coefficients.

**Ans :** [Board 2020 Delhi Basic]

Let, $p(x) = x^2+7x+10$

For zeroes of polynomial $p(x) = 0$,

$$x^2+7x+10 = 0$$

$$x^2 + 5x + 2x + 10 = 0$$
$$x(x+5) + 2(x+5) = 0$$
$$(x+5)(x+2) = 0$$

So, $x = -2$ and $x = -5$

Therefore, $\alpha = -2$ and $\beta = -5$ are the zeroes of the given polynomial.

Verification:

Sum of zeroes, $\alpha + \beta = -2 + (-5)$

$$= -7 = \frac{-7}{1}$$
$$= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

and product of zeroes $\alpha\beta = (-2)(-5) = 10$

$$= \frac{10}{1}$$
$$= \frac{\text{contant term}}{\text{coefficient of } x^2}$$

**91.** Solve for $x$ : $\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}$ $x \neq -4, \ -7$.

**Ans :** [Board 2020 OD Standard]

We have $\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}$

$$\frac{x+7-x-4}{(x+4)(x+7)} = \frac{11}{30}$$
$$\frac{3}{x^2+4x+7x+28} = \frac{11}{30}$$
$$\frac{3}{x^2+11x+28} = \frac{11}{30}$$
$$11x^2 + 121x + 308 = 90$$
$$11x^2 + 121x + 218 = 0$$

Comparing with $ax^2 + bx + c = 0$, we get $a = 11$, $b = 121$ and $c = 218$ we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-121 \pm \sqrt{14641 - 9592}}{22}$$
$$x = \frac{-121 \pm \sqrt{5049}}{22}$$
$$= \frac{-121 \pm 71.06}{22}$$
$$x = \frac{-49.94}{22}, \frac{-192.06}{22}$$
$$x = -2.27, -8.73.$$

**92.** Solve for $x$ :

$\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}; x \neq 1, -2, 2$

**Ans :** [Board Term-2 OD 2016]

We have $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$

$$\frac{x^2+3x+2+x^2-3x+2}{x^2+x-2} = \frac{4x-8-2x-3}{x-2}$$
$$\frac{2x^2+4}{x^2+x-2} = \frac{2x-11}{x-2}$$
$$(2x^2+4)(x-2) = (2x-11)(x^2+x-2)$$
$$5x^2 + 19x - 30 = 0$$
$$(5x-6)(x+5) = 0$$
$$x = -5, \frac{6}{5}$$

**93.** Solve for $x$ :

$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, -\frac{3}{2}$

**Ans :** [Board Term-2, Delhi 2016]

We have

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$$
$$2x(2x+3) + (x-3) + (3x+9) = 0$$
$$4x^2 + 6x + x - 3 + 3x + 9 = 0$$
$$4x^2 + 10x + 6 = 0$$
$$2x^2 + 5x + 3 = 0$$
$$(x+1)(2x+3) = 0$$

Thus $x = -1, x = -\frac{3}{2}$

**94.** Solve for $x$ : $\frac{1}{x} + \frac{2}{2x-3} = \frac{1}{x-2}, x \neq 0, \frac{2}{3}, 2.$

**Ans :** [Board Term-2, Foreign 2016]

We have $\frac{1}{x} + \frac{2}{2x-3} = \frac{1}{x-2}$

$$\frac{2x-3+2x}{x(2x-3)} = \frac{1}{x-2}$$
$$\frac{4x-3}{x(2x-3)} = \frac{1}{x-2}$$

$$\begin{aligned}(x-2)(4x-3) &= 2x^2-3x\\ 4x^2-11x+6 &= 2x^2-3x\\ 2x^2-8x+6 &= 0\\ x^2-4x+3 &= 0\\ (x-1)(x-3) &= 0\end{aligned}$$

Thus $x = 1,3$

**95.** Solve the following quadratic equation for $x$ :

$x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1=0$

**Ans :** [Board Term-2 OD 2016]

We have
$$\begin{aligned}x^2+\left(\frac{a}{a+b}+\frac{a+b}{a}\right)x+1 &= 0\\ x^2+\frac{a}{a+b}x+\frac{a+b}{a}x+1 &= 0\\ x\left(x+\frac{a}{a+b}\right)+\frac{a+b}{a}\left(x+\frac{a}{a+b}\right. &= 0\\ \left(x+\frac{a}{a+b}\right)\left(x+\frac{a+b}{a}\right) &= 0\end{aligned}$$

Thus $x = \frac{-a}{a+b},\frac{-(a+b)}{a}$

**96.** Solve for $x$ :

$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3};x\neq 1,2,3$

**Ans :** [Board Term-2 OD 2016]

We have
$$\begin{aligned}\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)} &= \frac{2}{3}\\ \frac{x-3+x-1}{(x-1)(x-2)(x-3)} &= \frac{2}{3}\\ \frac{2x-4}{(x-1)(x-2)(x-3)} &= \frac{2}{3}\\ \frac{2(x-2)}{(x-1)(x-2)(x-3)} &= \frac{2}{3}\\ \frac{2}{(x-1)(x-3)} &= \frac{2}{3}\\ 3 &= (x-1)(x-3)\\ x^2-4x+3 &= 3\\ x^2-4x &= 0\\ x(x-4) &= 0\end{aligned}$$

Thus $x = 0$ or $x=4$

**97.** Solve for $x$ : $\sqrt{3}\,x^2-2\sqrt{2}\,x-2\sqrt{3}=0$

**Ans :** [Board Term-2, OD 2015, Foreign 2014]

We have
$$\begin{aligned}\sqrt{3}\,x^2-2\sqrt{2}\,x-2\sqrt{3} &= 0\\ \sqrt{3}\,x^2-[3\sqrt{2}-\sqrt{2}]x-2\sqrt{3} &= 0\\ \sqrt{3}\,x^2-3\sqrt{2}\,x+\sqrt{2}\,x-2\sqrt{3} &= 0\\ \sqrt{3}\,x^2-\sqrt{3}\sqrt{3}\sqrt{2}\,x+\sqrt{2}\,x-\sqrt{2}\sqrt{2}\sqrt{3} &= 0\\ \sqrt{3}\,x(x-\sqrt{3}\sqrt{2})+\sqrt{2}\,(x-\sqrt{2}\sqrt{3}) &= 0\\ \sqrt{3}\,x[x-\sqrt{6}]+\sqrt{2}[x-\sqrt{6} &= 0\\ (x-\sqrt{6})(\sqrt{3}\,x+\sqrt{2} &= 0\end{aligned}$$

Thus $x = \sqrt{6} = -\sqrt{\frac{2}{3}}$

**98.** Solve for $x$ : $2x^2+6\sqrt{3}\,x-60=0$

**Ans :** [Board Term-2, OD 2015]

We have
$$\begin{aligned}2x^2+6\sqrt{3}\,x-60 &= 0\\ x^2+3\sqrt{3}\,x-30 &= 0\\ x^2+5\sqrt{3}\,x-2\sqrt{3}\,x-30 &= 0\\ x(x+5\sqrt{3})-2\sqrt{3}(x+5\sqrt{3} &= 0\\ (x+5\sqrt{3})(x-2\sqrt{3}) &= 0\end{aligned}$$

Thus $x = -5\sqrt{3},2\sqrt{3}$

**99.** Solve for $x$ : $x^2+5x-(a^2+a-6)=0$

**Ans :** [Board Term-2 Foreign Set I 2015]

We have
$$\begin{aligned}x^3+5x-(a^2+a-6) &= 0\\ x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}\end{aligned}$$

Thus
$$\begin{aligned}x &= \frac{-5\pm\sqrt{25+4(a^2+a-6}}{2}\\ &= \frac{-5\pm\sqrt{25+4a^2+4a-24}}{2}\\ &= \frac{-5\pm\sqrt{4a^2+4a+1}}{2}\\ &= \frac{-5\pm(2a+1)}{2}\\ &= \frac{2a-4}{2},\frac{-2a-6}{2}\end{aligned}$$

Thus $x= a-2,\ x=-(a+3)$

**100.** Solve for $x$ : $x^2-(2b-1)x+(b^2-b-20)=0$

**Ans :** [Board Term-2 Foreign 2015]

We have $x^2-(2b-1)x+(b^2-b-20)=0$

Comparing with $Ax^2+Bx+C=0$ we have

$$A=1, B=-(2b-1), C=(b^2-b-\ \ 0$$

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{(2b-1)\pm\sqrt{(2b-1)^2-4(b^2-b-20)}}{2}$$

$$=\frac{(2b-1)\pm\sqrt{4b^2-4b+1-4b^2+4b+80}}{2}$$

$$=\frac{(2b-1)\pm\sqrt{81}}{2}=\frac{(2b-1)\pm 9}{2}$$

$$=\frac{2b+8}{2},\frac{2b-10}{2}$$

$$=b+4,\ b-5$$

Thus $x=b+4$ and $x=b-5$

**101.** Solve for $x$ : $\frac{16}{x}-1=\frac{15}{x+1}$; $x\neq 0,-1$

**Ans :** [Board Term-2, OD 2014]

We have $$\frac{16}{x}-1=\frac{15}{x+1}$$

$$\frac{16}{x}-\frac{15}{x+1}=1$$

$$16(x+1)-15x=x(x+1)$$

$$16x+16-15x=x^2+x$$

$$x+16=x^2+x$$

$$x^2-16=0$$

$$x^2=16$$

$$x=\pm 4$$

Thus $x=-4$ and $x=+4$

**102.** Solve the quadratic equation $(x-1)^2-5(x-1)-6=0$

**Ans :** [Board Term-2, 2015]

We have $$(x-1)^2-5(x-1)-6=0$$

$$x^2-2x+1-5x+5-6=0$$

$$x^2-7x+6-6=0$$

$$x^2-7x=0$$

$$x(x-7)=0$$

Thus $x=0,7$

**103.** Solve the equation for $x$ : $\frac{4}{x}-3=\frac{5}{2x+3}$; $x\neq 0,\frac{-3}{2}$

**Ans :** [Board Term-2 Delhi 2014]

We have $$\frac{4}{x}-3=\frac{5}{2x+3}$$

$$\frac{4}{x}-\frac{5}{2x+3}=3$$

$$\frac{4(2x+3)-5x}{x(2x+3)}=3$$

$$8x+12-5x=3x(2x+3)$$

$$3x+12=6x^2+9x$$

$$6x^2+6x-12=0$$

$$x^2+x-2=0$$

$$x^2+2x-x-2=0$$

$$x(x+2)-(x+2)=0$$

$$(x+2)(x-1)=0$$

Thus $x=1,-2$

**104.** Find the roots of the equation $2x^2+x-4=0$, by the method of completing the squares.

**Ans :** [Board Term-2, OD 2014]

We have $$2x^2+x-4=0$$

$$x^2+\frac{x}{2}-2=0$$

$$x^2+2x\left(\frac{1}{4}\right)-2=0$$

Adding and subtracting $\left(\frac{1}{4}\right)^2$, we get

$$x^2+2x\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2-2=0$$

$$\left(x+\frac{1}{4}\right)^2-\left(\frac{1}{16}+2\right)=0$$

$$\left(x+\frac{1}{4}\right)^2-\left(\frac{1+32}{16}\right)=0$$

$$\left(x+\frac{1}{4}\right)^2-\frac{33}{16}=0$$

$$\left(x+\frac{1}{4}\right)^2=\frac{33}{16}$$

$$\left(x+\frac{1}{4}\right)=\pm\frac{\sqrt{33}}{4}$$

Thus roots are $x = \frac{-1+\sqrt{33}}{4}, \frac{-1-\sqrt{33}}{4}$

**105.** Solve for $x$ : $9x^2 - 6ax + (a^2 - b^2) = 0$

**Ans :** [Board Term-2 2012]

We have $9x^2 - 6ax + a^2 - b^2 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we have

$$A = 9, B = -6a, C = (a^2 - b^2$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{6a \pm \sqrt{(-6a)^2 - 4 \times 9k(a^2 - b^2)}}{2 \times 9}$$

$$= \frac{6a \pm \sqrt{36a^2 - 36a^2 + 36b^2}}{18}$$

$$= \frac{6a \pm \sqrt{36b^2}}{18} = \frac{6a \pm 6b}{18}$$

$$= \frac{a \pm b}{3}$$

$$x = \frac{(a+b)}{3}, \frac{(a-b)}{3}$$

Thus $x = \frac{a+b}{3}, x = \frac{a-b}{3}$

**106.** Solve the equation $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$, $x \neq -4, 7$ for $x$.

**Ans :** [Board Term-2, 2012]

We have, $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$$\frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{-1}{(x+4)(x-7)} = \frac{1}{30}$$

$$(x+4)(x-7) = -30$$

$$x^2 - 3x - 28 = -30$$

$$x^2 - 3x + 2 = 0$$

$$x^2 - 2x - x + 2 = 0$$

$$(x-1)(x-2) = 0$$

Thus $x = 1, 2$.

**107.** Find the roots of the quadratic equation :

$$a^2b^2x^2 + b^2x - a^2x - 1 = 0$$

**Ans :** [Board Term-2, 2012]

We have $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

$$b^2x(a^2x+1) - 1(a^2x+1 = 0$$

$$(b^2x - 1)(a^2x+1) = 0$$

$$x = \frac{1}{b^2} \text{ or } x = -\frac{1}{a^2}$$

Hence, roots are $\frac{1}{b^2}$ and $-\frac{1}{a^2}$.

**108.** If $(x^2 + y^2)(a^2 + b^2) = (ax + by)^2$, prove that $\frac{x}{a} = \frac{y}{b}$

**Ans :** [Board Term-2, 2014]

We have $(x^2 + y^2)(a^2 + b^2) = (ax + by)^2$

$$x^2a^2 + x^2b^2 + y^2a^2 + y^2b^2 = a^2x^2 + b^2y^2 + 2abxy$$

$$x^2b^2 + y^2a^2 - 2abxy = 0$$

$$(xb - ya)^2 = 0$$

$$xb = ya$$

Thus $\frac{x}{a} = \frac{y}{b}$

Hence Proved.

**109.** Solve the following quadratic equation for $x$ :

$p^2x^2 + (p^2 - q^2)x - q^2 = 0$

**Ans :** [Board Term-2, 2012]

We have $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = p^2, b = p^2 - q^2, \ c = -q$$

The roots are given by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(p^2 - q^2) - \sqrt{(p^2 - q^2)^2 - 4(p^2)(-q^2)}}{2p^2}$$

$$= \frac{-(p^2 - q^2) - \sqrt{p^4 + q^4 - 2p^2q^2 + 4p^2q^2}}{2p^2}$$

$$= \frac{-(p^2 - q^2) - \sqrt{p^4 + q^4 + 2p^2q^2}}{2p^2}$$

$$= \frac{-(p^2 - q^2) - \sqrt{(p^2+q^2)^2}}{2p^2}$$

$$= \frac{-(p^2-q^2) \pm (p^2+q^2)}{2p^2}$$

Thus $\quad x = \frac{-(p^2-q^2)+(p^2+q^2)}{2p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$

and $\quad x = \frac{-(p^2-q^2)-(p^2+q^2)}{2p^2} = \frac{-2p^2}{2p^2} = -1$

Hence, roots are $\frac{q^2}{p^2}$ and $-1$.

**110.** Solve the following quadratic equation for $x$ :

$$9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$$

**Ans :** [Board Term-2, Foreign 2016]

We have $\quad 9x^2 - 9(a+b)x + 2a^2 + 5ab + 2b^2 = 0$

Now $\quad 2a^2 + 5ab + 2b^2 = 2a^2 + 4ab + ab + 2b^2$

$$= 2a[a+2b] + b[a+2b]$$

$$= (a+2b)(2a+b)$$

Hence the equation becomes

$$9x^2 - 9(a+b)x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3[3a+3b]x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3[(a+2b)+(2a+b)]x + (a+2b)(2a+b) = 0$$

$$9x^2 - 3(a+2b)x - 3(2a+b)x + (a+2b)(2a+b) = 0$$

$$3x[3x-(a+2b)] - (2a+b)[3x-(a+2b)] = 0$$

$$[3x-(a+2b)][3x-(2a+b)] = 0$$

$$3x - (2a+b) = 0$$

$$x = \frac{a+2b}{3}$$

$$3x - (2a+b) = 0$$

$$x = \frac{2a+b}{3}$$

Hence, roots are $\frac{a+2b}{3}$ and $\frac{2a+b}{3}$.

**111.** Solve for $x$ : $\; x^2 + 6x - (a^2 + 2a - 8)$

**Ans :** [Board Term-2, Foreign 2015]

We have $\quad x^2 + 6x - (a^2+2a-8) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = 1, B = 6, C = (a^2 + 2a - 8)$$

The roots are given by the quadratic formula

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-6 \pm \sqrt{36 + 4(a^2 + 2a -}}{2}$$

$$= \frac{-6 \pm (2a+2)}{2}$$

Thus $\quad x = \frac{-6 + (2a+2)}{2} = a - 2$

and $\quad x = \frac{-6-(2a+2)}{2} = -a-4$

Thus $x = a-2, -a-4$

**112.** If the roots of the equation $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

**Ans :** [Board Term-2 2016]

We have $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$A = (a^2+b^2), B = -2(ac+bd), C = (c^2+d^2)$

If roots are equal, $D = B^2 - 4AC = 0$

or $\quad B^2 = 4AC$

Now $\quad [-2(ac+bd)]^2 = 4(a^2+b^2)(c^2+d^2$

$$4(a^2c^2 + 2abcd + b^2d^2) = 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$$

$$a^2c^2 + 2abcd + b^2d^2 = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$$

$$2abcd = a^2d^2 + b^2c^2$$

$$0 = a^2d^2 - 2abcd + b^2c^2$$

$$0 = (ad-bc)^2$$

$$0 = ad - bc$$

Thus $\quad ad = bc$

$$\frac{a}{b} = \frac{c}{d}$$

Hence Proved

**113.** If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find $k$.

**Ans :** [Board Term-2 Foreign 2014]

We have $\quad 3x^2 + px - 8 = 0$

Since 2 is a root of above equation, it must satisfy it.

Substituting $x = 2$ in $3x^2 + px - 8 = 0$ we have

$$12 + 2p - 8 = 0$$

$$p = -2$$

Since $4x^2 - 2px + k = 0$ has equal roots,

or $4x^2 + 4x + k = 0$ has equal roots,

$$D = b^2 - 4ac = 0$$

$$4^2 - 4(4)(k) = 0$$

$$16 - 16k = 0$$

$$16k = 16$$

Thus $k = 1$

**114.** For what value of $k$, the roots of the quadratic equation $kx(x - 2\sqrt{5}) + 10 = 0$ are equal ?

**Ans :** [Board Term-2 Delhi 2014, 2013]

We have $kx(x - 2\sqrt{5}) + 10 = 0$

or, $kx^2 - 2\sqrt{5}\,kx + 10 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = k, b = -2\sqrt{5}\,k \text{ and } c = 10$$

Since, roots are equal, $D = b^2 - 4ac = 0$

$$(-2\sqrt{5}\,k)^2 - 4 \times k \times 10 = 0$$

$$20k^2 - 40k = 0$$

$$20k(k-2) = 0$$

$$k(k-2) = 0$$

Since $k \neq 0$, we get $k = 2$

**115.** Find the nature of the roots of the following quadratic equation. If the real roots exist, find them : $3x^2 - 4\sqrt{3}\,x + 4 = 0$

**Ans :** [Board Term-2, 2012]

We have $3x^2 - 4\sqrt{3}\,x + 4 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 3,\ b = -4\sqrt{3},\ c = 4$$

$$b^2 - 4ac = (-4\sqrt{3})^2 - 4(3)(4)$$

$$= 48 - 48 = 0$$

Thus roots are real and equal.

Roots are $\left(-\frac{b}{2a}\right), \left(-\frac{b}{2a}\right)$ or $\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$

**116.** Determine the positive value of $k$ for which the equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will both have real and equal roots.

**Ans :** [Board Term-2, 2012, 2014]

We have $x^2 + kx + 64 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = 1, b = k, c = 64$$

For real and equal roots, $b^2 - 4ac = 0$

Thus $k^2 - 4 \times 1 \times 64 = 0$

$$k^2 - 256 = 0$$

$$k = \pm 16 \qquad (1)$$

Now for equation $x^2 - 8x + k = 0$ we have

$$b^2 - 4ac = 0$$

$$(-8)^2 - 4 \times 1 \times k = 0$$

$$64 = 4k$$

$$k = \frac{64}{4} = 16 \qquad (2)$$

From (1) and (2), we get $k = 16$. Thus for $k = 16$, given equations have equal roots.

**117.** Find that non-zero value of $k$, for which the quadratic equation $kx^2 + 1 - 2(k-1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation.

**Ans :** [ Board Term-2 Delhi 2015]

We have $kx^2 + 1 - 2(k-1)x + x^2 = 0$

$$(k+1)x^2 - 2(k-1)x + 1 = 0$$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = k+1, b = -2(k-1), c = 1$$

For real and equal roots, $b^2 - 4ac = 0$

$$4(k-1)^2 - 4(k+1) \times 1 = 0$$

$$4k^2 - 8k + 4 - 4k - 4 = 0$$

$$4k^2 - 12k = 0$$

$$4k(k-3) = 0$$

As $k$ can't be zero, thus $k = 3$.

**118.** Find the value of $k$ for which the quadratic equation $(k-2)x^2 + 2(2k-3)x + (5k-6) = 0$ has equal roots.

**Ans :** [Board Term-2, 2015]

We have $(k-2)x^2 + 2(2k-3)x + (5k-6) = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = k-2, b = 2(2k-3), c = (5k-6)$$

For real and equal roots, $b^2 - 4ac = 0$

$$\{2(2k-3)\}^2 - 4(k-2)(5k-6) = 0$$

$$4(4k^2 - 12k+9) - 4(k-2)(5k-6) = 0$$

$$4k^2 - 12k + 9 - 5k^2 + 6k + 10k - 12 = 0$$

$$k^2 - 4k + 3 = 0$$

$$k^2 - 3k - k + 3 = 0$$

$$k(k-3) - 1(k-3) = 0$$

$$(k-3)(k-1) = 0$$

Thus $k = 1,3$

**119.** If the roots of the quadratic equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b+c$.

**Ans :** [Board Term-2 Delhi 2016]

We have $(a-b)x^2 + (b-c)x + (c-a) = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = (a-b,),\ b = (b-c),\ c = c-a$$

For real and equal roots, $b^2 - 4ac = 0$

$$(b-c)^2 - 4(a-b)(c-a) = 0$$

$$b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$$

$$b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$$

$$4a^2 + b^2 + c^2 + 2bc - 4ab - 4ac = 0$$

Using $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2$,

$$(-2a + b + c)^2 = 0$$

$$-2a + b + c = 0$$

Hence, $b + c = 2a$

**120.** If the quadratic equation, $(1+a^2)b^2x^2 + 2abcx + (c^2 - m^2) = 0$ in $x$ has equal roots, prove that $c^2 = m^2(1+a^2)$

**Ans :** [Board Term-2, 2014]

We have $(1+a^2)b^2x^2 + 2abcx + (c^2 - m^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (1+a^2)b^2, B = 2abc, C = (c^2 - m^2)$$

If roots are equal, $B^2 - 4AC = 0$

$$(2abc)^2 - 4(1+a^2)b^2(c^2 - m^2 = 0$$

$$4a^2b^2c^2 - (4b^2 + 4a^2b^2)(c^2 - m^2) = 0$$

$$4a^2b^2c^2 - [4b^2c^2 - 4b^2m^2 + 4a^2b^2c^2 - 4a^2b^2m^2] = 0$$

$$4a^2b^2c^2 - 4b^2c^2 + 4b^2m^2 - 4a^2b^2c^2 + 4a^2b^2m^2 = 0$$

$$4b^2[a^2m^2 + m^2 - c^2] = 0$$

$$c^2 = a^2m^2 + m^2$$

$$c^2 = m^2(1+a^2)$$

**121.** If $-3$ is a root of quadratic equation $2x^2 + px - 15 = 0$, while the quadratic equation $x^2 - 4px + k = 0$ has equal roots. Find the value of $k$.

**Ans :** [Board Term-2 OD Compt. 2017]

Given $-3$ is a root of quadratic equation.

We have $2x^2 + px - 15 = 0$

Since 3 is a root of above equation, it must satisfy it. Substituting $x = 3$ in above equation we have

$$2(-3)^2 + p(-3) - 15 = 0$$

$$2 \times 9 - 3p - 15 = 0 \Rightarrow p = 1$$

Since $x^2 - 4px + k = 0$ has equal roots,

or $x^2 - 4x + k = 0$ has equal roots,

$$b^2 - 4ac = 0$$

$$(-4)^2 - 4k = 0$$

$$16 - 4k = 0$$

$$4k = 16 \Rightarrow k = 4$$

**122.** If $ad \neq bc$, then prove that the equation $(a^2+b^2)x^2 + 2(ac+bd)x + (c^2+d^2) = 0$ has no real roots.

**Ans :** [Board Term-2 OD 2017]

We have $(a^2+b^2)x^2 + 2(ac+bd)x + (c^2+d^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (a^2+b^2), B = 2(ac+bd) \text{ and } C = c^2 + d^2$$

For no real roots, $D = B^2 - 4AC < 0$

$$D = B^2 - 4AC$$

$$= [2(ac+bd)]^2 - 4(a^2+b^2)(c^2+d^2)$$

$$= 4[a^2c^2 + 2abcd + b^2d^2] - 4[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$$

$$= 4[a^2c^2 + 2abcd + b^2d^2 - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2]$$

$$= -4[a^2d^2 + b^2c^2 - 2abcd]$$

$$= -4(ad - bc)^2$$

Since $ad \neq bc$, therefore $D \neq 0$ and always negative. Hence the equation has no real roots.

**123.** Find the value of $c$ for which the quadratic equation $4x^2 - 2(c+1)x + (c+1) = 0$ has equal roots.

**Ans :** [Board Term-2 Delhi 2017]

We have $4x^2 - 2(c+1)x + (x+1) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = 4, B = 2(c+1), C = c+$$

If roots are equal, $B^2 - 4AC = 0$

$$[2(c+1)]^2 - 4\times 4(c+1) = 0$$
$$4(c^2 + 2c + 1) - 4(4c+4) = 0$$
$$4(c^2 + 2c + 1 - 4c - 4) = 0$$
$$c^2 - 2c - 3 = 0$$
$$c^2 - 3c + c - 3 = 0$$
$$c(c-3) + 1(c-3) = 0$$
$$(c-3)(c+1) = 0$$
$$c = 3, -1$$

Hence for equal roots $c = 3, -1$.

**124.** Show that if the roots of the following equation are equal then $ad = bc$ or $\frac{a}{b} = \frac{c}{d}$.

$$x^2(a^2 + b^2) + 2(ac + bd)x + c^2 + d^2 = 0$$

**Ans :** [Board Term-2 OD Compt. 2017]

We have $x^2(a^2 + b^2) + 2(ac + bd)x + c^2 + d^2 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = a^2 + b^2, B = 2(ac + bd)\ C = c^2 + d$$

If roots are equal, $B^2 - 4AC = 0$

$$[2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$$
$$4(a^2c^2 + 2abcd + b^2d^2) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) = 0$$
$$4(a^2c^2 + 2abcd + b^2d^2 - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2) = 0$$
$$-4(a^2d^2 + b^2c^2 - 2abcd) = 0$$
$$(ad - bc)^2 = 0$$

Thus $ad = bc$

$$\frac{a}{b} = \frac{c}{d}$$ Hence Proved.

**125.** Solve $\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, $a + b \neq 0$.

**Ans :** [Board Term-2 SQP 2016]

We have $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

$$\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$
$$\frac{x - (a+b+x)}{x(a+b+x)} = \frac{a+b}{ab}$$
$$\frac{x - a - b - x}{x(a+b+x)} = \frac{a+b}{ab}$$
$$\frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$$
$$x(a+b+x) = -ab$$
$$x^2 + (a+b)x + ab = 0$$
$$(x+a)(x+b) = 0$$
$$x = -a \text{ or } x = -b$$

## FOUR MARKS QUESTIONS

**126.** Solve for $x$ : $\left(\frac{2x}{x-5}\right)^2 + \left(\frac{2x}{x-5}\right) - 24 = 0, x \neq 5$

**Ans :** [Board Term-2 2016]

We have $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$

Let $\frac{2x}{x-5} = y$ then we have

$$y^2 + 5y - 24 = 0$$
$$(y+8)(y-3) = 0$$
$$y = 3, -8$$

Taking $y = 3$ we have

$$\frac{2x}{x-5} = 3$$
$$2x = 3x - 15 \Rightarrow x = 15$$

Taking $y = -8$ we have

$$\frac{2x}{x-5} = -8$$
$$2x = -8x + 40$$
$$10x = 40 \Rightarrow x = 4$$

Hence, $x = 15, 4$

**127.** Solve for $x$ : $\frac{1}{x+1}+\frac{2}{x+2} = \frac{4}{x+4} \quad x \neq -1, -2, -4$

**Ans :** [Board Term-2 OD 2016]

We have
$$\frac{1}{x+1}+\frac{2}{x+2} = \frac{4}{x+4}$$
$$\frac{x+2+2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4}$$
$$\frac{3x+4}{x^2+3x+2} = \frac{4}{x+4}$$
$$(3x+4)(x+4) = 4(x^2+3x+2)$$
$$3x^2+16x+16 = 4x^2+12x+8$$
$$x^2-4x-8 = 0$$

Now
$$x = \frac{-b\sqrt{b^2+4ac}}{2a}$$
$$= \frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-8)}}{2\times 1}$$
$$= \frac{4\pm\sqrt{16+32}}{2}$$
$$= \frac{4\pm\sqrt{48}}{2} = \frac{4\pm 4\sqrt{3}}{2}$$
$$= 2\pm 2\sqrt{3}$$

Hence, $x = 2+2\sqrt{3}$ and $2-2\sqrt{3}$

**128.** Find $x$ in terms of $a, b$ and $c$ :
$$\frac{a}{x-a}+\frac{b}{x-b} = \frac{2c}{x-c}, x \neq a, b, c$$

**Ans :**

We have
$$\frac{a}{x-a}+\frac{b}{x-b} = \frac{2c}{x-c}$$
$$a(x-b)(x-c)+b(x-a)(x-c) = 2c(x-a)(x-b)$$
$$ax^2-abx-acx+abc+bx^2-bax-bcx+abc$$
$$= 2cx^2-2cxb-2cxa+2abc$$
$$ax^2+bx^2-2cx^2-abx-acx-bax-bcx+2cbx+2acx$$
$$= 0$$
$$x^2(a+b-2c)-2abx+acx+bcx = 0$$
$$x^2(a+b-2c)+x(-2ab+ac+bc) = 0$$

Thus $x = -\left(\frac{ac+bc-2ab}{a+b-2c}\right)$

**129.** Solve for $x$ : $\frac{3}{x+1}+\frac{4}{x-1} = \frac{29}{4x-1};\ x \neq -1,\ 1,\ \frac{1}{4}$

**Ans :** [Board Term-2 Delhi 2015]

We have
$$\frac{3}{x+1}+\frac{4}{x-1} = \frac{29}{4x-1}$$
$$\frac{3x-3+4x+4}{x^2-1} = \frac{29}{4x-1}$$
$$\frac{7x+1}{x^2-1} = \frac{29}{4x-1}$$
$$(7x+1)(4x-1) = 29x^2-29$$
$$28x^2-7x+4x-1 = 29x^2-29$$
$$-3x = x^2-28$$
$$x^2+3x-28 = 0$$
$$x^2+7x-4x-28 = 0$$
$$x(x+7)-4(x+7) = 0$$
$$(x+7)(x-4) = 0$$

Hence, $x = 4, -7$

**130.** Solve for $x$ : $\frac{x-1}{2x+1}+\frac{2x+1}{x-1} = 2$ where $x \neq -\frac{1}{2}, 1$

**Ans :** [Board Term-2, OD 2015]

We have
$$\frac{x-1}{2x+1}+\frac{2x+1}{x-1} = 2$$

Let $\frac{x-1}{2x+1}$ be $y$ so $\frac{2x+1}{x-1} = \frac{1}{y}$

Substituting this value we obtain
$$y+\frac{1}{y} = 2$$
$$y^2+1 = 2y$$
$$y^2-2y+1 = 0$$
$$(y-1)^2 = 0$$
$$y = 1$$

Substituting $y = \frac{x-1}{2x+1}$ we have
$$\frac{x-1}{2x+1} = 1 \text{ or } x-1 = 2x+1$$

or $\qquad x = -2$

**131.** Find for $x$ : $\frac{1}{x-2}+\frac{2}{x-1} = \frac{6}{x};\ \ x \neq 0,\ 1,\ 2$

**Ans :** [Board Term-2 OD 2017]

We have $\dfrac{1}{x-2}+\dfrac{2}{x-1} = \dfrac{6}{x}$

$$\frac{x-1+2x-4}{(x-2)(x-1)} = \frac{6}{x}$$

$$3x^2-5x = 6x^2-18x+12$$

$$3x^2-13x+12 = 0$$

$$3x^2-4x-9x+12 = 0$$

$$x(3x-4)-3(3x-4) = 0$$

$$(3x-4)(x-3) = 0$$

$$x = \frac{4}{3} \text{ and } 3$$

Hence, $x = 3,\ \dfrac{4}{3}$

**132.** Solve, for $x$ : $\sqrt{3}\,x^2+10x+7\sqrt{3}=0$

**Ans :** [Board Term-2 Foreign 2017]

We have $\sqrt{3}\,x^2+10x+7\sqrt{3} = 0$

$$\sqrt{3}\,x^2+3x+7x+7\sqrt{3} = 0$$

$$(x+\sqrt{3})(\sqrt{3}\,x \quad 7 = 0$$

$$(x+\sqrt{3})(\sqrt{3}\,x \quad 7 = 0$$

$$x = -\sqrt{3} \text{ and } x = \frac{-7}{\sqrt{3}}$$

Hence roots $x=-\sqrt{3}$ and $x=\dfrac{-7}{\sqrt{3}}$

**133.** Solve for $x$ : $\dfrac{x+3}{x-2}-\dfrac{1-x}{x} = \dfrac{17}{4};\ \ x \neq 0,\ 2$

**Ans :** [Board Term -2 Delhi Compt. 2017]

We have $\dfrac{x+3}{x-2}-\dfrac{1-x}{x} = \dfrac{17}{4}$

$$\frac{x(x+3)-(1-x)(x-2)}{x(x-2)} = \frac{17}{4}$$

$$\frac{(x^2+3x)-(-x^2+3x-2}{x^2-2x} = \frac{17}{4}$$

$$\frac{2x^2+2}{x^2-2x} = \frac{17}{4}$$

$$8x^2+8 = 17x^2-34x$$

$$9x^2-34x-8 = 0$$

$$9x^2-36x+2x-8 = 0$$

$$9x(x-4)+2(x-4) = 0$$

$$(x-4)(9x+2) = 0$$

$$x = 4 \text{ or } x=-\frac{2}{9}$$

Hence, $x=4,\ -\dfrac{2}{9}$

**134.** Solve for $x$ : $4x^2+4bx-(a^2-b^2)=0$

**Ans :** [Board Term-2 Foreign 2017]

We have $4x^2+4bx-(a^2-b^2)=0$

Comparing with $Ax^2+Bx+C = 0$ we get

$$A=4, B=4b \text{ and } C=b^2-a^2$$

$$x = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$= \frac{-4b\pm\sqrt{(4b)^2-4.4(b^2-a\ )}}{2.4}$$

$$= \frac{-4b\pm\sqrt{16b^2-16b^2+16a^2}}{8}$$

$$= \frac{-4b\pm4a}{8}$$

$$= -\frac{(a+b)}{2}, \frac{(a-b)}{2}$$

Hence the roots are $-\dfrac{(a+b)}{2}$ and $\dfrac{(a-b)}{2}$

**135.** Find the zeroes of the quadratic polynomial $7y^2-\frac{11}{3}y-\frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

**Ans :** [Board 2019 OD]

We have $7y^2-\dfrac{11}{3}y-\dfrac{2}{3} = 0$

$$21y^2-11y-2 = 0 \quad \text{...(1)}$$

$$21y^2-14y+3y-2 = 0$$

$$7y(3y-2)+(3y-2) = 0$$

$$(3y-2)(7y+1) = 0$$

$$y = \frac{2}{3}, \frac{-1}{7}$$

Hence, zeros of given polynomial are,

$$y = \frac{2}{3} \text{ and } y = \frac{-1}{7}$$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 21$, $b = -11$ and $c = -2$

Now, sum of roots, $\alpha + \beta = \frac{2}{3} + \left(-\frac{1}{7}\right)$

$$= \frac{2}{3} - \frac{1}{7} = \frac{11}{21}$$

Thus $\alpha + \beta = -\frac{b}{a}$ Hence verified

and product of roots, $\alpha\beta = \frac{2}{3} \times \left(-\frac{1}{7}\right) = \frac{-2}{21}$

Thus $\alpha\beta = \frac{c}{a}$ Hence verified

**136.** Write all the values of $p$ for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.

**Ans :** [Board 2019 OD]

We have $x^2 + px + 16 = 0$ ...(1)

If this equation has equal roots, then discriminant $b^2 - 4ac$ must be zero.

i.e., $b^2 - 4ac = 0$ ...(2)

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 1$, $b = p$ and $c = 16$

Substituting above in equation (2) we have

$$p^2 - 4 \times 1 \times 16 = 0$$

$$p^2 = 64 \Rightarrow p = \pm 8$$

When $p = 8$, from equation (1) we have

$$x^2 + 8x + 16 = 0$$

$$x^2 + 2 \times 4x + 4^2 = 0$$

$$(x + 4)^2 = 0 \Rightarrow x = -4, -4$$

Hence, roots are $-4$ and $-4$.

When $p = -8$ from equation (1) we have

$$x^2 - 8x + 16 = 0$$

$$x^2 - 2 \times 4x + 4^2 = 0$$

$$(x - 4)^2 = 0 \Rightarrow x = 4, 4$$

Hence, the required roots are either $-4, -4$ or $4, 4$

**137.** Solve for $x : x^2 + 5x - (a^2 + a - 6) = 0$

**Ans :** [Board 2019 OD]

We have $x^2 + 5x - (a^2 + a - \;\;) = 0$

$$x^2 + 5x - [a^2 + 3a - 2a - 6 = 0$$

$$x^2 + 5x - [a(a + 3) - 2(a + 3) = 0$$

$$x^2 + 5x - (a + 3)(a - 2) = 0$$

$$x^2 + [a + 3 - (a - 2)]x - (a + 3)(a - 2) = 0$$

$$x^2 + (a + 3)x - (a - 2)x - (a + 3)(a - 2) = 0$$

$$x[x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$$

$$[x + (a + 3)][x - (a - 2)] = 0$$

Thus $x = -(a + 3)$ and $x = (a - 2)$

Hence, roots of given equations are $x = -(a + 3)$ and $x = a - 2$.

**138.** Find the nature of the roots of the quadratic equation $4x^2 + 4\sqrt{3x} + 3 = 0$.

**Ans :** [Board 2019 OD]

We have $4x^2 + 4\sqrt{3x} + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$ we get $a = 4$, $b = 4\sqrt{3}$ and $c = 3$.

Now, $D = b^2 - 4ac$

$$= (4\sqrt{3})^2 - 4 \times 4 \times$$

$$= 48 - 48 = 0$$

Since, $b^2 - 4ac = 0$, then roots of the given equation are real and equal.

**139.** If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of $k$.

**Ans :** [Board 2018]

If $x = 3$ is one root of the equation $x^2 - 2kx - 6 = 0$, it must satisfy it.

Thus substituting $x = 3$ in given equation we have

$$9 - 6k - 6 = 0$$

$$k = \frac{1}{2}$$

**140.** Find the positive values of $k$ for which quadratic equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ both will have the real roots.

**Ans :** [Board Term-2 Foreign 2016]

(1) For $x^2 + kx + 64 = 0$ to have real roots

$$k^2 - 256 \geq 0$$

$$k^2 \geq 256$$

$$k \geq 16 \text{ or } k < -16$$

(2) For $x^2 - 8x + k = 0$ to have real roots

$$64 - 4k \geq 0$$

$$16 - k \geq 0$$

$$16 \geq k$$

For (1) and (2) to hold simultaneously

$$k = 16$$

**141.** Find the values of $k$ for which the equation $(3k+1)^2 + 2(k+1)x + 1$ has equal roots. Also find the roots.

**Ans :** [Board Term-2, 2014]

We have $(3k+1)^2 + 2(k+1)x + 1$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (3k+1), B = 2(k+1), C = 1$$

If roots are equal, $B^2 - 4AC = 0$

$$[2(k+1)]^2 - 4(3k+1)(1) = 0$$

$$4(k^2+2k+1) - (12k+4) = 0$$

$$4k^2 + 8k + 4 - 12k - 4 = 0$$

$$4k^2 - 4k = 0$$

$$4k(k-1) = 0$$

$$k = 0, 1.$$

Substituting $k = 0$, in the given equation,

$$x^2 + 2x + 1 = 0$$

$$(x+1)^2 = 0$$

$$x = -1$$

Again substituting $k = 1$, in the given equation,

$$4x^2 + 4x + 1 = 0$$

$$(2x+1)^2 = 0$$

or, $$x = -\frac{1}{2}$$

Hence, roots $= -1, -\frac{1}{2}$

**142.** Find the values of $k$ for which the quadratic equations $(k+4)x^2 + (k+1)x + 1 = 0$ has equal roots. Also, find the roots.

**Ans :** [Board Term-2 Delhi 2014]

We have $(k+4)x^2 + (k+1)x + 1 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = (k+4), B = (k+1), C = 1$$

If roots are equal, $B^2 - 4AC = 0$

$$(k+1)^2 - 4(k+4)(1) = 0$$

$$k^2 + 1 + 2k - 4k - 16 = 0$$

$$k^2 - 2k - 15 = 0$$

$$(k-5)(k+3) = 0$$

$$k = 5, -3$$

For $k = 5$, equation becomes

$$9x^2 + 6x + 1 = 0$$

$$(3x+1)^2 = 0$$

or $$x = -\frac{1}{3}$$

For $k = -3$, equation becomes

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x = 1$$

Hence roots are 1 and $-\frac{1}{3}$.

**143.** If $x = -2$ is a root of the equation $3x^2 + 7x + p = 0$, find the value of $k$ so that the roots of the equation $x^2 + k(4x + k - 1) + p = 0$ are equal.

**Ans :** [Board Term-2 Foreign 2015]

We have $3x^2 + 7x + p = 0$

Since $x = -2$ is the root of above equation, it must satisfy it.

Thus $$3(-2) + 7(-2) + p = 0$$

$$p = 2$$

Since roots of the equation $x^2 + 4kx + k^2 - k + 2 = 0$ are equal,

$$16k^2 - 4(k^2 - k + 2) = 0$$

$$16k^2 - 4k^2 + 4k - 8 = 0$$

$$12k^2 + 4k - 8 = 0$$

$$3k^2 + k - 2 = 0$$

$$(3k-2)(k+1) = 0$$

$$k = \frac{2}{3}, -1$$

Hence, roots $= \frac{2}{3}, -1$

**144.** If $x = -4$ is a root of the equation $x^2 + 2x + 4p = 0$

, find the values of $k$ for which the equation $x^2 + px(1+3k) + 7(3+2k) = 0$ has equal roots.

**Ans :** [Board Term-2 Foreign 2015]

We have $x^2 + 2x + 4p = 0$

Since $x = -4$ is the root of above equation. It must satisfy it.

$$(-4)^2 + (2 \times -4) + 4p = 0$$

$$p = -2$$

Since equation $x^2 - 2(1+3k)x + 7(3+2k) = 0$ has equal roots.

$$4(1+3k)^2 - 28(3+2k) = 0$$

$$9k^2 - 8k - 20 = 0$$

$$(9k+10)(k-2) = 0$$

$$k \frac{-10}{9}, 2$$

Hence, the value of $k$ are $-\frac{10}{9}$ and 2.

**145.** Find the value of $p$ for which the quadratic equation $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0$, $p \neq -1$ has equal roots. Hence find the roots of the equation.

**Ans :** [Board Term-2, 2015]

We have $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$$a = p+1, b = -6(p+1), c = 3(p+9)$$

For real and equal roots, $b^2 - 4ac = 0$

$$36(p+1)^2 - 4(p+1) \times 3(p+9) = 0$$

$$3(p^2 + 2p + 1) - (p+1)(p+9) = 0$$

$$3p^2 + 6p + 3 - (p^2 + 9p + p + 9) = 0$$

$$2p^2 - 4p - 6 = 0$$

$$p^2 - 2p - 3 = 0$$

$$p^2 - 3p + p - 3 = 0$$

$$p(p-3) + 1(p-3) = 0$$

$$(p-3)(p+1) = 0$$

$$p = -1, 3$$

Neglecting $p \neq -1$ we get $p = 3$

Now the equation becomes

$$4x^2 - 24x + 36 = 0$$

or $$x^2 - 6x + 9 = 0$$

or, $$(x-3)(x-3) = 0$$

$$x = 3, 3$$

Thus roots are 3 and 3.

**146.** If the equation $(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1+m^2)$

**Ans :** [Board Term-2 Delhi 2015]

We have $(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get

$$A = 1 + m^2, B = 2mc, C = (c^2 - a^2)$$

If roots are equal, $B^2 - 4AC = 0$

$$(2mc)^2 - 4(1+m^2)(c^2 - a^2) = 0$$

$$4m^2c^2 - 4(1+m^2)(c^2 - a^2) = 0$$

$$m^2c^2 - (c^2 - a^2 + m^2c^2 - m^2a^2 = 0$$

$$m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$$

$$-c^2 + a^2 + m^2a^2 = 0$$

, $$c^2 = a^2(1+m^2)$$

Hence Proved.

**147.** If $(-5)$ is a root of the quadratic equation $2x^2 + px + 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, then find the values of $p$ and $k$.

**Ans :** [Board Term-2 Delhi 2015]

We have $2x^2 + px - 15 = 0$

Since $x = -5$ is the root of above equation. It must satisfy it.

$$2(-5)^2 + p(-5) - 15 = 0$$

$$50 - 5p - 15 = 0$$

$$5p = 35 \Rightarrow p = 7$$

Now $$p(x^2 + x) + k = 0$$ has equal roots

or $$7x^2 + 7x + k = 0$$

Taking $b^2 - 4ac = 0$ we have

$$7^2 - 4 \times 7 \times k = 0$$

$$7 - 4k = 0$$

$$k = \frac{7}{4}$$

Hence $p = 7$ and $k = \frac{7}{4}$.

**148.** If the roots of the quadratic equation

$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are equal. Then show that $a=b=c$.

**Ans :** [Board Term-2 Delhi 2015]

We have

$$(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a) = 0$$
$$x^2-ax-bx+ab+$$
$$+x^2-bx-cx+bc+$$
$$+x^2-cx-ax+ac = 0$$
$$3x^2-2ac-2bx-2cx+ab+bc+ca = 0$$

For equal roots $B^2-4AC=0$

$$\{-2(a+b+c)\}^2-4\times 3(ab+bc+ca) = 0$$
$$4(a+b+c)^2-12(ab+bc+ca) = 0$$
$$a^2+b^2+c^2-3(ab+bc+ca) = 0$$
$$a^2+b^2+c^2+2ab+2bc+2ac-3ab-3bc-3ac = 0$$
$$a^2+b^2+c^2-ab-ac-bc = 0$$
$$\tfrac{1}{2}[2a^2+2b^2+2c^2-2ab-2ac-2bc] = 0$$
$$\tfrac{1}{2}[(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ac)]=0$$
$$\tfrac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2] = 0$$

or, $$(a-b)^2+(b-c)^2+(c-a)^2 = 0$$

If $a\neq b\neq c$

$(a-b)^2>0,\ (b-c)^2>0,\ (c-a)^2>0$

If $(a-b)^2 = 0\Rightarrow a=b$

$(a-c)^2 = 0\Rightarrow b=c$

$(c-a)^2 = 0\Rightarrow c=a$

Thus $a=b=c$ Hence Proved

**149.** If the roots of the quadratic equation $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$ in $x$ are equal then show that either $a=0$ or $a^3+b^3+c^3=3abc$

**Ans :** [Board Term 2Outside Delhi 2017]

We have $(c^2-ab)x^2-2(a^2-bc)x+b^2-ac=0$

Comparing with $Ax^2+Bx+C = 0$ we get

$A=(c^2-ab), B=(a^2-bc), C=(b^2-ac)$

If roots are equal, $B^2-4AC=0$

$$[2(a^2-bc)]^2-4(c^2-ab)(b^2-ac) = 0$$
$$4[a^4+b^2c^2-2a^2bc]-4(b^2c^2-c^3a-ab^3-a^2bc) = 0$$
$$4[a^4+b^2c^2-2a^2bc-b^2c^2+c^3a+ab^3-a^2bc] = 0$$
$$4[a^4+ac^3+ab^3-3a^2bc = 0$$
$$a(a^3+c^3+b^3-3abc = 0$$
$$a=0 \text{ or } a^3+b^3+c^3 = 3abc$$

**150.** Solve for $x$ : $\dfrac{1}{a+b+x}=\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{x}$

where $a+b+x\neq 0$ and $a,b,x\neq 0$

**Ans :** [Board Term-2 Foreign 2017]

We have $$\frac{1}{a+b+x}-\frac{1}{x} = \frac{1}{a}+\frac{1}{b}$$
$$\frac{-(a+b)}{x^2+(a+b)x} = \frac{b+a}{ab}$$
$$x^2+(a+b)x+ab = 0$$
$$(x+a)(x+b) = 0$$
$$x = -a, x=-b$$

Hence $x=-a,-b$

**151.** Check whether the equation $5x^2-6x-2=0$ has real roots if it has, find them by the method of completing the square. Also verify that roots obtained satisfy the given equation.

**Ans :** [Board Term-2 SQP 2017]

We have $5x^2-6x-2=0$

Comparing with $ax^2+bx+c = 0$ we get

$$a=5, b=(-6) \text{ and } c=(-2)$$
$$b^2-4ac = (-6)^2-4\times 5\times -2$$
$$= 36+40=76>0$$

So the equation has real and two distinct roots.

$$5x^2-6x = 2$$

Dividing both the sides by 5 we get

$$\frac{x^2}{5}-\frac{6}{5}x = \frac{2}{5}$$
$$x^2-2x\left(\frac{3}{5}\right) = \frac{2}{5}$$

Adding square of the half of coefficient of $x$

$$x^2-2x\left(\frac{3}{5}\right)+\frac{9}{25} = \frac{2}{5}+\frac{9}{25}$$
$$\left(x-\frac{3}{5}\right)^2 = \frac{19}{25}$$
$$x-\frac{3}{5} = \pm\frac{\sqrt{19}}{5}$$

$$x = \frac{3+\sqrt{19}}{5} \text{ or } \frac{3-\sqrt{19}}{5}$$

Verification :

$$5\left[\frac{3+\sqrt{19}}{5}\right]^2 - 6\left[\frac{3+\sqrt{19}}{5}\right] - 2$$

$$= \frac{9+6\sqrt{19}+19}{5} - \left(\frac{18+6\sqrt{19}}{5}\right) - 2$$

$$= \frac{28+6\sqrt{19}}{5} - \frac{18+6\sqrt{19}}{5} - 2$$

$$= \frac{28+6\sqrt{19}-18-6\sqrt{19}-10}{5}$$

$$= 0$$

Similarly

$$5\left[\frac{3-\sqrt{19}}{5}\right]^2 - 6\left[\frac{3-\sqrt{19}}{5}\right] - 2 = 0$$

Hence verified.