CHAPTER 2

POLYNOMIALS

ONE MARK QUESTIONS

Multiple Choice Questions

1. If one zero of a quadratic polynomial $(kx^2 + 3x + k)$ is 2, then the value of k is

(a) $\frac{5}{6}$ (b) $-\frac{5}{6}$

(c) $\frac{6}{5}$ (d) $-\frac{6}{5}$

Ans : [Board 2020 Delhi Basic]

We have $p(x) = kx^2 + 3x + k$

Since, 2 is a zero of the quadratic polynomial

$$p(2) = 0$$

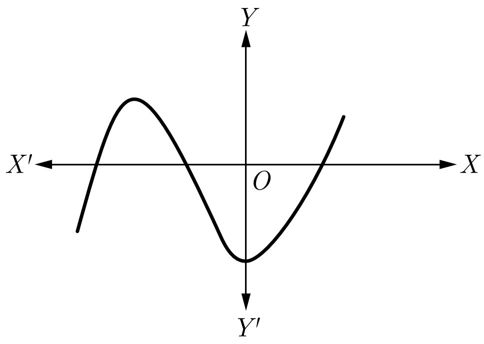
$$k(2)^2 + 3(2) + k = 0$$

$$4k + 6 + k = 0$$

$$5k = -6 \Rightarrow k = -\frac{6}{5}$$

Thus (d) is correct option.

2. The graph of a polynomial is shown in Figure, then the number of its zeroes is



(a) 3 (b) 1

(c) 2 (d) 4

Ans : [Board 2020 Delhi Basic]

Since, the graph cuts the x-axis at 3 points, the number of zeroes of polynomial $p(x)$ is 3.

Thus (a) is correct option.

3. The maximum number of zeroes a cubic polynomial can have, is

(a) 1 (b) 4

(c) 2 (d) 3

Ans : [Board 2020 OD Basic]

A cubic polynomial has maximum 3 zeroes because its degree is 3.

Thus (d) is correct option.

4. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10 (b) -10

(c) -7 (d) -2

Ans : [Board 2020 Delhi Standard]

We have $p(x) = x^2 + 3x + k$

If 2 is a zero of $p(x)$, then we have

$$p(2) = 0$$

$$(2)^2 + 3(2) + k = 0$$

$$4 + 6 + k = 0$$

$$10 + k = 0 \Rightarrow k = -10$$

Thus (b) is correct option.

5. The quadratic polynomial, the sum of whose zeroes is -5 and their product is 6, is

(a) $x^2 + 5x + 6$ (b) $x^2 - 5x + 6$

(c) $x^2 - 5x - 6$ (d) $-x^2 + 5x + 6$

Ans : [Board 2020 Delhi Standard]

Let α and β be the zeroes of the quadratic polynomial, then we have

$$\alpha + \beta = -5$$

and $$\alpha\beta = 6$$

Now $$p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

$$= x^2 - (-5)x + 6$$

$$= x^2 + 5x + 6$$

Thus (a) is correct option.

6. If one zero of the polynomial $(3x^2 + 8x + k)$ is the reciprocal of the other, then value of k is

(a) 3 (b) -3

(c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

Ans : [Board 2020 OD Basic]

Let the zeroes be α and $\frac{1}{\alpha}$.

Product of zeroes, $\alpha \cdot \frac{1}{\alpha} = \frac{\text{constant}}{\text{coefficient of } x^2}$

$$1 = \frac{k}{3} \Rightarrow k = 3$$

Thus (a) is correct option.

7. The zeroes of the polynomial $x^2 - 3x - m(m+3)$ are

(a) $m, m+3$ (b) $-m, m+3$

(c) $m, -(m+3)$ (d) $-m, -(m+3)$

Ans : [Board 2020 OD Standard]

We have $p(x) = x^2 - 3x - m(m+$

Substituting $x = -m$ in $p(x)$ we have

$$p(-m) = (-m)^2 - 3(-m) - m(m+$$
$$= m^2 + 3m - m^2 - 3m = 0$$

Thus $x = -m$ is a zero of given polynomial.

Now substituting $x = m+3$ in given polynomial we have

$$p(x) = (m+3)^2 - 3(m+3) - m(m+3)$$
$$= (m+3)[m+3-3-m]$$
$$= (m+3)[0] = 0$$

Thus $x = m+3$ is also a zero of given polynomial.

Hence, $-m$ and $m+3$ are the zeroes of given polynomial.
Thus (b) is correct option.

8. The value of x, for which the polynomials $x^2 - 1$ and $x^2 - 2x + 1$ vanish simultaneously, is

(a) 2 (b) -2

(c) -1 (d) 1

Ans :

Both expression $(x-1)(x+1)$ and $(x-1)(x-1)$ have 1 as zero. This both vanish if $x = 1$.
Thus (d) is correct option.

9. If α and β are zeroes and the quadratic polynomial $f(x) = x^2 - x - 4$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ α

(a) $\frac{15}{4}$ (b) $\frac{-15}{4}$

(c) 4 (d) 15

Ans :

We have $f(x) = x^2 - x - 4$

$$\alpha + \beta = -\frac{-1}{1} = 1 \text{ and } \alpha\beta = \frac{-4}{1} - 4$$

Now $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta = \frac{\alpha+\beta}{\alpha\beta} - \alpha\beta$

$$= -\frac{1}{4} + 4 = \frac{15}{4}$$

Thus (a) is correct option.

10. The value of the polynomial $x^8 - x^5 + x^2 - x + 1$ is

(a) positive for all the real numbers

(b) negative for all the real numbers

(c) 0

(d) depends on value of x

Ans :

We have $f(x) = \mathrm{x}^8 - \mathrm{x}^5 + \mathrm{x}^2 - \mathrm{x} + 1$

$f(x)$ is always positive for all $x > 1$

For $x = 1$ or 0, $\mathrm{f(x)} = 1 > 0$

For $x < 0$ each term of $f(x)$ is positive, thus $f(x) > 0$. Hence, $f(x)$ is positive for all real x.
Thus (a) is correct option.

11. Lowest value of $x^2 + 4x + 2$ is

(a) 0 (b) -2

(c) 2 (d) 4

Ans :

$$x^2 + 4x + 2 = (x^2 + 4x + 4) - 2$$
$$= (x+2)^2 - 2$$

Here $(x+2)^2$ is always positive and its lowest value is zero. Thus lowest value of $(x+2)^2 - 2$ is -2 when $x + 2 = 0$.
Thus (b) is correct option.

12. If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of k is

(a) 2 (b) -2

(c) 4 (d) -4

Ans :

Sum of the zeroes, $6 = \frac{3k}{2}$

$$k = \frac{12}{3} = 4$$

Thus (c) is correct option.

13. If the square of difference of the zeroes of the quadratic polynomial $x^2 + px + 45$ is equal to 144, then the value of p is

(a) ± 9 (b) ± 12

(c) ± 15 (d) ± 18

Ans :

We have $f(x) = x^2 + px + 45$

Then, $\alpha + \beta = \frac{-p}{1} = -p$

and $\alpha\beta = \frac{45}{1} = 45$

According to given condition, we have

$$(\alpha - \beta)^2 = 144$$

$$(\alpha + \beta)^2 - 4\alpha\beta = 144$$

$$(-p)^2 - 4(45) = 144$$

$$p^2 = 144 + 180 = 324 \Rightarrow p = \pm 18$$

14. If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then the value of k is

(a) $\frac{4}{3}$ (b) $\frac{-4}{3}$

(c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

Ans :

If a is zero of quadratic polynomial $f(x)$, then

$$f(a) = 0$$

So, $f(-3) = (k-1)(-3)^2 + (-3)k + 1$

$$0 = (k-1)(9) - 3k + 1$$

$$0 = 9k - 9 - 3k + 1$$

$$0 = 6k - 8$$

$$k = \frac{8}{6} = \frac{4}{3}$$

Thus (a) is correct option.

15. A quadratic polynomial, whose zeroes are -3 and 4, is

(a) $x^2 - x + 12$ (b) $x^2 + x + 12$

(c) $\frac{x^2}{2} - \frac{x}{2} - 6$ (d) $2x^2 + 2x - 24$

Ans :

We have $\alpha = -3$ and $\beta = 4$.

Sum of zeros $\alpha + \beta = -3 + 4 = 1$

Product of zeros, $\alpha \cdot \beta = -3 \times 4 = -12$

So, the quadratic polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 1 \times x + (-12)$$

$$= x^2 - x - 12$$

$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

Thus (c) is correct option.

16. If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then

(a) $a = -7, b = -1$

(b) $a = 5, b = -1$

(c) $a = 2, b = -6$

(d) $a = 0, b = -6$

Ans :

If a is zero of the polynomial, then $f(a) = 0$.
Here, 2 and -3 are zeroes of the polynomial $x^2 + (a+1)x + b$

So, $f(2) = (2)^2 + (a+1)(-3) + b = 0$

$$4 + 2a + 2 + b = 0$$

$$6 + 2a + b = 0$$

$$2a + b = -6 \quad ...(1)$$

Again, $f(-3) = (-3)^2 + (a+1)2 + b = 0$

$$9 - 3(a+1) + b = 0$$

$$9 - 3a - 3 + b = 0$$

$$6 - 3a + b = 0$$

$$-3a + b = -6$$

$$3a - b = 6 \quad ...(2)$$

Adding equations (1) and (2), we get

$$5a = 0 \Rightarrow a = 0$$

Substituting value of a in equation (1), we get

$$b = -6$$

Hence, $a = 0$ and $b = -6$.

Thus (d) is correct option.

17. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

(a) both positive

(b) both negative

(c) one positive and one negative

(d) both equal

Ans :

Let $f(x) = x^2 + 99x + 127$

Comparing the given polynomial with $ax^2 + bx + c$, we get $a = 1$, $b = 99$ and $c = 127$.

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = -99$

Product of zeroes $\alpha\beta = \frac{c}{a} = 127$

Now, product is positive and the sum is negative, so both of the numbers must be negative.

Alternative Method :

Let $f(x) = x^2 + 99x + 127$

Comparing the given polynomial with $ax^2 + bx + c$, we get $a = 1$, $b = 99$ and $c = 127$.

Now by discriminant rule,

$$D = \sqrt{b^2 - 4ac}$$
$$= \sqrt{(99)^2 - 4 \times 1 \times 127}$$
$$= \sqrt{9801 - 508} = \sqrt{9293}$$
$$= 96.4$$

So, the zeroes of given polynomial,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-99 \pm \sqrt{96.4}}{2}$$

Now,as $99 > 96.4$

So, both zeroes are negative.

Thus (b) is correct option.

18. The zeroes of the quadratic polynomial $x^2 + kx + k$ where $k \neq 0$,

(a) cannot both be positive

(b) cannot both be negative

(c) are always unequal

(d) are always equal

Ans :

Let $f(x) = x^2 + kx + k,\ k \neq 0$

Comparing the given polynomial with $ax^2 + bx + c$, we get $a = 1$, $b = k$ and $c = k$.

Again, let if α, β be the zeroes of given polynomial then,

$$\alpha + \beta = -k$$
$$\alpha\beta = k$$

Case 1: If k is negative, then $\alpha\beta$ is negative. It means α and β are of opposite sign.

Case 2: If k is positive, then $\alpha + \beta$ must be negative and $\alpha\beta$ must be positive and α and β both negative.

Hence, α and β cannot both positive.

Thus (a) is correct option.

19. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, where $c \neq 0$, are equal, then

(a) c and a have opposite signs

(b) c and b have opposite signs

(c) c and a have same sign

(d) c and b have the same sign

Ans :

Let $f(x) = ax^2 + bx + c$

Let α and β are zeroes of this polynomial

Then, $\alpha + \beta = -\frac{b}{a}$

and $\alpha\beta = \frac{c}{a}$

Since $\alpha = \beta$, then α and β must be of same sign i.e. either both are positive or both are negative. In both case

$$\alpha\beta > 0$$
$$\frac{c}{a} > 0$$

Both c and a are of same sign.

Thus (c) is correct option.

20. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

(a) has no linear term and the constant term is negative.

(b) has no linear term and the constant term is positive.

(c) can have a linear term but the constant term is negative.

(d) can have a linear term but the constant term is positive.

Ans :

Let $f(x) = x^2 + ax + b$

and let the zeroes of $f(x)$ are α and β,

As one of zeroes is negative of other,

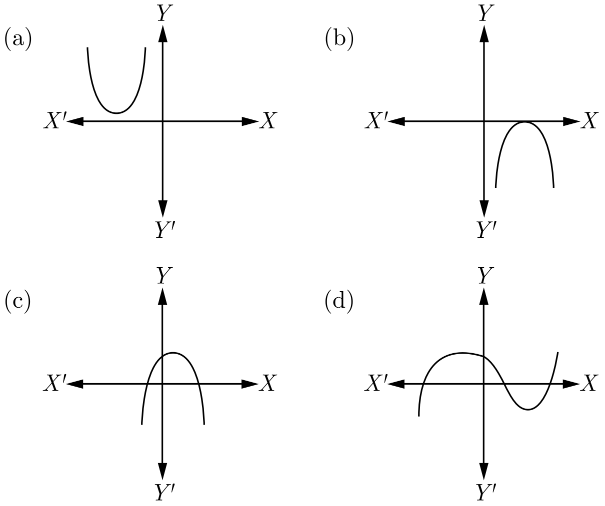
sum of zeroes $\alpha + \beta = \alpha + (-\alpha) = 0$ ...(1)

and $\alpha\beta = \alpha \cdot (-\alpha) = -\alpha^2$ ...(2)

Hence, the given quadratic polynomial has no linear term and the constant term is negative.

Thus (a) is correct option.

21. Which of the following is not the graph of a quadratic polynomial?



Ans :

As the graph of option (d) cuts x-axis at three points. So, it does not represent the graph of quadratic polynomial.

Thus (d) is correct option.

22. Assertion : $(2-\sqrt{3})$ is one zero of the quadratic polynomial then other zero will be $(2+\sqrt{3})$.

Reason : Irrational zeros (roots) always occurs in pairs.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

As irrational roots/zeros always occurs in pairs therefore, when one zero is $(2-\sqrt{3})$ then other will be $2+\sqrt{3}$. So, both A and R are correct and R explains A.

Thus (a) is correct option.

23. Assertion : If one zero of poly-nominal $p(x) = (k^2+4)x^2 + 13x + 4k$ is reciprocal of other, then $k = 2$.

Reason : If $(x - \alpha)$ is a factor of $p(x)$, then $p(\alpha) = 0$ i.e. α is a zero of $p(x)$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

Let $\alpha, \frac{1}{\alpha}$ be the zeroes of $p(x)$, then

$$\alpha \cdot \frac{1}{\alpha} = \frac{4k}{k^2+4}$$

$$1 = \frac{4k}{k^2+4}$$

$$k^2 - 4k + 4 = 0$$

$$(k-2)^2 = 0 \Rightarrow k = 2$$

Assertion is true Since, Reason is not correct for Assertion.

Thus (b) is correct option.

24. Assertion : $p(x) = 14x^3 - 2x^2 + 8x^4 + 7x - 8$ is a polynomial of degree 3.

Reason : The highest power of x in the polynomial $p(x)$ is the degree of the polynomial.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

The highest power of x in the polynomial $p(x) = 14x^3 - 2x^2 + 8x^4 + 7x - 8$ is 4. Degree is 4. So, A is incorrect but R is correct.

Thus (d) is correct option.

25. Assertion : $x^3 + x$ has only one real zero.

Reason : A polynomial of nth degree must have n real zeroes.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

A polynomial of nth degree at most can have n real zeroes. Thus reason is not true.

Again, $x^3 + x = x(x^2 + 1)$

which has only one real zero because $x^2 + 1 \neq 0$ for all $x \in R$.

Assertion (A) is true but reason (R) is false.

Thus (c) is correct option.

26. Assertion : If both zeros of the quadratic polynomial $x^2 - 2kx + 2$ are equal in magnitude but opposite in sign then value of k is ½.

Reason : Sum of zeros of a quadratic polynomial $ax^2 + bx + c$ is $\frac{-b}{a}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

As the polynomial is $x^2 - 2kx + 2$ and its zeros are equal but opposition sign, sum of zeroes must be zero.

$$\text{sum of zeros} = 0$$

$$\frac{-(-2k)}{1} = 0 \Rightarrow k = 0$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

Fill in the Blank Questions

27. A .... polynomial is of degree one.

Ans :

Linear

28. A cubic polynomial is of degree..........

Ans :

Three

29. Degree of remainder is always .... than degree of divisor.

Ans :

Smaller/less

30. Polynomials of degrees 1, 2 and 3 are called .... , ... and .... polynomials respectively.

Ans :

linear, quadratic, cubic

31. .... is not equal to zero when the divisor is not a factor of dividend.

Ans :

Remainder

32. The zeroes of a polynomial $p(x)$ are precisely the $x-$coordinates of the points, where the graph of $y = p\ x$ intersects the .... axis.

Ans :

x

33. The algebraic expression in which the variable has non-negative integral exponents only is called ....

Ans :

Polynomial

34. A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most .... zeroes.

Ans :

3

35. A .... is a polynomial of degree 0.

Ans :

Constant

36. The highest power of a variable in a polynomial is called its ....

Ans :

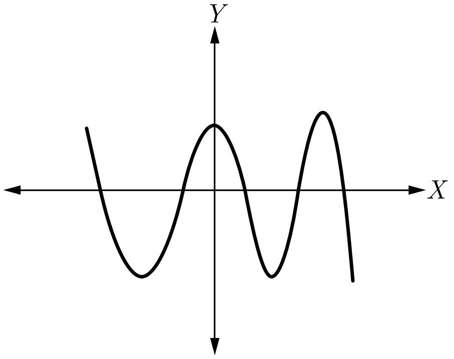
Degree

37. A polynomial of degree n has at the most .... zeroes.

Ans :

n

38. The graph of $y = p(x)$, where $p(x)$ is a polynomial in variable x, is as follows.



The number of zeroes of $p(x)$ is ....

Ans : [Board 2020 SQP Standard]

The graph of the given polynomial $p(x)$ crosses the x-axis at 5 points. So, number of zeroes of $p(x)$ is 5.

39. If one root of the equation $(k-1)x^2 - 10x + 3 = 0$ is the reciprocal of the other then the value of k is ....

Ans : [Board 2020 SQP Standard]

We have $(k-1)x^2 - 10x + 3 = 0$

Let one root be α, then another root will be $\frac{1}{\alpha}$

Now $$\alpha \cdot \frac{1}{\alpha} = \frac{c}{a} = \frac{3}{(k-1)}$$

$$1 = \frac{3}{(k-1)}$$

$$k - 1 = 3 \Rightarrow k = 4$$

Very Short Answer Questions

40. If α and β are the roots of $ax^2 - bx + c = 0 (a \neq 0)$, then calculate $\alpha + \beta$.

Ans : [Board Term-1 2014]

We know that

$$\text{Sum of the roots} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Thus $$\alpha + \beta = -\left(\frac{-b}{a}\right) = \frac{b}{a}$$

41. Calculate the zeroes of the polynomial $p(x) = 4x^2 - 12x + 9$.

Ans : [Board Term-1 2010]

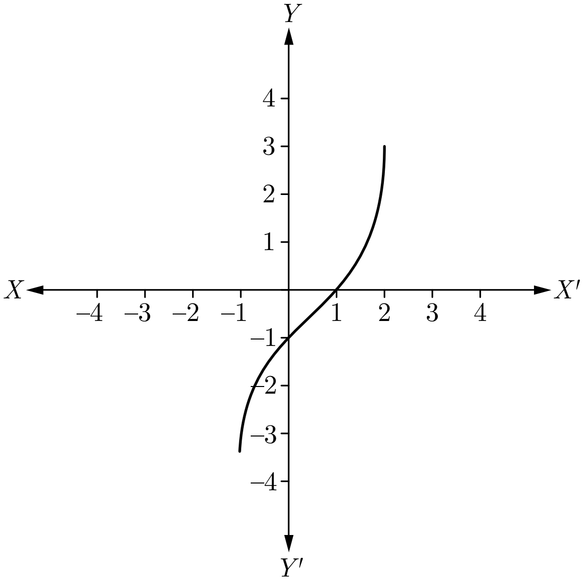
$$p(x) = 4x^2 - 12x + 9$$
$$= 4x^2 - 6x - 6x + 9$$
$$= 2x(2x-3) - 3(2x-3)$$
$$= (2x-3)(2x-3)$$

Substituting $p(x) = 0$, and solving we get $x = \frac{3}{2}, \frac{3}{2}$

$x = \frac{3}{2}, \frac{3}{2}$

Hence, zeroes of the polynomial are $\frac{3}{2}, \frac{3}{2}$.

42. In given figure, the graph of a polynomial $p(x)$ is shown. Calculate the number of zeroes of $p(x)$.



Ans : [Board Term-1 2013]

The graph intersects x-axis at one point $x = 1$. Thus the number of zeroes of $p(x)$ is 1.

43. If sum of the zeroes of the quadratic polynomial $3x^2 - kx + 6$ is 3, then find the value of k.

Ans : [Board 2009]

We have $$p(x) = 3x^2 - kx - 6$$

$$\text{Sum of the zeroes} = 3 = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Thus $$3 = -\frac{(-k)}{3} \Rightarrow k = 9$$

44. If -1 is a zero of the polynomial $f(x) = x^2 - 7x - 8$, then calculate the other zero.

Ans :

We have $$f(x) = x^2 - 7x -$$

Let other zero be k, then we have

Sum of zeroes, $$-1 + k = -\left(\frac{-7}{1}\right) = 7$$

or $$k = 8$$

TWO MARKS QUESTIONS

45. If zeroes of the polynomial $x^2 + 4x + 2a$ are a and $\frac{2}{a}$, then find the value of a.

Ans : [Board Term-1 2016]

Product of (zeroes) roots,

$$\frac{c}{a} = \frac{2a}{1} = \alpha \times \frac{}{\alpha} = 2$$

or, $\quad 2a = 2$

Thus $\quad a = 1$

46. Find all the zeroes of $f(x) = x^2 - 2x$.

Ans : [Board Term-1 2013]

We have $\quad f(x) = x^2 - 2x$

$$= x(x-2)$$

Substituting $f(x) = 0$, and solving we get $x = 0, 2$
Hence, zeroes are 0 and 2.

47. Find the zeroes of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$.

Ans : [Board Term-1 2013]

We have $\quad p(x) = \sqrt{3}x^2 - 8x + 4\sqrt{3}$

$$= \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3}$$

$$= \sqrt{3}x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3})$$

$$= (\sqrt{3}x - 2)(x - 2\sqrt{3})$$

Substituting $p(x) = 0$, we have

$$(\sqrt{3}x - 2)(x - 2\sqrt{3}) \;\; p(x) = 0$$

Solving we get $x = \frac{2}{\sqrt{3}}, \; 2\sqrt{3}$

Hence, zeroes are $\frac{2}{\sqrt{3}}$ and $2\sqrt{3}$.

48. Find a quadratic polynomial, the sum and product of whose zeroes are 6 and 9 respectively. Hence find the zeroes.

Ans : [Board Term-1 2016]

Sum of zeroes, $\quad \alpha + \beta = 6$

Product of zeroes $\quad \alpha\beta = 9$

Now $\quad p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

Thus $\quad = x^2 - 6x + 9$

Thus quadratic polynomial is $x^2 - 6x + 9$.

Now $\quad p(x) = x^2 - 6x + 9$

$$= (x-3)(x-3)$$

Substituting $p(x) = 0$, we get $x = 3, 3$

Hence zeroes are 3, 3

49. Find the quadratic polynomial whose sum and product of the zeroes are $\frac{21}{8}$ and $\frac{5}{16}$ respectively.

Ans : [Board Term-1 2012, Set-35]

Sum of zeroes, $\quad \alpha + \beta = \frac{21}{8}$

Product of zeroes $\quad \alpha\beta = \frac{5}{16}$

Now $\quad p(x) \;\; x^2 - (\alpha + \beta)x + \alpha\beta$

$$= x^2 - \frac{21}{8}x + \frac{5}{16}$$

or $\quad p(x) = \frac{1}{16}(16x^2 - 42x + 5)$

50. Form a quadratic polynomial $p(x)$ with 3 and $-\frac{2}{5}$ as sum and product of its zeroes, respectively.

Ans : [Board Term-1 2012]

Sum of zeroes, $\alpha + \beta = 3$

Product of zeroes $\alpha\beta = -\frac{2}{5}$

Now $\quad p(x) \;\; x^2 - (\alpha + \beta)x + \alpha\beta$

$$= x^2 - 3x - \frac{2}{5}$$

$$= \frac{1}{5}(5x^2 - 15x - 2)$$

The required quadratic polynomial is $\frac{1}{5}(5x^2 - 15x - 2)$

51. If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

Ans : [Board Term-1 2012]

We have $\quad \frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{mn} = \frac{(m+n)^2 - 2mn}{mn} \quad (1)$

Sum of zeroes $\quad m + n = -\frac{11}{3}$

Product of zeroes $mn = \frac{-4}{3}$

Substituting in (1) we have

$$\frac{m}{n} + \frac{n}{m} = \frac{(m+n)^2 - 2mn}{mn}$$

$$= \frac{(-\frac{11}{3})^2 - \frac{-4}{3} \times 2}{\frac{-4}{3}}$$

$$= \frac{121 + 4 \times 3 \times 2}{-4 \times 3}$$

or $$\frac{m}{n} + \frac{n}{m} = \frac{-145}{12}$$

52. If p and q are the zeroes of polynomial $f(x) = 2x^2 - 7x + 3$, find the value of $p^2 + q^2$.

Ans : [Board Term-1 2012]

We have $f(x) = 2x^2 - 7x + 3$

Sum of zeroes $p + q = -\frac{b}{a} = -\left(\frac{-7}{2}\right) = \frac{7}{2}$

Product of zeroes $pq = \frac{c}{a} = \frac{3}{2}$

Since, $(p+q)^2 = p^2 + q^2 + 2pq$

so, $p^2 + q^2 = (p+q)^2 - 2pq$

$$= \left(\frac{7}{2}\right)^2 - 3 = \frac{49}{4} - \frac{3}{1} = \frac{37}{4}$$

Hence $p^2 + q^2 = \frac{37}{4}$.

53. Find the condition that zeroes of polynomial $p(x) = ax^2 + bx + c$ are reciprocal of each other.

Ans : [Board Term-1 2012]

We have $p(x) = ax^2 + bx + c$

Let α and $\frac{1}{\alpha}$ be the zeroes of $p(x)$, then

Product of zeroes,

$$\frac{c}{a} = \alpha \times \frac{1}{\alpha} = 1 \text{ or } \frac{c}{a} = 1$$

So, required condition is, $c = a$

54. Find the value of k if -1 is a zero of the polynomial $p(x) = kx^2 - 4x + k$.

Ans : [Board Term-1 2012]

We have $p(x) = kx^2 - 4x + k$

Since, -1 is a zero of the polynomial, then

$$p(-1) = 0$$

$$k(-1)^2 - 4(-1) + k = 0$$

$$k + 4 + k = 0$$

$$2k + 4 = 0$$

$$2k = -4$$

Hence, $k = -2$

55. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}\,x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

Ans : [Board Term-1 2015]

We have $p(x) = x^2 - 4\sqrt{3}\,x + 3$

If α and β are the zeroes of $x^2 - 4\sqrt{3}\,x + 3$, then

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = -\frac{(-4\sqrt{3})}{1}$

or, $\alpha + \beta = 4\sqrt{3}$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{3}{1}$

or, $\alpha\beta = 3$

Now $\alpha + \beta - \alpha\beta = 4\sqrt{3} - 3.$

56. Find the values of a and b, if they are the zeroes of polynomial $x^2 + ax + b$.

Ans : [Board Term-1 2013]

We have $p(x) = x^2 + ax + b$

Since a and b, are the zeroes of polynomial, we get,

Product of zeroes, $ab = b \Rightarrow a = 1$

Sum of zeroes, $a + b = -a \Rightarrow b = -2a = -2$

57. If α and β are the zeroes of the polynomial $f(x) = x^2 - 6x + k$, find the value of k, such that $\alpha^2 + \beta^2 = 40$.

Ans : [Board Term-1 2015]

We have $f(x) = x^2 - 6x + k$

Sum of zeroes, $\alpha + \beta = -\frac{b}{a} = \frac{-(-6)}{1} = 6$

Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$

Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 40$

$$(6)^2 - 2k = 40$$

$$36 - 2k = 40$$

$$-2k = 4$$

Thus $k = -2$

58. If one of the zeroes of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of the other, find the value of 'k'.

Ans : [Board Term-1 2012]

We have $f(x) = 14x^2 - 42k^2x - 9$

Let one zero be α, then other zero will be $-\alpha$.

Sum of zeroes $\alpha + (-\alpha) = 0$.

Thus sum of zero will be 0.

Sum of zeroes $0 = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$0 = -\frac{42k^2}{14} = -3k^2$$

Thus $k = 0$.

59. If one zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and the other zero.

Ans : [Board Term-1 2012]

Let, the zero of $2x^2 + 3x + \lambda$ be $\dfrac{1}{2}$ and β.

Product of zeroes $\dfrac{c}{a}$, $\dfrac{1}{2}\beta = \dfrac{\lambda}{2}$

or, $\beta = \lambda$

and sum of zeroes $-\dfrac{b}{a}, \dfrac{1}{2} + \beta = -\dfrac{3}{2}$

or $\beta = -\dfrac{3}{2} - \dfrac{1}{2} = -2$

Hence $\lambda = \beta = -2$

Thus other zero is -2.

60. If α and β are zeroes of the polynomial $f(x) = x^2 - x - k$, such that $\alpha - \beta = 9$, find k.

Ans : [Board Term-1 2013, Set FFC]

We have $f(x) = x^2 - x - k$

Since α and β are the zeroes of the polynomial, then

Sum of zeroes, $\alpha + \beta = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$= -\left(\frac{-1}{1}\right) = 1$$

$$\alpha + \beta = 1 \qquad ...(1)$$

Given $\alpha - \beta = 9$ ...(2)

Solving (1) and (2) we get $\alpha = 5$ and $\beta = -4$

$$\alpha\beta = \frac{\text{Constan term}}{\text{Coefficient of } x^2}$$

or $\alpha\beta = -k$

Substituting $\alpha = 5$ and $\beta = -4$ we have

$$(5)(-4) = -k$$

Thus $k = 20$

61. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q.

Ans : [Board Term-1 2012, Set-39]

We have $f(x) = 2x^2 - 5x - 3$

Let the zeroes of polynomial be α and β, then

Sum of zeroes $\alpha + \beta = \dfrac{5}{2}$

Product of zeroes $\alpha\beta = -\dfrac{3}{2}$

According to the question, zeroes of $x^2 + px + q$ are 2α and 2β.

Sum of zeros, $2\alpha + 2\beta = \dfrac{-p}{1}$

$$2(\alpha + \beta) = -p$$

Substituting $\alpha + \beta = \frac{5}{2}$ we have

$$2 \times \frac{5}{2} = -p$$

or $p = -5$

Product of zeroes, $2\alpha 2\beta = \dfrac{q}{1}$

$$4\alpha\beta = q$$

Substituting $\alpha\beta = -\frac{3}{2}$ we have

$$4 \times \frac{-3}{2} = q$$

$$-6 = q$$

Thus $p = -5$ and $q = -6$.

62. If α and β are zeroes of $x^2 - (k-6)x + 2(2k-1)$, find the value of k if $\alpha + \beta = \dfrac{1}{2}\alpha\beta$.

Ans :

We have $p(x) = x^2 - (k-6)x + 2(2k-1)$

Since α, β are the zeroes of polynomial $p(x)$, we get

$$\alpha+\beta = -[-(k-6)] = k-6$$

$$\alpha\beta = 2(2k-1)$$

Now $\alpha+\beta = \frac{1}{2}\alpha\beta$

Thus $k+6 = \frac{2(2k-1)}{2}$

or, $k-6 = 2k-1$

$$k = -5$$

Hence the value of k is -5.

THREE MARKS QUESTIONS

63. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

Ans : [Board 2020 Delhi Standard]

Let α and β be zeros of the given polynomial $ax^2 + bx + c$.

$$\alpha+\beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

Let $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ be the zeros of new polynomial then we have

Sum of zeros, $s = \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta}$

$$= \frac{-\frac{b}{a}}{\frac{c}{a}} = \frac{-b}{c}$$

Product of zeros, $p = \frac{1}{\alpha}\cdot\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{a}{c}$

Required polynomial,

$$g(x) = x^2 - sx + p$$

$$g(x) = x^2 + \frac{b}{c}x + \frac{a}{c}$$

$$cg(x) = cx^2 + bx + a$$

$$g'(x) = cx^2 + bx + a$$

64. Verify whether 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$.

Ans : [Board Term-1 2013, LK-59]

If 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x) = 0$

(1) 2, $p(x) = 2x^2 - 11x^2 + 17x - 6$

$$p(2) = 2(2)^3 - 11(2)^2 + 17(2) - 6$$

$$= 16 - 44 + 34 - 6$$

$$= 50 - 50$$

or $p(2) = 0$

(2) 3, $p(3) = 2(3)^3 - 11(3)^2 + 17(3) - 6$

$$= 54 - 99 + 51 - 6$$

$$= 105 - 105$$

or $p(3) = 0$

(3) $\frac{1}{2}$ $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6$

$$= \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6$$

or $p\left(\frac{1}{2}\right) = 0$

Hence, 2, 3, and $\frac{1}{2}$ are the zeroes of $p(x)$.

65. If the sum and product of the zeroes of the polynomial $ax^2 - 5x + c$ are equal to 10 each, find the value of 'a' and 'c'.

Ans : [Board Term-1 2011, Set-25]

We have $f(x) = ax^2 - 5x + c$

Let the zeroes of $f(x)$ be α and β, then,

Sum of zeroes $\alpha+\beta = -\frac{-5}{a} = \frac{5}{a}$

Product of zeroes $\alpha\beta = \frac{c}{a}$

According to question, the sum and product of the zeroes of the polynomial $f(x)$ are equal to 10 each.

Thus $\frac{5}{a} = 10$ ...(1)

and $\frac{c}{a} = 10$ ...(2)

Dividing (2) by eq. (1) we have

$$\frac{c}{5} = 1 \Rightarrow c = 5$$

Substituting $c = 5$ in (2) we get $a = \frac{1}{2}$

Hence $a = \frac{1}{2}$ and $c = 5$.

66. If one the zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

Ans : [Board Term-1 2011, Set-40]

We have $f(x) = 3x^2 - 8x + 2k + 1$

Let α and β be the zeroes of the polynomial, then

$$\beta = 7\alpha$$

Sum of zeroes, $\alpha + \beta = -\left(-\frac{8}{3}\right)$

$$\alpha + 7\alpha = 8\alpha = \frac{8}{3}$$

So $\alpha = \frac{1}{3}$

Product of zeroes, $\alpha \times 7\alpha = \frac{2k+1}{3}$

$$7\alpha^2 = \frac{2k+1}{3}$$

$$7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$$

$$7 \times \frac{1}{9} = \frac{2k+1}{1}$$

$$\frac{7}{3} - 1 = 2k$$

$$\frac{4}{3} = 2k \Rightarrow k = \frac{2}{3}$$

67. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

Ans : [Board Term-2 2015]

We have $f(x) = 2x^2 - 3x + 1$

If α and β are the zeroes of $2x^2 - 3x + 1$, then

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = \frac{3}{2}$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{1}{2}$

New quadratic polynomial whose zeroes are 3α and 3β is,

$$p(x) = x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$$

$$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$$

$$= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$$

$$= x^2 - \frac{9}{2}x + \frac{9}{2}$$

$$= \frac{1}{2}(2x^2 - 9x + 9)$$

Hence, required quadratic polynomial is $\frac{1}{2}(2x^2 - 9x + 9)$

68. If α and β are the zeroes of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Ans : [Board Term-1 2011]

We have $p(y) = 6y^2 - 7y + 2$

Sum of zeroes $\alpha + \beta = -\left(-\frac{7}{6}\right) = \frac{7}{6}$

Product of zeroes $\alpha\beta = \frac{2}{6} = \frac{1}{3}$

Sum of zeroes of new polynomial $g(y)$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{7/6}{2/6} = \frac{7}{2}$$

and product of zeroes of new polynomial $g(y)$,

$$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{1/3} = 3$$

The required polynomial is

$$g(x) = y^2 - \frac{7}{2}y + 3$$

$$= \frac{1}{2}[2y^2 - 7y + 6]$$

69. Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify relationship between zeroes and coefficients of the polynomial.

Ans : [Board Term-1 2011]

We have $p(x) = 4x^2 + 4x - 3$

If $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $p(x)$, then these must satisfy $p(x) = 0$

$$p\left(\frac{1}{2}\right) = 4\left(\frac{1}{4}\right) + 4\left(\frac{1}{2}\right) - 3$$

$$= 1 + 2 - 3 = 0$$

and $p\left(-\frac{3}{2}\right) = 4\left(\frac{9}{2}\right) + 4\left(-\frac{3}{2}\right) - 3$

$$= 9 - 6 - 3 = 0$$

Thus $\frac{1}{2}, -\frac{3}{2}$ are zeroes of polynomial $4x^2 + 4x - 3$.

Sum of zeroes $= \frac{1}{2} - \frac{3}{2} = -1 = \frac{-4}{4}$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \left(\frac{1}{2}\right)\left(-\frac{3}{2}\right) = \frac{-3}{4}$$

$$= \frac{\text{Constan term}}{\text{Coefficient of } x^2} \qquad \text{Verified}$$

70. A teacher asked 10 of his students to write a polynomial in one variable on a paper and then to handover the paper. The following were the answers given by the students :

$2x+3$, $3x^2+7x+2$, $4x^3+3x^2+2$, $x^3+\sqrt{3x}+7$, $7x+\sqrt{7}$, $5x^3-7x+2$, $2x^2+3-\frac{5}{x}$, $5x-\frac{1}{2}$, ax^3+bx^2+cx+d, $x+\frac{1}{x}$.

Answer the following question :

(i) How many of the above ten, are not polynomials?

(ii) How many of the above ten, are quadratic polynomials?

Ans : [Board 2020 OD Standard]

(i) $x^3+\sqrt{3x}+7, 2x^2+3-\dfrac{5}{x}$ and $x+\dfrac{1}{x}$ are not polynomials.

(ii) $3x^2+7x+2$ is only one quadratic polynomial.

71. Find the zeroes of the quadratic polynomial $x^2-2\sqrt{2}\,x$ and verify the relationship between the zeroes and the coefficients.

Ans : [Board Term-1 2015]

We have $\quad p(x)x^2-2\sqrt{2}\,x = 0$

$$x(x-2\sqrt{2}) = 0$$

Thus zeroes are 0 and $2\sqrt{2}$.

Sum of zeroes $\quad 2\sqrt{2} = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

and product of zeroes $\quad 0 = \dfrac{\text{Constan term}}{\text{Coefficient of } x^2}$

Hence verified

72. Find the zeroes of the quadratic polynomial $5x^2+8x-4$ and verify the relationship between the zeroes and the coefficients of the polynomial.

Ans : [Board Term-1 2013, Set LK-59]

We have $\quad p(x) = 5x^2+8x-4 = 0$

$$= 5x^2+10x-2x-4 = 0$$

$$= 5x(x+2)-2(x+2) = 0$$

$$= (x+2)(5x-2)$$

Substituting $p(x) = 0$ we get zeroes as -2 and $\frac{2}{5}$.

Verification :

$$\text{Sum of zeroes} = -2+\frac{2}{5} = \frac{-8}{5}$$

$$\text{Product of zeroes} = (-2)\times\left(\frac{2}{5}\right) = \frac{-4}{5}$$

Now from polynomial we have

Sum of zeroes $\quad -\dfrac{b}{a} = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \dfrac{-8}{5}$

Product of zeroes $\quad \dfrac{c}{a} = \dfrac{\text{Constan term}}{\text{Coefficient of } x^2} = -\dfrac{4}{5}$

Hence Verified.

73. If α and β are the zeroes of a quadratic polynomial such that $\alpha+\beta = 24$ and $\alpha-\beta = 8$. Find the quadratic polynomial having α and β as its zeroes.

Ans : [Board Term-1 2011, Set-44]

We have $\quad \alpha+\beta = 24 \quad$ ...(1)

$\qquad \alpha-\beta = 8 \quad$ ...(2)

Adding equations (1) and (2) we have

$$2\alpha = 32 \Rightarrow \alpha = 16$$

Subtracting (1) from (2) we have

$$2\beta = 16 \Rightarrow \beta = 8$$

Hence, the quadratic polynomial

$$p(x) = x^2-(\alpha+\beta)x+\alpha\beta$$

$$= x^2-(16+8)x+(16)(8)$$

$$= x^2-24x+128$$

74. If α,β and γ are zeroes of the polynomial $6x^3+3x^2-5x+1$, then find the value of $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$.

Ans :

We have $\quad p(x) = 6x^3+3x^2-5x+1$

Since α,β and γ are zeroes polynomial $p(x)$, we have

$$\alpha+\beta+\gamma = -\frac{b}{c} = -\frac{3}{6} = -\frac{1}{2}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a} = -\frac{5}{6}$$

and $$\alpha\beta\gamma = -\frac{d}{a} = -\frac{1}{6}$$

Now $$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}$$

$$= \frac{-5/6}{-1/6} = \frac{-5}{6}\times\frac{6}{-1} = 5$$

Hence $\alpha^{-1}+\beta^{-1}+\gamma^{-1} = 5$.

75. When $p(x) = x^2+7x+9$ is divisible by $g(x)$, we get $(x+2)$ and -1 as the quotient and remainder respectively, find $g(x)$.

Ans : [Board Term-1 2011]

We have $$p(x) = x^2+7x+9$$

$$q(x) = x+2$$

$$r(x) = -1$$

Now $$p(x) = g(x)q(x)+r(x)$$

$$x^2+7x+9 = g(x)(x+2)-1$$

or, $$g(x) = \frac{x^2+7x+10}{x+2}$$

$$= \frac{(x+2)(x+5)}{(x+2)} = x+5$$

Thus $g(x) = x+5$

76. Find the value for k for which $x^4+10x^3+25x^2+15x+k$ is exactly divisible by $x+7$.

Ans : [Board Term 2010]

We have $$f(x) = x^4+10x^3+25x^2+15x+k$$

If $x+7$ is a factor then -7 is a zero of $f(x)$ and $x = -7$ satisfy $f(x) = 0$.

Thus substituting $x = -7$ in $f(x)$ and equating to zero we have,

$$(-7)^4+10(-7)^3+25(-7)^2+15(-7)+k = 0$$

$$2401-3430+1225-105+k = 0$$

$$3626-3535+k = 0$$

$$91+k = 0$$

$$k = -91$$

77. On dividing the polynomial $4x^4-5x^3-39\ ^2$ by the polynomial $g(x)$, the quotient is x^2-3x-5 and the remainder is $-5x+$. Find the polynomial $g(x)$.

Ans : [Board Term 2009]

Dividend = (Divisor × Quotient) + Remainder

$$4x^4-5x^3-39x^3-46x-2$$

$$= g(x)(x^2-3x-5)+(-5x+8)$$

$$4x^2-5x^3-39x^2-46x-2+5x-8$$

$$= g(x)(x^2-3x-5)$$

$$4x^4-5x^3-39x^2-41x-10 = g(x)(x^2-3x-5)$$

$$g(x) = \frac{4x^4-5x^3-39x^2-41x-10}{(x^2-3x-5)}$$

Hence, $$g(x) = 4x^2+7x+2$$

78. If the squared difference of the zeroes of the quadratic polynomial $f(x) = x^2+px+45$ is equal to 144, find the value of p.

Ans : [Board 2008]

We have $$f(x) = x^2+px+45$$

Let α and β be the zeroes of the given quadratic polynomial.

Sum of zeroes, $$\alpha+\beta = -p$$

Product of zeroes $$\alpha\beta = 45$$

Given, $$(\alpha-\beta)^2 = 144$$

$$(\alpha+\beta)^2-4\alpha\beta = 144$$

Substituting value of $\alpha+\beta$ and $\alpha\beta$ we get

$$(-p)^2-4\times45 = 144$$

$$p^2-180 = 144$$

$$p^2 = 144+180 = 324$$

Thus $$p = \pm\sqrt{324} = \pm18$$

Hence, the value of p is ±18.

FOUR MARKS QUESTIONS

79. Polynomial $x^4+7x^3+7x^2+px+q$ is exactly divisible by $x^2+7x+12$, then find the value of p and q.

Ans : [Board Term-1 2015]

We have $$f(x) = x^4+7x^3+7x^2+px+q$$

Now $$x^2+7x+12 = 0$$

$$x^2+4x+3x+12 = 0$$
$$x(x+4)+3(x+4) = 0$$
$$(x+4)(x+3) = 0$$
$$x = -4, -3$$

Since $f(x) = x^4+7x^3+7x^2+px+q$ is exactly divisible by $x^2+7x+12$, then $x=-4$ and $x=-3$ must be its zeroes and these must satisfy $f(x) = 0$

So putting $x=-4$ and $x=-3$ in $f(x)$ and equating to zero we get

$$f(-4): \ (-4)^4+7(-4)^3+7(-4)^2+p(-4)+q = 0$$
$$256-448+112-4p+q = 0$$
$$-4p+q-80 = 0$$
$$4p-q = -80 \qquad ...(1)$$
$$f(-3): \ (-3)^4+7(-3)^3+7(-3)^2+p(-3)+q = 0$$
$$81-189+63-3p+q = 0$$
$$-3p+q-45 = 0$$
$$3p-q = -45 \qquad ...(2)$$

Subtracting equation (2) from (1) we have

$$p = -35$$

Substituting the value of p in equation (1) we have

$$4(-35)-q = -80$$
$$-140-q = -80$$
$$-q = 140-80$$

or
$$-q = 60$$
$$q = -60$$

Hence, $p=-35$ and $q=-60$.

80. If α and β are the zeroes of the polynomial $p(x) = 2x^2+5x+k$ satisfying the relation, $\alpha^2+\beta^2+\alpha\beta = \frac{21}{4}$, then find the value of k.

Ans : [Board Term-1 2012]

We have $\quad p(x) = 2x^2+5x+k$

Sum of zeroes, $\quad \alpha+\beta = -\frac{b}{a} = -\left(\frac{5}{2}\right)$

Product of zeroes $\quad \alpha\beta = \frac{c}{a} = \frac{k}{2}$

According to the question,

$$\alpha^2+\beta^2+\alpha\beta = \frac{21}{4}$$
$$\alpha^2+\beta^2+2\alpha\beta-\alpha\beta = \frac{21}{4}$$
$$(\alpha+b)^2-\alpha\beta = \frac{21}{4}$$

Substituting values we have

$$\left(\frac{-5}{2}\right)^2-\frac{k}{2} = \frac{21}{4}$$
$$\frac{k}{2} = \frac{25}{4}-\frac{21}{4}$$
$$\frac{k}{2} = \frac{4}{4} = 1$$

Hence, $k = 2$

81. If α and β are the zeroes of polynomial $p(x) = 3x^2+2x+1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

Ans : [Board Term-1 2010, 2012]

We have $\quad p(x) = 3x^2+2x+1$

Since α and β are the zeroes of polynomial $3x^2+2x+1$, we have

$$\alpha+\beta = -\frac{2}{3}$$

and
$$\alpha\beta = \frac{1}{3}$$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\alpha_1+\beta_1 = \frac{1-\alpha}{1+\alpha}+\frac{1-\beta}{1+\beta}$$
$$= \frac{(1-\alpha+\beta-\alpha\beta)+(1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)}$$
$$= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$
$$= \frac{\frac{4}{3}}{\frac{2}{3}} = 2$$

For $q(x)$, product of the zeroes,

$$\alpha_1\beta_1 = \left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right]$$
$$= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$
$$= \frac{1-\alpha-\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta}$$

$$= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$$= \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{2}{3}} = 3$$

Hence, Required polynomial

$$q(x) = x^2 - (\alpha_1+\beta_1)2x + \alpha_1\beta_1$$

$$= x^2 - 2x + 3$$

82. If α and β are the zeroes of the polynomial x^2+4x+3, find the polynomial whose zeroes are $1+\frac{\beta}{\alpha}$ and $1+\frac{\alpha}{\beta}$.

Ans : [Board Term-1 2013]

We have $p(x) = x^2+4x+3$

Since α and β are the zeroes of the quadratic polynomial x^2+4x+3,

So, $\alpha+\beta = -4$

and $\alpha\beta = 3$

Let α_1 and β_1 be zeros of new polynomial $q(x)$.

Then for $q(x)$, sum of the zeroes,

$$\alpha_1+\beta_1 = 1+\frac{\beta}{\alpha}+1+\frac{\alpha}{\beta}$$

$$= \frac{\alpha\beta+\beta^2+\alpha\beta+\alpha^2}{\alpha\beta}$$

$$= \frac{\alpha^2+\beta^2+2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(-4)}{3} = \frac{16}{3}$$

For $q(x)$, product of the zeroes,

$$\alpha_1\beta_1 = \left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right)$$

$$= \left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\beta+\alpha}{\beta}\right)$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta}$$

$$= \frac{(-4)^2}{3} = \frac{16}{3}$$

Hence, required polynomial

$$q(x) = x^2 - (\alpha_1+\beta_1)x + \alpha_1\beta_1$$

$$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3}$$

$$= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$$

$$= \frac{1}{3}(3x^2 - 16x + 16)$$

83. If α and β are zeroes of the polynomial $p(x) = 6x^2 - 5x + k$ such that $\alpha - \beta = \frac{1}{6}$, Find the value of k.

Ans : [Board 2007]

We have $p(x) = 6x^2 - 5x + k$

Since α and β are zeroes of

$$p(x) = 6x^2 - 5x + k,$$

Sum of zeroes, $\alpha+\beta = -\left(\frac{-5}{6}\right) = \frac{5}{6}$ ...(1)

Product of zeroes $\alpha\beta = \frac{k}{6}$ ...(2)

Given $\alpha-\beta = \frac{1}{6}$ ...(3)

Solving (1) and (3) we get $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ and substituting the values of (2) we have

$$\alpha\beta = \frac{k}{6} = \frac{1}{2}\times\frac{1}{3}$$

Hence, $k = 1$.

84. If β and $\frac{1}{\beta}$ are zeroes of the polynomial $(a^2+a)x^2+61x+6a$. Find the value of β and α.

Ans :

We have $p(x) = (a^2+a)x^2+61x+6$

Since β and $\frac{1}{\beta}$ are the zeroes of polynomial, $p(x)$

Sum of zeroes, $\beta + \frac{1}{\beta} = -\frac{61}{a^2 + a}$

or, $\frac{\beta^2 + 1}{\beta} = \frac{-61}{a^2 + a}$ ...(1)

Product of zeroes $\beta\frac{1}{\beta} = \frac{6a}{a^2 + a}$

or, $1 = \frac{6}{a+1}$

$$a + 1 = 6$$

$$a = 5$$

Substituting this value of a in (1) we get

$$\frac{\beta^2 + 1}{\beta} = \frac{-61}{5^2 + 5} = -\frac{61}{30}$$

$$30\beta^2 + 30 = -61\beta$$

$$30\beta^2 + 61\beta + 30 = 0$$

Now $\beta \; \frac{-61 \pm \sqrt{(-61)^2 - 4 \times 30 \times 30}}{2 \times 30}$

$$= \frac{-61 \pm \sqrt{3721 - 3600}}{60}$$

$$\frac{-61 \mp 11}{60}$$

Thus $\beta = \frac{-5}{6}$ or $\frac{-6}{5}$

Hence, $\alpha = 5, \beta = \frac{-5}{6}, \frac{-6}{5}$

85. If α and β are the zeroes the polynomial $2x^2 - 4x + 5$, find the values of

(i) $\alpha^2 + \beta^2$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

(iii) $(\alpha - \beta)^2$ (iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(v) $\alpha^2 + \beta^2$

Ans : [Board 2007]

We have $p(x) = 2x^2 - 4x + 5$

If α and β are then zeroes of $p(x) = 2x^2 - 4x + 5$, then

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-4)}{2} = 2$$

and $\alpha\beta = \frac{c}{a} = \frac{5}{2}$

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= 2^2 - 2 \times \frac{5}{2}$$

$$= 4 - 5 = -1$$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{2}{\frac{5}{2}} = \frac{4}{5}$

(iii) $(\alpha - \beta)^2 = (\alpha - \beta)^2 - 4\alpha\beta$

$$= 2^2 - \frac{4 \times 5}{2}$$

$$4 - 10 = -6$$

(iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{-1}{(\frac{5}{2})^2} = \frac{-4}{25}$

(v) $(\alpha^3 + \beta^3) = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$$= 2^3 - 3 \times \frac{5}{2} \times 2 = 8 - 15 = -7$$