CLASS 12^{TH} MATHS CHAPTER :3

MATRICES

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CHAPTER OVERVIEW

- INTRODUCTON
- Types of Matrices
- Equality of Matrices
- Operations on Matrices
- Transpose of a matrix
- Symmetric and Skew Symmetric Matrix
- Invertible Matrices

KYA HAI YEH "MATRIX?"

→ Matrix has 6 elements :

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

6×1

1×6

2×3

3×2

Ex 3. Construct a 3×2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i - 3j|$.

$$A = [a_{ij}]_{3\times 2}$$

$$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$\longrightarrow a_{11} = \frac{1}{2}|1-3| = 1$

$a_{12} = \frac{1}{2}|1-6| = \frac{5}{2}$

$a_{21} = \frac{1}{2}|2-3| = \frac{1}{2}$

$a_{22} = \frac{1}{2}|2-6| = 2$

$a_{31} = \frac{1}{2}|3-3| = 0$

$a_{32} = \frac{1}{2}|3-6| = \frac{3}{2}$

$$\therefore A = \begin{bmatrix} 1 & 5/2 \\ \frac{1}{2} & 2 \\ 0 & \frac{3}{2} \end{bmatrix}$$

$$A = [a_{ij}]_{\underline{\underline{2\times 3}}} \quad , \quad a_{ij} = i + j$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

$$a_{ij} = \frac{(i+j)^2}{2}$$

$$B = \begin{bmatrix} 2 & 9/2 & 8 \\ 9/2 & 8 & 25/2 \end{bmatrix}$$

EXERCISE 3.1

1. In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, write:

   (i) The order of the matrix, (ii) The number of elements,

   (iii) Write the elements a_{13}, a_{21}, a_{33}, a_{24}, a_{23}.

2. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

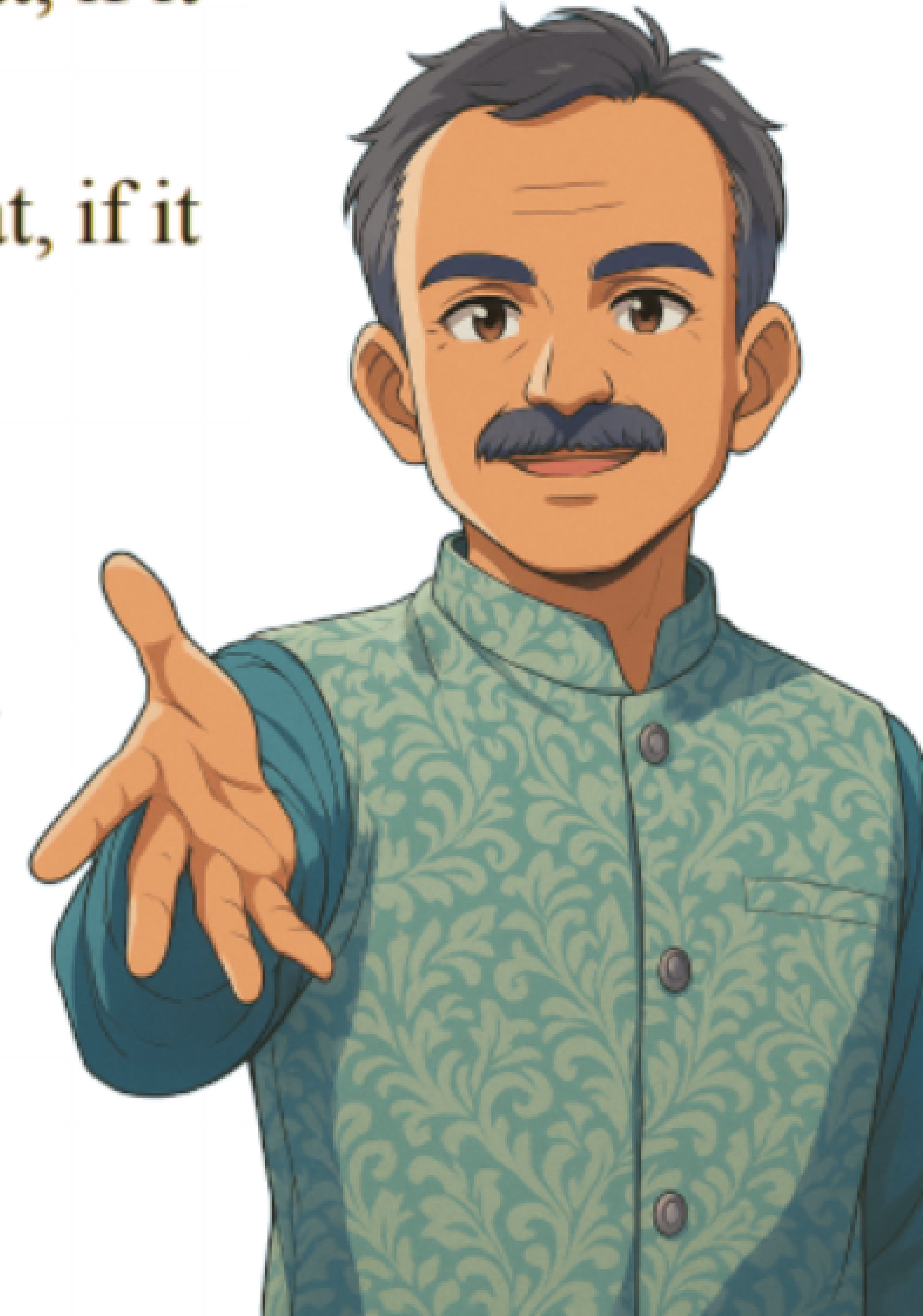
3. If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

4. Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by:

   (i) $a_{ij} = \dfrac{(i+j)^2}{2}$ (ii) $a_{ij} = \dfrac{i}{j}$ (iii) $a_{ij} = \dfrac{(i+2j)^2}{2}$

5. Construct a 3×4 matrix, whose elements are given by:

   (i) $a_{ij} = \dfrac{1}{2}|-3i + j|$ (ii) $a_{ij} = 2i - j$



6. Find the values of x, y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ (ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ (iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

7. Find the value of a, b, c and d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

8. $A = [a_{ij}]_{m \times n}$ is a square matrix, if

(A) $m < n$ (B) $m > n$ (C) $m = n$ (D) None of these

9. Which of the given values of x and y make the following pair of matrices equal

$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$, $\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

(A) $x = \frac{-1}{3}$, $y = 7$ (B) Not possible to find

(C) $y = 7$, $x = \frac{-2}{3}$ (D) $x = \frac{-1}{3}$, $y = \frac{-2}{3}$

10. The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

(A) 27 (B) 18 (C) 81 (D) 512

Q2. **If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?**

→ 24 elements

→ possible order

1×24 , 24×1

2×12 , 12×2

3×8 , 8×3

4×6 , 6×4

→ 13 elements

→ possible order

1×13 , 13×1

Q4. Construct a 2 × 2 matrix, A = [a_{ij}], whose elements are given by:

(iii) $a_{ij} = (i + 2j)^2 / 2$

$$A = [a_{ij}]_{2\times 2}$$

$$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$\rightarrow$ $a_{11} = (1+2)^2/2 = 9/2$ $\quad$ $a_{21} = (2+2)^2/2 = 8$

$a_{12} = (1+4)^2/2 = 25/2$ $\quad$ $a_{22} = (2+4)^2/2 = 18$

$$\therefore A = \begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$$

Q7. Find the value of a, b, c and d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 2c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

$\therefore \quad a - b = -1$

and $\quad 2a - b = 0$

$\quad - \quad + \quad -$

$-a = -1$

$\therefore \boxed{a = 1}$

$\rightarrow 2a - b = 0$

$2 - b = 0$

$\boxed{b = 2}$

$\rightarrow 2a + c = 5$

$2 + c = 5$

$\therefore \boxed{c = 3}$

$\rightarrow 2c + d = 13$

$6 + d = 13$

$\boxed{d = 7}$

Q10. The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

(A) 27 (B) 18 (C) 81 (D) 512

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{22} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

No. of matrices $= 2^9$

$= 512$

OPERATIONS ON MATRICES

* **Multiplication**

$A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{p \times q}$

Condition : i) $n = p$

ii) Resultent matrix

$AB = [(ab)_{ij}]$ $m \times q$

iii) $AB \neq BA$

A	B	AB
2×2	2×3	2×3
2×3	3×2	2×2
2×3	2×3	not possible

--> KUCH KAAM KI BAAT

✓ $\sin 2\theta = 2\sin\theta \cdot \cos\theta$

$= \dfrac{2\tan\theta}{1+\tan^2\theta}$

✓ $\sin\theta = 2\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}$

$= \dfrac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

✓ $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$= \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

✓ $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

✓ $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

✓ $1 - \cos 2\theta = 2\sin^2\theta$

$1 + \cos 2\theta = 2\cos^2\theta$

✓ $1 - \cos\theta = 2\sin^2\frac{\theta}{2}$

$1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

→ <u>Diagonal matrix</u>

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A = \operatorname{diag}\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

→ <u>Scalar matrix</u>

$$B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

→ <u>Identity matrix</u>

o) for 2×2 : $\quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

o) for 3×3 : $\quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

EXERCISE 3.2

1. Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

   Find each of the following:

   (i) $A + B$ (ii) $A - B$ (iii) $3A - C$

   (iv) AB (v) BA

2. Compute the following:

   (i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ (ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

   (iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$ (iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

3. Compute the indicated products.

   (i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ (ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

   (iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ (v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

   (vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

4. If $A=\begin{bmatrix}1 & 2 & -3\\ 5 & 0 & 2\\ 1 & -1 & 1\end{bmatrix}$, $B=\begin{bmatrix}3 & -1 & 2\\ 4 & 2 & 5\\ 2 & 0 & 3\end{bmatrix}$ and $C=\begin{bmatrix}4 & 1 & 2\\ 0 & 3 & 2\\ 1 & -2 & 3\end{bmatrix}$, then compute $(A+B)$ and $(B-C)$. Also, verify that $A+(B-C)=(A+B)-C$.

5. If $A=\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}$ and $B=\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$, then compute $3A-5B$.

6. Simplify $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

7. Find X and Y, if

   (i) $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

   (ii) $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ and $3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$

8. Find X, if $Y=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$ and $2X+Y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$

9. Find x and y, if $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$

10. Solve the equation for x, y, z and t, if $2\begin{bmatrix}x & z\\y & t\end{bmatrix}+3\begin{bmatrix}1 & -1\\0 & 2\end{bmatrix}=3\begin{bmatrix}3 & 5\\4 & 6\end{bmatrix}$

11. If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$, find the values of x and y.

12. Given $3\begin{bmatrix}x & y\\z & w\end{bmatrix}=\begin{bmatrix}x & 6\\-1 & 2w\end{bmatrix}+\begin{bmatrix}4 & x+y\\z+w & 3\end{bmatrix}$, find the values of x, y, z and w.

13. If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\\sin x & \cos x & 0\\0 & 0 & 1\end{bmatrix}$, show that $F(x)\,F(y) = F(x+y)$.

14. Show that

(i) $\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\neq\begin{bmatrix}2 & 1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & -1\\6 & 7\end{bmatrix}$

(ii) $\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\neq\begin{bmatrix}-1 & 1 & 0\\0 & -1 & 1\\2 & 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{bmatrix}$

15. Find $A^2 - 5A + 6I$, if $A=\begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}$

16. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$

17. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$

18. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

19. A trust fund has ₹ 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹ 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(a) ₹ 1800 (b) ₹ 2000

20. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹ 80, ₹ 60 and ₹ 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. Choose the correct answer in Exercises 21 and 22.

21. The restriction on n, k and p so that PY + WY will be defined are:

(A) $k = 3, p = n$ (B) k is arbitrary, $p = 2$

(C) p is arbitrary, $k = 3$ (D) $k = 2, p = 3$

22. If $n = p$, then the order of the matrix $7X - 5Z$ is:

(A) $p \times 2$ (B) $2 \times n$ (C) $n \times 3$ (D) $p \times n$

Q3. **Compute the indicated products.**

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$2 \times 2 \qquad 2 \times 2$

$$= \begin{bmatrix} a^2 + b^2 & -ab + ab \\ -ab + ab & b^2 + a^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

Q3. Compute the indicated products.

$$\underset{3\times \underline{1}}{\begin{bmatrix} \boxed{1} \\ \boxed{2} \\ \boxed{3} \end{bmatrix}} \underset{\underline{1}\times 3}{\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$$

Q3. **Compute the indicated products.**

$$\underset{2\times 2}{\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

Q3. Compute the indicated products.

$$\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}_{2\times 3} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} 6-1+9 & -9+3 \\ -2+6 & 3+2 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

Q7. Find X and Y, if

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \quad X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Q7. Find X and Y, if

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \quad 3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

Q13. **Show that F(x)F(y)=F(x+y), if** $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\text{LHS} = F(x)\cdot F(y)$$

$$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x\cos y - \sin x\sin y & -\cos x\sin y - \sin x\cos y & 0 \\ \sin x\cos y + \cos x\sin y & -\sin x\sin y + \cos x\cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 \end{bmatrix} = F(x+y) = \text{RHS}.$$

Q15. Find $A^2 - 5A + 6I$, if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

$\rightarrow A^2 = A \cdot A$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$\therefore A^2 - 5A + 6I$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

Q17. Find k so that $A^2 = kA - 2I$, if $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\rightarrow A^2 = A \cdot A$

$= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

$\therefore A^2 = kA - 2I$

$\therefore \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$

$\therefore 1 = 3k - 2$

$\therefore 3 = 3k$

$\therefore \boxed{k = 1}$

18. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

→ Let $\tan\alpha/2 = t$

→ LHS = $I + A$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} \quad \text{— (1)}$$

RHS : $(I - A) \cdot \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2} & -\frac{2\tan\alpha/2}{1+\tan^2\alpha/2} \\ \frac{2\tan\alpha/2}{1+\tan^2\alpha/2} & \frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \begin{bmatrix} \frac{1-t^2}{1+t^2} & \frac{-2t}{1+t^2} \\ \frac{2t}{1+t^2} & \frac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix} \begin{bmatrix} \dfrac{1-t^2}{1+t^2} & \dfrac{-2t}{1+t^2} \\[2ex] \dfrac{2t}{1+t^2} & \dfrac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1-t^2}{1+t^2} + \dfrac{2t^2}{1+t^2} & \dfrac{-2t}{1+t^2} + \dfrac{t(1-t^2)}{1+t^2} \\[2ex] \dfrac{-t(1-t^2)}{1+t^2} + \dfrac{2t}{1+t^2} & \dfrac{2t^2}{1+t^2} + \dfrac{1-t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1+t^2}{1+t^2} & \dfrac{-t(1+t^2)}{1+t^2} \\[2ex] \dfrac{t(1+t^2)}{1+t^2} & \dfrac{1+t^2}{1+t^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} \quad \text{——} \ (2)$$

from (1) and (2)

$$I + A = (I - A) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

✓

Q19. A trust fund has ₹ 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹ 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(A) ₹ 1800 (B) ₹ 2000

Let investment in bond : 1 is Rs. x

" " " : 2 is Rs. $(30000 - x)$

$$\therefore A : \begin{array}{cc} B1 & B2 \\ \end{array}$$
$$A = \begin{bmatrix} x & 30000 - x \end{bmatrix}$$

→ Bond : 1 pays 5% interest

" : 2 " 7% "

$$B : \begin{array}{c} I_1 \\ I_2 \end{array} \begin{bmatrix} 5/100 \\ 7/100 \end{bmatrix}$$

(A) Interest earned = Rs. 1800

$$A \cdot B = [1800]$$

$$\therefore \begin{bmatrix} x & 30000 - x \end{bmatrix} \begin{bmatrix} 5/100 \\ 7/100 \end{bmatrix} = [1800]$$

$$\therefore \frac{5x}{100} + \frac{210000 - 7x}{100} = 1800$$

$$\therefore 210000 - 2x = 180000$$

$$\therefore 2x = 30000$$

$\therefore x = 15000$

→ Investment in

bond : 1 Rs. 15000

bond : 2 Rs. 15000

MATRIX'S TRANSPOSE

eg. $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times 3}$

$A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}_{3\times 2}$

→ Properties:

1) $(A^T)^T = A$
2) $(A \pm B)^T = A^T \pm B^T$
3) $(AB)^T = B^T \cdot A^T$

→ Symmetric matrix : $A^T = A$

→ Skew-symmetric matrix : $A^T = -A$

→ $A + A^T$: symmetric matrix

→ $A - A^T$: skew-symmetric matrix

→ Que: Express matrix (A) as the sum of symmetric and skew-symmetric matrices.

Ans:

$$A = \underbrace{\frac{1}{2}(A + A^T)}_{\text{Symmetric}} + \underbrace{\frac{1}{2}(A - A^T)}_{\text{Skew-symmetric}}$$

Ex 22. Express this matrix as the sum of a symmetric and a skew symmetric matrix.

$$B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$\rightarrow \quad B^T = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$

$\rightarrow \quad C = B + B^T = \begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$

$\rightarrow \quad C^T = \begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$

$\rightarrow$ Here, $C^T = C \Rightarrow C$ is symmetric.

$\rightarrow \quad D = B - B^T = \begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$

$D^T = \begin{bmatrix} 0 & 1 & 5 \\ -1 & 0 & -6 \\ -5 & 6 & 0 \end{bmatrix} = -D$

Here, $D^T = -D \Rightarrow D$ is skew-symmetric.

$\rightarrow \quad \frac{1}{2} \cdot C + \frac{1}{2} \cdot D = \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix}$

$= \begin{bmatrix} \quad \end{bmatrix} = B$

EXERCISE 3.3

1. Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

2. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A + B)' = A' + B'$, (ii) $(A - B)' = A' - B'$

3. If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

(i) $(A + B)' = A' + B'$ (ii) $(A - B)' = A' - B'$

4. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$

5. For the matrices A and B, verify that $(AB)' = B'A'$, where

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

6. If (i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A'A = I$

(ii) If $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$, then verify that $A'A = I$

7. (i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

8. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix

(ii) $(A - A')$ is a skew symmetric matrix

9. Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$, when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

10. Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

(i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Choose the correct answer in the Exercises 11 and 12.

11. If A, B are symmetric matrices of same order, then AB – BA is a

(A) Skew symmetric matrix (B) Symmetric matrix

(C) Zero matrix (D) Identity matrix

12. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, and $A + A' = I$, then the value of α is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) π (D) $\frac{3\pi}{2}$

Q6. (i) Verify that A′ A = I if

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\rightarrow \quad A^T = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\rightarrow \quad A^T \cdot A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha \cdot \sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \cos\alpha\sin\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Q8. **Verify these for the matrix** $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

(i)(A + A′) is a symmetric matrix
(ii) (A – A′) is a skew symmetric matrix

$\rightarrow A^T = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

$\rightarrow B = A + A^T = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$B^T = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

Here $B^T = B \Rightarrow B$ is symmetric

$\Rightarrow (A + A^T)$ is symmetric.

$\rightarrow C = A - A^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$C^T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -C$

Here, $C^T = -C \Rightarrow C$ is skew-symmetric

$\Rightarrow (A - A^T)$ is skew-symmetric.

Q11. If A, B are symmetric matrices of same order, then AB – BA is a

(A) Skew symmetric matrix (B) Symmetric matrix

(C) Zero matrix (D) Identity matrix

$\rightarrow \quad A^T = A \quad \text{and} \quad B^T = B$

$$\rightarrow \quad (AB - BA)^T = (AB)^T - (BA)^T$$

$$= B^T A^T - A^T B^T$$

$$= BA - AB$$

$$= -(AB - BA)$$

$\therefore (AB - BA)$ is skew-symmetric.

Q12. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ **and A + A′ = I, then value of α is**

(A) π / 6 **(B) π / 3**

(C) π **(D) 3π / 2**

$$\rightarrow \quad A^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\rightarrow \quad A + A^T = I$$

$$\therefore \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2\cos\alpha = 1$$

$$\cos\alpha = \frac{1}{2}$$

$$\alpha = \pi/3$$

INVERTIBLE MATRICES

$\longrightarrow \quad A^{-1} = \frac{1}{|A|} \cdot \text{adj } A \quad , \quad |A| \neq 0$

$\longrightarrow \quad A \cdot A^{-1} = A^{-1} \cdot A = I$

$\longrightarrow \quad A^2 \cdot A^{-1} = A$

$A^3 \cdot A^{-1} = A^2$

$I \cdot A^{-1} = A^{-1} \cdot I = A^{-1}$

EXERCISE 3.4

1. Matrices A and B will be inverse of each other only if

   (A) $AB = BA$ (B) $AB = BA = 0$

   (C) $AB = 0, BA = I$ (D) $AB = BA = I$

YEH TUM LOG SOLVE KARNA!
HUM ABHI MISC. PART SOLVE KARTE HAI

Ex 24. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute, that is AB = BA

$\rightarrow \quad A^T = A \quad$ and $\quad B^T = B$

$\rightarrow \quad \underline{(AB)^T} = B^T A^T$

$= \underline{BA}$

$= \underline{AB}$

$\therefore$ AB is symmetric if and only if A and B commute.

Miscellaneous Exercise on Chapter 3

1. If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.
2. Show that the matrix B′AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.
3. Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation $A'A = I$.
4. For what values of x : $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$?
5. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$.
6. Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

7. A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market	**Products**		
I	10,000	2,000	18,000
II	6,000	20,000	8,000

(a) If unit sale prices of x, y and z are ₹ 2.50, ₹ 1.50 and ₹ 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are ₹ 2.00, ₹ 1.00 and 50 paise respectively. Find the gross profit.

8. Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Choose the correct answer in the following questions:

9. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

(A) $1 + \alpha^2 + \beta\gamma = 0$ (B) $1 - \alpha^2 + \beta\gamma = 0$

(C) $1 - \alpha^2 - \beta\gamma = 0$ (D) $1 + \alpha^2 - \beta\gamma = 0$

10. If the matrix A is both symmetric and skew symmetric, then

(A) A is a diagonal matrix (B) A is a zero matrix

(C) A is a square matrix (D) None of these

11. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(A) A (B) I – A (C) I (D) 3A

Q2. **Show that the matrix B′AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.**

→ If A is symmetric

$A' = A$

→ $(B'AB)' = (AB)'(B')'$

$= B'A'B$

$= B'AB$

$\therefore$ $B'AB$ is symmetric

→ If A is skew-symmetric

$\therefore A' = -A$

→ $(B'AB)' = (AB)'(B')'$

$= B'A'B$

$= -(B'AB)$

$\therefore$ $B'AB$ is skew-symmetric

Q3. **Find the values of x, y, z if the matrix** $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ **satisfy the equation A′A = I.**

$A' \cdot A = I$

$\therefore \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore 2x^2 = 1 \quad , \quad 6y^2 = 1 \quad , \quad 3z^2 = 1$

$x = \pm \frac{1}{\sqrt{2}} \qquad y = \pm \frac{1}{\sqrt{6}} \qquad z = \pm \frac{1}{\sqrt{3}}$

Q6. **Find x, if**

$$O = \underbrace{\begin{bmatrix} x & -5 & -1 \end{bmatrix}}_{1\times 3} \underbrace{\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}}_{3\times 3} \underbrace{\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}}_{3\times 1}$$

$$\therefore \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\therefore x^2 - \cancel{2x} - 40 + \cancel{2x} - 8 = 0$$

$$\therefore x^2 - 48 = 0$$

$$\therefore x^2 = 48$$

$$\therefore x = \pm 4\sqrt{3}$$

Q7. A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

MARKET	PRODUCT x	y	z
I	10000	2000	18000
II	6000	20000	8000

(a) If unit sale prices of x, y and z are ₹ 2.50, ₹ 1.50 and ₹ 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are ₹ 2.00, ₹ 1.00 and 50 paise respectively. Find the gross profit.

$$\rightarrow A = \begin{array}{c c} & \begin{array}{ccc} x & y & z \end{array} \\ \begin{array}{c} mI \\ mII \end{array} & \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c c} \begin{array}{c} x \\ y \\ z \end{array} & \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} \end{array}$$

(i) Total revenue

$C = AB$

$$\therefore C = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

$$\therefore C = \begin{bmatrix} 25000 + 3000 + 18000 \\ 15000 + 30000 + 8000 \end{bmatrix}$$

$$\therefore C = \begin{array}{c c} \begin{array}{c} mI \\ mII \end{array} & \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} \end{array}$$

$$D = \begin{array}{c} x \\ y \\ z \end{array} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

(ii) Total cost

$$E = A \cdot D$$

$$= \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

$$= \begin{bmatrix} 20000 + 2000 + 9000 \\ 12000 + 20000 + 4000 \end{bmatrix} = \begin{array}{c} m\mathrm{I} \\ m\mathrm{II} \end{array} \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$$

$\therefore$ Total profit

$$= C - E$$

$$= \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} - \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$$

$$= \begin{array}{c} m\mathrm{I} \\ m\mathrm{II} \end{array} \begin{bmatrix} 15000 \\ 17000 \end{bmatrix}$$

Q8. Find the matrix X so that $\underset{2\times 2}{X}\underset{2\times 3}{\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}} = \underset{2\times 3}{\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}}$

Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$\therefore \quad a + 4b = -7$

and $\quad 2a + 5b = -8$

$\therefore \quad \cancel{2a} + 8b = -14$

$\quad \underset{-}{\cancel{2a}} \underset{-}{+} 5b = \underset{+}{-} 8$

$\quad 3b = -6$

$\boxed{b = -2}$

$\Rightarrow a - 8 = -7$

$\boxed{a = 1}$

and $\quad c + 4d = 2$

$\quad 2c + 5d = 4$

$\therefore \quad \cancel{2c} + 8d = 4$

$\quad \underset{-}{\cancel{2c}} \underset{-}{+} 5d = \underset{-}{4}$

$\quad 3d = 0$

$\therefore \boxed{d = 0}$

$\Rightarrow c + 0 = 2$

$\boxed{c = 2}$

$$\therefore X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$$

Q9. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

(A) $1 + \alpha^2 + \beta\gamma = 0$
(B) $1 - \alpha^2 + \beta\gamma = 0$
✓(C) $1 - \alpha^2 - \beta\gamma = 0$
(D) $1 + \alpha^2 - \beta\gamma = 0$

$$A^2 = I$$

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\alpha^2 + \beta\gamma = 1$$

Q11. If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(A) A (B) I – A (C) I (D) 3A

$$(I + A)^3 - 7A$$

$$= I^3 + A^3 + 3I^2A + 3IA^2 - 7A$$

$$= I + A^2 \cdot A + 3IA + 3IA - 7A$$

$$= I + A \cdot A + 3A + 3A - 7A$$

$$= I + A + 3A + 3A - 7A$$

$$= I$$


IT'S PYQ
TIME!

Q1. If A and B are square matrices of same order such that AB = A and BA = B, then $A^2 + B^2$ is equal to ________________. (CBSE 2025)

a) A + B b) 2BA c) BA d) 2(A + B)

Q2. This matrix is ________________ matrix. $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -7 \\ 2 & 7 & 0 \end{bmatrix}$ (CBSE 2025)

Q3. Let both AB′ and B′A be defined for matrices A and B. If order of A is n × m, then the order of B is n×m (CBSE 2025)

Q4. Sum of two skew-symmetric matrices of same order is always a/an ________________. (CBSE 2025)

Q5. If $A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$ is a symmetric matrix, then (2x + y) is ________________. (CBSE 2025)

Q6. ________________ matrix can be both a symmetric and skew-symmetric matrix? (CBSE 2025)

Q7. Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify 4AB + 3(AB + BA) - 4BA, where A and B are both matrices of order 2 × 2. It is known that Aeq BeqI and A^{-1}eqB.

Their answers are given as :

- Abhay : 6AB
- Bina : 7AB - BA
- Chhaya : 8AB
- Devesh : 7BA-AB

Who answered it correctly ? (CBSE 2025)

Q8. What is the total number of possible matrices of order 3 × 3 with each entry as $\sqrt{2}$ or $\sqrt{3}$? (CBSE 2025)

Q9. The value of (x - y) if $\begin{bmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{bmatrix} = \begin{bmatrix} x+3 & 12 \\ 0 & 35 \end{bmatrix}$ (CBSE 2025)

Q10. Value of $a + \cancel{2b + 3c} + 4d$ will be ________ if matrix is $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is scalar matrix. (CBSE 2024)

Q11. Value of $I - A + A^2 - A^3 \ldots$ will be ________ if matrix is $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$ (CBSE 2024)

$A^2 = O$

Q12. If a matrix has 36 elements, the number of possible orders it can have, is ____________. (CBSE 2024)

Q13. If A is a square matrix of order 3 such that the value of $|adj \cdot A| = 8$, then the value of $|A^T|$ is ____________. (CBSE 2024)

Q14. If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is ______0______. (CBSE 2024)

$$\begin{matrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{matrix}$$

Q15. Write the number of all possible matrices of order 2 × 2 with each entry 1, 2 or 3. (CBSE AI 2016)

$A^3 - \cancel{I^3} - 3\cancel{A^2}I + 3AI^2$

$+ A^3 + \cancel{I^3} + 3\cancel{A^2}I + 3AI^2 - 7A$

$= 2A + 6A - 7A = A$

Q16. If A is a square matrix such that $A\overset{2}{\cancel{2}} = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$. (NCERT Exemplar, CBSE Delhi 2016)

Q17. If A is a square matrix such that $A\overset{2}{\cancel{2}} = A$, then write the value of $7A - (I + A)^3$, where lis an identity matrix. (CBSE AI 2014)

Q18. Show that $A\overset{3}{\cancel{3}} - 23A - 40I = O$ if A is $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ (CBSE AI 2014)

Q19. Show that all the diagonal elements of a skew symmetric matrix are zero. (CBSE Delhi 2017)

Q20. A trust fund, ₹ 35,000 is to be invested in two different types of bonds. The first bond pays 8% interest per annum which will be given to orphanage and second bond pays 10% interest per annum which will be given to an N.G.O. (Cancer Aid Society). Using matrix multiplication, determine how to divide ₹ 35,000 among two types of bonds if the trust fund obtains an annual total interest of ₹ 3,200. What are the values reflected in this question? (CBSE AI 2015C)

SUMMARY

$$A^2 + B^2$$

$$= (AB)^2 + (BA)^2$$

$$= ABAB + BABA$$

$$= ABB + BAA$$

$$= AB + BA$$

$$= \underline{A + B}$$

CHAPTER KHATAM!!

(KUCH ZYAADA HI JALDI KHATAM NAHI KIYA NA?)