Relations and Functions

(2025)

Q.1 For real x, let $f(x) = x^3 + 5x + 1$. Then :

(1 Mark) (CBSE 2025 - 65/4/1)

A. f is one-one but not onto on R

B. f is one-one and onto on R

C. f is neither one-one nor onto on R

D. f is onto on R but not one-one

Q.2

If $f : N \to W$ is defined as

$$f(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$$

then f is :

(1 Mark) (CBSE 2025 - 65/4/1)

A. a bijection

B. neither surjective nor injective

C. surjective only

D. injective only

Q.3 Assertion (A) : Let $f(x) = e^x$ and $g(x) = \log x$.

Then $(f + g)\,x = e^x + \log x$ $(f + g)x = e^x + \log x$ where domain of $(f + g)$ is R.

Reason (R) : $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.

(1 Mark) (CBSE 2025 - 65/6/1)

A. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

B. Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

C. Assertion (A) is false, but Reason (R) is true.

D. Assertion (A) is true, but Reason (R) is false.

Q.4 Domain of $f(x) = \cos^{-1}x + \sin x$ is :

(1 Mark) (CBSE 2025 - 65/7/1)

A. ϕ

B. $(-1,1)$

C. R

D. $[-1,1]$

Q.5 Assertion (A) : Let $A = \{x \in R: -1 \leq x \leq 1\}$. If $f : A \to A$ be defined as $f(x) = x^2$, then f is not an onto function.

Reason (R) : If $y = -1 \in A$, then $x = \pm \sqrt{-1} otin$ A.

(1 Mark) (CBSE 2025 - 65/5/1)

A. Assertion (A) is false, but Reason (R) is true.

B. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

C. Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

D. Assertion (A) is true, but Reason (R) is false.

Q.6 Assertion (A) : Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5, \forall Z$ is a bijective.

Reason (R) : A function is a bijective if it is both surjective and injective.

(1 Mark) (CBSE 2025 - 65/1/1)

A. Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

B. Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

C. Assertion (A) is true, but Reason (R) is false.

D. Assertion (A) is false, but Reason (R) is true.

Q.7

Find domain of $\sin^{-1}\sqrt{x-1}$.

(2 Mark) (CBSE 2025 - 65/4/1)

Q.8 Find the domain of $f(x) = \sin^{-1}(-x^2)$.

(2 Mark) (CBSE 2025 - 65/6/1)

Q.9 Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = R - \{3\}$ and $B = R - \{1\}$. Discuss the bijectivity of the function.

(2 Mark) (CBSE 2025 - 65/7/1)

Q.10 Find the domain of the function $f(x) = \cos^{-1}(x^2 - 4)$.

(2 Mark) (CBSE 2025 - 65/5/1)

Q.11

If $f : R^+ \to R$ is defined as $f(x) = \log_a x (a > 0$ and $a \neq 1)$, prove that f is a bijection. ($\mathrm{R}^+$is a set of all positive real numbers.)

(3 Mark) (CBSE 2025 - 65/4/1)

Q.12 Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$.

(3 Mark) (CBSE 2025 - 65/4/1)

(i) Write all elements of R.
(ii) Is R a function ? Justify.
(iii) Determine domain and range of R.

Q.13 A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

(3 Mark) (CBSE 2025 - 65/6/1)

Q.14

Show that the function
$f : N \rightarrow N$
, where N is a set of natural numbers, given by

$$f(n) = \begin{cases} n-1, & \text{if } n \text{ is even} \\ n+1, & \text{if } n \text{ is odd} \end{cases}$$

is a bijection.

(3 Mark) (CBSE 2025 - 65/6/1)

Q.15 Let R be a relation defined over N, where N is set of natural numbers, defined as " mRn if and only if mm is a multiple of n, m, n ∈ N." Find whether R is reflexive, symmetric and transitive or not.

(3 Mark) (CBSE 2025 - 65/2/1)

Q.16 Show that the function $f : R \rightarrow R$ defined by $f(x) = 4x^3 - 5$, $\forall x \in R$ is one-one and onto.

(3 Mark) (CBSE 2025 - 65/7/1)

Q.17 Let R be a relation defined on a set N of natural numbers such that R = {(x, y) : xy is a square of a natural number, x, y ∈ N}. Determine if the relation RR is an equivalence relation.

(3 Mark) (CBSE 2025 - 65/7/1)

Q.18 A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A = {1, 2, 3}:

(4 Mark) (CBSE 2025 - 65/1/1)

$$R_1 = \{(2,3),(3,2)\}$$
$$R_2 = \{(1,2),(1,3),(3,2)\}$$
$$R_3 = \{(1,2),(2,1),(1,1)\}$$
$$R_4 = \{(1,1),(1,2),(3,3),(2,2)\}$$
$$R_5 = \{(1,1),(1,2),(3,3),(2,2),(2,1),(2,3),(3,2)\}$$

The students are asked to answer the following questions about the above relations:

(i) Identify the relation which is reflexive, transitive but not symmetric.

(ii) Identify the relation which is reflexive and symmetric but not transitive.

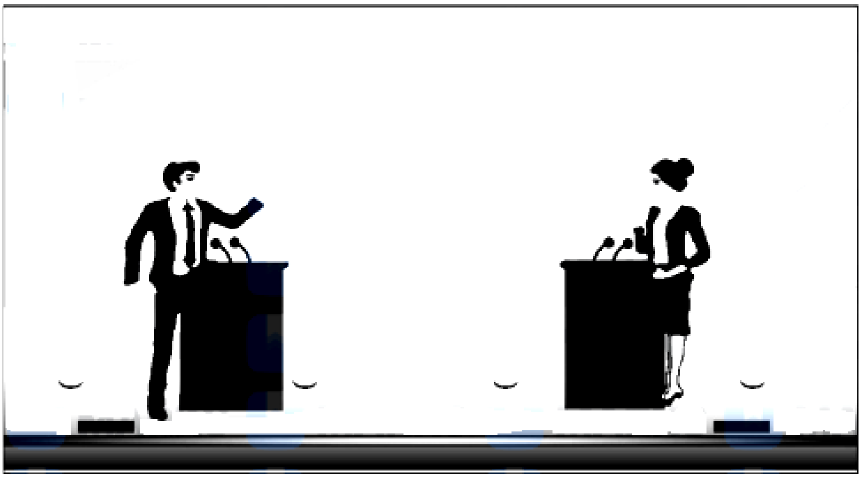
(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive.

(iii)(b) (iii) (b) What pairs should be added to the relation R_2
to make it an equivalence relation?

Q.19 A school is organizing a debate competition with participants as speakers S = {S_1, S_2, S_3, S_4} and these are judged by judges J = {J_1, J_2, J_3}. Each speaker can be assigned one judge. Let RR be a relation from set S to J defined as R = {(x, y) : speaker x is judged by judge

$\mathrm{y}, x \in \mathrm{S}, \mathrm{y} \in J\}$.

(4 Mark) (CBSE 2025 - 65/2/1)



Based on the above, answer the following :

(i) How many relations can be there from S to J ?

(ii) A student identifies a function from S to J as f = {(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)} Check if it is bijective.

(iii) (a) How many one-one functions can be there from set S to set J ?

(iii)(b) Another student considers a relation R1 = {(S_1, S_2), (S_2, S_4)} in set S. Write minimum ordered pairs to be included in R_1 so that R1R1 is reflexive but not symmetric.

Q.20 Let A be the set of 30 students of class XII in a school. Let f : A → N, N is a set of natural numbers such that function f(x)=f(x)= Roll Number of student x. On the basis of the given information, answer the following :

(4 Mark) (CBSE 2025 - 65/5/1)

(i) Is f a bijective function ?

(ii) Give reasons to support your answer to (i).

(iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where R = {(x, y) : x, y are Roll Numbers of students such that y = 3x}. List the elements of RR. Is the relation R reflexive, symmetric and transitive? Justify your answer.

(iii) (b) Let RR be a relation defined by $R = \{(x, y) : x, y$ are Roll Numbers of students such that $y = x^3\}$. List the elements of R. Is R a function ? Justify your answer.

Answer

Q.1 B Q.2 C Q.3 C Q.4 D Q.5 B Q.6 D

Q.7

Here $-1 \leq \sqrt{x-1} \leq 1$

$\Rightarrow 0 \leq x-1 \leq 1 \Rightarrow 1 \leq x \leq 2$

Hence, domain is
$x \in [1,2]$

Q.8

$-1 \leq -\mathrm{x}^2 \leq 1 \Rightarrow -1 \leq -\mathrm{x}^2 \leq 0$
$\Rightarrow 0 \leq \mathrm{x}^2 \leq 1 \Rightarrow -1 \leq \mathrm{x} \leq 1$

Q.9

Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \Rightarrow x_1 = x_2, \therefore$ ' f ' is one-one. For each $y \in B$, there exists $x = \frac{3y-2}{y-1} \in R - \{3\}$, such that $f(x) = y, \therefore$ ' f ' is onto $\Rightarrow$ ' f ' is one-one \& onto, or ' f ' is a bijective function.

Q.10

Domain of
$\cos^{-1} \mathrm{x}$
is
$[-1,1]$

$\Rightarrow -1 \leq x^2 - 4 \leq 1 \Rightarrow 3 \leq x^2 \leq 5$

$\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

Q.11

$f(x) = \log_a x \quad (a > 0, a \neq 1)$

Let
$x_1, x_2 \in R^+$
such that
$f(x_1) = f(x_2)$

$\Rightarrow \log_a x_1 = \log_a x_2$
$\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.

Let
$f(x) = y \Rightarrow \log_a x = y \Rightarrow a^y = x$

$\therefore$
for every
$y \in R$
, there exists
$x \in R^+$

$\therefore f$
is onto.

f is a bijection.

Q.12

(i) $R = \{(1,5),(2,4)\}$
(ii) R is not a function as $3 \in A$ do not have an image in co-domain.
(iii) Domain of $R = \{1,2\}$, Range of $R = \{4,5\}$

Q.13

$\mathrm{R} = \{(1,39),(2,37),\_\_\_,(20,1)\}$
Domain $= \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20\}$
Range $= \{1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39\}$
$(1,1)$ does not belong to R hence not reflexive
$(1,39)$ belongs to R but $(39,1)$ does not belong to R hence not symmetric
$(11,19)$ and $(19,3)$ belong to R but $(11,3)$ does not belong to R hence not transitive.
Hence R is not an equivalence relation.

Q.14

Let $f(x)=f(y)$

Let x and y are both odd or both even

Then either $\mathrm{x}+1=\mathrm{y}+1$ or $\mathrm{x}-1=\mathrm{y}-1$ gives

$x=y$

x odd and y even is rejected as

$\mathrm{x}+1=\mathrm{y}-1$ gives $\mathrm{x}-\mathrm{y}=-2$ not possible as odd number and even number cannot differ by 2

Hence f is one-one

For onto: Let $\mathrm{f}(\mathrm{x})=\mathrm{y}$ gives $\mathrm{x}=\mathrm{y}+1$ or $\mathrm{x}=\mathrm{y}-1$

If y is odd, x is even and if y is even, x is odd.

$\text{Range } =\mathrm{N}= \text{ co-domain, hence onto}$

As f is both one-one and onto hence bijective

Q.15

Let $x \in N$. Then we know that x is a multiple of itself.

$\Rightarrow xRx$

Hence, R is reflexive.

We have $2,8 \in N$ such that 8 is a multiple of 2

$\Rightarrow 8R2$

But, 2 is not a multiple of 8 . Hence, 2 is not R-related to 8 .

Therefore, R is not symmetric.

Let $x, y, z \in N$ such that xRy, yRz

Then $x=my, y=nz$ for some $\mathrm{m}, n \in N$

$\Rightarrow x=mnz \Rightarrow x=pz$, where $p=mn \in N$. Hence, xRz

Therefore, R is transitive.

Q.16

One-One: Let $x_1, x_2 \in R$ such that

$\mathbf{f}(\mathbf{x}_1)=\mathbf{f}(\mathbf{x}_2) \Rightarrow \mathbf{4x_1}^3-\mathbf{5}=\mathbf{4x_2}^3-\mathbf{5} \Rightarrow \mathbf{x_1}^3=\mathbf{x_2}^3 \Rightarrow \mathbf{x}_1=\mathbf{x}_2, \therefore$ ' $\mathbf{f}$ ' is one-one Onto:
$x \in R\left(D_f\right) \Rightarrow x^3 \in R \Rightarrow 4x^3-5 \in R \Rightarrow f(x) \in R, \therefore R_f=$ Co-domain (f)

$\therefore$ ' $\mathbf{f}$ ' is an onto function
$\Rightarrow$ ' f ' is one-one \& onto both

Q.17

Reflexive: For any
$\mathbf{x} \in \mathbf{N}, \mathbf{x} \cdot \mathbf{x} = \mathbf{x}^2$
, which is square of the natural number '
$\mathbf{x}$
'.
$\Rightarrow (\mathbf{x}, \mathbf{x}) \in \mathbf{R}$
$\therefore$
'
$\mathbf{R}$
' is a Reflexive relation.

Symmetric: Let
$(x, y) \in R \Rightarrow xy$
is a square of a natural number

$\Rightarrow \mathbf{yx}$
is a square of a natural number,
$\because \mathbf{xy} = \mathbf{yx}$
$\Rightarrow (\mathbf{y}, \mathbf{x}) \in \mathbf{R}$
$\therefore$
'
$\mathbf{R}$
' is a Symmetric relation.

Transitive: Let
$(x, y), (y, z) \in R \Rightarrow xy = a^2, yz = b^2$
for some
$a, b \in N$

$$\therefore \frac{a^2}{y} = x, \frac{b^2}{y} = z \in N$$

$$\Rightarrow xz = \frac{a^2}{y} \cdot \frac{b^2}{y} = \left(\frac{ab}{y}\right)^2, \frac{ab}{y} \in N$$

$$\Rightarrow (x, z) \in R$$

$\therefore$

$\mathbf{R}$

' is a Transitive relation.

Hence, **R** is an Equivalence relation

Q.18

(i) R_4

(ii) R_5

(iii)(a) R_1 and R_3

(iii) (b) Required pairs to be added to make the relation R_2 as an equivalence relation are:

$$(1,1),(2,2),(3,3),(2,1),(3,1) \text{ and } (2,3)$$

Q.19 The number of relations $= 2^{4 \times 3} = 2^{12}$

Since, S_2 and S_3 have been assigned the same judge J_2, the function is not one-one.

Hence, it is not bijective.

There cannot exist any one-one function from S to J as n(S) > n(J). Hence, the number of one-one functions from S to J is 0.

(iii)(b) To make R_1 reflexive and not symmetric we need to add the following ordered pairs:

$$v\left(S_1, S_1\right),\left(S_2, S_2\right),\left(S_3, S_3\right),\left(S_4, S_4\right)$$

Q.20

(i) No, f is not bijective function

(ii) Range $= \{1, 2, 3, 4, \ldots\ldots\ldots\ldots, 30\}$ and codomain $= N$

Since, Range eq codomain $\Rightarrow \mathbf{f}$ is not onto and hence $\mathbf{f}$ is not bijective.

(iii) (a)

$R = \{(1,3), (2,6), (3,9), (4,12), (5,15), (6,18), (7,21), (8,24), (9,27), (10,30)\}$

Since $(1,1) otin R \Longrightarrow R$ is not reflexive.

$(1,3) \in R$ but $(3,1) otin R \Rightarrow R$ is not symmetric

$(1,3) \in R, (3,9) \in R$ but $(1,9) otin R \Rightarrow R$ is not transitive.

(iii) (b)

$R = \{(1,1), (2,8), (3,27)\}$

$\because$ elements $4, 5, 6 \ldots 30$ do not have an image. Hence the above relation is not a function.

(2024)

Q.1 Let f : R + → [−5, ∞) be defined as $f(x) = 9x^2 + 6x - 5$, where R_+ is the set of all non-negative real numbers. Then, f is :

(1 Mark) (CBSE 2024 - 65/2/1)

A. bijective

B. neither one-one nor onto

C. onto

D. one-one

Q.2 Assertion (A) : The relation $R = \{(x, y) : (x + y)$ is a prime number and $x, y \in N\}$ is not a reflexive relation.

Reason (R) : The number '2n' is composite for all natural numbers n.

(1 Mark) (CBSE 2024 - 65/4/1)

A. Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

B. Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).

C. Assertion (A) is true, but Reason (R) is false.

D. Assertion (A) is false, but Reason (R) is true.

Q.3 A function $f : R_+ \rightarrow R$ (where R_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is :

(1 Mark) (CBSE 2024 - 65/1/1)

A. one-one but not onto

B. both one-one and onto

C. neither one-one nor onto

D. onto but not one-one

Q.4 Let R_+ denote the set of all non-negative real numbers. Then the function $f: R_+ \to R_+$ defined as $f(x) = x^2 + 1$ is :

(1 Mark) (CBSE 2024 - 65/3/1)

A. onto but not one-one

B. neither one-one nor onto

C. one-one but not onto

D. both one-one and onto

Q.5 A function $f: R \to R$ defined as $f(x) = x^2 - 4x + 5$ is :

(1 Mark) (CBSE 2024 - 65/5/1)

A. both injective and surjective.

B. injective but not surjective.

C. surjective but not injective.

D. neither injective nor surjective.

Q.6 Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

(2 Mark) (CBSE 2024 - 65/5/1)

Q.7 A relation R on set A = {1, 2, 3, 4, 5} is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check...

(3 Mark) (CBSE 2024 - 65/1/1)

Q.8 A function f is defined from $R \to R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.

(3 Mark) (CBSE 2024 - 65/1/1)

Q.9 Students of a school are taken to a railway museum to learn about railways heritage and its history.

(4 Mark) (CBSE 2024 - 65/5/1)

An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1$ is parallel to $l_2\}$

On the basis of the above information, answer the following questions :

(i) Find whether the relation R is symmetric or not.
(ii) Find whether the relation R is transitive or not.
(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.
(iii)(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1$ is perpendicular to $l_2\}$ check whether the relation S is symmetric and transitive.

Q.10

Show that a function $f : R \to R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.

(5 Mark) (CBSE 2024 - 65/2/1)

Q.11 A relation R is defined on N × N (where NN is the set of natural numbers) as :

(5 Mark) (CBSE 2024 - 65/2/1)

$(a, b) R (c, d) \Leftrightarrow a - c = b - d$

Show that R is an equivalence relation.

(5 Mark) (CBSE 2024 - 65/4/1)

Q.12 Let A = R – {5} and B = R–{1}. Consider the function f : A → B, defined by $f(x) = \frac{x-3}{x-5}$.

Show that f is one-one and onto.

Q.13 Check whether the relation S in the set of real numbers R defined by S = {(a, b): where a – b + √2 is an irrational number} is reflexive, symmetric or transitive.

(5 Mark) (CBSE 2024 - 65/4/1)

Q.14 A relation R on set A = {–4, –3, –2, –1, 0, 1, 2, 3, 4} be defined as R = {(x, y) : x + y is an integer divisible by 2}. Show that R is an equivalence relation. Also, write the equivalence class [2].

Answer

Q.1 A **Q.2 C** **Q.3 A** **Q.4 C** **Q.5 D**

Q.6

$-1 \leqslant x^2 - 4 \leqslant 1$

$\Rightarrow 3 \leqslant x^2 \leqslant 5$

Domain $= [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

Range

$= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Q.7

(a) Reflexive:

$\because \left|x^2 - x^2\right| < \mathbf{8} \forall x \in A \Rightarrow (x, x) \in R \quad \therefore R$ is reflexive.

(b) Symmetric:

Let $(\mathrm{x}, \mathrm{y}) \in R$ for some $\mathrm{x}, \mathrm{y} \in \mathrm{A}$

$\therefore \left|x^2 - y^2\right| < 8 \Rightarrow \left|y^2 - x^2\right| < 8 \Rightarrow (y, x) \in R$

Hence R is symmetric.

(c) Transitive:

$(1,2),(2,3) \in R$ as $\left|1^2 - 2^2\right| < 8, \left|2^2 - 3^2\right| < 8$ respectively

But $\left|1^2 - 3^2\right| less 8 \Rightarrow (1,3) otin R$

Hence R is not transitive.

Q.8

$f(x) = ax + b$

Solving $\mathrm{a} + \mathrm{b} = 1$ and $2\mathrm{a} + \mathrm{b} = 3$ to get $\mathrm{a} = 2,\ \mathrm{b} = -1$

$f(x) = 2x - 1$

Let f $(x_1) =$ f (x_2) for some $x_1, x_2 \in R$

$2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2$

Hence f is one - one.

Let $\mathrm{y} = 2\mathrm{x} - 1, \mathrm{y} \in R$ (Codomain)

$\Rightarrow \mathrm{x} = \frac{y+1}{2} \in R$ (domain)

Also, $\mathrm{f(x)} = \mathrm{f}\left(\frac{y+1}{2}\right) = \mathrm{y}$

$\therefore$ f is onto.

Q.9

(i) Let $(l_1, l_2) \in R \Rightarrow l_1 \| l_2 \Rightarrow l_2 \| l_1 \Rightarrow (l_2, l_1) \in R, \therefore R$ is a symmetric relation

(ii) Let $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \| l_2, l_2 \| l_3 \Rightarrow l_1 \| l_3 \Rightarrow (l_1, l_3) \in R, \therefore R$ is a transitive relation

(iii) The set is $\{1 : 1$ is a line of type $y = 3x + c, c \in R\}$

(iii)(b)

Let $(1_1, 1_2) \in R \Rightarrow 1_1 \perp 1_2 \Rightarrow 1_2 \perp 1_1 \Rightarrow (1_2, 1_1) \in R, \therefore R$ is a symmetric relation Let $(1_1, 1_2), (1_2, 1_3) \in R \Rightarrow 1_1 \perp 1_2, 1_2 \perp 1_3 \Rightarrow 1_1 \| 1_3 \Rightarrow (1_1, 1_3)\, otin R, \therefore R$ is not a transitive relation

Q.10

Let

$f(x_1) = f(x_2)$

for some

$x_1, x_2 \in R$

Then

$\frac{2x_1}{1+x_1{}^2} = \frac{2x_2}{1+x_2{}^2}$

$\Rightarrow x_1 + x_1 x_2^2 = x_2 + x_1^2 x_2$

$\Rightarrow (x_1 - x_2) - x_1 x_2 (x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)(1 - x_1 x_2) = 0$

$\Rightarrow x_1 - x_2 = 0$ or $1 - x_1 x_2 = 0$

$\Rightarrow x_1 = x_2$ or $x_1 x_2 = 1,$ so if $x_1 x_2 = 1, x_1 \neq x_2$

Hence f is not one -one

Let y = f(x) where x ∈ R

If
$y \neq 0$
, then
$y = \frac{2x}{1+x^2}$

$$\Rightarrow yx^2 - 2x + y = 0$$

$$\Rightarrow x = \frac{2 \pm \sqrt{4 - 4y^2}}{2y}$$

For x to be real,
$4 - 4y^2 \geq 0$

$$\Rightarrow y^2 \leq 1$$
$$\Rightarrow -1 \leq y \leq 1$$

Hence, range
$= [-1, 1] \neq$
codomain Hence, f is not onto.

For the given function to become onto, A = [-1, 1]

Q.11

Let
$(a, b) \in N \times N$

We have

This implies that
$(\mathrm{a}, \mathrm{b})\mathrm{R}(\mathrm{a}, \mathrm{b}) \forall (a, b) \in N \times N$

Hence R is reflexive

Let (a, b) R (c, d) for some
$(a, b), (c, d) \in N \times N$

Then
$a - c = b - d$

$\Rightarrow c - a = d - b$
$\Rightarrow (\mathrm{c},\ \mathrm{d}) \mathrm{R} (\mathrm{a},\ \mathrm{b})$

Hence, **R** is symmetric.

Let
$(\mathrm{a}, \mathrm{b}) \mathrm{R} (\mathrm{c}, \mathrm{d}), (\mathrm{c}, \mathrm{d}) \mathrm{R} (\mathrm{e}, \mathrm{f})$
for some
$(a, b), (c, d), (e, f) \in N \times N$

Then
$a - c = b - d, c - e = d - f$

$\Rightarrow a - c + c - e = b - d + d - f$
$\Rightarrow a - e = b - f$
$\Rightarrow (\mathrm{a},\ \mathrm{b}) \mathrm{R} (\mathrm{e}, \mathrm{f})$

Hence, R is transitive

Thus, **R** is an equivalence relation.

Q.12

Let $f(x_1) = f(x_2)$, for some $x_1, x_2 \in A$
$\Rightarrow \frac{x_1 - 3}{x_1 - 5} = \frac{x_2 - 3}{x_2 - 5}$
$\Rightarrow (x_1 - 3)(x_2 - 5) = (x_2 - 3)(x_1 - 5)$
$\Rightarrow x_1 = x_2$, So f is one-one Function.
Let $y = f(x) = \frac{x-3}{x-5} \Rightarrow y(x - 5) = x - 3$
$\Rightarrow yx - 5y = x - 3$
$\Rightarrow x = \frac{5y-3}{y-1}$, We observe that x is defined for all values of y except $y = 1$,
So, Range $= R - \{1\}$ and Co-domain is Given $R - \{1\}$ $\quad$ [As, $f : A \rightarrow B$]
Since, Range $=$ Co-domain, f is onto Function.
Thus, f is one-one \& onto function.

Q.13

$$\left.\begin{array}{l}\text{Reflexive: For } a \in S \\ \Rightarrow a-a+\sqrt{2} \text{ is irrational number} \\ \Rightarrow \sqrt{2} \text{ is irrational number} \\ \Rightarrow (a, a) \in S \\ \text{Thus, S is Reflexive Relation.}\end{array}\right\}$$

Symmetric: Let

$(a, b) \in S \Rightarrow a-b+\sqrt{2}$

is irrational number but

$b-a+\sqrt{2}$

may not be irrational number

For example,

$(\sqrt{2}, 1) \in S \Rightarrow \sqrt{2}-1+\sqrt{2}=2\sqrt{2}-1$

is irrational number

$(1, \sqrt{2}) \notin S$

as

$1-\sqrt{2}+\sqrt{2}=1$

is not irrational number

$\therefore (b, a) \notin S$

, So S is NOT Symmetric Relation.

Transitive : Let

$(a, b) \in S \Rightarrow a-b+\sqrt{2}$

is irrational number

$\&(b, c) \in S \Rightarrow b-c+\sqrt{2}$

is irrational number but

$a-c+\sqrt{2}$

may not be irrational number

For example,

$(1, \sqrt{3}) \in S \Rightarrow 1 - \sqrt{3} + \sqrt{2}$

is irrational number

$(\sqrt{3}, \sqrt{2}) \in S \Rightarrow \sqrt{3} - \sqrt{2} + \sqrt{2} = \sqrt{3}$

is irrational number But

$(1, \sqrt{2}) \notin S$

as

$1 - \sqrt{2} + \sqrt{2} = 1$

is not irrational number

$\therefore (a, c) \notin S, SoS$

is NOT Transitive Relation. Thus, S is Reflexive But Neither Symmetric nor Transitive Relation.

Q.14

For reflexive: clearly $x + x$ i.e. $2x$ is integer divisible by 2 . $\Rightarrow (x, x) \in R \Rightarrow R$ is reflexive. For symmetric: $(x, y) \in R \Rightarrow x + y$ is integer divisible by 2 . $\Rightarrow y + x$ is integer divisible by $2 \Rightarrow (y, x) \in R$ For transitive: $(x, y) \in R \Rightarrow x + y$ is integer divisible by 2 . and $(y, z) \in R \Rightarrow y + z$ is integer divisible by 2 . so, $(x + z) + 2y$ is integer divisible by 2 . $\Rightarrow x + z$ is integer divisible by $2 \Rightarrow (x, z) \in R$ Equivalence class $[2] = \{-4, -2, 0, 2, 4\}$

Previous Years' CBSE Board Questions

1.2 Types of Relations

MCQ

1. Let $A = \{3, 5\}$. Then number of reflexive relations on A is
(a) 2 (b) 4
(c) 0 (d) 8 *(2023)*

2. Let R be a relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$. Then
(a) $(8, 7) \in R$ (b) $(6, 8) \in R$
(c) $(3, 8) \in R$ (d) $(2, 4) \in R$ *(2023)*

3. A relation R is defined on N. Which of the following is the reflexive relation?
(a) $R = \{(x, y): x > y, x, y \in N\}$
(b) $R = \{(x, y): x + y = 10, x, y \in N\}$
(c) $R = \{(x, y): xy$ is the square number, $x, y \in N\}$
(d) $R = \{(x, y): x + 4y = 10, x, y \in N\}$
(Term I, 2021-22) An

4. The number of equivalence relations in the set $\{1, 2, 3\}$ containing the elements $(1, 2)$ and $(2, 1)$ is
(a) 0 (b) 1
(c) 2 (d) 3 *(Term I, 2021-22)*

5. A relation R is defined on Z as aRb if and only if $a^2 - 7ab + 6b^2 = 0$. Then, R is
(a) reflexive and symmetric
(b) symmetric but not reflexive
(c) transitive but not reflexive
(d) reflexive but not symmetric
(Term I, 2021-22) Ap

6. Let $A = \{1, 3, 5\}$. Then the number of equivalence relations in A containing $(1, 3)$ is
(a) 1 (b) 2
(c) 3 (d) 4 *(2020)*

7. The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1), (1, 1)\}$ is
(a) symmetric and transitive, but not reflexive
(b) reflexive and symmetric, but not transitive
(c) symmetric, but neither reflexive nor transitive
(d) an equivalence relation *(2020)*

VSA (1 mark)

8. Write the smallest reflexive relation on set $A = \{a, b, c\}$. *(2021 C)*

9. A relation R in a set A is called _______, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$. *(2020)* R

10. A relation in a set A is called ________ relation, if each element of A is related to itself. *(2020)* R

11. If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N, write the range of R. *(AI 2014)*

12. Let $R = \{(a, a^3) : a$ is a prime number less than $5\}$ be a relation. Find the range of R. *(Foreign 2014)*

13. Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2$ divides $(a - b)\}$. Write the equivalence class $[0]$. *(Delhi 2014 C)*

SA I (2 marks)

14. Check if the relation R in the set $\mathbb{R}$ of real numbers defined as $R = \{(a, b) : a < b\}$ is (i) symmetric, (ii) transitive. *(2020)*

15. Let W denote the set of words in the English dictionary. Define the relation R by
$R = \{(x, y) \in W \times W$ such that x and y have at least one letter in common$\}$.
Show that this relation R is reflexive and symmetric, but not transitive. *(2020)*

LA I (4 marks)

16. Show that the relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ given by $R = \{(a, b) : |a - b|$ is divisible by $2\}$ is an equivalence relation. *(2020)*

17. Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as
$R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive. *(2019)*

18. Show that the relation R on the set Z of all integers, given by
$R = \{(a, b) : 2$ divides $(a - b)\}$ is an equivalence relation. *(2019)*

19. Show that the relation R on $\mathbb{R}$ defined as $R = \{(a, b): a \le b\}$, is reflexive and transitive but not symmetric. *(NCERT, Delhi 2019)*

20. Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by $3\}$ is an equivalence relation. *(AI 2019)* Ap

21. Let $A = \{1, 2, 3, ..., 9\}$ and R be the relation in $A \times A$ defined by $(a, b)\ R\ (c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation. Also obtain the equivalence class $[(2, 5)]$. *(Delhi 2014)*

22. Let R be a relation defined on the set of natural numbers N as follow :
$R = \{(x, y) \mid x \in N, y \in N$ and $2x + y = 24\}$
Find the domain and range of the relation R.
Also, find if R is an equivalence relation or not. *(Delhi 2014 C)* An

LA II (5/6 marks)

23. If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by $(a, b)\ R\ (c, d)$, if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation. *(2023, Delhi 2015)*

24. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b|$ is divisible by 4$\}$, is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2]. *(2018)*

25. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|$ is divisible by 2$\}$ is an equivalence relation. Write all the equivalence classes of R. *(AI 2015 C)*

1.3 Types of Functions

MCQ

26. The function $f : R \to R$ defined by $f(x) = 4 + 3\cos x$ is
(a) bijective (b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto *(Term I, 2021-22)* An

27. The number of functions defined from $\{1, 2, 3, 4, 5\} \to \{a, b\}$ which are one-one is
(a) 5 (b) 3
(c) 2 (d) 0 *(Term I, 2021-22)*

28. Let $f : R \to R$ be defined by $f(x) = 1/x$, for all $x \in R$, Then, f is
(a) one-one (b) onto
(c) bijective (d) not defined *(Term I, 2021-22)*

29. The function $f : N \to N$ is defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

The function f is
(a) bijective
(b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto *(Term I, 2021-22)* Ev

VSA (1 mark)

30. If $f = \{(1, 2), (2, 4), (3, 1), (4, k)\}$ is a one-one function from set A to A, where $A = \{1, 2, 3, 4\}$, then find the value of k. *(2021 C)*

LA I (4 marks)

31. **Case Study :** An organization conducted bike race under two different categories - Boys and girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.
Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.

Based on the above information, answer the following questions.

(i) How many relations are possible from B to G?
(ii) Among all the possible relations from B to G, how many functions can be formed from B to G?
(iii) Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective, justify your answer. *(2023)* Ap

32. Let $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not. *(2023)*

33. Show that the function $f : (-\infty, 0) \to (-1, 0)$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in (-\infty, 0)$ is one-one and onto. *(2020)*

CBSE Sample Questions

1.2 Types of Relations

MCQ

1. A relation R in set $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A?
(a) (1, 1) (b) (1, 2) (c) (2, 2) (d) (3, 3) *(Term I, 2021-22)* An

2. Let the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a, b) : |a - b|$ is a multiple of 4.$\}$ Then [1], the equivalence class containing 1, is
(a) $\{1, 5, 9\}$ (b) $\{0, 1, 2, 5\}$
(c) ϕ (d) A *(Term I, 2021-22)* Ev

VSA (1 mark)

3. How many reflexive relations are possible in a set A whose $n(A) = 3$? *(2020-21)* Ap

4. A relation R in $S = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which element(s) of relation R be removed to make R an equivalence relation? *(2020-21)*

5. An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3. What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$. *(2020-21)*

SA I (2 marks)

6. Let R be the relation in the set Z of integers given by $R = \{(a, b) : 2 \text{ divides } a - b\}$. Show that the relation R is transitive? Write the equivalence class of 0. *(2020-21)* Ap

SA II (3 marks)

7. Check whether the relation R in the set Z of integers defined as $R = \{(a, b) : a + b$ is "divisible by 2"$\}$ is reflexive, symmetric or transitive. Write the equivalence class containing 0, *i.e.*, [0]. *(2020-21)*

LA II (5/6 marks)

8. Given a non-empty set X, define the relation R on $P(X)$ as :
For $A, B \in P(X)$, $(A, B) \in R$ iff $A \subset B$. Prove that R is reflexive, transitive, and not symmetric. *(2022-23)*

9. Define the relation R in the set $N \times N$ as follows:
For $(a, b), (c, d) \in N \times N$, $(a, b)\, R\, (c, d)$ iff $ad = bc$. Prove that R is an equivalence relation in $N \times N$. *(2022-23)*

1.3 Types of Functions

MCQ

10. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Based on the given information, f is best defined as
(a) Surjective function (b) Injective function
(c) Bijective function (d) Function
(Term I, 2021-22) Ev

11. The function $f : R \to R$ defined as $f(x) = x^3$ is
(a) One-one but not onto
(b) Not one-one but onto
(c) Neither one-one nor onto
(d) One-one and onto *(Term I, 2021-22)*

VSA (1 mark)

12. Check whether the function $f : R \to R$ defined as $f(x) = x^3$ is one-one or not. *(2020-21)*

13. A relation R in the set of real numbers **R** defined as $R = \{(a, b) : \sqrt{a} = b\}$ is a function or not. Justify *(2020-21)*

Detailed SOLUTIONS

Previous Years' CBSE Board Questions

1. (b): Total number of reflexive relations on a set having n number of elements $= 2^{n^2 - n}$
Here, $n = 2$
$\therefore$ Required number of reflexive relations $= 2^{2^2 - 2}$
$= 2^{4-2} = 2^2 = 4$

2. (b): Given, $R = \{(a, b) : a = b - 2, b > 6\}$
Since, $b > 6$, so $(2, 4) \notin R$
Also, $(3, 8) \notin R$ as $3 \neq 8 - 2$
and $(8, 7) \notin R$ as $8 \neq 7 - 2$
Now, for $(6, 8)$, we have
$8 > 6$ and $6 = 8 - 2$, which is true
$\therefore$ $(6, 8) \in R$

3. (c) : Consider, $R = \{(x, y) : xy$ is the square number, $x, y \in N\}$
As, $xx = x^2$, which is the square of natural number x.
$\Rightarrow$ $(x, x) \in R$. So, R is reflexive.

Concept Applied
- A relation R in a set A is called reflexive, if $(a, a) \in R$, for all $a \in A$.

4. (c) : Equivalence relations in the set $\{1, 2, 3\}$ containing the elements $(1, 2)$ and $(2, 1)$ are
$R_1 = \{(1, 2), (2, 1), (1, 1), (2, 2), (3, 3)\}$
$R_2 = \{(1, 2), (2, 1), (1, 3), (3, 1) (2, 3), (3, 2), (1, 1), (2, 2), (3, 3)\}$
$\therefore$ Number of equivalence relations is 2.

Concept Applied
- A relation R in a set A is called an equivalence relation, if R is reflexive, symmetric and transitive.

5. (d): Given, aRb, $a, b \in Z$
Reflexive : For $a \in Z$, we have
$a^2 - 7a{\cdot}a + 6a^2 = a^2 - 7a^2 + 6a^2 = 0 \Rightarrow (a, a) \in R$
$\therefore$ Relation is reflexive.
Symmetric : Since, $(6, 1) \in R$
As, $6^2 - 7 \times 6 \times 1 + 6 \times 1^2 = 36 - 42 + 6 = 0$
But $(1, 6) \notin R$. $\therefore$ Relation is not symmetric.

6. (b): Equivalence relations in the set containing the element $(1, 3)$ are
$R_1 = \{(1, 1), (3, 3), (1, 3), (3, 1), (5, 5)\}$
$R_2 = \{(1, 1), (3, 3), (5, 5), (1, 5), (5, 1), (3, 5), (5, 3), (1, 3), (3, 1)\}$
$\therefore$ There are 2 possible equivalence relations.

7. (c) : Given $R = \{(1, 2), (2, 1), (1, 1)\}$ is a relation on set $\{1, 2, 3\}$
Reflexive : Clearly $(2, 2), (3, 3) \notin R$
$\therefore$ R is not a reflexive relation.
Symmetric : Now, $(1, 2) \in R$ and $(2, 1) \in R$ $\therefore$ R is symmetric.
Transitive : Now, $(2, 1) \in R$ and $(1, 2) \in R$ but $(2, 2) \notin R$
$\therefore$ R is not transitive relation.
R is symmetric, but neither reflexive nor transitive.

8. We have, $A = \{a, b, c\}$
A relation R on the set A is said to be reflexive if $(a, a) \in R$, $\forall a \in A$

$\therefore$ $R = \{(a, a), (b, b), (c, c)\}$ is the required smallest reflexive relation on A.

9. A relation R in a set A is called symmetric, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

10. A relation in a set A is called reflexive relation, if each element of A is related to itself.

11. Here, $R = \{(x, y) : x + 2y = 8x, y \in N\}$.
For $x = 1, 3, 5, ...$
$x + 2y = 8$ has no solution in N.
For $x = 2$, we have $2 + 2y = 8 \Rightarrow y = 3$
For $x = 4$, we have $4 + 2y = 8 \Rightarrow y = 2$
For $x = 6$, we have $6 + 2y = 8 \Rightarrow y = 1$
For $x = 8, 10, ...$
$x + 2y = 8$ has no solution in N.
$\therefore$ Range of $R = \{y : (x, y) \in R\} = \{1, 2, 3\}$

12. Given relation is
$R = \{(a, a^3) : a$ is a prime number less than 5$\}$.
$\therefore$ $R = \{(2, 8), (3, 27)\}$. So, the range of R is $\{8, 27\}$.

13. Here, $R = \{(a, b) : 2 \text{ divides } (a - b)\}$
$\therefore$ Equivalence class of $[0] = \{a \in A : (a, 0) \in R\}$.
$\Rightarrow$ $(a - 0)$ is divisible by 2 and $a \in A \Rightarrow a = 0, 2, 4$
Thus $[0] = \{0, 2, 4\}$.

14. We have, $R = \{(a, b) : a < b\}$, where $a, b \in \mathbb{R}$
(i) Symmetric : Let $(x, y) \in R$, *i.e.*, $x R y \Rightarrow x < y$
But $y \not< x$, so $(x, y) \in R \Rightarrow (y, x) \notin R$
Thus, R is not symmetric.
(ii) Transitive : Let $(x, y), (y, z) \in R$
$\Rightarrow$ $x < y$ and $y < z \Rightarrow x < z$
$\Rightarrow$ $(x, z) \in R$. Thus, R is transitive.

15. We have, $R = \{(x, y) \in W \times W : x$ and y have at least one letter in common$\}$
Reflexive : Clearly $(x, x) \in R$, because same words will contains all common letters.
$\Rightarrow$ R is reflexive.
Symmetric : Let $(x, y) \in R$ *i.e.*, x and y have at least one letter in common.
$\Rightarrow$ y and x will also have at least one letter in common.
$\Rightarrow$ $(y, x) \in R$
$\Rightarrow$ R is symmetric.
Transitive : Let, x = LAND, y = NOT and z = HOT
Clearly $(x, y) \in R$ as x and y have a common letter and $(y, z) \in R$ as y and z have 2 common letters.
but $(x, z) \notin R$ as x and z have no letter in common.
Hence, R is not transitive.

Concept Applied

- A relation R in a set A is not transitive if for $(a, b) \in R$ and $(b, c) \in R$ but $(a, c) \notin R$

16. We have, $A = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(a, b) : |a - b|$ is divisible by 2$\}$
(i) Reflexive : For any $a \in A$
$|a - a| = 0$, which is divisible by 2.
Thus, $(a, a) \in R$. So, R is reflexive.
(ii) Symmetric : For any $a, b \in A$
Let $(a, b) \in R$
$\Rightarrow$ $|a - b|$ is divisible by $2 \Rightarrow |b - a|$ is divisible by 2
$\Rightarrow$ $(b, a) \in R \therefore (a, b) \in R \Rightarrow (b, a) \in R \therefore R$ is symmetric.
(iii) Transitive : For any $a, b, c \in A$
Let $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow$ $|a - b|$ is divisible by 2 and $|b - c|$ is divisible by 2.
$\Rightarrow$ $a - b = \pm 2 k_1$ and $b - c = \pm 2k_2 \ \forall k_1, k_2 \in N$
$\Rightarrow$ $a - b + b - c = \pm 2(k_1 + k_2) \Rightarrow a - c = \pm 2 k_3 \ \forall k_3 \in N$
$\Rightarrow$ $|a - c|$ is divisible by $2 \Rightarrow (a, c) \in R \therefore R$ is transitive.
Hence, R is an equivalence relation.

17. We have, $A = \{1, 2, 3, 4, 5, 6\}$ and a relation R on A defined as $R = \{(a, b) : b = a + 1\}$
Reflexive : Let $(a, a) \in R$
$\Rightarrow$ $a = a + 1 \Rightarrow a - a = 1 \Rightarrow 0 = 1$, which is not possible.
$\therefore$ $(a, a) \notin R \Rightarrow R$ is not reflexive.
Symmetric : Let $(a, b) \in R \Rightarrow b = a + 1$ ...(i)
Now, if $(b, a) \in R$
$\Rightarrow$ $a = b + 1 \Rightarrow b = b + 1 + 1$ (using (i))
$\Rightarrow$ $b = b + 2 \Rightarrow b - b = 2 \Rightarrow 0 = 2$, which is not possible
$\Rightarrow$ $(b, a) \notin R \Rightarrow R$ is not symmetric.
Transitive : Let $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow$ $b = a + 1$ and $c = b + 1 \Rightarrow c = a + 1 + 1$
$\Rightarrow$ $c = a + 2 \neq a + 1 \Rightarrow (a, c) \notin R \Rightarrow R$ is not transitive.

18. We have, $R = \{(a, b) : 2 \text{ divides } (a - b)\}$
Reflexive : For any $a \in Z$, $a - a = 0$ and 2 divides 0.
$\Rightarrow$ $(a, a) \in R$ for every $a \in Z \therefore R$ is a reflexive.
Symmetric : Let $(a, b) \in R$
$\Rightarrow$ 2 divides $(a - b)$
$\Rightarrow$ $a - b = 2m$, for some $m \in Z$
$\Rightarrow$ $b - a = 2m$
$\Rightarrow$ 2 divides $b - a$
$\Rightarrow$ $(b, a) \in R$
$\therefore$ R is symmetric.
Transitive : Let $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow$ 2 divides $(a - b)$ and 2 divides $(b - c)$
$\Rightarrow$ $a - b = 2m$ and $b - c = 2n$ for some $m, n \in Z$
$\Rightarrow$ $a - b + b - c = 2m + 2n$
$\Rightarrow$ $a - c = 2(m + n)$
$\Rightarrow$ 2 divides $a - c$
$\Rightarrow$ $(a, c) \in R$
$\therefore$ R is transitive.
Hence, R is an equivalence relation.

19. We have, $R = \{(a,b) : a \le b, \ a, b \in \mathbb{R}\}$
(i) Reflexive : Since $a \le a \ \therefore aRa \ \forall a \in R$
Hence, R is reflexive.
(ii) Symmetric : $(a,b) \in R$ such that $aRb \Rightarrow a \le b \not\Rightarrow b \le a$
So, $(b, a) \notin R$.
Hence, R is not symmetric.
(iii) Transitive : Let $a, b, c \in R$ such that aRb and bRc
Now, $aRb \Rightarrow a \le b$ ... (i) and $bRc \Rightarrow b \le c$ ... (ii)
From (i) and (ii), we have $a \le b \le c \Rightarrow a \le c \ \therefore aRc$
Hence, relation R is transitive.

20. We have, $A = \{x \in Z : 0 \le x \le 12\}$
$\therefore$ $A = \{0, 1, 2, 3, ..., 12\}$
Also, $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by 3$\}$

(i) Reflexive : For any $a \in A$,

$|a - a| = 0$, which is divisible by 3

Thus, $(a, a) \in S \therefore S$ is reflexive.

(ii) Symmetric : Let $(a, b) \in S$

$\Rightarrow |a-b|$ is divisible by 3.

$\Rightarrow |b-a|$ is divisible by 3 $\Rightarrow (b,a) \in S$ *i.e.* $(a,b) \in S \Rightarrow (b,a) \in S$

$\therefore$ S is symmetric.

(iii) Transitive :

Let $(a, b) \in S$ and $(b, c) \in S$

$\Rightarrow$ $|a - b|$ is divisible by 3 and $|b - c|$ is divisible by 3.

$\Rightarrow$ $(a - b) = \pm 3k_1$ and $(b - c) = \pm 3k_2$; $\forall k_1, k_2 \in N$

$\Rightarrow$ $(a - b) + (b - c) = \pm 3(k_1 + k_2)$

$\Rightarrow$ $(a - c) = \pm 3(k_1 + k_2)$; $\forall k_1, k_2 \in N$

$\Rightarrow$ $|a - c|$ is divisible by 3 $\Rightarrow (a, c) \in S \therefore S$ is Transitive.

Hence, S is an equivalence relation.

Concept Applied

- A relation R in a set A is called
  (i) reflexive, if $(a, a) \in R$, for all $a \in A$
  (ii) symmetric, if $(a, b) \in R \Rightarrow (b, a) \in R$, for all $a, b \in A$
  (iii) transitive, if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$, for all $a, b, c \in A$

21. Given $A = \{1, 2, 3, 4, ..., 9\}$

To show : R is an equivalence relation.

(i) Reflexive : Let (a, b) be an arbitrary element of $A \times A$. Then, we have $(a, b) \in A \times A \Rightarrow a, b \in A$

$\Rightarrow$ $a + b = b + a$ (by commutativity of addition on $A \subset N$)

$\Rightarrow$ $(a, b) R (a, b)$

Thus, $(a, b) R (a, b)$ for all $(a, b) \in A \times A$. So, R is reflexive.

(ii) Symmetric : Let $(a, b), (c, d) \in A \times A$ such that $(a, b) R (c, d)$

$\Rightarrow$ $a + d = b + c \Rightarrow b + c = a + d$

$\Rightarrow$ $c + b = d + a$ (by commutativity of addition on $A \subset N$)

$\Rightarrow$ $(c, d) R (a, b)$.

Thus, $(a, b) R (c, d) \Rightarrow (c, d) R (a, b)$ for all $(a, b), (c, d) \in A \times A$.

So, R is symmetric.

(iii) Transitive : Let $(a, b), (c, d), (e, f) \in A \times A$ such that $(a, b) R (c, d)$ and $(c, d) R (e, f)$

Now, $(a, b) R (c, d) \Rightarrow a + d = b + c$ ...(i)

and $(c, d) R (e, f) \Rightarrow c + f = d + e$ ...(ii)

Adding (i) and (ii), we get $(a + d) + (c + f) = (b + c) + (d + e)$

$\Rightarrow$ $a + f = b + e \Rightarrow (a, b) R (e, f)$

Thus, $(a, b) R (c, d)$ and $(c, d) R (e, f) \Rightarrow (a, b) R (e, f)$.

So, R is transitive. $\therefore$ R is an equivalence relation.

Equivalence class of $[(2, 5)] = \{(x, y) \in N \times N : (x, y) R (2, 5)\}$

$= \{(x, y) \in N \times N : x + 5 = y + 2\}$

$= \{(x, y) \in N \times N : y = x + 3\} = \{(x, x + 3) : x \in A\}$

$= \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$.

Answer Tips

- First, prove the given relation is an equivalence relation and then find the equivalence class by using the given relation.

22. Here, $R = \{(x, y) \mid x \in N, y \in N$ and $2x + y = 24\}$

$R = \{(1, 22), (2, 20), (3, 18), ..., (11, 2)\}$

Domain of $R = \{1, 2, 3, 4, ..., 11\}$

Range of $R = \{2, 4, 6, 8, 10, 12, ..., 22\}$

R is not reflexive as if $(2, 2) \in R \Rightarrow 2 \times 2 + 2 = 6 \neq 24$

In fact R is neither symmetric nor transitive.

$\Rightarrow$ R is not an equivalence relation.

23. (i) Reflexive : Let (a, b) be an arbitrary element of $N \times N$. Then, $(a, b) \in N \times N$

$\Rightarrow$ $ab(b + a) = ba(a + b)$

[by commutativity of addition and multiplication on N]

$\Rightarrow$ $(a, b) R (a, b)$

So, R is reflexive on $N \times N$.

(ii) Symmetric : Let $(a, b), (c, d) \in N \times N$ such that $(a, b) R (c, d)$.

$\Rightarrow$ $ad(b + c) = bc(a + d) \Rightarrow cb(d + a) = da(c + b)$

[by commutativity of addition and multiplication on N]

Thus, $(a, b) R (c, d) \Rightarrow (c, d) R (a, b)$ for all $(a, b), (c, d) \in N \times N$.

So, R is symmetric on $N \times N$.

(iii) Transitive : Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b) R (c, d)$ and $(c, d) R (e, f)$. Then,

$(a, b) R (c, d) \Rightarrow ad(b + c) = bc(a + d)$

$$\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad} \Rightarrow \frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d} \quad ...(i)$$

and $(c, d) R (e, f) \Rightarrow cf(d + e) = de(c + f)$

$$\Rightarrow \frac{d+e}{de} = \frac{c+f}{cf} \Rightarrow \frac{1}{d} + \frac{1}{e} = \frac{1}{c} + \frac{1}{f} \quad ...(ii)$$

Adding (i) and (ii), we get

$$\left(\frac{1}{b}+\frac{1}{c}\right)+\left(\frac{1}{d}+\frac{1}{e}\right)=\left(\frac{1}{a}+\frac{1}{d}\right)+\left(\frac{1}{c}+\frac{1}{f}\right)$$

$$\Rightarrow \frac{1}{b}+\frac{1}{e}=\frac{1}{a}+\frac{1}{f} \Rightarrow \frac{b+e}{be}=\frac{a+f}{af}$$

$\Rightarrow$ $af(b + e) = be(a + f) \Rightarrow (a, b) R (e, f)$

So, R is transitive on $N \times N$.

Hence, R is an equivalence relation.

24. We have, $A = \{x \in Z : 0 \le x \le 12\}$

$\therefore$ $A = \{0, 1, 2, 3, ..., 12\}$

and $S = \{(a, b) : |a - b|$ is divisible by 4$\}$

(i) Reflexive : For any $a \in A$, $|a - a| = 0$, which is divisible by 4. Thus, $(a, a) \in R \therefore R$ is reflexive.

(ii) Symmetric : Let $(a, b) \in R$

$\Rightarrow$ $|a - b|$ is divisible by 4

$\Rightarrow$ $|b - a|$ is divisible by 4 $\Rightarrow (b, a) \in R$

i.e., $(a, b) \in R \Rightarrow (b, a) \in R \therefore R$ is symmetric.

(iii) Transitive : Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow$ $|a - b|$ is divisible by 4 and $|b - c|$ is divisible by 4

$\Rightarrow$ $a - b = \pm 4k_1$ and $b - c = \pm 4k_2$; $\forall k_1, k_2 \in N$

$\Rightarrow$ $(a - b) + (b - c) = \pm 4 (k_1 + k_2)$; $\forall k_1, k_2 \in N$

$\Rightarrow$ $a - c = \pm 4 (k_1 + k_2)$; $\forall k_1, k_2 \in N$

$\Rightarrow$ $|a - c|$ is divisible by $4 \Rightarrow (a, c) \in R \therefore R$ is transitive.

Hence, R is an equivalence relation.

The set of elements related to 1 is $\{1, 5, 9\}$.

Equivalence class for [2] is $\{2, 6, 10\}$.

Concept Applied

- In a relation R in a set A, the set of all elements related to any element $a \in A$ is denoted by $[a]$
  i.e., $[a] = \{x \in A : (x, a) \in R\}$
  Here, $[a]$ is called an equivalence class of $a \in A$.

25. We have, $A = \{1, 2, 3, 4, 5\}$
and $R = \{(a, b) : |a - b|$ is divisible by 2$\}$
(i) Reflexive : For any $a \in A$,
$|a - a| = 0$, which is divisible by 2
Thus, $(a, a) \in R$ $\therefore$ R is reflexive.
(ii) Symmetric : Let $(a, b) \in R$
$\Rightarrow$ $|a - b|$ is divisible by 2
$\Rightarrow$ $|b - a|$ is divisible by 2 $\Rightarrow$ $(b, a) \in R$
i.e., $(a, b) \in R \Rightarrow (b, a) \in R$ $\therefore$ R is symmetric.
(iii) Transitive : Let $(a, b) \in R$ and $(b, c) \in R$
$\Rightarrow$ $|a - b|$ is divisible by 2 and $|b - c|$ is divisible by 2
$\Rightarrow$ $a - b = \pm 2k_1$ and $b - c = \pm 2k_2$; $\forall\, k_1, k_2 \in N$
$\Rightarrow$ $(a - b) + (b - c) = \pm 2(k_1 + k_2)$; $\forall\, k_1, k_2 \in N$
$\Rightarrow$ $(a - c) = \pm 2(k_1 + k_2)$; $\forall\, k_1, k_2 \in N$
$\Rightarrow$ $|a - c|$ is divisible by 2 $\Rightarrow (a, c) \in R$ $\therefore$ R is transitive.
Hence, R is an equivalence relation.
Further R has only two equivalence classes, namely $[1] = [3] = [5] = \{1, 3, 5\}$ and $[2] = [4] = \{2, 4\}$.

26. (d): We have, $f(x) = 4 + 3\cos x$, $\forall\, x \in R$

At $x = \frac{\pi}{2}, f\left(\frac{\pi}{2}\right) = 4 + 3\cos\frac{\pi}{2} = 4 \Rightarrow f\left(-\frac{\pi}{2}\right) = 4 + 3\cos\left(-\frac{\pi}{2}\right) = 4$

Since, $f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$, But $\frac{\pi}{2} \neq -\frac{\pi}{2}$

Therefore, f is not one-one.
As $-1 \le \cos x \le 1, \forall\, x \in R \Rightarrow 1 \le 4 + 3\cos x \le 7, \forall\, x \in R$
$\Rightarrow$ $f(x) \in [1, 7]$, where $[1, 7]$ is subset of R. $\therefore$ f is not onto.

Concept Applied

- Range of $\cos x$ is $[-1, 1]$.

27. (d): $\because f : X \to Y$ is one-one, if different element of X have different image in Y under f. But here, no such situation is possible.

28. (d): Given $f(x) = \frac{1}{x}$, for all $x \in R$

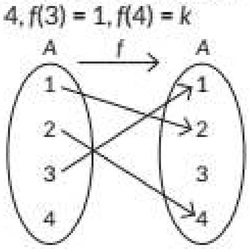
At $x = 0 \in R$, $f(x)$ is not defined.

29. (c): Given, $f(x) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

Now, $f(1) = \frac{1+1}{2} = 1, f(2) = \frac{2}{2} = 1$

$\Rightarrow$ $f(1) = f(2)$ but $1 \neq 2$ $\therefore$ f is not one-one.
But f is onto ($\because$ range of f is N.)

30. We have, $A = \{1, 2, 3, 4\}$ function $f : A \to A$ is one-one and $f(1) = 2, f(2) = 4, f(3) = 1, f(4) = k$



As f is one-one, so no two element of A has same image in A.
$\therefore$ $f(4) = 3 \Rightarrow k = 3$

Concept Applied

- For a function to be one-one, no two elements should have the same image in A.

31. (i) Here $n(B) = 3$ and $n(G) = 2$
$\therefore$ Number of relation from B to $G = 2^{3 \times 2} = 2^6$
(ii) Number of functions formed from B to $G = 2^3 = 8$
(iii) We have, $R = \{(x, y) = x$ and y are students of the same sex$\}$
$\therefore$ R is reflexive as $(x, x) \in R$.
R is symmetric as $(x, y) \in R \Rightarrow (y, x) \in R$.
Since, $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$
Hence, R is an equivalence relations.

OR

We have $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$
Since, elements b_1 and b_3 have the same image, therefore, the functions is not one-one but it is many one functions.
Since, every element in G has its pre-image in B, so the functions is onto.
For bijection, function should be one-one and onto both.
Hence, the function is surjective but not injective.

32. The function $f : R - \left\{-\frac{4}{3}\right\} \to R$ is given by $f(x) = \frac{4x}{3x+4}$.

One-one : Let $x, y \in R - \left\{-\frac{4}{3}\right\}$ such that $f(x) = f(y)$

$\Rightarrow$ $\frac{4x}{3x+4} = \frac{4y}{3y+4}$
$\Rightarrow$ $4x(3y + 4) = 4y(3x + 4) \Rightarrow 12xy + 16x = 12xy + 16y$
$\Rightarrow$ $16x = 16y \Rightarrow x = y$
$\therefore$ f is one-one.
Onto : Let y be an arbitrary element of R. Then $f(x) = y$

$\Rightarrow \frac{4x}{3x+4} = y \Rightarrow 4x = 3xy + 4y \Rightarrow 4x - 3xy = 4y \Rightarrow x = \frac{4y}{4-3y}$

As $y \in R - \left\{\frac{4}{3}\right\}, \frac{4y}{4-3y} \in R$

Also, $\frac{4y}{4-3y} \neq \frac{-4}{3}$ as if

$\frac{4y}{4-3y} = -\frac{4}{3} \Rightarrow 12y = 12y - 16$, which is not possible.

Thus, $x = \frac{4y}{4-3y} \in R - \left\{-\frac{4}{3}\right\}$ such that

$$f(x) = f\left(\frac{4x}{3x+4}\right) = \frac{4\left(\frac{4y}{4-3y}\right)}{3\left(\frac{4y}{4-3y}\right)+4} = \frac{16y}{12y + 16 - 12y} = \frac{16y}{16} = y$$

So, every element in $R - \left\{\frac{4}{3}\right\}$ has pre-image in $R - \left\{-\frac{4}{3}\right\}$

$\therefore$ f is not onto.

33. Given, $f(x) = \frac{x}{1+|x|}, x \in (-\infty, 0)$

$= \frac{x}{1-x}$ $\qquad (\because x \in (-\infty, 0), |x| = -x)$

For one-one : Let $f(x_1) = f(x_2)$, $x_1, x_2 \in (-\infty, 0)$

$\Rightarrow \quad \frac{x_1}{1-x_1} = \frac{x_2}{1-x_2} \Rightarrow x_1(1 - x_2) = x_2(1 - x_1)$

$\Rightarrow \quad x_1 - x_1x_2 = x_2 - x_1x_2 \Rightarrow x_1 = x_2$

Thus, $f(x_1) = f(x_2)$, $\Rightarrow x_1 = x_2$

$\therefore$ f is one-one

For onto : Let $f(x) = y$

$\Rightarrow \quad y = \frac{x}{1-x} \Rightarrow y(1 - x) = x \Rightarrow y - xy = x$

$\Rightarrow \quad x + xy = y \Rightarrow x(1 + y) = y \Rightarrow x = \frac{y}{1+y}$

Here, $y \in (-1, 0)$

So, x is defined for all values of y in codomain. $\therefore$ f is onto.

Concept Applied

- A function $f : A \rightarrow B$ is called
  - (i) one-one or injective function, if distinct elements of A have distinct images in B. *i.e.*, for $a, b \in A$, $f(a) = f(b) \Rightarrow a = b$
  - (ii) onto or surjective function, if for every element $b \in B$, there exists some $a \in A$ such that $f(a) = b$.

CBSE Sample Questions

1. (b): We have, $(1, 2) \in R$ but $(2, 1) \notin R$
So, (1, 2) should be removed from R to make it an equivalence relation. **(1)**

2. (a): We have, $R = \{(a, b) : |a - b|$ is a multiple of $4\}$
$\therefore$ The set of elements related to 1 is $\{1, 5, 9\}$.
So, equivalence class for [1] is $\{1, 5, 9\}$ **(1)**

3. Number of reflexive relations on a set having n elements $= 2^{n(n-1)}$
So, required number of reflexive relations $= 2^{3(3-1)} = 2^6$ **(1)**

4. We have, $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$
which is reflexive and transitive.
For R to be symmetric (1, 2) should be removed from R. **(1)**

5. As we know that, union of all equivalence classes of a set is the set itself.
$\therefore \quad A_1 \cup A_2 \cup A_3 = A$
Also, $\quad A_1 \cap A_2 \cap A_3 = \phi$
[$\because$ Equivalence classes are either equal or disjoint] **(1)**

6. Let $(a, b) \in R$ and $(b, c) \in R$. Then, 2 divides $(a - b)$ and 2 divides $(b - c)$: where $a, b, c \in Z$
So, 2 divides $[(a - b) + (b - c)]$
$\Rightarrow$ 2 divides $(a - c) \Rightarrow (a, c) \in R$. So, relation R is transitive. **(1)**
Equivalence class of $0 = \{0, \pm 2, \pm 4, \pm 6, ...\}$ **(1)**

7. (i) Reflexive : Since, $a + a = 2a$ which is even.
$\therefore \quad (a, a) \in R \;\forall\; a \in Z$
Hence, R is reflexive. **(1/2)**
(ii) Symmetric : If $(a, b) \in R$, then $a + b = 2\lambda \Rightarrow b + a = 2\lambda$
$\Rightarrow \quad (b, a) \in R$. Hence, R is symmetric. **(1)**
(iii) Transitive : If $(a, b) \in R$ and $(b, c) \in R$
then $a + b = 2\lambda$ .... (i) and $b + c = 2\mu$ ... (ii)
Adding (i) and (ii), we get
$a + 2b + c = 2(\lambda + \mu) \Rightarrow a + c = 2(\lambda + \mu - b)$
$\Rightarrow \quad a + c = 2k$, where $k = \lambda + \mu - b \Rightarrow (a, c) \in R$
Hence, R is transitive. **(1)**
Equivalence class containing 0 *i.e.*,
$[0] = \{..., -4, -2, 0, 2, 4, ...\}$ **(1/2)**

8. We have, a relation R on X such that, $(A, B) \in R$ iff $A \subset B$ for $A, B \in P(X)$. **(1/2)**
Reflexive : Clearly every set is a subset of itself.
$\Rightarrow \quad (A, A) \in R$
$\therefore \quad R$ is reflexive. **(1)**
Symmetric : Let $(A, B) \in R$
$\Rightarrow \quad A \subset B$
$\Rightarrow \quad B$ is a super set of A. **(1/2)**
$\Rightarrow \quad B \not\subset A \Rightarrow (B, A) \notin R$
$\therefore \quad R$ is not symmetric. **(1)**
Transitive : Let $(A, B) \in R$ and $(B, C) \in R$, for all $A, B, C \in P(X)$
$\Rightarrow \quad A \subset B$ and $B \subset C \Rightarrow A \subset B \subset C$ **(1/2)**
$\Rightarrow \quad A \subset C \Rightarrow (A, C) \in R$
$\therefore \quad R$ is transitive. **(1)**
Hence, R is reflexive and transitive but not symmetric. **(1/2)**

9. Reflexive : Let $(a, b) \in N \times N$. Then $ab = ba$
(By commutativity of multiplication of natural number)
$\Rightarrow \quad (a, b) \, R \, (b, a)$
Thus, $(a, b) \, R \, (b, a)$ for all $(a, b) \in N \times N$
So, R is reflexive. **(1)**
Symmetric : Let $(a, b), (c, d) \in N \times N$ such that $(a, b) \, R \, (c, d)$
$\Rightarrow ad = bc \Rightarrow bc = ad \Rightarrow cb = da$
(By commutativity of multiplication of natural numbers)
$\Rightarrow \quad (c, d) \, R \, (a, b)$
Thus, $(a, b) \, R \, (c, d) = (c, d) \, R \, (a, b)$ for $(a, b), (c, d) \in N \times N$
So, R is symmetric. **(1)**
Transitive : Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b) \, R \, (c, d)$ and $(c, d) \, R \, (e, f)$
Now, $(a, b) \, R \, (c, d) \Rightarrow ad = bc$ ...(i)
and $(c, d) \, R \, (e, f) \Rightarrow cf = de$ ...(ii)
Multiplying (i) and (ii), we get $ad \cdot cf = bc \cdot de$ **(1)**
$\Rightarrow \quad af = be \Rightarrow (a, b) \, R \, (e, f)$
Thus, $(a, b) \, R \, (c, d)$ and $(c, d) \, R \, (e, f) \Rightarrow (a, b) \, R \, (e, f)$ **(1)**
So, R is transitive.
$\therefore \quad R$ is an equivalence relation. **(1)**

10. (b): As every pre-image $x \in A$, has a unique image $y \in B$.
$\Rightarrow \quad f$ is injective function. **(1)**

11. (d): Let $x_1, x_2 \in R$ be such that $f(x_1) = f(x_2)$
$\Rightarrow \quad x_1^3 = x_2^3 \Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.
Let $f(x) = x^3 = y$ for some arbitrary element $y \in R \Rightarrow x = y^{1/3}$
$\Rightarrow \quad f(y^{1/3}) = y$
Every image $y \in R$ has a unique pre-image in R.
$\Rightarrow \quad f$ is onto
$\therefore \quad f$ is one-one and onto. **(1)**

12. Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R$.
$\Rightarrow \quad (x_1)^3 = (x_2)^3$
$\Rightarrow \quad x_1 = x_2$, hence $f(x)$ is one-one. **(1)**

13. Since $\sqrt{a}$ is not defined for $a \in (-\infty, 0)$
$\therefore \quad R = \{(a, b) : \sqrt{a} = b\}$ is not a function. **(1)**